ATLANTIS PRESS Journal of Statistical Theory and Applications, Vol. 16, No. 1 (March 2017) 26–37

An Efficient Estimation Procedure in Two-Occasion Successive Sampling

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Received 11 November 2015

Accepted 5 July 2016

Abstract

In this paper an efficient estimation procedure to estimate the current population mean in two-occasion successive sampling has been developed. An exponential regression type estimator of current population mean is proposed and corresponding optimum replacement strategy has been suggested. The superiority of the proposed estimator is empirically established over sample mean estimator and natural successive sampling estimator. Results are interpreted and suitable recommendations have been made.

Key words: Successive sampling, auxiliary information, bias, mean square error, optimum replacement strategy.

2000 Mathematics Subject Classification: 62D05.

1. Introduction

When character under study of a finite population changes over time, one time survey carried out on a single occasion provides information about the characteristic of the surveyed population for the given occasion only and does not give any information about the nature or pattern of change of characteristic over different occasions and the precise estimates of the characteristic over all occasions or on the most recent occasion. To overcome this situation, sampling is done on successive occasions for generating reliable estimates of population parameters on different occasions.

Theory of successive sampling appears to have started with the work of Jessen (1942), he was pioneered in using the entire information collected during previous investigations to make current estimates more precise. This theory was extended by Pattersons (1950), Rao and Graham (1964), Gupta (1979), Das (1982), among others. Sen (1971) developed estimators of the population mean on the current occasion using information on two auxiliary variables which were readily available on previous occasion. Sen (1972, 1973) extended his work for several auxiliary variables. Singh *et al.* (1991), and Singh and Singh (2001) used the auxiliary information on current occasion for estimating the current population mean in two occasion successive sampling. Singh (2003) extended the theory for h-occasion successive sampling.

In many situations, information on an auxiliary variable may be readily available on the first as well as on the second occasion, for example, tonnage (or seat capacity) of each vehicle or ship is known in transportation survey, more examples may be cited where the information on auxiliary variables are available on both the occasions of two-occasion successive sampling. Utilizing the auxiliary information on both the occasions , Feng and Zou (1997), Birader and Singh (2001), Singh (2005), Singh and Priyanka (2006, 2007), Singh and Priyanka (2008, 2010), Singh and Karna (2009), Singh and Vishwakarma (2009), Singh and Prasad (2010), Singh *et al.* (2011), Singh and Prasad (2013) and Singh and Homa (2013) among others have proposed varieties of estimators of population mean on current (second) occasions in two occasion successive sampling.

In follow up of the above arguments, the objective of the present work is to propose a more precise estimator of current population mean in two-occasion successive sampling using the information on two stable auxiliary variables which are readily available on both the occasions. Utilising the information on two auxiliary variables an exponential regression type estimator of current population mean in two-occasion successive sampling has been proposed. Properties of the proposed estimator are examined and relative comparison of the efficiencies have been made with sample mean estimator, when there is no matching from the previous occasion and the natural successive sampling estimator, when no auxiliary information is used. Empirical studies are carried out which show the highly significant improvements in the performances of the proposed estimator. Results have been nicely interpreted and suitable recommendations are made.

2. Formulation of Estimator

Let $U = (U_1, U_2, \text{-}, \text{-}, \text{-}, U_N)$ be the finite population of N units, which has been sampled over two occasions. The character under study be denoted by $x(y)$ on the first (second) occasion respectively. It is assumed that the information on two stable auxiliary variables z_1 and z_2 whose population means are known and closely related to x and y are readily available on first (second) occasion respectively. Let a simple random sample (without replacement) of size n be drawn on the first occasion. A random sub-sample of size $m = n\lambda$ is retained (matched) from the sample on first occasion for its use on the second occasion, while a fresh simple random sample (without replacement) of size $u = (n\text{-}m) = n\mu$ is drawn on the second occasion from the entire population so that the total sample size on this occasion is also n. Here λ and μ $(\lambda+\mu =1)$ are the fractions of the matched and fresh samples, respectively, on the current (second) occasion. The values of λ or μ would be chosen optimally.

The following notations have been considered for further use:

$\bar{X}$ $(\bar{Y})$: The population mean of the study variable x (y) on the first (second) occasion respectively.

$\bar{Z}_1$, $\bar{Z}_2$: Population means of the auxiliary variables z_1 and z_2 respectively.

$\bar{x}_n$, $\bar{x}_m$, $\bar{y}_u$, $\bar{y}_m$, $\bar{z}_{jn}$, $\bar{z}_{ju}$, $\bar{z}_{jm}$, $(j = 1, 2)$: The sample means of the respective variables based on the sample sizes shown in suffices.

ρ_{yx} , ρ_{yz_1}, ρ_{yz_2}, ρ_{xz_1}, ρ_{xz_2}, $\rho_{z_1z_2}$: Population correlation coefficients between the variables shown in suffices.

$b_{yx}^{(m)}$, $b_{yz_1}^{(m)}$ and $b_{yz_2}^{(m)}$: Sample regression coefficient between the variables shown in suffices and based on the sample size shows in braces

$S_x^2 = (N-1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})^2$: Population variance of the variable x.

