An Efficient QR-based Selection Criterion for Selecting an Optimal Precoding Matrix Employed in a Simplistic MIMO Detection

Chien-Hung Pan
Department of Communication Engineering
National Chiao Tung University
1001 Ta-Hsueh Rd., Hsinchu city, Hsinchu 300, Taiwan.
Taiwan (ROC)
pch.cm92g@nctu.edu.tw

Abstract: - In multiple-input multiple-output (MIMO) channel (**H**) communication, when channel status information (CSI) is known to the receiver but not to the transmitter, the precoding technique can achieve a highly reliable communication link, when the receiver informs an optimal precoding matrix index to the transmitter based on current CSI. To select an optimal precoding matrix (**F**), the maximum capacity selection criterion and the maximum minimum singular value selection criterion are developed. However, with QR-decomposition detection (**HF** = **QR**) in the precoding system, these two selection criteria may involve high complexity and poor detection performance due to the full matrix multiplication and inaccurate detection of the first layer, respectively. In this paper, to simplify the QR-decomposition processes, the real and imaginary parts of channel elements are rearranged to achieve a column-wise orthogonal structure to reduce the repeated computation. In precoding systems, to achieve 1) low-complexity and 2) performance enhancement, the efficient QR-based selection (QR-selection) criterion is proposed to select an optimal precoding matrix by maximizing the absolute value of the lowest layer of the upper triangular matrix **R**. For low-complexity, to reduce the multiplication complexity of computing **R**, we prove that the absolute value of **R** is equal to the absolute value of $\overline{\mathbf{R}}$ ($\dot{\mathbf{R}}\mathbf{F} = \overline{\mathbf{QR}}$), where $\mathbf{H} = \dot{\mathbf{Q}}\dot{\mathbf{R}}$. Based on this equivalence, we can reduce the multiplication complexity because the number of multiplications for computing **RF** is less than the number of multiplications for computing **HF**. For performance enhancement, the proposed QR-selection criterion can effectively mitigate the impact of error-propagation because the probability of an early error in the sequence of decisions is lower. Simulation results show that the proposed scheme with a low-complexity level has a better performance than others, and that it can improve detection performance as the codebook size increases.

Key-Words: Multiple-input multiple-output, precoding, QR-decomposition, capacity selection, minimum singular value selection, QR selection, column-wise orthogonal structure, codebook.

1 Introduction

In multiple-input multiple-output (MIMO) wireless communications [1]-[5], to make sure of an efficient transmission, a spatial multiplexing (SM) system is often employed [6]. In SM systems, the successive interference cancellation (SIC) detection in the receiver can separate the streams one by one via applying sequential interference nulling and cancellation [7]-[10]. Recently, QR-based MIMO detection with reduced complexity has been devised to realize the SIC detection [11]-[19]. Therefore, due to implementation simplicity and numerical stability, the QR-based MIMO detection has been applied in many places such as the precoding system. In the limited feedback system [20]-[22], channel status information (CSI) is not always known to the transmitter because the forward and reverse channels lack reciprocity. When CSI is known to the receiver but not to the transmitter, the precoding technique can achieve a highly reliable communication link, where the receiver informs an optimal precoding matrix index to the transmitter under assumption that the transmitter and receiver have the same codebook. Most of the existing works on limited feedback precoding systems focus on the development of quantization scheme and the related performance analysis [23]-[24]. In practice, the precoding matrix (**F**) with quantization scheme was also mentioned in the technical reports of IEEE 802.16-2005 standard [25]-[30]. Other important issues, the optimal codebook design with different

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dimensions and sizes are also constructed in this standard. For the performance analysis, the optimal precoding selection criteria in terms of maximum MIMO channel (**H**) capacity and maximum minimum singular value (MSV) were reported in [31]-[33]. Assuming the limited, zero-delay, and error-free feedback precoding system, these two selection criteria can dramatically improve the detection performance in MIMO communication. However, with QR-decomposition detection (**HF** = **QR**) in the precoding system, these two selection criteria may involve high complexity and poor detection performance due to the full matrix multiplication and inaccurate detection of the first layer, respectively.

In this paper, for the implementation simplicity, we develop an efficient precoding selection criterion employed in MIMO QR-based detection. To simplify the QR-decomposition processes, we propose a simplistic QR-based detection where the real and imaginary parts of the channel elements are rearranged to achieve a column-wise orthogonal structure. This column-wise orthogonal structure means that the neighboring columns of **HF** are orthogonal. By using this orthogonal structure, the proposed QR-decomposition processes are less complex than the conventional QR-decomposition processes [12]-[14] because elements in the neighboring columns of the upper triangular matrix are repeated, and thus only half of the elements are computed. This upper triangular matrix is obtained from QR-decomposition of the equivalent MIMO channel (**HF**). That is, **HF**, the equivalent MIMO channel, is composed of the MIMO channel (**H**) and the precoding matrix (**F**). Then, in the precoding system, to achieve 1) low-complexity and 2) performance enhancement, the efficient QR-based selection (QR-selection) criterion is proposed to select an optimal precoding matrix by maximizing the absolute value of the lowest layer of the upper triangular matrix **R**. Namely, this proposed QR-selection criterion maximizes the absolute value of the (N, N) entry of **R** under the assumption that **R**'s size is N-by-N. For low-complexity, when the codebook size is large, both capacity-selection criterion and MSV-selection criterion have high computational complexity due to the full matrix multiplication. Hence, to reduce the multiplication complexity of computing **R**, we prove that the absolute value of **R** is equal to the absolute value of the upper triangular matrix $\overline{\mathbf{R}}$ ($\dot{\mathbf{R}}\mathbf{F} = \overline{\mathbf{Q}\mathbf{R}}$), where $\mathbf{H} = \dot{\mathbf{Q}}\dot{\mathbf{R}}$ is QR-decomposition of **H**. In other words, we prove that $|\mathbf{R}| = |\overline{\mathbf{R}}|$. This upper triangular matrix $\overline{\mathbf{R}}$ is obtained from QR-decomposition jointly with respect to $\dot{\mathbf{R}}$ and **F**. Based on this equivalence, we can reduce the multiplication complexity because the number of multiplications for computing $\dot{\mathbf{R}}\mathbf{F}$ is less than the number of multiplications for computing **HF**. From this point of view, the multiplication ($\dot{\mathbf{R}}\mathbf{F}$) of the upper triangular matrix ($\dot{\mathbf{R}}$) and the precoding matrix (**F**) is proposed in the QR-selection processes due to involving a lot of zeros in $\dot{\mathbf{R}}$. Additionally, the QR-based MIMO detection performance is limited due to error-propagation. To reduce the error-propagation effect, the ordered SIC (OSIC) [9] and sort QR-decomposition (SQRD) detection schemes [11]-[19] show that the accurate detection of the first layer is very import as the detection performance of all remaining sub-streams is influenced with the detection accuracy of the first detected sub-stream. That is, the first sub-stream with the smallest post-detection SNR will dominate the system performance. Based on this knowledge, for performance enhancement, to enhance the QR-based detection performance, the proposed QR-selection criterion can effectively mitigate the impact of error-propagation because the probability of an early error in the sequence of decisions is lower [11]-[19]. Namely, by referring to the OSIC knowledge, the proposed QR-selection criterion has largest post-detection SNR in the first sub-stream and thus detection accuracy in each layer, leading to a better performance against spatial correlation. By using IEEE 802.16-2005 standard [30], simulation results show that the proposed scheme has better performance than others [31]-[33] but at a low-complexity level. In addition, we claim that the proposed QR-selection criterion can improve detection performance as the codebook size increases in simulations.

