An Efficient Rule-Based Constructive Heuristic to Solve Dynamic Weapon-Target Assignment Problem

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***Abstract*—In this paper, we propose an efficient rule-based heuristic to solve asset-based dynamic weapon-target assignment (DWTA) problems. The main idea of the proposed heuristic is to utilize the domain knowledge of DWTA problems to directly achieve weapon assignment, without large number of function evaluations. We update the saturation states of constraints in the assignment process to guarantee the feasibility of generated solutions. For the purpose of testing the performance of the proposed heuristic, we build a general Monte Carlo simulation-based DWTA framework. For comparison, we also employ a Monte Carlo method (MCM) to make DWTA decisions in different defense scenarios. From simulations with DWTA instances under different scales, the heuristic has obvious advantages over the MCM with regard to solution quality and computation time. The proposed method can solve large-scale DWTA problems (e.g., those including 100 weapons, 100 targets, and four defense stages) within only a few seconds.**

***Index Terms*—Combinatorial optimization, constraint handling, decision making, dynamic weapon-target assignment (DWTA), heuristic, military operations.**

I. Introduction

The weapon-target assignment (WTA) problem mainly stems from the requirement of command and control (C2) automation [1]. It is a fundamental subject in defense-related applications of military operations research. Its objective is to minimize the expected damage of own-force assets by assigning weapons to offensive targets at appropriate occasions. This problem can be categorized as a combinatorial optimization problem proved to be nondeterministic polynomial time-complete [2]. There are two versions of the WTA problem, namely: static WTA (SWTA) and dynamic WTA (DWTA) [3], [4]. In SWTA, all weapons engage targets at a single stage, and decision makers know all parameters of the problem. Thus, the goal of solving the SWTA problem is to find the optimal assignment for a temporary defense task. In contrast, DWTA is a multistage problem, and decision makers assess the outcome of each engagement for subsequent decisions. In a sense, DWTA is regarded as a repetition of SWTA which will terminate if the defender has destroyed all targets or used up all weapons. However, DWTA is much more complicated than SWTA. On one hand, this is due to the increase of actual constraints in DWTA (e.g., the time windows of weapons and targets [5]). Complicated constraints tend to make it harder to generate feasible solutions. On the other hand, anterior decisions have chain reactions on posterior decisions in DWTA. Typically, the number of available missiles or shells decreases with engagement times. If a weapon, for example, can be used only once, the defender has to consider its engagement occasion carefully in order to make the best of the weapon. In other words, a logical DWTA model should incorporate the choice of the engagement occasion of weapons.

Most of the previous researches on WTA focus on SWTA [5]–[19]. With respect to SWTA models, Hosein and Athans [3] proposed an asset-based SWTA model which was also used in [6] and [7]. Karasakal [8] used the probability of shooting down all incoming targets as the objective function of the air defense WTA model for a naval task group. Some scholars adopted target-based SWTA models which do not employ the value of assets directly [9]–[18]. In this case, researchers often assume each target to have certain value of threat, and the objective is to minimize the total threat of all targets. In fact, the probability model in [8] and the target-based models are just special cases of the asset-based model established by Hosein and Athans [3]. Besides, some WTA models also take into account the cost of weapons. For examples, Kwon *et al.* [19] employed the overall firing cost as the objective function for optimization. Interested readers can find a more complicated model which considers the function of special assets in the work of Hosein *et al.* [20].

Based on the aforementioned models, various algorithms have been proposed to solve SWTA problems since the middle of the last century. At early stages, solving SWTA problems depends on traditional approaches, such as implicit enumeration algorithms, branch-and-bound algorithms, and dynamic programming [5]. With the evolution of computer technology, some novel algorithms, such as neural networks [9], genetic algorithms (GAs) [6], [10], [11], tabu search [12], simulated annealing (SA) [13], ant colony optimization [14], and particle swarm optimization [15], have been developed. Some scholars also tried hybrid algorithms [7], [16], [17]. For example, Lee *et al.* [16] designed a memetic algorithm which combines the advantages of global search (GA) and local search (greedy eugenics) to solve target-based SWTA problems. Besides, Ahuja *et al.* [18] developed several branch-and-bound algorithms and a very-large-scale neighborhood search algorithm to solve target-based SWTA problems.