$S_y^2, S_{z_1}^2, S_{z_2}^2$: Population variances of the variables y, z_1, z_2 respectively.

To estimate the population mean $\bar{Y}$ on the current (second) occasion, two independent estimators are suggested. One is exponential type estimator based on a sample of size u $(= n\mu)$ drawn afresh on the second occasion.

$$T_u = \bar{y}_u \exp \sum_{j=1}^{2} \left\{ \frac{\bar{Z}_j - \bar{z}_{ju}}{\bar{Z}_j + \bar{z}_{ju}} \right\}; (j = 1, 2) \tag{1}$$

Second estimator is an exponential regression type estimator based on the sample of size m (= $n\lambda$) common to both the occasions and is defined as

$$T_m = \bar{y}_m + b_{yx}^{(m)}(\bar{x}_n^* - \bar{x}_m^*) + b_{yz_1}^{(m)}(\bar{Z}_1 - \bar{z}_{1m}) + b_{yz_2}^{(m)}(\bar{Z}_2 - \bar{z}_{2m}) \tag{2}$$

where

$$\bar{x}_n^* = \bar{x}_n \exp\sum_{j=1}^{2}\left\{\frac{\bar{Z}_j - \bar{z}_{jn}}{\bar{Z}_j + \bar{z}_{jn}}\right\}; (j = 1,2)$$

and

$$\bar{x}_m^* = \bar{x}_m \exp\sum_{j=1}^{2}\left\{\frac{\bar{Z}_j - \bar{z}_{jm}}{\bar{Z}_j + \bar{z}_{jm}}\right\}; (j = 1,2).$$

Combining the estimators T_u and T_m, we have the final estimator of $\bar{Y}$ as

$$T = \varphi T_u + (1-\varphi)T_m \tag{3}$$

where $\varphi(0 \le \varphi \le 1)$ is an unknown constant (scalar) to be determined under certain criterion.

3. Properties of the Proposed Estimator

3.1 *Bias and Mean Square Error*

Since the estimators T_u and T_m are exponential and exponential regression type estimators, they are biased estimators of the population mean $\bar{Y}$. Therefore, the resulting estimator T is also a biased estimator of $\bar{Y}$. The bias $B(.)$ and mean square error $M(.)$ of the estimator T is derived up to the first order of approximations under large sample assumption and shown in the following theorems:

Theorem 1. *Bias of the estimator T to the first order of approximations is obtained as*

$$B(T) = \varphi B(T_u) + (1-\varphi)B(T_m) \tag{4}$$

where

$$B(T_u) = \left(\frac{1}{u} - \frac{1}{N}\right)\left\{\begin{matrix}\left(\alpha_{0100} - \frac{1}{2}\bar{Y}\left(\frac{\alpha_{0010}}{\bar{Z}_1} + \frac{\alpha_{0001}}{\bar{Z}_2}\right)\right) - \frac{1}{2}\frac{\alpha_{0110}}{\bar{Z}_1} - \frac{1}{2}\frac{\bar{Y}\alpha_{0102}}{\bar{Z}_1\bar{Z}_2} + \\ \frac{3}{8}\left(\frac{\alpha_{0020}}{\bar{Z}_1^2} + \frac{\alpha_{0002}}{\bar{Z}_2^2}\right)\end{matrix}\right\} \tag{5}$$

and

$$B(T_m) = \left(\frac{1}{m} - \frac{1}{N}\right)\left\{\left(\alpha_{0100} - \frac{\alpha_{0010}\alpha_{0110}}{\alpha_{0020}} + \frac{\alpha_{0120}}{\alpha_{0020}} - \frac{\alpha_{0030}\alpha_{0110}}{\alpha_{0020}^2} - \frac{\alpha_{0001}\alpha_{0101}}{\alpha_{0002}} - \frac{\alpha_{0102}}{\alpha_{0002}} + \frac{\alpha_{0003}\alpha_{0101}}{\alpha_{0002}^2}\right)\right\} +$$
$$\left(\frac{1}{m} - \frac{1}{n}\right)\left\{\frac{1}{\bar{Z}_1}\frac{\alpha_{0010}\alpha_{1100}}{\alpha_{2000}} + \frac{1}{\bar{Z}_2}\frac{\alpha_{0001}\,\alpha_{1100}}{\alpha_{2000}} + \frac{\bar{X}}{\bar{Z}_1}\frac{\alpha_{1110}}{\alpha_{2000}} + \frac{\bar{X}}{\bar{Z}_1}\frac{\alpha_{1101}}{\alpha_{2000}} - \frac{\bar{X}}{\bar{Z}_1}\frac{\alpha_{2010}\alpha_{1100}}{\alpha_{2000}^2} - \frac{\bar{X}}{\bar{Z}_2}\frac{\alpha_{2001}\alpha_{1100}}{\alpha_{2000}}\right\} - \frac{1}{4}\frac{\alpha_{0011}\alpha_{1100}}{\bar{Z}_1\bar{Z}_2} -$$
$$\frac{3}{8}\left(\frac{\bar{X}}{\bar{Z}_1^2}\frac{\alpha_{0020}\alpha_{1100}}{\alpha_{2000}} + \frac{\bar{X}}{\bar{Z}_2^2}\frac{\alpha_{0002}\alpha_{1100}}{\alpha_{2000}}\right) + \frac{\alpha_{3000}\alpha_{1100}}{\alpha_{2000}^2} - \frac{\alpha_{1000}\alpha_{1100}}{\alpha_{2000}} - \frac{\alpha_{2100}}{\alpha_{2000}} \tag{6}$$