This paper is organized as follows. In Section II, the system model is described. In Section III, various precoding selection criteria are depicted. In Section IV, the computational complexity of the proposed scheme is analyzed. In Section V, we conduct the computer simulation to confirm the effectiveness of the proposed algorithm. In Section VI, we give a conclusion and suggest future work. Finally, we collect all proofs in the Appendix in order to enhance the flow of the paper.

2 System Model Problem Formulation

In MIMO wireless communication [1]-[6], the precoding matrix is often used to improve the channel capacity and reliability link. In many MIMO wireless systems, the transmitter has no current channel state information (CSI) because the forward and reverse channels lack reciprocity [20]-[24]. The basic idea of limited feedback systems is to select an optimal precoding matrix from a given codebook at the receiver and transmitter based on the current CSI. Specifically, in this paper, considering various receiver selection criteria, an optimal precoding matrix is selected according to CSI, and then the receiver informs the transmitter about the codebook and codeword index via a limited feedback channel (zero-delay and error-free is assumed), as depicted in Fig. 1. Furthermore, we consider the precoding system [16], [20]-[22] with N transmitting antennas and M receiving antennas ($M \geq N$ is assumed) in the MIMO channel as

$$\mathbf{y} = \mathbf{HFx} + \mathbf{v}, \tag{1}$$

where $\mathbf{H} \in C^{MxN}$ is a channel matrix, the data symbol vector $\mathbf{x} = [x_1, x_2, \ldots, x_S]^T \in C^{1\times S}$ is multiplied by the $N \times S$ precoding matrix $\mathbf{F}$, $\mathbf{y} \in C^{M\times 1}$ is the received signal vector, the nose vector $\mathbf{v}$ is independent identically distributed (*i.i.d*) with $\mathcal{CN}(0, \sigma_v^2)$, and the equivalent MIMO channel $\mathbf{HF}$ is composed of the MIMO channel ($\mathbf{H}$) and the precoding matrix ($\mathbf{F}$). After the precoding operation, the transimitting signal vector $\mathbf{z} = [z_1, z_2, \ldots, z_N]^T$ in (1) can be given as

$$\mathbf{z} = \mathbf{Fx}. \tag{2}$$

In this paper, to simulate a correlated channel for the precoding system, we introduce the well-know Ricean channel as follows. $\mathbf{H}_{\text{sp}}$ denotes the specular component that illuminates the entire array and is spatially deterministic from antenna to antenna. $\mathbf{H}_{\text{sc}}$ denotes the scattered component that varies randomly from antenna to antenna (Rayleigh-distributed) [34]. Thus, the channel response can be given as

$$\mathbf{H} = \sqrt{\frac{\kappa}{\kappa+1}}\mathbf{H}_{\text{sp}} + \sqrt{\frac{1}{\kappa+1}}\mathbf{H}_{\text{sc}}, \tag{3}$$

where Ricean factor κ is defined as the ratio of deterministic-to-scattered power; when $\kappa \to \infty$, the channel is fully correlated, but as $\kappa \to 0$, the channel is rich-scattered. A large κ implies severe correlations; the extreme selections $\kappa = 0$ and $\kappa = 100$, respectively, render the channel independently fading and being almost light-of-sight. Then, to achieve low-complexity, we propose a simplistic QR-based detection [17]-[19] as following

$$\tilde{\mathbf{y}} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_N \end{bmatrix} = \mathbf{Q}^H\mathbf{y} = \mathbf{Rx} + \tilde{\mathbf{v}} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N} \\ 0 & r_{2,2} & \cdots & r_{2,N} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots\ 0 & \cdots & r_{N,N} \end{bmatrix} \mathbf{x} + \tilde{\mathbf{v}}, \tag{4}$$

where $\mathbf{R} = \mathbf{Q}^H\mathbf{HF}$ and $\tilde{\mathbf{v}} = \mathbf{Q}^H\mathbf{v}$, and $r_{i,j}^2$, $1 \leq i, j \leq N$, is a chi-square random variable with $2(M - i + 1)$. The i^{th} element of modified received signals is detected as follows

$$\hat{y}_i = \tilde{y}_i - \sum_{j=i+1}^{N} r_{i,j}\hat{x}_j = r_{i,i}\hat{x}_j + \sum_{j=i+1}^{N} r_{i,j}(s_j - \hat{s}_j) + \tilde{v}_i, \tag{5}$$

where $\hat{x}_j$ is the j^{th} element of detected transmit signals. Assuming the previous symbol detection is error-free [17]-[19], we can obtain

$$\hat{x}_i = Decision\left(\frac{\hat{y}_i}{r_{i,i}}\right), \text{ when } \hat{y}_i = r_{i,i}s_i + \tilde{v}_i. \tag{6}$$

To achieve low-complexity for simplistic QR-based detection, we propose a column-wise orthogonal structure for QR-decomposition. This column-wise orthogonal structure means that the neighboring columns of $\mathbf{HF}$ are orthogonal. To achieve this, the real and imaginary parts in the elements of $\mathbf{HF}$ are rearranged as

$$\mathbf{HF}_r := \begin{bmatrix} \text{Re}\{\mathbf{hf}_1\} & -\text{Im}\{\mathbf{hf}_1\} & \text{Re}\{\mathbf{hf}_2\} & -\text{Im}\{\mathbf{hf}_2\},\ldots,\text{Re}\{\mathbf{hf}_N\} & -\text{Im}\{\mathbf{hf}_N\} \\ \text{Im}\{\mathbf{hf}_1\} & \text{Re}\{\mathbf{hf}_1\} & \text{Im}\{\mathbf{hf}_2\} & \text{Re}\{\mathbf{hf}_2\},\ldots,\text{Im}\{\mathbf{hf}_N\} & \text{Re}\{\mathbf{hf}_N\} \end{bmatrix} \in \Re^{2(M\times S)} \tag{7}$$