Although nearly two decades has passed since Hosein and Athans put forward the DWTA problem [4], there are fewer research results on DWTA reported in the literature in contrast to SWTA [5], [20]–[24]. Cai *et al.* [5] introduced some basic concepts regarding DWTA and provided a systematic survey on WTA problem. Hosein and Athans [4] completed an early research on a two-stage asset-based DWTA problem and proposed a suboptimal algorithm for finding a good solution. Hosein *et al.* also considered the special function of C2 nodes in the formulation of WTA problems and provided analytical solutions to several simple cases of DWTA [20], [21]. Khosla [22] used a hybrid GA which incorporates an SA-type heuristic to solve a target-based DWTA problem. In particular, the objective function of this DWTA model employs a weighted combination of threat value and option weight. Havens [23] models DWTA by means of simulation. However, it is in fact the repetition of SWTA [5]. Li *et al.* [24] proposed a target-based DWTA model, with the objective of minimizing the total threat of the targets which survive the final stage of air defense operation. As far as we know, no algorithm has been proposed in the literature to solve asset-based DWTA problems incorporating complicated constraints caused by the issue of engagement feasibility (e.g., the limit of time windows).

The goal of this paper is to develop an efficient algorithm to solve asset-based DWTA problems incorporating engagement feasibility. This paper is structured as follows. In Section II, we present a formulation of the mathematical model for asset-based DWTA problem. In order to solve the problem, we propose a rule-based heuristic in Section III with its computational complexity being analyzed. Section IV includes a simulation framework to verify the performance of the proposed algorithm. Accordingly, we compare the heuristic

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TABLE I
NOTATION DECLARATION

$K(t)$: the number of existing assets at stage t, $t = 1, 2, \cdots, S$; $K(1) = K$;
$T(t)$: the number of existing targets at stage t, $t = 1, 2, \cdots, S$; $T(1) = T$;
$W(t)$: the number of available weapons at stage t, $t = 1, 2, \cdots, S$; $W(1) = W$;
$\mathbf{V} = [v_k]_{1\times K}$: the asset value vector (v_k denotes the value of asset k, $k = 1, 2, \cdots, K$);
$U_j(t)$: the set of weapons available to intercept target j at stage t, $j = 1, 2, \cdots, T$; $t = 1, 2, \cdots, S$;
$\mathbf{Q} = [q_{jk}]_{T\times K}$: the target lethality matrix (q_{jk} denotes the probability that target j destroys asset k, $j = 1, 2, \cdots, T$; $k = 1, 2, \cdots, K$);
$\mathbf{P}^t = [p_{ij}(t)]_{W\times T}$: the weapon lethality matrix ($p_{ij}(t)$ denotes the probability that weapon i destroys target j when assigned to it at stage t, $i = 1, 2, \cdots, W$; $j = 1, 2, \cdots, T$; $t = 1, 2, \cdots, S$);
n_i: the maximal number of targets that weapon i can shoot at each stage, $i = 1, 2, \cdots, W$;
m_j: the maximal number of weapons that can be assigned to target j at the same time, $j = 1, 2, \cdots, T$;
N_i: the maximal number of times that weapon i can be used, $i = 1, 2, \cdots, W$;
$\mathbf{F}^t = [f_{ij}(t)]_{W\times T}$: the engagement feasibility matrix ($f_{ij}(t) = 0$ if weapon i cannot shoot target j at stage t with any potential reason, and $f_{ij}(t) = 1$ otherwise. The time windows of targets and weapons are the primary factors that affect engagement feasibility [5],[24]);
$\mathbf{S}_{AS} = [s_{as}(k)]_{1\times K}$: the state vector of assets ($s_{as}(k) = 0$ if asset k has been destroyed, and $s_{as}(k) = 1$ otherwise);
$\mathbf{S}_{TA} = [s_{ta}(j)]_{1\times T}$: the state vector of targets ($s_{ta}(j) = 0$ if target j has been destroyed, and $s_{ta}(j) = 1$ otherwise);
$\mathbf{S}_{AT} = [s_{at}(j)]_{1\times T}$: the state vector of targets' attack ($s_{at}(j) = 1$ if target j has completed its attack, and $s_{at}(j) = 0$ otherwise);
$\mathbf{O}$: zero matrix whose elements are all zeros;
$rand$: a random number generated in the interval (0,1);
$J_t(\bullet)$: the expected total value of assets surviving through t stages (adopted as the objective function for DWTA decision-making)