where

$$\alpha_{pqrs} = E[(x_i - \bar{X})^p(y_i - \bar{Y})^q(z_{1i} - \bar{Z}_1)^r(z_{2i} - \bar{Z}_2)^s]; \quad ((p,q,r,s) \ge 0 \text{ are integers}).$$

Theorem 2. *Mean square error of the estimator T to the first degree of approximations is obtained as*

$$M(T) = \varphi^2 M(T_u) + (1-\varphi)^2 M(T_m) + 2\varphi(1-\varphi)C(T_u, T_m) \tag{7}$$

where

$$M(T_u) = \left(\frac{1}{u} - \frac{1}{N}\right) S_y^2 \left[\frac{3}{2} - \rho_{yz_1} - \rho_{yz_2} + \frac{1}{2}\rho_{z_1 z_2}\right] \quad (8)$$

$$M(T_m) = \left(\frac{1}{m} - \frac{1}{N}\right) S_y^2 \left[1 - \rho_{yz_1}^2 - \rho_{yz_2}^2 + 2\rho_{yz_1}\rho_{yz_2}\rho_{z_1 z_2}\right] + \left(\frac{1}{m} - \frac{1}{n}\right) S_y^2 \left[\rho_{yx}^2 \left(\frac{3}{2} - \rho_{xz_1} - \rho_{xz_2} + \frac{1}{2}\rho_{z_1 z_2}\right) + 2\rho_{yx}\left(\rho_{xz_1}\rho_{yz_1} + \rho_{xz_2}\rho_{yz_2}\right) - \rho_{yx}\rho_{z_1 z_2}\left(\rho_{yz_1} + \rho_{yz_2}\right) - 2\rho_{yx}^2\right] \quad (9)$$

and

$$C(T_u, T_m) = -\frac{1}{N} S_y^2 \left[1 - \rho_{yz_1}^2 - \rho_{yz_2}^2 + \frac{1}{2}\rho_{z_1 z_2}\left(\rho_{yz_1} + \rho_{yz_2}\right)\right] \quad (10)$$

Remark 1. The above results are derived under the assumptions that the coefficients of variation of variables x, y, z_1 and z_2 are approximately equal.

3.2 *Minimum mean square errors of the estimator T*

Since the mean square error of the estimator *T* in equation (7) is the function of the unknown constant (scalar) φ, therefore, it is minimized with respect to φ and subsequently the optimum value of φ is obtained as

$$\varphi_{opt} = \frac{M(T_m) - C(T_u, T_m)}{M(T_u) + M(T_m) - 2C(T_u, T_m)}. \quad (11)$$

From equation (11), substituting the value of φ_{opt} in equation (7) we get the optimum mean square error of the estimator *T* as

$$M(T)_{opt.} = \frac{M(T_u)M(T_m) - \{C(T_u, T_m)\}^2}{M(T_u) + M(T_m^*) - 2C(T_u, T_m)}. \quad (12)$$

Further substituting the values from equations (8)-(10) in equations (11) and (12), the simplified values of φ_{opt} and $M(T)_{opt.}$ are obtained as

$$\varphi_{opt} = \frac{\mu(A_{12} + \mu A_{11})}{A_1 - \mu A_{14} + \mu^2 A_{15}} \quad (13)$$

and

$$M(T)_{opt.} = \left[\frac{A_{22} + \mu^2 A_{21} - \mu A_{23}}{A_1 + \mu^2 A_{15} - \mu A_{14}}\right] \frac{S_y^2}{n} \quad (14)$$

where

$A_1 = \left[\frac{3}{2} - \rho_{yz_1} - \rho_{yz_2} + \frac{1}{2}\rho_{z_1 z_2}\right]$, $A_2 = \left[1 - \rho_{yz_1}^2 - \rho_{yz_2}^2 + 2\rho_{yz_1}\rho_{yz_2}\rho_{z_1 z_2}\right]$

$A_3 = \left[\rho_{yx}^2 \left\{\frac{3}{2} - \rho_{xz_1} - \rho_{xz_2} + \frac{1}{2}\rho_{z_1 z_2}\right\}\right]$, $A_4 = \left[2\rho_{yx}^2 + \rho_{yx}\rho_{z_1 z_2}(\rho_{yz_1} + \rho_{yz_2})\right]$,

$A_5 = \left[2\rho_{yx}\left(\rho_{yz_1}\rho_{xz_1} + \rho_{yz_2}\rho_{xz_2}\right)\right]$, $A_6 = \left[1 - \rho_{yz_1}^2 - \rho_{yz_2}^2 + \frac{1}{2}\rho_{z_1 z_2}(\rho_{yz_1} + \rho_{yz_2})\right]$