where $\mathbf{hf}_i$, $i \in \{1, \ldots, S\}$, is the i^{th} column of $\mathbf{HF}$ and thus (1) can be rewritten as

$$\mathbf{y}_r = \mathbf{HF}_r\mathbf{x}_r + \mathbf{v}_r, \tag{8}$$

where $\mathbf{y}_r$ =[Re{y_1} Im{y_1} Re{y_2} Im{y_2} … Re{y_M} Im{y_M}]$^T \in \mathbf{R}^{2M\times 1}$ is the equivalent received signal vector, $\mathbf{x}_r$ =[Re{x_1} Im{x_1} Re{x_2} Im{x_2} … Re{x_S} Im{x_S}]$^T \in \mathbf{R}^{2S\times 1}$ is the equivalent transmitted signal vector, and $\mathbf{v}_r$ =[Re{v_1} Im{v_1} Re{v_2} Im{v_2} … Re{v_M} Im{y_M}]$^T \in \mathbf{R}^{2M\times 1}$ is the equivalent noise vector. By exploiting this column-wise orthogonal structure, the QR-decomposition (QRD) of $\mathbf{HF}_r$ can be formulated as follows.

Property 1: We Assume $\mathbf{HF}_r(:,c)$ is the c^{th} column of $\mathbf{HF}_r$, $\mathbf{HF}_r \in \Re^{2(M\times S)}$, where $(\mathbf{HF}_r(:,c))^H\mathbf{HF}_r(:,c+1) = \mathbf{0}$ for $c \in \{1,3,5,\ldots,2S-1\}$, and let $\mathbf{Q}$ and $\mathbf{R}$ be QRD of $\mathbf{HF}_r$ ($\mathbf{HF}_r = \mathbf{QR}$). Then, we have $\mathbf{R} \in \mathbf{B}_{2^n\times 2^n}$, where

$$\mathbf{B}_{2^n\times 2^n} \equiv \{\mathbf{B} \in \Re^{2^n\times 2^n};\ \forall i,\ j \in \left\{1, 3, 5, \ldots, 2^{n-2}+1\right\},$$
$$\mathbf{B}(i,i+1;j,j+1) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} or \begin{pmatrix} a & b \\ b & -a \end{pmatrix} \textit{ for some } a,b \in \Re\}. \tag{9}$$

[*Proof*]: See appendix I.

Therefore, due to the involvement of the column-wise orthogonal structure in (7)-(9), the proposed simplistic QR-decomposition processes have half of the computational complexity compared to the conventional QR-decomposition process [14 Eq. (7)].

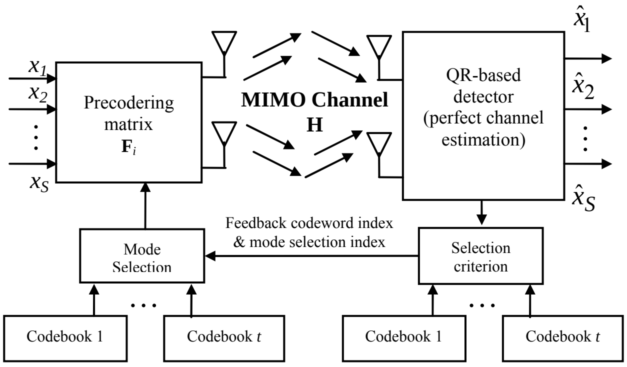


Fig. 1. The transmitter switches between various precoding codebooks via precoding selection criterion at the receiver for QR-based detection in MIMO communication.

3 Precoding Selection Criteria

In this section, to select an optimal precoding matrix in the MIMO transmission with the QR-based detection, three selection criteria for 1) maximum capacity (capacity-selection), 2) maximum minimum singular value (MSV-selection), and 3) the proposed QR-based selection criterion (QR-selection) are introduced as follows. For capacity-selection criterion, considering a precoding matrix ($\mathbf{F}$) in the transmitter, the equivalent channel is $\mathbf{HF}$. Then, the mutual information in the equivalent channel by using $\mathbf{H}$ and a fixed $\mathbf{F}$ is

$$I(\mathbf{F}) = \log_2\det(\mathbf{I}_S + \frac{P_T}{N\sigma_v^2}\mathbf{F}^H\mathbf{H}^H\mathbf{HF}), \tag{10}$$

where P_T is limited, irrespective of the number of transmitting antennas and $\mathbf{I}_S$ is an identity matrix. By using (10), the capacity-selection criterion is used to select an optimal precoding matrix for maximizing the channel capacity [31]-[33] by selecting $\mathbf{F}$ in

$$\mathbf{F}_{\text{capacity}} = \arg\max_{\mathbf{F}_i \in \Omega} I(\mathbf{F}_i), \tag{11}$$

where Ω is the precoding set for containing all possible precoding matrices in the codebook. For MSV-selection criterion, the maximum minimum singular value selection criterion provides a close approximation to avoid the rank deficient channel. Thus, selecting an optimal precoding matrices by maximizing the minimum singular value $\lambda_{\min}$ in the precoding subset [31]-[33] is given as

$$\mathbf{F}_{\text{MSV}} = \arg\max_{\mathbf{F}_i \in \Omega} \lambda_{\min}(\mathbf{HF}_i), \tag{12}$$

where $\lambda_{\min}(\mathbf{HF})$ is the minimum singular value of $\mathbf{HF}$. For the proposed QR-selection criterion, with the OSIC detection, the probability of an early error in the sequence of decisions is lower and thus the QR-based detection has better performance. Based on this knowledge, we develop a QR-based selection criterion to select an optimal precoding matrix for improving the QR-based detection performance. By maximizing the lowest layer of the upper triangular matrix ($\mathbf{R}$), the proposed QR-based selection criterion is given as

$$\mathbf{F}_{\text{QR}} = \arg\max_{\mathbf{F}_i \in \Omega} |r_{N,N}| \text{ with } \mathbf{HF}_i = \mathbf{QR}, \tag{13}$$

where $r_{N,N}$ is the entry (N, N) of $\mathbf{R}$ for $\mathbf{R} \in C^{N\times N}$. To achieve low-complexity via reducing the full matrix multiplication ($\mathbf{HF}_i$), the triangular matrix multiplication ($\dot{\mathbf{R}}\mathbf{F}_i$) in the proposed QR-selection criterion is proposed as

$$\mathbf{F}_{\mathrm{QR}} = \arg\max_{\mathbf{F}_i \in \Omega} \left|\bar{r}_{N,N}\right| \text{ with } \dot{\mathbf{R}}\mathbf{F}_i = \overline{\mathbf{QR}}, \qquad (14)$$

where $\bar{r}_{N,N}$ is the entry (N, N) of $\overline{\mathbf{R}}$ for $\overline{\mathbf{R}} \in C^{N\times N}$, and $\mathbf{H} = \dot{\mathbf{Q}}\dot{\mathbf{R}}$ is QR-decomposition of the MIMO channel ($\mathbf{H}$). Furthermore, (14) has approximately the half computational complexity compared to (13) because the upper triangular matrix multiplication ($\dot{\mathbf{R}}\mathbf{F}_i$) is less complex than the full matrix multiplication ($\mathbf{HF}_i$). That is, in the upper triangular matrix multiplication ($\dot{\mathbf{R}}\mathbf{F}_i$), the upper triangular matrix $\dot{\mathbf{R}}$ involves a lot of zero that can reduce the multiplication complexity as depicted in (4). The equality at (14) holds because $|r_{N,N}|$ in (13) is equal to $\left|\bar{r}_{N,N}\right|$ in (14). To achieve this, we claim $|\mathbf{R}| = |\overline{\mathbf{R}}|$ as following Theorem 1.