algorithm with the Monte Carlo method (MCM) with regard to different performance indexes. Some important issues which have impact on the effectiveness of DWTA model and decision making are also discussed in this section. Section V concludes the paper.

II. Problem Formulation

DWTA models depend on many factors, such as defense strategies, features of targets and weapons, as well as actual combat situations. Different defense scenarios may require different models. The scenario considered in this paper is narrated as follows. At certain time, the defender detects T offensive targets, with their attack aims exposed, which threaten K assets of the defender. There are W weapons available for the defender to intercept the targets. Before these targets break through the defense, there are at most S stages in which the defender can use its weapons to strike the targets. The value of S depends on the distance between targets and their aims, target's flight speed, weapon's regulation, launch and flight time, the delay of data analysis and decision making, and so on [8]. In the DWTA literature, most researchers often adopt a "Shoot-Look-Shoot (SLS)" engagement policy which is a tradeoff between defense effect and cost [4], [8], [20]–[22]. In fact, this engagement policy fits very well with the well-known Observe-Orient-Decide-Act information processing loop [25], [26]. It is widely employed in practical combating scenarios like air-defense-oriented naval group combating [8].

The notation employed in the context is listed in Table I.

A. Objective Function

We employ the total expected value of surviving assets after the final stage as the objective of WTA for the current stage. This objective is somewhat similar to the one in [24], taking into account the optimization of the whole defense effect. The formulation of the objective function for stage t is shown as follows:

$$J_t(\mathbf{X}^t) = \sum_{k=1}^{K(t)} v_k \prod_{j=1}^{T(t)} \left[1 - q_{jk} \prod_{h=t}^{S} \prod_{i=1}^{W(t)} (1 - p_{ij}(h))^{x_{ij}(h)} \right] \quad \text{with } t \in \{1, 2, \ldots, S\} \tag{1}$$

where $J_t(\cdot)$ is the objective function (see Table I); $\mathbf{X}^t = [X_t, X_{t+1}, \ldots, X_S]$ with $X_t = [x_{ij}(t)]_{W\times T}$ is the decision matrix at stage t ($x_{ij}(t) = 1$ if weapon i is assigned to target j at stage t and $x_{ij}(t) = 0$ otherwise); and h is an index of stages.

Note that the defender only carries out the decision component X_t for the current stage t. If the defender observes that one or more targets have been destroyed at stage t, then it is necessary to reevaluate the objective for stage $t + 1$. This is due to the alleviation of defense pressure and the requirement of economizing the use of weapons. In this case, the weapons preassigned to destroyed targets have to be reassigned. Without destruction of the targets, the defender can directly execute the decision component X_{t+1} at the new stage. In order to distinguish these notions, we provide three crucial concepts as follows.

Definition 1: Global Decision—The complete decision matrix $\mathbf{X}^t$ ($t \in \{1, 2, \ldots, S\}$) is termed as global decision at stage t.