$A_7 = [A_6 - A_2]$, $A_8 = [A_2 + A_3 - A_4 + A_5]$, $A_9 = [A_3 - A_4 + A_5]$, $A_{10} = [A_8 - A_9]$

$A_{11} = [A_9 - fA_7]$, $A_{12} = [A_{10} + fA_7]$, $A_{13} = [A_6 - A_1]$, $A_{14} = [A_1 - A_2 - fA_{13}]$,

$A_{15} = [A_{11} - fA_{13}]$, $A_{16} = [A_1 A_2 + A_1 A_9]$, $A_{17} = [A_1 A_2]$, $A_{18} = [A_1 A_9]$

$A_{19} = [A_6^2 - A_1 A_2]$, $A_{20} = [fA_{16} - fA_{17} - fA_{18} - A_{18}]$, $A_{21} = [fA_{18} + f^2 A_{19}]$,

$A_{22} = [A_{16} - A_{18} - fA_{17}]$, $A_{23} = [f^2 A_{19} + A_{20}]$ *and* $f = \frac{n}{N}$.

4. Optimum Replacement strategy of the estimator T

The optimum mean square error $M(T)_{opt.}$ in equation (14) is a function μ (fraction of sample to be drawn afresh on the second occasion). It is an important factor in reducing the cost of the survey, therefore, to determine the optimum value of μ so that $\bar{Y}$ may be estimated with maximum precision and minimum cost, we minimize $M(T)_{opt.}$ with respect to μ which results in a quadratic equation in μ, which is shown as

$$\mu^2 D_1 + 2\mu D_2 + D_3 = 0 \tag{15}$$

Solving the equation (15) for μ, the solutions of μ (say $\hat{\mu}$) are given as

$$\hat{\mu} = \frac{-D_2 \pm \sqrt{D_2^2 - D_1 D_3}}{D_1} \tag{16}$$

where

$$D_1 = (A_{15}A_{23} - A_{14}A_{21}),\ D_2 = (A_1 A_{21} - A_{15}A_{22}),\ D_3 = (A_{14}A_{22} - A_1 A_{23})\ .$$

From equation (16) it is clear that the real values of $\hat{\mu}$ exist, iff, the quantities under square root is greater than or equal to zero. For any combination of correlations, which satisfy the condition of real solutions, two real values of $\hat{\mu}$ are possible. Hence, while choosing the values of $\hat{\mu}$, it should be remembered that $0 \le \hat{\mu} \le 1$, and all other values of $\hat{\mu}$ are said to be inadmissible. If both the values of $\hat{\mu}$ are admissible, the lowest one is the best choice as it reduces the cost of the survey. From equation (16), substituting the admissible value of $\hat{\mu}$ (say μ_0) in equation (14), we have the optimum value of mean square error of the estimator *T*, which is shown below:

$$M(T)^*_{opt} = \left[\frac{A_{22} + \mu_0^2 A_{21} - \mu_0 A_{23}}{A_1 + \mu_0^2 A_{15} - \mu_0 A_{14}}\right]\frac{S_y^2}{n}. \tag{17}$$

5. Efficiency Comparison

The percent relative efficiencies of the estimator *T* with respect to (i) sample mean estimator $\bar{y}_n$ when there is no matching and (ii) natural successive sampling estimator $\hat{\hat{Y}} = \varphi^* \bar{y}_u + (1 - \varphi^*)\bar{y}_m^*$, where $\bar{y}_m^* = \bar{y}_m + b_{yx}^{(m)}(\bar{x}_n - \bar{x}_m)$, when no auxiliary information is used on any occasion have been computed for different choices of correlations and presented in Tables 1-4. Following Sukhatme *et.al* (1984) the variance of $\bar{y}_n$ and optimum mean square error of $\hat{\hat{Y}}$ are given by

$$V(\bar{y}_n) = \left(\frac{1}{n} - \frac{1}{N}\right) S_y^2 \tag{18}$$

$$M(\hat{\hat{Y}})_{opt} = \left[1 + \sqrt{1 - \rho_{yx}^2}\right]\frac{S_y^2}{2n} - \frac{S_y^2}{N}. \tag{19}$$

Since, the optimum mean square error of the estimator *T* derived in equation (17) involve six correlations $\rho_{yx}, \rho_{xz_1}, \rho_{xz_2}, \rho_{yz_1}, \rho_{yz_2}, \rho_{z_1 z_2}$, therefore, for simplifying the expressions and to show the empirical results in tabular form we have considered the assumptions $\rho_{xz_1} = \rho_{yz_1} = \rho_0$ *and* $\rho_{xz_2} = \rho_{yz_2} = \rho_1$. These assumptions are intuitive which were also considered by Cochran (1977) and Feng and Zou (1997). Under the above assumptions, finally, we have only three correlations ρ_{yx}, ρ_0 and ρ_1 and subsequently, the values of A_1, A_2, A_3, A_4, A_5 and A_6take the following forms:

$$A_1 = \left[\frac{3}{2} - \rho_0 - \rho_1 + \frac{1}{2}\rho_{z_1 z_2}\right],\ A_2 = \left[1 - \rho_0^2 - \rho_1^2 + 2\rho_0\rho_1\rho_{z_1 z_2}\right]$$

$$A_3 = \left[\rho_{yx}^2\left\{\frac{3}{2} - \rho_0 - \rho_1 + \frac{1}{2}\rho_{z_1 z_2}\right\}\right],\ A_4 = \left[2\rho_{yx}^2 + \rho_{yx}\rho_{z_1 z_2}(\rho_0 + \rho_1)\right],$$

$$A_5 = \left[2\rho_{yx}(\rho_0^2 + \rho_1^2)\right],\ A_6 = \left[1 - \rho_0^2 - \rho_1^2 + \frac{1}{2}\rho_{z_1 z_2}(\rho_0 + \rho_1)\right]$$

Table 1. Optimum values of μ_0 and PRE's of T with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for f=0.1 and $\rho_{z_1 z_2} = 0.3$.

ρ_0	ρ_1	ρ_{yx}	0.6	0.7	0.8	0.9
0.4	0.4	μ_0	0.3663	0.4658	0.5425	0.6326
		E_1	132.78	139.32	149.92	168.88
		E_2	118.03	117.19	116.61	115.957
	0.5	μ_0	0.3881	0.4843	0.5535	0.6404
		E_1	144.61	152.08	164.33	186.81
		E_2	128.54	127.93	127.81	128.26
	0.6	μ_0	0.2343	0.4617	0.5395	0.6258
		E_1	160.87	168.63	182.22	207.56
		E_2	142.99	141.85	141.73	142.51
	0.8	μ_0	0.6109	0.7156	*	0.4037
		E_1	209.73	217.57		268.87
		E_2	186.43	183.02		184.61
0.5	0.4	μ_0	0.3881	0.4843	0.5535	0.6404
		E_1	144.61	152.08	164.33	186.81
		E_2	128.54	127.93	127.81	128.26
	0.5	μ_0	0.5134	0.5382	0.5782	0.6512
		E_1	158.43	167.02	181.04	207.33
		E_2	140.83	140.49	140.81	142.35
	0.6	μ_0	*	0.6170	0.5921	0.6440
		E_1		187.14	202.46	231.69
		E_2		157.42	157.46	159.08
	0.8	μ_0	0.4351	0.4520	0.4815	0.5350
		E_1	243.59	253.93	272.12	305.72
		E_2	216.53	213.60	211.65	209.91
0.6	0.4	μ_0	0.2343	0.4617	0.5395	0.6258
		E_1	160.87	168.63	182.22	207.56
		E_2	142.99	141.85	141.73	142.51
	0.5	μ_0	*	0.6170	0.5921	0.6440
		E_1		187.14	202.46	231.69
		E_2		157.42	157.46	159.08
	0.6	μ_0	0.2284	*	0.6820	0.6528
		E_1	199.23		230.76	262.27
		E_2	177.09		179.48	180.08
	0.8	μ_0	0.3284	0.3282	0.3084	*
		E_1	293.80	304.99	324.79	
		E_2	261.16	256.56	252.61	
0.7	0.4	μ_0	0.7220	**0.0152**	0.4638	0.5773
		E_1	180.09	193.05	205.64	232.83
		E_2	160.08	162.39	159.94	159.86
	0.5	μ_0	0.3863	0.1655	0.6163	0.6176
		E_1	203.59	212.35	231.19	262.44
		E_2	180.97	178.63	179.82	180.19
	0.6	μ_0	0.3008	0.2300	*	0.6855
		E_1	234.84	244.03		305.28
		E_2	208.75	205.28		209.60
	0.8	μ_0	0.2521	0.2527	0.2479	0.1987
		E_1	389.79	402.30	425.68	465.21
		E_2	346.48	338.41	331.08	319.41
0.8	0.4	μ_0	0.6109	0.7156	*	0.4037
		E_1	209.73	217.57		268.87
		E_2	186.43	183.02		184.61
	0.5	μ_0	0.4351	0.4520	0.4815	0.5350
		E_1	243.59	253.93	272.12	305.72
		E_2	216.53	213.60	211.65	209.91
	0.6	μ_0	0.3284	0.3282	0.3084	*
		E_1	293.80	304.99	324.79	
		E_2	261.16	256.56	252.61	
	0.8	μ_0	0.1530	0.1543	0.1559	0.1541
		E_1	695.75	712.60	747.82	811.09
		E_2	618.44	599.43	581.63	556.90

Note: "*" indicate μ_0 do not exist.

Table 2. Optimum values of μ_0 and PRE's of T with respect to $\bar{y}_n$ and $\hat{\hat{\bar{Y}}}$ for f=0.1 and $\rho_{z_1z_2}$= 0.5.