Theorem 1: Let channel matrix $\mathbf{H} \in C^{M\times N}$, and let $\mathbf{H} = \dot{\mathbf{Q}}\dot{\mathbf{R}}$ be QR-decomposition of $\mathbf{H}$. The precoding matrix is $\mathbf{F} \in C^{N\times N}$, and let $\mathbf{HF} = \mathbf{QR}$ be QR-decomposition of $\mathbf{HF}$. Then, we have $|\mathbf{R}| = |\overline{\mathbf{R}}|$ where $\dot{\mathbf{R}}\mathbf{F} = \overline{\mathbf{QR}}$ is QR-decomposition of $\dot{\mathbf{R}}\mathbf{F}$.

[*Proof*]*:* See Appendix II.

Table I. MIMO precoding codebook $V(2,2,3)$

Matrix index	Column1	Column2
0b000	1	0
	0	1
0b001	0.794	-0.5801-0.1818i
	-0.5801+0.1818i	-0.794
0b010	0.794	0.0576-0.6051i
	0.0576+0.6051i	-0.794
0b011	0.7491	-0.2978+0.5298i
	-0.2978-0.5298i	-0.7491
0b100	0.7941	0.6038-0.0689i
	0.6038+0.0689i	-0.7941
0b101	0.3289	0.6614-0.6740i
	0.6614+0.6740i	-0.3289
0b110	0.5112	0.4754+0.7160i
	0.4754 -0.7160i	-0.5112
0b111	0.3289	-0.8779+0.3481i
	-0.8779-0.3481i	-0.3289

Additionally, in this paper, we consider that codebooks with different dimensions and sizes are defined in IEEE 802.16-2005 standard [30]. All codebooks are divided into two kinds: 3-bit codebooks and 6-bit codebooks. The notation $V(N,S,L)$ denotes the matrix codebook, which consists of 2^L complex, unit matrices of a dimension N-by-N and the number of substreams S. The interger L is the number of bits required for the index to indicate any matrix in the codebook. For example, in the precoding scheme with eight 2-by-2 matrices, the 3-bit 2-by-2 codebook $V(2,2,3)$ is listed in Table I.

4 Computational Complexity

In this section, we analyze the computational efficiency (CE) to address the comparison of computational complexity with capacity-selection criterion, MSV-selection criterion, and QR-selection criterion. For simplicity, the computational complexity is only in terms of the number of complex multiplications [35]. Considering 2^L precoding matrixes in the codebook, the computational complexity of the capacity-selection criterion can be approximated as

$$Comp_{\mathrm{capacity}} \approx 2^L(2SMN + S!), \qquad (15)$$

where the multiplication complexity of $\mathbf{F}^H\mathbf{H}^H\mathbf{HF}$ is $2SMN$ and the complexity of the determine process of (10) is approximately $O(S!)$. Then, the computational complexity of the MSV-selection criterion can be approximated as

$$Comp_{\mathrm{MSV}} \approx 2^L(MNS + M^2 + 4M^3/3), \qquad (16)$$

where the multiplication complexity of $\mathbf{HF}$ is MNS and the complexity of the singular value decomposition for $\mathbf{HF}$ is about $O(M^2 + 4M^3/3)$ [35]. Then, the computational complexity of the QR-selection criterion can be approximated as

$$Comp_{\mathrm{QR}} \approx 2^L(S(N + N^2)/2 + MN^2 - N^3/3), \qquad (17)$$

where the multiplication complexity of $\dot{\mathbf{R}}\mathbf{F}$ is $S(N+N^2)/2$ and the QR complexity for $\dot{\mathbf{R}}\mathbf{F}$ is about $O(MN^2 - N^3/3)$ via (7)-(9). Based on (15) and (17), assuming large L and S in the precoding system, the computational requirements of capacity-selection criterion and QR-selection criterion are compared by the CE ratio:

$$\mu_{\text{QR-capacity}} = \frac{Comp_{\text{QR}}}{Comp_{\text{Capacity}}} = \frac{S(N+N^2)/2 + MN^2 - N^3/3}{2SMN + S!}$$

$$\approx 0, \text{ when } S \text{ is large enough.} \tag{18}$$

Based on (16) and (17), and assuming large L and M in the precoding system, the computational requirements of capacity-selection criterion and QR-selection criterion are compared by the CE ratio:

$$\mu_{\text{QR-SVD}} = \frac{Comp_{\text{QR}}}{Comp_{\text{SVD}}} = \frac{S(N+N^2)/2 + MN^2 - N^3/3}{MNS + M^2 + 4M^3/3} \leq 1, \text{ when } M \geq N. \tag{19}$$

Clearly, with (18) and (19), the proposed QR-selection criterion has less computational complexity than others because of the matrix multiplication reduction in (14) and a low-complexity QR-decomposition processes in (7)-(9).

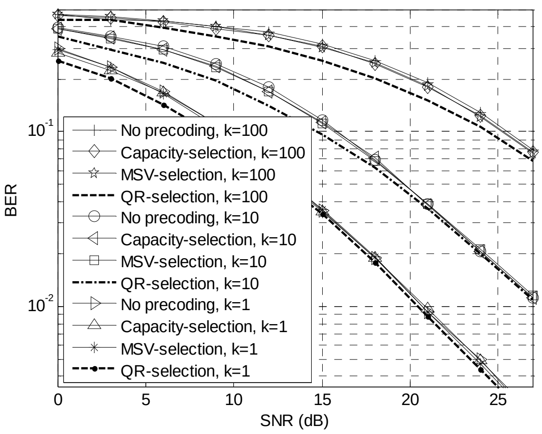
5 Simulation Result

This section uses several numerical examples to illustrate the performance of the proposed scheme. This proposed scheme is the QR-section criterion used to select an optimal precoding matrix in QR-based MIMO detection. With $M \geq N$, we compare three selection criteria (capacity-selection, MSV-selection and QR-selection) [31]-[33] for the precoding system in the Monte Carlo simulations. By using the previous development, the performance is measured in terms of BER for various QAM constellations averaged over 10^7 trials. In all simulations, we assume that CSI is given to the receiver but not to the transmitter, and we define SNR := P_T / σ_v^2 . The proposed QR-selection criterion in the QR-based detection evaluates the effects of A) precoding with single-mode codebook, B) precoding with multiple-mode codebook and C) computational complexity as follows.