Definition 2: Local Decision—Any decision component X_s ($t \leq s \leq S$) in global decision is termed as local decision at stage t.

Definition 3: Executive Decision—The first component of global decision $\mathbf{X}^t$ ($t \in \{1, 2, \ldots, S\}$), i.e., X_t, is termed as executive decision at stage t. Obviously, executive decisions are special cases of local decisions.

B. Constraints

The following four categories of constraints are included in the DWTA model:

$$\sum_{j=1}^{T} x_{ij}(t) \leq n_i,\ \forall t \in \{1,2,\ldots,S\},\ \forall i \in \{1,2,\ldots,W\} \quad (2)$$

$$\sum_{i=1}^{W} x_{ij}(t) \leq m_j,\ \forall t \in \{1,2,\ldots,S\},\ \forall j \in \{1,2,\ldots,T\} \quad (3)$$

$$\sum_{t=1}^{S} \sum_{j=1}^{T} x_{ij}(t) \leq N_i,\ \forall i \in \{1,2,\ldots,W\} \quad (4)$$

$$x_{ij}(t) \leq f_{ij}(t),\ \forall t \in \{1,2,\ldots,S\},\ \forall i \in \{1,2,\ldots,W\},\ \forall j \in \{1,2,\ldots,T\}. \quad (5)$$

Constraint set (2) reflects the capability of weapons of firing at multiple targets at the same time. Most of actual weapons can shoot only one target at a time. Besides, a special weapon that is capable of engaging multiple targets simultaneously can be viewed as multiple separate weapons. In view of these facts, we set $n_i = 1$ for $\forall i \in \{1,2,\ldots,W\}$. Constraint set (3) limits the weapon cost for each target at each stage. The setting of $m_j\ (j = 1,2,\ldots,T)$ usually depends on the combat performance of available weapons. In this paper, we assume that $m_j = 1$ for $\forall j \in \{1,2,\ldots,T\}$. This is a reasonable setting for missile-based defense systems and the SLS engagement policy [8], [24]. For artillery-based defense systems, the value of $m_j\ (j = 1,2,\ldots,T)$ may increase greatly under the same demand on defense strength. Constraint set (4) reflects in essence the amount of ammunition equipped for weapons. Constraint set (5) is very important to actual dynamic WTA problems since it takes into account the influence of time windows on the engagement feasibility of weapons. Besides, it also increases the complexity of DWTA problems and the difficulty of generating feasible solutions. In this case, it is hard to design a desirable operator, which can generate new solutions and guarantee their feasibility at the same time.

C. Optimization Model for DWTA

The optimization model for the dynamic WTA problem aforementioned can be formulated as follows:

$$\max J_t(\mathbf{X}^t),\ s.t.\ (2), (3), (4) \text{ and } (5), \text{ with } t \in \{1,2,\ldots,S\}. \quad (6)$$

The DWTA model here is a typical constrained nonlinear 0–1 programming problem. The dynamic characteristics of this model are mainly embodied by the choice of engagement stages, the change of engagement feasibility, and the uncertainty of the damage of targets and assets at each stage before it is confirmed by observation.

In contrast to target-based DWTA models [22], [24], this asset-based model stresses on the protection of own-force assets. Besides, this model directly embodies the threats of all targets while target-based models depend on the evaluation of the threats of targets.

III. Rule-Based Heuristic for DWTA

A. Rules for DWTA Problem Solver

The use of domain knowledge can reduce the complexity of problems to be solved, which is one of the main reasons for the popularity of heuristics. Problem-specific heuristics, for example, have been successfully applied to solve the optimization problems in the field of public transport management [27], grid computing [28], stock market forecasting [29], and so forth. Here, we try to make the best of available knowledge contained in the structure and parameters of DWTA problems. We incorporate the following knowledge into the proposed heuristic algorithm.

