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ABSTRACT

An efficient algorithm has been devised to compute the crossovers in satellite altimetry. The significance of the crossovers is twofold. First, they are needed to perform the crossover adjustment to remove the orbit error.Secondly, they yield important insight to the oceanic variability. Nevertheless there is no published algorithm to make this very time consuming task easier, which is the goal of this note. The success of the algorithm is predicated on the ability to predict (by analytical means) the crossover coordinates to within 6 km and 1 second of the true values. Hence, only one interpolation/extrapolation step on the data is needed to derive the crossover coordinates in contrast to the many interpolation/extrapolation operations that are usually needed to arrive at the same accuracy level if deprived of this information.

1. INTRODUCTION

In satellite altimetry (in which the sea level relative to a reference ellipsoid that best approximates the shape of the earth is measured along the satellite ground track, e.g., Wunsch and Gaposchkin, 1980), the term "crossover" refers to the intersection of two ground tracks. The coordinates of a crossover point are comprised of the location (i.e., latitude and longitude) and the two times when the satellite passes over the crossover point. The crossover difference is the difference between the two sea level measurements. The crossovers are important in two aspects.

First, the crossover difference reveals the temporal variability of the sea level and also avoids the geoid uncertainty (the geoid is an equipotential surface of the earth gravity field, to which a motionless ocean would conform). The sea level varies with respect to the reference ellipsoid primarily because of the earth gravity field and secondarily due to the dynamical effects of the ocean. Presently the uncertainty of the geoid is at least comparable if not larger then the effects of ocean dynamics (e.g., Tai, 1983). Analyses based on crossover differences have shown and are continuing to reveal valuable insight on the sea level temporal variability (e.g., Cheney and Marsh, 1981; Fu and Chelton, 1985; Cheney *et al.*, 1986.)

Secondly, the crossovers play a pivotal role in removing the satellite orbit error (i.e., the uncertainty of the satellite's altitude) by the so-called crossover adjustment method (e.g., Tai and Fu, 1986; Tai, 1987). The orbit error, which usually causes the crossover difference to be over 1 m in size, cannot be taken lightly lest the crossover difference should reveal not the sea level temporal variability but the variability produced by the orbit error. Furthermore, the basin-scale geoid uncertainty is small enough for the basin-scale circulation to be investigated (e.g., Tai and Wunsch 1983, 1984) and the orbit error reduction is of paramont importance in this case.

For the exact repeat orbit, one can use along-track differences instead of crossover differences to reduce the orbit error if one is only interested in the temporal variability. However, the crossover adjustment has to be performed if one is interested in the absolute topography because ascending and descending tracks may have different orbit error characteristics which are only evident in crossover differences.

There are many error sources in satellite altimetry (e.g., Tapley *et al.*, 1982). Thus, it is imperative that any estimate be deduced from many samples so as to minimize the effect of a particular error source or a particularly bad error realization because the errors are more or less independent of each other; and it is not unusual to find thousands or even millions of crossovers being treated in a single problem (e.g., Marsh *et al.*, 1982; Rapp, 1983). However, the simple prelude of finding the crossovers can consume more computing resources than solving the problem itself if an inefficient algorithm is used. Realizing the importance of a good algorithm, yet one would search in vain in the literature for a published algorithm to compute the crossovers. Since the launch of Geosat (March, 1985), the matter has taken on added urgency. The initial 18 months of the Geosat Mission is classified, but the crossover differences in this period are unclassified and would be available to the public if it were not such a cumbersome job.

The purpose of this note is to make one good and tested algorithm available to the public. Hopefully a better algorithm will be published as the result of this note. In the following, it will be demonstrated that one can derive the coordinates of a crossover point to within 1 second and 6 km of the true values (1 km if an ad hoc formula is used) from the circular orbit approximation while compensating for the earth's oblateness. Thus only one interpolation/extrapolation step is needed to derive the coordinates of the crossover from the data as oppose to three or four interpolation/extrapolation steps that are usually needed if deprived of this information.

2. CIRCULAR ORBIT APPROXIMATION

The orbit of the satellite is better described by an ellipse. However for the purpose of satellite altimetry, the orbit is made so circular that the circular approximation would greatly simplify the problem while incur little error. For example, the first eccentricity (see definitions in Section 3) of the Seasat's orbit is merely 0.001, while its counterpart for the earth is 0.0818 (i.e., one incurs much bigger error by assuming the earth is spherical. The effect of the earth's oblateness will be discussed in Section 3).