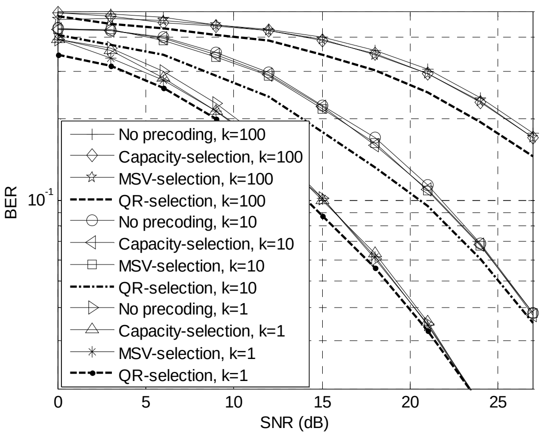
5.1 Precoding with single-mode codebook

In this subsection, the single-mode codebook in the precoding system indicates that the transmitter and receiver use a codebook in the MIMO transmission [36]-[38]. In the following simulations, we use the precoding system with various correlated channels (Ricean factors: κ = 1, 10, 100) to evaluate the performances of the proposed QR-based selection (QR-selection) criterion of (14) in the QR-based MIMO detection. The proposed QR-selection criterion is compared with the following capacity selection criterion (capacity-selection) in (11), with the minimum singular value selection criterion (MSV-selection) in (12) and with no precoding system as shown in Fig. 2 and Fig. 3. In Fig. 2(a), the proposed QR-selection criterion in this precoding system has a better performance than others [31]-[33]. This is because the probability of an early error in the sequence of decisions in (5) is lower, and thus the QR-based MIMO detection performs the better. That is, in the QR-based MIMO detection, our proposed QR-selection criterion can achieve the best performance due to enlargement of the entry (N, N) of **R**. Then, the MSV-selection criterion is compared with capacity-selection criterion. Fig. 2(a) shows that the MSV-selection criterion [33] has a better performance than the capacity-selection criterion [33]. This is because the MSV-selection criterion can avoid the *ill-conditioned* channel in the QR-based MIMO detection. Furthermore, the capacity-selection criterion is compared with no precoding scheme. In Fig. 2(a), the performance of the capacity-selection criterion is slightly better than that of the no precoding scheme. This is because the capacity-selection criterion is based on a general capacity formula and is not specific for the QR-based MIMO detection. In this simulation environment, $N = M = 2$, 4QAM and the 3-bit 2-by-2 codebook $V(2,2,3)$ (see Table I) [30] are considered in Fig. 2(a).

Obviously, it can be seen in Fig. 2 that the performances in all cases deteriorate as κ increases. This is expected since the spatial multiplexing scheme may lose the diversity gain over the correlated channels, and large κ factor thus incurs large BER degradation. The performance decline of the QR-based detection can result from the increasing amount of error-propagation. Namely, the poor detected streams in the SM systems are caused by the loss in diversity gain over low-rank channels. In addition, the proposed QR-selection criterion seems to incur less BER spread as κ increases. This phenomenon could benefit from the OSIC mechanism, in which the detect-and-cancel process improves detection accuracy in an early detected layer, leading to a better average performance against a severe spatial correlation. Similarly, with 16QAM, Fig. 2(b) shows the proposed QR-selection criterion can achieve the best performance. Then, considering $N = M = 3$ and the single-mode codebook with $V(3,3,6)$ for precoding system in Fig. 3. With 4QAM and 16QAM in Fig. 3(a) and Fig. 3(b), respectively, these figures could affirm that the proposed QR-selection criterion can achieve the best performance in all cases.



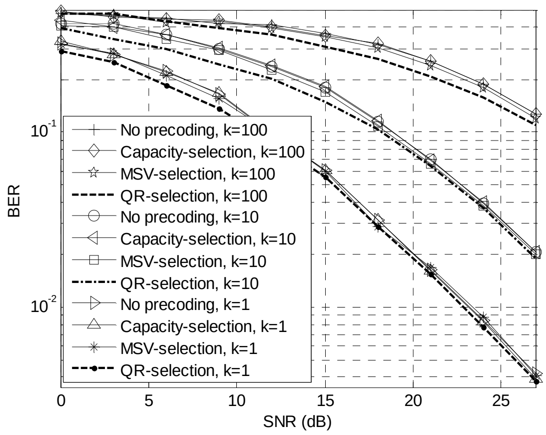
(a)



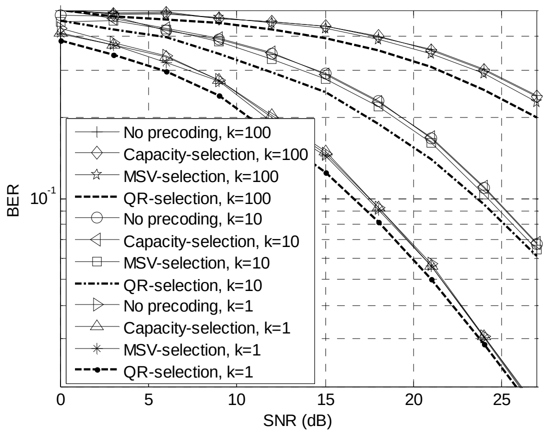
(b)

Fig. 2. BER performance of various selection criteria with $N = M = 2$, various SNR conditions, various Ricean factors ($\kappa = 1, 10, 100$) and the 3-bit 2-by-2 codebook $V(2,2,3)$ system for the QR-based detection in MIMO communication employing at (a) 4QAM and (b) 16QAM.

5.2 Precoding with multiple-mode codebook



(a)



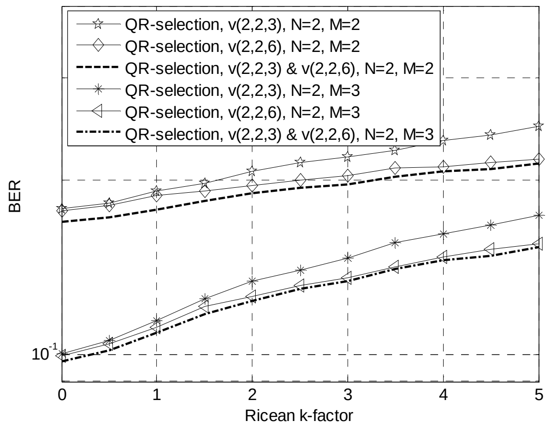
(b)

Fig. 3. BER performance of various selection criteria with $N = 3$ $M = 3$, various SNR conditions, various Ricean factors ($\kappa = 1, 10, 100$) and the 6-bit 3-by-3 codebook $V(3,3,6)$ system for the QR-based detection in MIMO communication employing at (a) 4QAM and (b) 16QAM.