- The more damage a target can cause to assets, the higher priority it should be given in terms of interception.
- The more threat a weapon can reduce at certain stage, the higher priority it should be given to be used at that stage.
- If any effective weapon is assigned to a target, the target's threat (i.e., the potential damage it may cause) will be reduced.

The threat of a target can be expressed by $v_{k(j)} q_{jk(j)} \hat{p}_j$ where $\hat{p}_j$ denotes the surviving probability of target j, and $k(j)$ the index of the asset aimed by the target. The probability $\hat{p}_j\ (j = 1,2,\ldots,T)$ will change in the process of weapon assignment. It is clear that $\hat{p}_j = 1\ (j = 1,2,\ldots,T)$ if no weapons are assigned.

Denote by *VQP* the value of a weapon-target-stage pair. We can determine *VQP* by multiplying the threat value of the target and the probability that the weapon destroys the target at the stage. The *VQP* value of assignment pairs is a crucial factor that determines the final decision scheme.

B. Constraints Handling

In order to guarantee the feasibility of the solution generated by the heuristic algorithm, we make some preliminary treatment to the constraint sets (2)–(5). First, we utilize the engagement feasibility matrices $\mathbf{F}^t\ (t = 1,2,\ldots,S)$ to obtain the sets of available weapons $U_j(t)\ (j = 1,2,\ldots,T; t = 1,2,\ldots,S)$. The weapon assignment will be implemented within the confines of these sets. Thus, those unusable weapons will not be considered, and the constraints in (5) are satisfied. For constraints in (2)–(4), we use the corresponding variables $C2\#F$, $C3\#F$, and $C4\#F$ to mark whether these constraints are saturated or not. Note that a constraint $c(\mathbf{X}) \leq 0$ is said to be saturated if $c(\mathbf{X}) = 0$. The following are the instructions for $C2\#F$, $C3\#F$, and $C4\#F$, namely:

$C2\#F = [c2_{it}]_{W \times S}$ — $c2_{it} = 1$ if the constraint in (2) corresponding to weapon i and stage t is saturated, $c2_{it} = 0$ otherwise;

$C3\#F = [c3_{jt}]_{T \times S}$ — $c3_{jt} = 1$ if the constraint in (3) corresponding to target j and stage t is saturated, $c3_{jt} = 0$ otherwise; and

$C4\#F = [c4_i]_{1 \times W}$ — $c4_i = 1$ if the constraint in (4) corresponding to weapon i is saturated, $c4_i = 0$ otherwise.

Each time a weapon is assigned to a target, their corresponding variables in the aforementioned three matrices are updated. The rules for handling the constraint sets (2)–(4) in the process of weapon assignment are presented later.

- If $c2_{it} = 1$, weapon i will not be used again at stage t;
- If $c3_{jt} = 1$, no more weapons will be assigned to target j at stage t;
- If $c4_i = 1$, weapon i will not be used later.

C. Processing Procedure

The processing procedure of the rule-based heuristic is presented in Fig. 1. At each stage, the DWTA problem solver runs to find a global WTA decision, and the defender carries out the corresponding executive decision. If no target is destroyed at the previous stage, the defender will use the global decision made at the previous stage, and take the second component (local decision) in it as the executive decision for the current stage. In this case, the main part of the algorithm, that is, the process of sorting all available assignment pairs,