ρ_0	ρ_1	ρ_{yx}	0.6	0.7	0.8	0.9
0.4	0.4	μ_0	0.3668	0.4611	0.5403	0.6331
		E_1	123.45	129.45	139.06	155.99
		E_2	109.73	108.89	108.16	107.10
	0.5	μ_0	0.4153	0.4932	0.5612	0.6491
		E_1	132.09	139.01	150.09	170.09
		E_2	117.41	116.94	116.73	116.79
	0.6	μ_0	0.4276	0.5027	0.5636	0.6459
		E_1	143.62	151.19	163.41	185.77
		E_2	127.66	127.18	127.10	127.55
	0.8	μ_0	0.6513	0.6576	*	0.2622
		E_1	178.33	213.77		260.02
		E_2	158.51	179.82		178.53
0.5	0.4	μ_0	0.4153	0.4932	0.5612	0.6491
		E_1	132.09	139.01	150.09	170.09
		E_2	117.42	116.94	116.73	116.79
	0.5	μ_0	0.5279	0.5536	0.5941	0.6667
		E_1	141.64	149.41	161.81	184.61
		E_2	125.90	125.68	125.85	126.75
	0.6	μ_0	0.7220	0.6212	0.6198	0.6698
		E_1	155.65	163.34	176.50	200.99
		E_2	138.36	137.40	137.27	138.00
	0.8	μ_0	0.2317	0.3386	0.2639	0.9230
		E_1	192.77	237.92	251.03	285.95
		E_2	171.35	200.14	195.25	196.33
0.6	0.4	μ_0	0.4276	0.5027	0.5636	0.6459
		E_1	143.62	151.19	163.41	185.77
		E_2	127.66	127.18	127.10	127.55
	0.5	μ_0	0.7220	0.6212	0.6198	0.6698
		E_1	155.65	163.34	176.50	200.99
		E_2	138.36	137.40	137.27	138.00
	0.6	μ_0	*	0.8416	0.6908	0.6877
		E_1		182.98	194.63	219.55
		E_2		153.92	151.38	150.75
	0.8	μ_0	0.0794	0.1503	0.0443	*
		E_1	207.52	263.02	269.87	
		E_2	184.46	221.25	209.90	
0.7	0.4	μ_0	0.2456	0.4717	0.5418	0.6211
		E_1	159.27	166.86	179.87	203.62
		E_2	141.57	140.36	139.90	139.81
	0.5	μ_0	*	0.8017	0.6561	0.6645
		E_1		183.21	195.53	220.45
		E_2		154.11	152.08	151.36
	0.6	μ_0	*	*	0.8750	0.7223
		E_1			223.56	244.72
		E_2			173.88	168.02
	0.8	μ_0	**0.0292**	0.0341	*	*
		E_1	232.30	302.09		
		E_2	206.49	254.11		
0.8	0.4	μ_0	0.6513	*	0.4407	0.5593
		E_1	178.33		201.54	225.60
		E_2	158.51		156.75	154.90
	0.5	μ_0	0.2317	*	0.8441	0.6655
		E_1	192.77		223.34	245.81
		E_2	171.35		173.71	168.78
	0.6	μ_0	0.0794	*	*	0.8844
		E_1	207.52			287.79
		E_2	184.46			197.60

Note: "*" indicate μ_0 do not exist.

Table 3. Optimum values of μ_0 and PRE's of T with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for f=0.1 and $\rho_{z_1z_2}$ = 0.7.

ρ_0	ρ_1	ρ_{yx}	0.6	0.7	0.8	0.9
0.4	0.4	μ_0	**0.3733**	0.4623	0.5416	0.6350
		E_1	115.46	121.05	129.85	145.10
		E_2	102.63	101.83	100.99	100.63
	0.5	μ_0	0.4348	0.5036	0.5701	0.6580
		E_1	121.73	128.20	138.33	156.35
		E_2	108.20	107.84	107.59	107.35
	0.6	μ_0	0.4754	0.5277	0.5826	0.6630
		E_1	130.21	137.28	148.40	168.45
		E_2	115.74	115.48	115.42	115.65
	0.8	μ_0	0.4697	0.5214	0.5597	0.6211
		E_1	155.73	163.24	175.28	196.54
		E_2	138.42	137.32	136.33	134.94
0.5	0.4	μ_0	0.4348	0.5036	0.5701	0.6580
		E_1	121.73	128.20	138.33	156.35
		E_2	108.20	107.84	107.59	107.35
	0.5	μ_0	0.5402	0.5668	0.6078	0.6798
		E_1	128.24	135.37	146.51	166.62
		E_2	113.99	113.87	113.95	114.40
	0.6	μ_0	0.6662	0.6300	0.6383	0.6886
		E_1	138.03	145.17	156.74	177.80
		E_2	122.70	122.11	121.90	122.08
	0.8	μ_0	*	*	0.7570	0.7018
		E_1			187.91	206.83
		E_2			146.15	142.01
0.6	0.4	μ_0	0.4754	0.5277	0.5826	0.6630
		E_1	130.21	137.28	148.40	168.45
		E_2	115.74	115.48	115.42	115.65
	0.5	μ_0	0.6662	0.6300	0.6383	0.6886
		E_1	138.03	145.17	156.74	177.80
		E_2	122.70	122.11	121.90	122.08
	0.6	μ_0	0.9917	0.7648	0.7001	0.7089
		E_1	153.84	158.00	168.61	189.21
		E_2	136.75	132.91	131.14	129.91
	0.8	μ_0	*	*	*	0.8236
		E_1				229.67
		E_2				157.69
0.7	0.4	μ_0	0.4995	0.5369	0.5808	0.6512
		E_1	141.27	148.70	160.46	181.56
		E_2	125.57	125.08	124.80	124.66
	0.5	μ_0	0.9592	0.7287	0.6748	0.6915
		E_1	153.88	158.77	169.82	190.63
		E_2	136.79	133.55	132.08	130.89
	0.6	μ_0	*	*	0.8054	0.7403
		E_1			186.82	204.73
		E_2			145.30	140.56
	0.8	μ_0	*	*	*	*
		E_1				
		E_2				
0.8	0.4	μ_0	0.4697	0.5214	0.5597	0.6211
		E_1	155.73	163.24	175.28	196.54
		E_2	138.42	137.32	136.33	134.94
	0.5	μ_0	*	*	0.7570	0.7018
		E_1			187.91	206.83
		E_2			146.15	142.01
	0.6	μ_0	*	*	*	0.8236
		E_1				229.67
		E_2				157.69