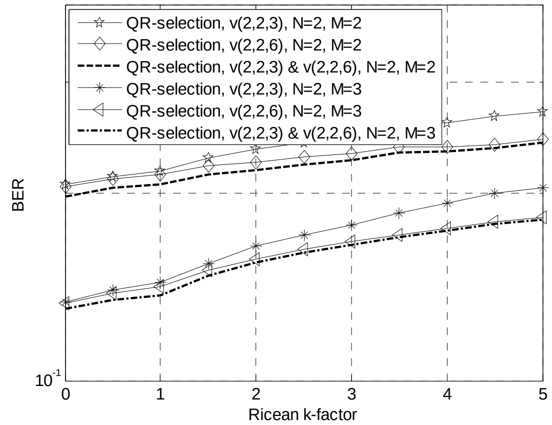
In IEEE 802.16-2005 standard [30], all codebooks can be divided into two kinds: 3-bit codebooks and 6-bit codebooks. In this subsection, to achieve an optimal transmission via increasing the codebook size, the multiple-mode codebook is proposed in the precoding system. This multiple-mode codebook, which the transmitter and receiver use a lot of codebooks, can enhance the detection performance due to selection diversity. Specifically, the proposed QR-based selection criterion with multiple-mode codebook can be denoted as

$$\mathbf{F}_{\mathrm{QR}} = \underset{\mathbf{F}_i \in \Omega_T}{\arg\max} \left| r_{N,N} \right| \text{ with } \dot{\mathbf{R}}\mathbf{F}_i = \overline{\mathbf{QR}}, \qquad (20)$$

where $\Omega_T = \Omega_1 + \ldots + \Omega_t$ $(1 \le t)$, the multiple-mode codebook, is composed by different codebooks, and Ω_t is t^{th} codebook. In the following simulations, considering $t = 2$ for the multiple-mode codebook in (20), this multiple-mode codebook contains two join 2-by-2 codebooks with respect to $V(2,2,3)$ and $V(2,2,6)$ [30]. Then, considering $V(2,2,3)$, $V(2,2,6)$ and multiple-mode codebook, with 32QAM and $N = 2$ in various Ricean κ-factors, Fig. 4(a) shows that the multiple-mode codebook ($V(2,2,3)$ & $V(2,2,6)$) has a better performance than the single-mode codebook employed in the proposed QR-selection criterion when $M = 3$. This is because the large-size codebook is suitable for the temporal-correlation channel by enhancing the selection diversity. Obviously, in Fig. 4(a), the QR-based detection could benefit from increasing M in all cases. Specifically, the detect-and-cancel process in the QR-based detection can induce more diversity gain. This diversity gain can improve detection accuracy in each layer, thus leading to a better average performance against spatial correlation. Similarly, with 64QAM in the precoding system, Fig. 4(b) shows that the multiple-mode codebook has a better performance than the single-mode in the QR-selection criterion when $M = 3$.



(a)



(b)

Fig. 4. BER performance of various MIMO precoding codebooks (v(2,2,3), v(2,2,6) and v(2,2,3) & v(2,2,6)) with $M \ge N$, and various Ricean factors (κ is form 0 to 5) for the QR-based detection in MIMO communication employing at (a) 32QAM and (b) 64QAM.

5.3 Computational complexity

To compare the computational complexities of the QR-selection criteria in ratio to capacity-selection and MSV-selection [33], the precoding system is considered with the $V(3,3,6)$ codebook [30] for the simulation analysis and the theoretical analysis (CE ratio) when $\kappa = 10$, $N = 3$ and 16QAM. This simulation analysis for the complexity comparison via the program executing-time can be derived by

$$\Delta = \frac{\text{QR selection executing time}}{\text{Capacity (or MSV) selection executing time}}. \quad (21)$$

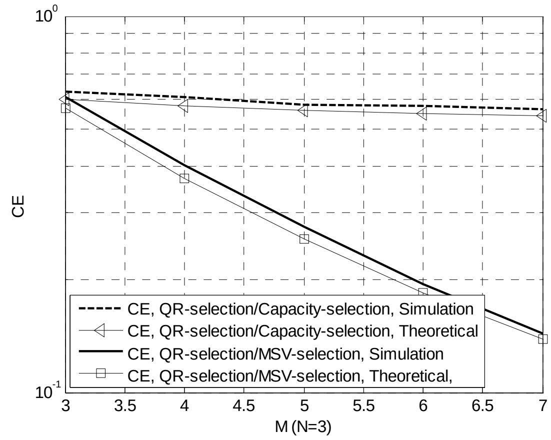


Fig. 5. CE ratio for simulation and theoretical analysis versus various number of receive antennas for $N = 3$.

In the theoretical analysis, based on the CE ratio in (18) and (19), the ratio of the QR-selection complexity and conventional selection (capacity-selection and MSV-selection) complexity [31]-[33] are computed and shown in Fig. 5. As observed in Fig. 5, when M = 7, the CE ratio of QR-selection and MSV-selection is about 0.1, and the CE ratio of QR-selection and capacity-selection is about 0.5. That is, the proposed QR-selection criterion can reduce the computational complexity by 90% and 50% compared with MSV-selection criterion [31]-[33] and capacity-selection [33], respectively. The MSV-selection criterion of (12) has a larger complexity than others due to a large value of M in the SVD processes as depicted in (16). Particularly, when M is increased, Fig. 5 demonstrates that the proposed QR-selection complexity decreases much more because of the matrix multiplication reduction via (14) and the low-complexity QR-decomposition processes via (7)-(9). For the simulation analysis, Fig. 5 demonstrates that the proposed QR-selection criterion has a lower complexity than others. Obviously, it is the sorting operation that causes the small gap between the theoretical analysis and the simulation analysis. This sorting operation indicates that the optimization of (11), (12) and (14) has sorted the 2^L precoding matrices in the ascending or descending order to find an optimal precoding matrix by using the maximum mutual information, the maximum minimum singular value or maximum $\left|\bar{r}_{N,N}\right|$, respectively.

6 Conclusion

In this paper, to achieve the implementation simplicity, we proposed a simplistic QR-decomposition by using the column-wise orthogonal structure to reduce the repeated computation. Furthermore, in the precoding system, the proposed QR-selection criterion has achieved low-complexity and performance enhancement due to the matrix multiplication reduction and an accurate detection of the first layer, respectively. By using IEEE 802.16-2005 standard, simulation results showed that the proposed scheme with a low-complexity level has better performance than others, and that it has improved detection performance as the codebook size increases. Our future work will investigate MIMO techniques in accordance with precoding scheme and STBC for mobile broadband wireless access applications.