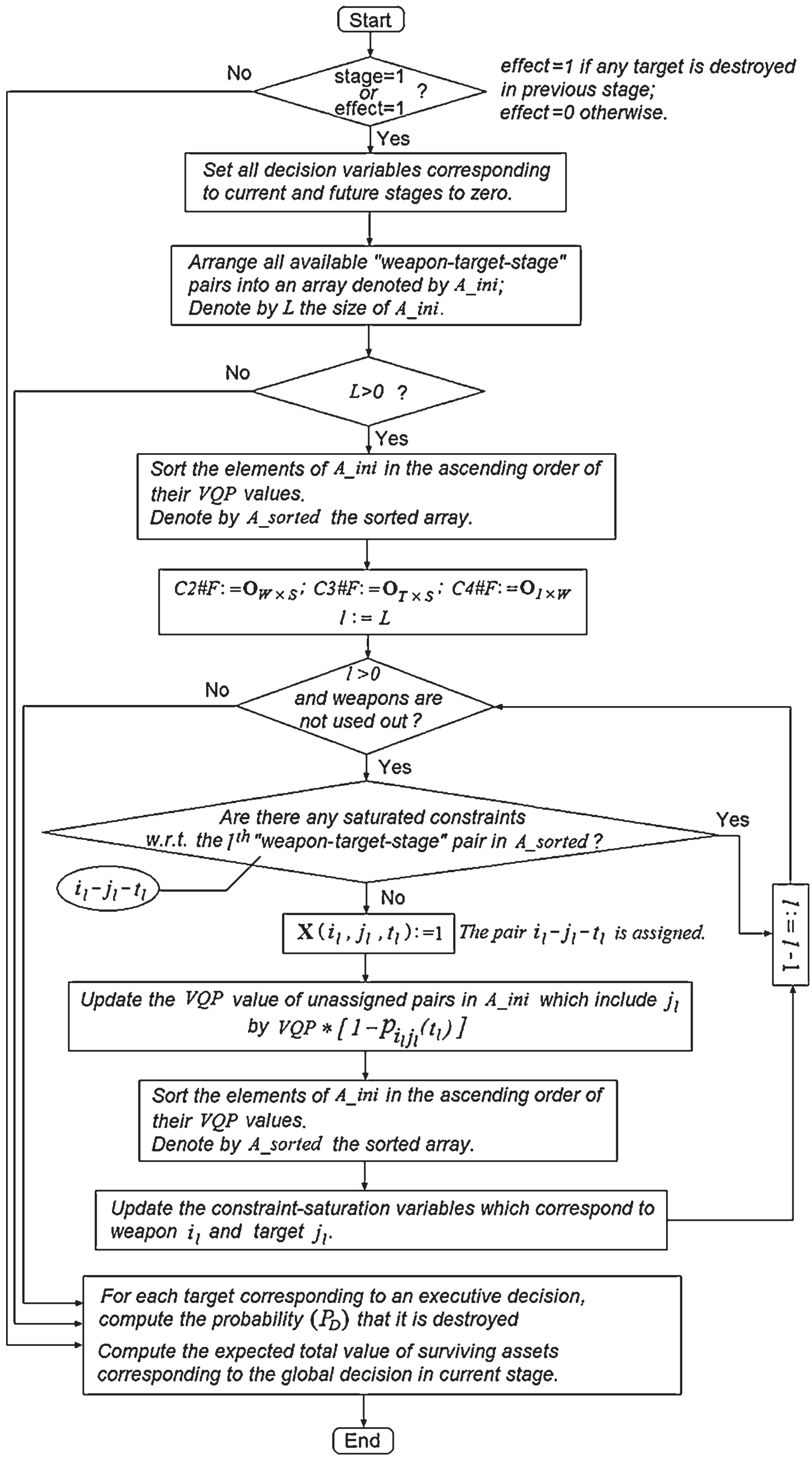

Fig. 1. Rule-based heuristic for DWTA decision making.

will not be executed. Once any target is confirmed to be destroyed at the previous stage, the decision maker will make a new sorting-based decision to reassign all available weapons to surviving targets.

All available assignment pairs are sorted according to their *VQP* values. The bigger the *VQP* value of an assignment pair is, the earlier the corresponding assignment is performed, which is consistent with the anterior two crucial assignment rules presented in Section III-A. In a sense, such a constructive heuristic is much similar to a greedy strategy. Following each assignment, the decision maker will update the *VQP* values of all unassigned pairs which share the same target with the performed assignment. This agrees with the last rule presented in Section III-A.