If one assumes the orbit is circular and the earth is spherical, the location of the ground track can be easily derived as a function of time. This has been done in Tai and Fu (1986, see their eq. (3), Appendix 1, and Figure 1). We can adapt their formulation to the present case; and the relevant equations are

$$\bar{\phi} = j \sin^{-1}(\sin \Delta t \sin i) \;, \tag{1}$$

$$\Delta\lambda = \tan^{-1}(\tan \Delta t \cos i) - \Omega \Delta t \,, \tag{2}$$

where the time, t, is nondimensionalized to make 2π correspond to the duration of one revolution; $\bar{\phi}$ is the latitude (the overhead bar conveys the fact that it is the geocentric latitude, see Figure 1); λ is the longitude and defined to be in the range of $[0, 2\pi)$; i is the inclination angle; Ω is the earth rotation rate relative to the orbital plane. Quantities with a subscript o are related to the equator crossing, and $\Delta t = t - t_o$, $\Delta\lambda = \lambda - \lambda_o$ (i.e., t_o and λ_o are the equator crossing time and longitude respectively). The adaptation is done so that equations (1) and (2) are valid for either an ascending or a descending track with j=1 if ascending and j=-1 if descending. Thus, the valid range of each variable is: $-i \le \bar{\phi} \le i$, $|\Delta t| \le \pi/2$, and $|\Delta\lambda| \le (1+k\Omega)\, \pi/2$, where k=1 if $i > \pi/2$ (i.e., retrograde), and k= -1 if $i < \pi/2$ (i.e., prograde).

2a. Determine the equator crossing time and longitude

As a first step, the algorithm requires that the data be sorted into ascending and descending tracks, and the equator crossing time and longitude be determined for each track. Because data gaps (e.g., over land) often prevent the direct determination of these coordinates from data, there is a need to determine them analytically. From a point (with cordinates t, $\bar{\phi}$, λ) along the track, one can determine from (1) that for the equator crossing time,

$$\Delta t = j \sin^{-1}(\sin \bar{\phi} / \sin i) \,, \tag{3}$$

$$t_o = t - \Delta t \;, \tag{4}$$

and for the equator crossing longitude, from (2) and (3),

$$\lambda_o = \lambda - \tan^{-1}(\tan \Delta t \cos i) + \Omega \Delta t \;. \tag{5}$$

Note that if λ_o should lie outside $[0, 2\pi)$, one can add or substract 2π from λ_o to make it fall in this range.

2b. Determine the coordinates of a crossover point

(1). Longitude

Let a subscript a (or d) convey the meaning of ascending (or descending). Then from the geometrical symmetry entailed in equations (1) and (2), it is easy to see that the longitude of the crossover point must lie right in the middle between the two equator crossing longitudes of the two tracks. To be more specific,

$$\lambda = \frac{\lambda_{oa} + \lambda_{od}}{2} \qquad \text{if } |\lambda_{oa} - \lambda_{od}| \le (1-\Omega)\pi \ , \tag{6a}$$

$$\lambda = \frac{\lambda_{oa} + \lambda_{od}}{2} \pm \pi \qquad \text{if } |\lambda_{oa} - \lambda_{od}| \ge (1+\Omega)\pi \ , \tag{6b}$$

where the sign in (6b) is such that $0 \le \lambda < 2\pi$. And if $(1-\Omega)\pi < |\lambda_{oa} - \lambda_{od}| < (1+\Omega)\pi$, we have two possibilities:

(i) two crossover points if $i > \pi/2$, i.e.,

$$\lambda_1 = \frac{\lambda_{oa} + \lambda_{od}}{2} \ , \quad \lambda_2 = \lambda_1 \pm \pi \tag{6c}$$

(ii) no crossover point if $i < \pi/2$.

(2) Time

Knowing λ (therefore $\Delta\lambda$), one can solve for Δt using eq. (2), which can be transformed to a more convenient form that avoids the evaluation of $\tan^{-1}$, i.e.,

$$f(\Delta t) = \tan(\Delta\lambda + \Omega\Delta t) - \tan\Delta t \cos i = 0 \ . \tag{7}$$

Hence, it becomes a problem of finding the zero of the transcendental function f. One can use either the ascending or the descending equator crossing longitude to form $\Delta\lambda$. Care must be taken (i.e., add or substract 2π) to make sure that $|\Delta\lambda| \le (1+k\Omega)\pi/2$. Also note that if $\Delta\lambda < 0$ (or $\Delta\lambda > 0$), Δt lies between 0 and $k\pi/2$ (or $-k\pi/2$). Remember that $k=1$ if $i > \pi/2$ and $k=-1$ if $i<\pi/2$.

