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AN EFFICIENT FINITE-ELEMENT METHOD FOR THE ANALYSIS OF PHOTONIC BAND-GAP MATERIALS

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Abstract

An efficient finite-element method (FEM) is presented in this paper to calculate the bandgap information of photonic bandgap (PBG) Materials. A uniaxial anisotropic absorber is used to enclose the computational domain of the finite-element method. The presented method is very efficient in the bandgap calculation, which is essential for the design of various practical applications using PBG materials.

1 Introduction

The bandgap properties of photonic band-gap (PBG) materials provide basic understanding of PBG structures, for example, one can predict which kind of waves can be propagated or prohibited along certain directions, which can be used to direct antenna design using PBG material substrates. The PBG materials considered in this paper are periodic in two dimensions and conductor backed.

Many electromagnetic simulation techniques can be applied to calculate the bandgap information of PBG materials, such as the FDTD method in [1], the periodic volume integral equation method(VIEM) in [2], and the finite element-integral equation method(FE-IEM) in [3]. In the calculation, basically a pair of propagation constants along the surface is given, then the numerical method is used to find the related frequencies. When using FDTD, we let a time domain source pass through the PBG material, and we examine the peaks of the Fourier transform of the response. In this method, the computation time for each pair of specified propagation constants might be very long, which is typical for periodic structures. For the VIEM and FE-IEM, given the propagation constants, one needs to solve a matrix equation repeatedly to search for the desired frequency, which is very time consuming.

In this paper we present an efficient finite element method. In this method, the bandgap information is obtained by solving an eigenvalue problem. In our previous paper[3], we used a FE-IEM approach where the integral equation is used to truncate the computational domain. Here, we use a uniaxial anisotropic perfectly matched layer (PML) as an absorber [4]. In this way, we eliminate the bottleneck coming from the integral equation, and the final matrix in the eigenvalue problem is sparse. Also, the matrix elements are not function of frequency, therefore the iteration steps in [2][3] are no longer needed.

2 Basic Formulation

Shown in Fig. 1 is the computational domain of the FEM. The PBG material is periodic along both the x and y directions but finite in the z direction. In the z direction at the bottom, it is conductor backed, so that it can be used as the substrate for printed antenna applications. Our goal is to study the properties of the eigenmodes of such a PBG structure and therefore to direct our antenna design. The computational domain is chosen to be one unit cell of the PBG material. A uniaxial anisotropic absorber presented in is used as a perfect matched layer. Application of such a absorber as a perfect matched layer has been extensively discussed in [4]. By placing a PEC at the end of the PML the computational domain is enclosed.

Using the variational principle for non-self-adjoint operators [5][6], the equivalent functional for this periodic problem is:

$$\begin{aligned} F(\vec{E},\vec{E}_a) &= \iiint(\nabla\times\vec{E}_a)\cdot([\mu_r]^{-1}\nabla\times\vec{E})dv \\ &\quad -k_0^2\iiint\vec{E}_a\cdot([\epsilon_r]\vec{E})dv \qquad (1) \end{aligned}$$

Where $\vec{E}$ and $\vec{E}_a$ are the electromagnetic fields satisfying the original and adjoint problems respectively. After going through the standard finite element procedure, the following eigenvalue equation can be obtained:

$$[A][E] = k_0^2[B][E] \qquad (2)$$

For different specified pair of k_x and k_y, we can find the corresponding frequencies, repeat it for typical propagation constants in one irreducible Brouinne zone, we can obtain the bandgap information of the PBG material.

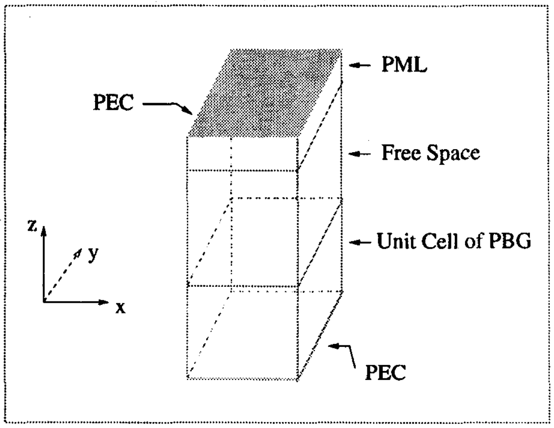

Figure 1: Computational domain of FEM. A uniaxial anisotropic absorber with a PEC is used to enclose the computational domain.

3 Numerical Examples

Using the present method, we calculate the bandgap information of the PBG material shown in Fig. 2, the result is plotted in Fig. 3. In [3], we obtained the bandgap of this material using FE-IEM which is shown in Fig. 4, where only the first two modes have been plotted there. It can be concluded that the agreement between these two methods is very good.