D. Computational Complexity

The time complexity of the proposed algorithm can be approximated by that of the sorting algorithm embedded in the heuristic. A desirable sorting algorithm has the complexity of $O(n \log n)$, where n is the size of the array to be sorted [30]. In the worst case, n is equal to $W \cdot T \cdot S$ in the proposed algorithm. Since the sorting algorithm runs at most $W \cdot T \cdot S$ times, the worst time complexity of the heuristic can be expressed by $O(L^2 \log L)$ with $L = W \cdot T \cdot S$. Besides, it is easy to see that the worst space complexity of the heuristic is $O(W \cdot T \cdot S)$.

IV. DWTA Simulations

A. Test-Case Generator

1) Parameters' Generator: Given W, T, K, and S, the generator will provide the essential parameters for a DWTA instance which include $\mathbf{V}$, $\mathbf{Q}$, $\mathbf{P}^t$ $(t = 1, 2, \ldots, S)$, $\mathbf{F}^t$ $(t = 1, 2, \ldots, S)$, and N_i $(i = 1, 2, \ldots, W)$. The value of assets is randomly generated in the interval (10,100). Since the number of targets is not less than that of threatened assets, that is to say $T \geq K$, we assume that the aim of the kth target is the kth asset $(k = 1, 2, \ldots, K)$ to ensure that each asset is threatened by at least one target. The aims of the remaining targets will be randomly selected from K assets. Any target has no threat to the assets which are not its aim. We divide the generation of $\mathbf{Q}$ into two steps as follows.

Step 1: $q_{jk} := q_L + (q_H - q_L) \cdot rand$ for $j \in G_k$ $(j = 1, 2, \ldots, T; k = 1, 2, \ldots, K)$ where q_L and q_H are predefined constants with $0 < q_L < q_H < 1$, and G_k is the set of targets aiming at asset k, $k = 1, 2, \ldots, K$; $q_{jk} := 0$ for $j \notin G_k$ $(j = 1, 2, \ldots, T; k = 1, 2, \ldots, K)$;

Step 2: $r := a_L + (a_H - a_L) \cdot rand$, $q_{jk} := q_{jk} \cdot r$ $(j = 1, 2, \ldots, T; k = 1, 2, \ldots, K)$ where a_L and a_H are predefined constants with $0 < a_L < a_H < 1$.

Remark 1: q_L and q_H reflect the lower and upper limits of targets' performance, respectively. a_L and a_H reflect the upper and lower limits of assets' tolerance, respectively. Note that a smaller tolerance value (r) corresponds to a better tolerance performance. The first step provides the performance parameters of targets. The second step provides the tolerance parameters of assets and finally generates composite probabilities that targets kill their aims.

The probabilities that weapons kill targets at different stages (i.e., $\mathbf{P}^t$ $(t = 1, 2, \ldots, S)$) are generated in a similar but simpler way as follows:

$$p_{ij}(t) := p_L + (p_H - p_L) \cdot \ rand$$
$$\text{for } i = 1, 2, \ldots, W.\ ;\ j = 1, 2, \ldots, T\ ;\ t = 1, 2, \ldots, S$$

where p_L and p_H are predefined constants with $0 < p_L < p_H < 1$ which reflect the lower and upper limits of weapons' performance, respectively.

The engagement feasibility parameters $\mathbf{F}^t$ $(t = 1, 2, \ldots, S)$ are also generated in a similar way. However, more zeros will appear in $\mathbf{F}^t$ corresponding to later stages, which accords with the fact that fewer weapons can be used at later stages. Note that $f_{ij}(t) = 0$ implies weapon i cannot engage target j at stage t. The generation of $\mathbf{F}^t$ $(t = 1, 2, \ldots, S)$ is shown as follows:

$$ratio(t) := f_L + (f_H - f_L) \cdot (t - 1)/(S - 1)$$
$$\text{for } t = 1, 2, \ldots, S;$$
$$f_{ij}(t) := [sign\,(rand - ratio(t)) + 1]\,/2;$$
$$\text{for } i = 1, 2, \ldots, W\ ;\ j = 1, 2, \ldots, T;$$
$$t = 1, 2, \ldots, S$$

where $ratio(t)$ is the probability that $f_{ij}(t)$ is set to 0; f_L and f_H are predefined constants with $0 < f_L < f_H < 1$ which reflect the lower and upper limits of $ratio(t)$, respectively; and the function $sign(\cdot)$ is equal to 1 if its argument is positive and -1 otherwise.