(3) Latitude

The geocentric latitude can be derived immediately from eq. (1).

3. EFFECTS OF THE EARTH'S OBLATENESS

As mentioned in the previous section, the earth is far more elliptical than the orbit. The effects are twofold (see Fig. 1). First, the subsatellite point (marked as g in Fig. 1, where the line joining it with the satellite is perpendicular to the local earth surface) is different from the point (marked s in Fig. 1) where the line joining the satellite with the earth center intersects the earth surface. Secondly, the coordinate data are given in terms of the geographic latitude (defined as the angle between the local vertical and the equatorial plane, i.e., the latitude that can be determined from local astronomical observations), which is to be distinguished from the geocentric latitude because of the earth's oblateness (see Fig. 1, varibles with an overhead bar are geocentric, while those without it are geographic). The subscript g (or s) distinguishes quantities related to point g (or s).

The relation between ϕ and $\bar{\phi}$ can be derived from (see Appendix) the following equation.

$$\frac{\tan\bar{\phi}}{\tan\phi} = \frac{b^2}{a^2} = (1-e^2) = \frac{1}{1+e'^2} , \tag{8}$$

where a and b are respectiely the major and minor axis of the earth; e is the first eccentricity [defined as $e=(1-b^2/a^2)^{1/2}$]; e′ is the second eccentricity [defined as $e'=(a^2/b^2-1)^{1/2}$]. These values are given in Table 1.

The derivations in Section 2 all refer to $\bar{\phi}_s$, but the data refer to ϕ_g. Their relationship can be derived by solving the following transcendental equation (see Appendix),

$$\frac{c\sin\bar{\phi}_s - b\sin\beta}{c\cos\bar{\phi}_s - b\cos\beta} = \frac{a}{b}\tan\beta , \tag{9}$$

where c is the distance of the satellite to the center of the earth and β is related to ϕ_g [see eq. (10) below]. Given β, $\bar{\phi}_s$ can be solved, or vice versa; and the relation between β and ϕ_g is given by

$$\tan\phi_g = \frac{a}{b} tan\,\beta . \tag{10}$$

Two inequalities are very helpful in specifying the range that the solution of equation (9) must fall in. First, to determine the equator crossing time and longitude, ϕ_g is given, but $\bar{\phi}_s$ is needed in equations (3) and (4). Thus, to determine $\bar{\phi}_s$ from ϕ_g, $\bar{\phi}_s$ is in the range (see Fig. 1),

$$\bar{\phi}_g < \bar{\phi}_s < \phi_g . \tag{11}$$

Secondly, to determine the latitude of a crossover point, equation (1) yields the value of $\bar{\phi}_s$, while ϕ_g is the final answer. Hence, to determine ϕ_g from $\bar{\phi}_s$, ϕ_g is in the range (see Fig. 1),

$$\bar{\phi}_s < \phi_g < \phi_s . \tag{12}$$

It has been discovered that there is no need to solve equations (9) and (10). The following two approximations, representing the midpoints of the ranges specified by equations (11) and (12), actuall yield better results than equations (9) and (10).

$$\bar{\phi}_s = (\bar{\phi}_g + \phi_g)/2 , \tag{13}$$

$$\phi_g = (\bar{\phi}_s + \phi_s)/2 . \tag{14}$$

The better accuracy (1 km versus 6 km, see Section 5) does not portent physical consistency. Actually, the 6 km accuracy in location is consistent with 1 second accuracy in time (i.e., the ad-hoc approximation is simply a lucky guess). The orbit eccentricity (~~with~~ which can cause deviations from the circular orbit by typically a few parts per thousand) is perhaps the cause here.

To be physically more consistent, one could take the orbit eccentricity and the movement of the perigee (i.e., the point on the orbit ellipse which is closest to the center of the earth) into account. But this would defeat the purpose of an efficient algorithm because one only needs to get close enough so that one interpolation/extrapolation step would give us the precise values.

In summary, the oblateness only affects the latitude. Hence, it has no effect on the determination of the times and longitude of a crossover point (Section 2b) except that it may affect the equator crossing time and longitude if these coordinates have to be derived from the latitude (Section 2a) in lieu of the data. However, the oblateness effects can be accounted for by equations (9) and (10), or equations (13) and (14). As such, an accuracy to within 1 km can be achieved.