Note: "*" indicate μ_0 do not exist.

Table 4. Optimum values of μ_0 and PRE's of T with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for f=0.1 and $\rho_{z_1z_2} = 0$.

ρ_0	ρ_1	ρ_{yx}	0.6	0.7	0.8	0.9
0.4	0.4	μ_0	0.4149	0.5004	0.5588	0.6367
		E_1	150.37	158.10	170.68	193.61
		E_2	133.66	132.99	132.75	132.93
	0.5	μ_0	0.1001	0.4902	0.5507	0.6293
		E_1	169.52	177.91	192.59	220.15
		E_2	150.68	149.66	149.79	151.16
	0.6	μ_0	0.5926	*	0.4787	0.5868
		E_1	194.39		221.85	253.68
		E_2	172.79		172.55	174.18
	0.8	μ_0	0.5683	0.6080	0.7026	*
		E_1	288.88	299.01	316.56	
		E_2	256.78	251.52	246.21	
0.5	0.4	μ_0	**0.1001**	0.4902	0.5507	0.6293
		E_1	169.52	177.91	192.59	220.15
		E_2	150.68	149.66	149.79	151.16
	0.5	μ_0	0.4858	0.5091	0.5481	0.6218
		E_1	193.72	204.03	221.59	255.63
		E_2	172.20	171.63	172.34	175.51
	0.6	μ_0	0.4920	0.5481	0.4277	0.5721
		E_1	229.55	241.27	262.56	303.30
		E_2	204.05	202.95	204.21	208.25
	0.8	μ_0	0.4960	0.5237	0.5850	0.7486
		E_1	388.39	402.84	430.69	478.98
		E_2	345.24	338.87	334.98	328.87
0.6	0.4	μ_0	0.5926	*	0.4787	0.5868
		E_1	194.39		221.85	253.68
		E_2	172.79		172.55	174.18
	0.5	μ_0	0.4920	0.5481	0.4277	0.5721
		E_1	229.55	241.27	262.56	303.30
		E_2	204.05	202.95	204.21	208.25
	0.6	μ_0	0.4692	0.5041	0.6236	0.4493
		E_1	284.25	298.27	323.29	378.53
		E_2	252.67	250.90	251.45	259.90
	0.8	μ_0	0.4217	0.4414	0.4855	0.5829
		E_1	635.42	658.45	707.62	803.82
		E_2	564.82	553.88	550.37	551.91
0.7	0.4	μ_0	0.5753	0.6569	**	0.4144
		E_1	231.24	241.03		302.56
		E_2	205.55	202.75		207.74
	0.5	μ_0	0.5012	0.5419	0.6656	0.1449
		E_1	285.88	298.94	321.74	385.86
		E_2	254.11	251.47	250.24	264.93
	0.6	μ_0	0.4555	0.4837	0.5477	0.7857
		E_1	383.81	400.85	433.31	491.18
		E_2	341.17	337.19	337.01	337.24
	0.8	μ_0	*	*	*	*
		E_1				
		E_2				
0.8	0.4	μ_0	0.5683	0.6080	0.7026	*
		E_1	288.88	299.01	316.56	
		E_2	256.78	251.52	246.21	
	0.5	μ_0	0.4960	0.5237	0.5850	0.7486
		E_1	388.39	402.84	430.69	478.98
		E_2	345.24	338.87	334.98	328.87
	0.6	μ_0	0.4217	0.4414	0.4855	0.5829
		E_1	635.42	658.45	707.62	803.82
		E_2	564.82	553.88	550.37	551.91

Note: "*" indicate μ_0 do not exist.

For different choices of correlations ρ_{yx}, ρ_0, ρ_1 , $\rho_{z_1z_2}$ and f, Tables 1-4 present the optimum values of μ and the percent relative efficiencies E_1 and E_2 of T with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ respectively, where $E_1 = \frac{V(\bar{y}_n)}{M(T)^*_{opt}} \times 100$ and $E_2 = \frac{M(\hat{\hat{Y}})_{opt}}{M(T)^*_{opt}} \times 100$.