APPENDIX I

DETAILED PROOF OF PROPERTY 1

Proof: For $N = M = 1$, we assume $\mathbf{HF} := \left[a+bi\right] \in C^{1x1}$ and hence the real-value channel is $\mathbf{HF}_r := \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \in \Re^{2\times 2}$. We have $\mathbf{u}_1 = \mathbf{v}_1$ and $\mathbf{u}_2 = \mathbf{v}_2 - \frac{\langle \mathbf{v}_2, \mathbf{u}_1 \rangle}{\langle \mathbf{u}_1, \mathbf{u}_1 \rangle}\mathbf{u}_1 = \mathbf{v}_2$, where $\langle \mathbf{v}_2, \mathbf{u}_1 \rangle = -ab + ba = 0$. Thus, the QR-decomposition of $\mathbf{HF}_r$ is given as

$$\mathbf{HF}_r = \begin{bmatrix} \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|} \end{bmatrix} \begin{bmatrix} \|\mathbf{u}_1\| & 0 \\ 0 & \|\mathbf{u}_2\| \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_2}{\|\mathbf{u}_1\|} \end{bmatrix} \begin{bmatrix} \|\mathbf{u}_1\| & 0 \\ 0 & \|\mathbf{u}_1\| \end{bmatrix}, \tag{A.1}$$

For $N = 1$ and $M = 2$, we assume $\mathbf{HF} := \begin{bmatrix} a+bi \\ c+di \end{bmatrix} \in C^{2x1}$ and hence the real-value channel is $\mathbf{HF}_r := \begin{bmatrix} a & -b \\ c & -d \\ b & a \\ d & c \end{bmatrix} \in \Re^{4\times 2}$. We have $\mathbf{u}_1 = \mathbf{v}_1$ and $\mathbf{u}_2 = \mathbf{v}_2 - \frac{\langle \mathbf{v}_2, \mathbf{u}_1 \rangle}{\langle \mathbf{u}_1, \mathbf{u}_1 \rangle}\mathbf{u}_1 = \mathbf{v}_2$, where $\langle \mathbf{v}_2, \mathbf{u}_1 \rangle = -ab - cd + ba + cd = 0$. Thus, the QR-decomposition of $\mathbf{HF}_r$ is given as

$$\mathbf{HF}_r = \begin{bmatrix} \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_2}{\|\mathbf{u}_1\|} \end{bmatrix} \begin{bmatrix} \|\mathbf{u}_1\| & 0 \\ 0 & \|\mathbf{u}_1\| \end{bmatrix}, \tag{A.2}$$

For $N = 2$ and $M = 1$, we assume $\mathbf{HF} := \left[a+bi \quad c+di\right] \in C^{1x2}$ and hence the real-value channel is $\mathbf{HF}_r := \begin{bmatrix} a & -b & c & -d \\ b & a & d & c \end{bmatrix} \in \Re^{2\times 4}$, then $\mathbf{u}_1 = \mathbf{v}_1$ and $\mathbf{u}_2 = \mathbf{v}_2 - \frac{\langle \mathbf{v}_2, \mathbf{u}_1 \rangle}{\langle \mathbf{u}_1, \mathbf{u}_1 \rangle}\mathbf{u}_1 = \mathbf{v}_2$, where $\langle \mathbf{v}_2, \mathbf{u}_1 \rangle = -ab + ba = 0$. Then, we have $\mathbf{u}_3 = \mathbf{v}_3 - \frac{\langle \mathbf{v}_3, \mathbf{u}_1 \rangle}{\langle \mathbf{u}_1, \mathbf{u}_1 \rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_3, \mathbf{u}_2 \rangle}{\langle \mathbf{u}_2, \mathbf{u}_2 \rangle}\mathbf{u}_2$, where $\langle \mathbf{v}_3, \mathbf{u}_1 \rangle = ac + bd$ and $\langle \mathbf{v}_3, \mathbf{u}_2 \rangle = -$

$bc + ad$. To obtain $\mathbf{u}_4$, we have $\langle \mathbf{v}_4, \mathbf{u}_1\rangle = -ad - bc$ and $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = bd + ac$. Thus, we have $\langle \mathbf{v}_4, \mathbf{u}_3\rangle = \langle \mathbf{v}_4, \mathbf{v}_3\rangle - \langle \mathbf{v}_4, \mathbf{u}_1\rangle \frac{\langle \mathbf{v}_3, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle} - \langle \mathbf{v}_4, \mathbf{u}_2\rangle \frac{\langle \mathbf{v}_3, \mathbf{u}_2\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle} = 0$, where $\langle \mathbf{v}_4, \mathbf{u}_1\rangle = -\langle \mathbf{v}_3, \mathbf{u}_2\rangle$, $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = \langle \mathbf{v}_3, \mathbf{u}_1\rangle$ and $\langle \mathbf{v}_4, \mathbf{v}_3\rangle = 0$. Then, we have

$$\mathbf{u}_4 = \mathbf{v}_4 - \frac{\langle \mathbf{v}_4, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_4, \mathbf{u}_2\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle}\mathbf{u}_2 - \frac{\langle \mathbf{v}_4, \mathbf{u}_3\rangle}{\langle \mathbf{u}_3, \mathbf{u}_3\rangle}\mathbf{u}_3 = \mathbf{v}_4 - \frac{-\langle \mathbf{v}_3, \mathbf{u}_2\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_3, \mathbf{u}_1\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle}\mathbf{u}_2, \tag{A.3}$$

where $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = \langle \mathbf{v}_3, \mathbf{u}_1\rangle$. Thus, the QR-decomposition of $\mathbf{HF}_r$ is given as

$$\mathbf{HF}_r = \begin{bmatrix} \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_2}{\|\mathbf{u}_1\|} \end{bmatrix} \begin{bmatrix} \|\mathbf{u}_1\| & 0 & \langle \mathbf{v}_3, \mathbf{u}_1\rangle & -\langle \mathbf{v}_3, \mathbf{u}_2\rangle \\ 0 & \|\mathbf{u}_1\| & \langle \mathbf{v}_3, \mathbf{u}_2\rangle & \langle \mathbf{v}_3, \mathbf{u}_1\rangle \end{bmatrix}. \tag{A.4}$$