It would be interesting if a thin PBG material has a complete bandgap since such a PBG material is very well suited for practical printed circuit and antenna design applications. Shown in Fig. 5 is a PBG material with complete bandgaps which is presented by D. Sievenpiper and E. Yablonovitch

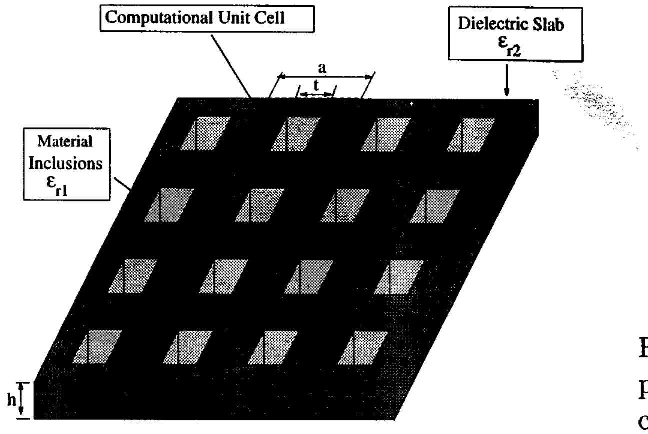

Figure 2: PBG material composed by placing periodic material blocks inside a host dielectric slab [2]. When this material is used as an antenna substrate, it is often backed by a conductor.

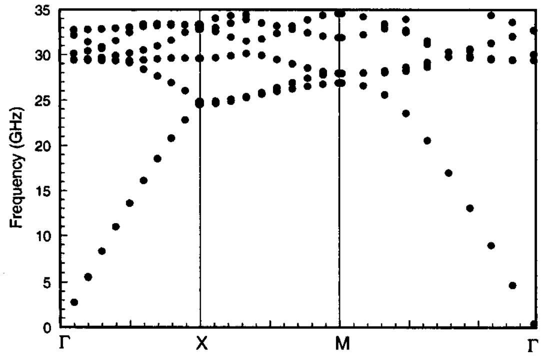

Figure 3: Photonic bandgap for materials composed of air blocks inside a dielectric slab calculated using present FEM. $a = 5.0mm$, $t = 3.0mm$, $h = 1.0mm$, $\epsilon_{r1} = 1.0$, $\epsilon_{r2} = 10.0$.

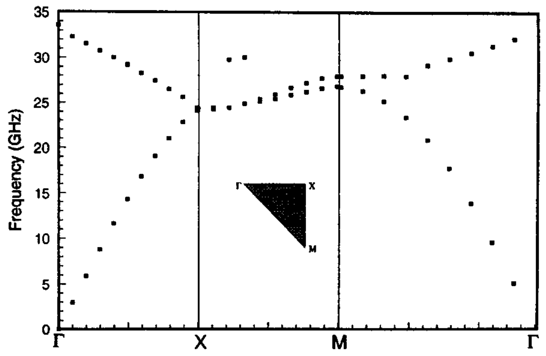

Figure 4: Photonic bandgap for materials composed of air blocks inside a dielectric slab calculated using the FE-IEM in [3].

[7][8][9]. It is composed of metallic patches with grounded vias supported by a conductor backed substrate. This class of structures will have complete bandgaps if properly designed, the material can be regarded as equivalent distributed transmission lines periodically loaded with inductors and capacitors. Using our FEM code, we first calculated the bandgap of such a material, which is whown in Fig. 6. The first two modes are TM and TE waves, respectively, which can be identified from the field distribution, the TE wave near the Γ points are attenuating which is labeled by hollow circles. We see that there is a complete bandgap around 11.0GHz, such a bandgap has been experimentally verified [7][9], which further proves the method we presented.

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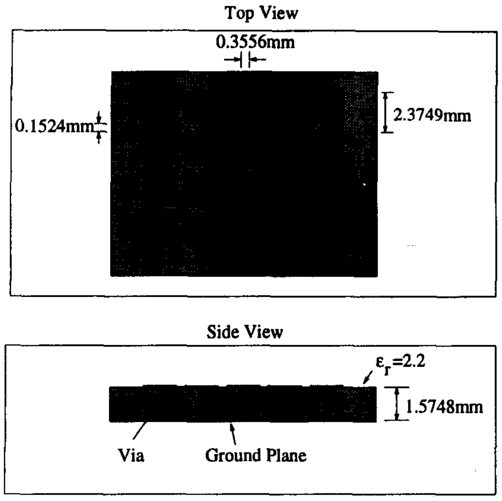

Figure 5: PBG material composed of metallic patches with grounded vias supported by a conductor backed substrate, presented by D. Sievenpiper and E. Yablonovitch [8].

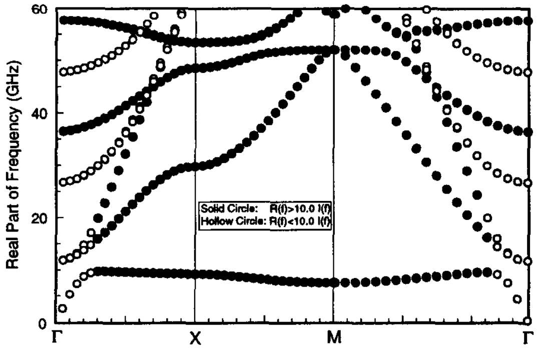

Figure 6: Bandgap information for the structure in Fig. 5 calculated using present FEM.

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