4. ALGORITHM

A. Sort data into ascending and descending tracks.

B. Determine the equator crossing time and longitude for each track.

It is recommended that each category of data be assembled into an array, e.g., all the latitudes of a track can form a latitude array. Then there is the so-called table look-up (look-down) subroutine, which searches a strictly increasing (decreasing) array for the location index that a specified level is penetrated for the first time. Thus, specifying zero for the latitude array, one can quickly find the closest points to the equator to facilitate the interpolation/extrapolation to the equator. However, if the nearest points are too far away, one could apply equations (3), (4), (5), and (13) to the nearest point to find the time and longitude of equator crossing.

C. From two tracks, compute λ (longitude of the crossover point) from eq. (6).

Note that λ is the perfect candidate for the specified level in the table look-up (look-down) subroutine in step E except that the longitude array is not strictly increasing (descreasing) because of the jump at zero longitude.

D. Compute Δt from eq. (7). Then compute t_a and t_d.

E. Use t_a and t_d as the specified levels in the table look-up to find the nearest points in the data to the crossover point.

F. Interpolation/extrapolation step.

However, if the nearest points are not close enough, this step should be abandoned because the crossover difference would be contaminated by the geoid.

5. RESULTS AND DISCUSSIONS

The algorithm outlined in the previous section has been tested using the Geosat exact repeat data. The first descending track of the second repeat period (17 days) is selected as the basis. Then crossovers between this and the subsequent 244 ascending tracks (i.e., to the end of the 17-day period) are computed using the algorithm. The results of the first day (actually the first file. There are 17 files for each repeat period) are tabulated in Table 2. Note that 260 crossovers are generated in 17 days because 16 ascending tracks form two crossovers each with the selected descending track (see Appendix 2 of Tai and Fu, 1986, for explanation). It would be rather cumbersome to list them all. In Table 3, statistics of the differences between predicted (i.e., analytic approximaton) and true (i.e., deduced from data) values are presented. Note that the data only yield 147 crossovers (out of a theoretically possible 260) because of data gaps.

The descending track descends from (72.054°N, 284.959°E) to (72.054°S, 92.910°E) and crosses the equator at 189.134°E on 59877839.064 seconds counting from the beginning of the Geosat Mission (hereafter 59 million seconds are substracted to make the presentation easier). In Table 2 and 3, all unprimed variables are derived from the data. Primed variables are derived from the equations. ϕ' is determined from equation (14) (i.e., a simple average), while ϕ'' is derived from equations (9) and (10). As discussed in Section 3, ϕ' approximates ϕ better than ϕ''. From Table 2, ϕ' is accurate to within 0.01° and ϕ'' is accurate to within 0.06°. The longitude is highly accurate except near high latitudes where the tracks are going more east-west than north-south. But even there, the maximum error is only about 0.01°. The time discrepancies are generally less than 1 second except for two cases where they are around 1.5 seconds. Note that the orbit eccentricity is partially accounted for here by adapting an orbit period of 6031.4 seconds north of the equator, while using 6043.6 seconds south of the equator. Because the purpose is to compute the coordinates of the crossover point, all available data have been used (i.e., the land values are included). These general remarks are substantiated by Table 3.

To compute all 260 crossovers, it takes about 0.5 second cpu time of the San Diego Supercomputer Center's Cray XMP-48. It translates to about two-minute cpu time to compute all the crossovers in the 17-day period. Because the predicted values are so close to the real values, only one interpolation/extrapolation step is needed versus the situation that many steps are needed if starting from scratch. Without any optimization, the prediction step takes less than one third the time that is needed for the search and interpolation/extrapolation step.

The transcendental equation in the prediction step is solved by a method, which combines the bisection and the secant rule. One could easily optimize the prediction step by solving and storing the solutions to be used later as the starting points for the interations. For instance, eq. (7) can be solved for Δt with $\Delta\lambda$ ranging from 1° to 192° with 1° increment. Then when one gets a $\Delta\lambda$ of 99.5°, he can use the solutions for 99° and 100° to start the interation. Furthermore, because the location can be predicted so accurately, when the crossover point is obviously over land, the extremely time consuming search step (i.e., table look-up, look-down) can be avoided altogether, for example, by setting up 1° by 1° land flags.