For $N = M = 2$, we assume $\mathbf{HF} := \begin{bmatrix} a+bi & e+fi \\ c+di & h+ji \end{bmatrix} \in C^{2x2}$ and hence the real-value channel is $\mathbf{HF}_r := \begin{bmatrix} a & -b & e & -f \\ c & -d & h & -j \\ b & a & f & e \\ d & c & j & h \end{bmatrix} \in \Re^{4\times 4}$, then $\mathbf{u}_1 = \mathbf{v}_1$ and $\mathbf{u}_2 = \mathbf{v}_2 - \frac{\langle \mathbf{v}_2, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 = \mathbf{v}_2$, where $\langle \mathbf{v}_2, \mathbf{u}_1\rangle = -ab - cd + ba + dc = 0$. Then, we have $\mathbf{u}_3 = \mathbf{v}_3 - \frac{\langle \mathbf{v}_3, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_3, \mathbf{u}_2\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle}\mathbf{u}_2$, where $\langle \mathbf{v}_3, \mathbf{u}_1\rangle = ae + ch + bf + dj$ and $\langle \mathbf{v}_3, \mathbf{u}_2\rangle = -be - dh + af + cj$. To obtain $\mathbf{u}_4$, we have $\langle \mathbf{v}_4, \mathbf{u}_1\rangle = -af - cj + be + dh$ and $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = bf + di + ae + ch$. Thus, we have $\langle \mathbf{v}_4, \mathbf{u}_3\rangle = \langle \mathbf{v}_4, \mathbf{v}_3\rangle - \langle \mathbf{v}_4, \mathbf{u}_1\rangle \frac{\langle \mathbf{v}_3, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle} - \langle \mathbf{v}_4, \mathbf{u}_2\rangle \frac{\langle \mathbf{v}_3, \mathbf{u}_2\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle} = 0$, where $\langle \mathbf{v}_4, \mathbf{u}_1\rangle = -\langle \mathbf{v}_3, \mathbf{u}_2\rangle$, $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = \langle \mathbf{v}_3, \mathbf{u}_1\rangle$ and $\langle \mathbf{v}_4, \mathbf{v}_3\rangle = 0$. Then, we have

$$\mathbf{u}_4 = \mathbf{v}_4 - \frac{\langle \mathbf{v}_4, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_4, \mathbf{u}_2\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle}\mathbf{u}_2 - \frac{\langle \mathbf{v}_4, \mathbf{u}_3\rangle}{\langle \mathbf{u}_3, \mathbf{u}_3\rangle}\mathbf{u}_3 = \mathbf{v}_4 - \frac{-\langle \mathbf{v}_3, \mathbf{u}_2\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_3, \mathbf{u}_1\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle}\mathbf{u}_2, \tag{A.5}$$

where $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = \langle \mathbf{v}_3, \mathbf{u}_1\rangle$. Thus, the QR-decomposition of $\mathbf{HF}_r$ is given as

$$\mathbf{H}_r = \begin{bmatrix} \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_2}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_3}{\|\mathbf{u}_3\|} & \frac{\mathbf{u}_4}{\|\mathbf{u}_4\|} \end{bmatrix} \begin{bmatrix} \|\mathbf{u}_1\| & 0 & \langle \mathbf{v}_3, \mathbf{u}_1\rangle & -\langle \mathbf{v}_3, \mathbf{u}_2\rangle \\ 0 & \|\mathbf{u}_1\| & \langle \mathbf{v}_3, \mathbf{u}_2\rangle & \langle \mathbf{v}_3, \mathbf{u}_1\rangle \\ 0 & 0 & \|\mathbf{u}_3\| & 0 \\ 0 & 0 & 0 & \|\mathbf{u}_3\| \end{bmatrix}. \tag{A.6}$$

$\vdots$

As it turns out, we have R $\in \mathbf{B}_{2^n \times 2^n}$ for any N and M.

APPENDIX II

DETAILED PROOF OF THEOREM 1

proof: For $N = M = 1$, we have $|\mathbf{R}| = |\overline{\mathbf{R}}|$ because $|\mathbf{Q}| = |\overline{\mathbf{Q}}| = |\dot{\mathbf{Q}}| = 1$. Then, for $N = 1$, $M = 2$, since

$$\mathbf{HF} = \mathbf{QR} = \dot{\mathbf{Q}}\overline{\mathbf{Q}}\overline{\mathbf{R}} = \dot{\mathbf{Q}}\dot{\mathbf{R}}\,\mathbf{F}, \tag{A.7}$$

Based on (A.7), we have

$$\overline{\mathbf{R}} = \overline{\mathbf{Q}}^{-1}\,\dot{\mathbf{Q}}^{-1}\,\mathbf{QR} = \tilde{\mathbf{Q}}\,\mathbf{R}, \tag{A.8}$$

where the 2-by-2 orthogonal matrix $\tilde{\mathbf{Q}} = \overline{\mathbf{Q}}^{-1}\,\dot{\mathbf{Q}}^{-1}\,\mathbf{Q}$ is a rotation matrix and thus we have

$$\tilde{\mathbf{Q}}\,\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} k_1 \\ 0 \end{bmatrix} = \begin{bmatrix} k_1\cos\theta \\ k_1\sin\theta \end{bmatrix} = \begin{bmatrix} m_1 \\ 0 \end{bmatrix} = \overline{\mathbf{R}}, \tag{A.9}$$

where $\tilde{\mathbf{Q}} = [\cos\theta\ \ -\sin\theta\,;\ \sin\theta\ \ \cos\theta\,]$, $\mathbf{R} = [k_1\ \ 0]^T$ and $\overline{\mathbf{R}} = [m_1\ \ 0]^T$. Since, $k_1 \neq 0$ and $k_1\sin\theta = 0$, we have $\theta = 0$ or π for $\mathbf{R} = \overline{\mathbf{R}}$ or $\mathbf{R} = -\overline{\mathbf{R}}$, respectively.

That is, $|\mathbf{R}| = |\overline{\mathbf{R}}|$. Then, for $N = 2$, $M = 2$, based on (A.8), we have

$$\tilde{\mathbf{Q}}\,\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} k_1 & k_2 \\ 0 & k_3 \end{bmatrix}$$

$$= \begin{bmatrix} k_1\cos\theta & k_2\cos\theta - k_3\sin\theta \\ k_1\sin\theta & k_2\sin\theta + k_3\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} m_1 & m_2 \\ 0 & m_3 \end{bmatrix} = \overline{\mathbf{R}}, \qquad \text{(A.10)}$$

where $k_1 \neq 0$, $k_2 \neq 0$, $k_3 \neq 0$, and $k_1\sin\theta = 0$ and thus we have $\theta = 0$ or π for $\mathbf{R} = \overline{\mathbf{R}}$ or $\mathbf{R} = -\overline{\mathbf{R}}$, respectively. That is, $|\mathbf{R}| = |\overline{\mathbf{R}}|$.

$$\vdots$$

As it turns out, we have $|\mathbf{R}| = |\overline{\mathbf{R}}|$ for any N and M.

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