6. Interpretations of Results

6.1 *Interpretations based on Table 1*

- **(a)** For fixed values of ρ_0 and ρ_1, the values of μ_0 and E_1 are increasing while the values of E_2 do not show any definite pattern with the increasing values of ρ_{yx}.This behaviour is in agreement with Sukhatme *et al.* (1984), results which explain that more the value of ρ_{yx}, more the fraction of fresh sample required on the current occasion.
- **(b)** For fixed values of ρ_1 and ρ_{yx} , the values of E_1 and E_2 are increasing while no definite patterns are seen in the values of μ_0 with the increasing values of ρ_0 .
- **(c)** For fixed values of ρ_0 and ρ_{yx} the values of E_1 and E_2 are increasing while no definite patterns are seen in the values of μ_0 with the increasing values of ρ_1 .
- **(d)** Minimum value of μ_0 is observed as 0.0152, which indicates that the fraction of sample to be replaced on the current occasion is as low as about 1 percent of the total sample size, which leads to huge reduction in cost of the survey, which is a highly desirable phenomenon.

6.2 *Interpretations based on Table 2*

- **(a)** For fixed values of ρ_0 and ρ_1, the values of $\mu_0$and E_1 are increasing while the values of E_2 do not follow any definite pattern with the increase in the values of ρ_{yx}.This behaviour is in agreement with the standard results in successive sampling which explain that the more fraction of fresh sample on the current occasion is required with the increase in the values ρ_{yx}.
- **(b)** For fixed values of ρ_1 and ρ_{yx} , the values of E_1 and E_2 are increasing while no definite patterns are visible in the values of μ_0 with the increasing values of ρ_0 .
- **(c)** For fixed values of ρ_0 and ρ_{yx} the values of E_1 and E_2 are increasing while no definite trends are seen in the values of μ_0 if we increase the values of ρ_1 .
- **(d)** Minimum value of μ_0 is found as 0.0292, which indicates that the fraction of sample to be replaced on the current occasion is as low as about 2 percent of the total sample size, which leads in reduction of survey cost, such behaviour is always desired in survey sampling.

6.3 *Interpretations based on Table 3*

- **(a)** For fixed choices of ρ_0 and ρ_1, the values of μ_0 and E_1 show the increasing pattern while the values of E_2 do not follow any trend when we increase the values of ρ_{yx}.These behaviours support the standard theory of successive sampling that more the value of ρ_{yx}, more the fraction of fresh sample is required on the current occasion.
- **(b)** For fixed choices of ρ_1 and ρ_{yx} , the values of E_1 and E_2 are increasing while no trends are seen in the values of μ_0 with the increasing values of ρ_0 .
- **(c)** For fixed choices of ρ_0 and ρ_{yx} the values of E_1 and E_2 are increasing while no definite patterns are seen in the values of μ_0 with the increasing values of ρ_1 .
- **(d)** Minimum value of μ_0 is observed as 0.3733, which indicates that the fraction to be replaced on the current occasion is as low as about 37 percent of the total sample size, which leads in reduction of the survey cost.

(e) If we compare the results of percent relative efficiencies with the results presented in Tables 1-2, it is clearly visible that the percent relative efficiencies are decreasing with the increase in the values of correlation coefficient between auxiliary variables z_1 and z_2. This finding generates curiosity to examine the behaviour of the proposed estimator when the auxiliary variables are independent. For such situation results are given in Table 4 and corresponding interpretations are given below in sub section 6.4.

6.4 *Interpretations based on Table 4*

(a) When auxiliary variables are uncorrelated, it has been observed that for fixed choices of ρ_0 and ρ_1 the values of μ_0 and E_1 increase with the increase in the values of ρ_{yx}, while no definite patterns are observed in the values of E_2.

(b) For fixed values of ρ_1 and ρ_{yx} , the values of E_1 and E_2 are increasing while no definite patterns are seen in the values of μ_0 with the increasing values of ρ_0 . Similar pattern are visible for the case when the values of ρ_0 and ρ_{yx} are fixed and increasing values of ρ_1 are observed.

7. Conclusions and Recommendations

From the above interpretations and discussions it has been observed that the use of information on two auxiliary variables on estimation stage is highly rewarding in terms of precision of the proposed estimator. The most important point, we have noticed in the present work is the percent relative efficiencies of the proposed estimator are decreasing with the increase in the values of correlation coefficient between auxiliary variables z_1 and z_2. This phenomenon suggests that if information on more number of mutually least correlated auxiliary variables is used at the estimation stage, more reliable estimates of population parameters may be generated. Looking on the nice behaviour of the proposed estimator the survey statisticians may be recommended for its practical applications in their real life problems.

Acknowledgements

Authors are thankful to the reviewer for their constructive suggestions. Authors are also thankful to the Indian Institute of Technology (Indian School of Mines), Dhanbad for providing financial and infrastructural support in completion of this research work.

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