APPENDIX

To get the relation between ϕ and $\bar{\phi}$, an ellipse can be expressed as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ ,$$

or as

$$x = a \cos\beta \ , \tag{15}$$

$$y = b \sin\beta \ . \tag{16}$$

Thus from (15), (16),

$$\tan\bar{\phi} = \frac{y}{x} = \frac{b}{a} \tan\beta \tag{17}$$

and

$$\frac{dx}{d\beta} = -a \ sin\beta \ , \tag{18}$$

$$\frac{dy}{d\beta} = b \ cos\beta \ , \tag{19}$$

i.e., the tangential unit vector at the point (x,y) and in the direction of increasing β is proportional to (-a sinβ, b cosβ). Thus the normal unit vector is proportional to (b cosβ, a sinβ). Hence,

$$\tan\phi = \frac{a}{b}\tan\beta \tag{20}$$

From (17) and (20), eq. (8) is the result. To derive eq. (9), let us consider Fig. 1. The coordinate of the satellite is (c cos $\bar{\phi}_s$, c sin $\bar{\phi}_s$). The coordinate of point g can be expressed as (a cosβ, b sinβ) [see equations (15), (16)]. thus, one can form the vector from point g to the satellite and eq. (9) would result if one considers the ratio of the y component to the x component of this vector.

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FIGURE LEGEND

Fig. 1. An exaggerated view of the earth and the satellite.

Table 1. The relevant Geodetic Reference System 1980 values adopted in this paper and certain orbit parameters

a = 6378137 m	semimajor axis
b = 6356752 m	semiminor axis
$e^2 = 0.0066943800229$	e = first eccentricity
$e'^2 = 0.00673949677548$	e′ = second essentricity
c = 7163 km	distance between the satellite and the center of the earth

Table 2. Computed and predicted crossover coordinates where the time is in seconds, latitude and longitude in degrees.

λ_{oa}	t_d	t'_d	t_a	t'_a	λ	λ'	ϕ	ϕ'	ϕ''
356.581	N.A.	876395.66	N.A.	882304.23	N.A.	272.858	N.A.	71.664	71.709
331.499	876465.92	876465.69	888271.47	888271.69	260.319	260.317	70.406	70.406	70.453
306.417	N.A.	876548.71	N.A.	894225.99	N.A.	247.776	N.A.	68.000	68.052
281.335	N.A.	876657.10	N.A.	900155.21	N.A.	235.235	N.A.	63.813	63.872
256.254	876815.08	876814.40	906034.89	906035.60	222.697	222.694	56.512	56.506	56.575
231.172	877067.34	877066.59	911820.25	911821.03	210.155	210.153	43.370	43.360	43.435
206.090	877477.72	877477.28	917447.45	917447.91	197.612	197.612	20.560	20.551	20.601
181.007	878017.13	878017.66	922945.60	922945.05	185.071	185.071	-10.149	-10.146	-10.172
155.925	878487.38	878488.84	928512.87	928511.37	172.529	172.530	-36.588	-36.584	-36.656
130.843	878789.53	878790.95	934248.23	934246.78	159.988	159.989	-52.739	-52.740	-52.812
105.761	878974.18	878975.28	940101.13	940100.01	147.444	147.448	-61.698	-61.700	-61.762
80.679	N.A.	879097.57	N.A.	946015.34	N.A.	134.907	N.A.	-66.778	-66.832
55.597	879187.41	879187.90	851963.24	951962.64	122.354	122.366	-69.712	-69.713	-69.762
30.515	879261.35	879261.63	957926.87	957926.58	109.813	109.825	-71.328	-71.330	-71.375
5.434	879327.39	879327.47	963898.32	963898.17	97.274	97.284	-72.006	-72.008	-72.052
5.434	N.A.	876372.60	N.A.	966853.04	N.A.	277.284	N.A.	71.894	71.938

Table 3. Statistics of the differences between predicted (primed) and real (unprimed) values. Time is in seconds. Latitudes and Longitudes are in degrees.

Statistics	$t'_a - t_a$	$t'_d - t_d$	$\lambda' - \lambda$	$\phi' - \phi$	$\phi'' - \phi$
Maximum	0.79	1.53	0.011	0.0058	0.0667
Minimum	-1.56	-0.78	0.078	-0.0101	-0.0746
Mean	-0.34	0.34	-0.0004	-0.0023	-0.0162
Standard Deviation	0.82	0.79	0.0098	0.0041	0.0532
RMS Value	0.88	0.86	0.0098	0.0047	0.0555

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Fig. 1