Remark 2: It can be expected that $\mathbf{F}^t$ will include more zero elements as t increases.

In order to simplify the operations of the heuristic algorithm, we utilize the engagement feasibility matrices $\mathbf{F}^t$ $(t = 1, 2, \ldots, S)$ to obtain the sets of available weapons $U_j(t)$ $(j = 1, 2, \ldots, T; t = 1, 2, \ldots, S)$. Besides, we categorize the generation of N_i $(i = 1, 2, \ldots, W)$ into the following four cases so as to cover different defense scenarios:

Case 1: One weapon, one shot. $N_i := 1$ for $i = 1, 2, \ldots, W$;

Case 2: No weapon can be used at all stages

$$N_i := \lceil S \cdot rand/2 \rceil \ \text{ for } i = 1, 2, \ldots, W;$$

Case 3: All weapons are available at all stages

$$N_i := S \text{ for } i = 1, 2, \ldots, W$$

Case 4: A hybrid case

$$N_i := \lceil 2S \cdot rand \rceil \ \text{ for } i = 1, 2, \ldots, W.$$

2) Damage's Generator: This generator follows DWTA decision making to determine whether a target or an asset is destroyed. If a target survives the final stage or it breaks through the defense at certain stage, its damage to aimed asset comes into effect, and the target will not be considered at subsequent stages. A target is confirmed to break through the defense if no weapons can be used to intercept it at current stage due to the restriction of engagement feasibility. We regard the destruction of targets as a random event and employ Monte Carlo Simulation to simulate it. For each target, its destruction probability denoted by p_D is computed first according to the corresponding weapon assignment scheme. Then, a pseudorandom number denoted by r and distributed in the interval (0,1) is generated by MCM. If $r < p_D$, then the target is confirmed to be destroyed. We simulate the destruction of assets in the same way.

B. Simulation Procedure and Parameter Setting

The loop of DWTA simulation process includes the following four steps:

Step 1. *Initialization*. All parameters for a DWTA instance are generated by DWTA parameters' generator and loaded to a DWTA problem solver. The states of assets, targets and targets' attacks, denoted by $\mathbf{S}_{AS}$, $\mathbf{S}_{TA}$ and $\mathbf{S}_{AT}$, respectively (see Table I), are initialized. Set the stage index $t = 1$.

Step 2. *Global decision-making and local decision execution*. The DWTA problem solver produces a global decision about weapon assignment, and the defender implements the corresponding executive decision.

Step 3. *Damage generation*. The Monte Carlo simulation generates the damage of assets and targets according to the damage probabilities in response to decision-making. Accordingly, the states of targets and assets are updated.

Step 4. *Termination Criteria*. A composite termination criterion constituted by two components $\mathbf{C1}$ and $\mathbf{C2}$ is executed. The instructions for $\mathbf{C1}$ and $\mathbf{C2}$ are stated as follows:

$\mathbf{C1} = 1$ if all targets are destroyed or complete their attacks; $\mathbf{C1} = 0$ otherwise;

$\mathbf{C2} = 1$ if all weapons available are used out; $\mathbf{C2} = 0$ otherwise.

If the value of at least one of the two components becomes 1, the decision maker will terminate the DWTA loop and evaluate the defense results; otherwise, let $t = t + 1$ and go to Step 2.

Remark 3: The evaluation includes asset damage evaluation (ADE), target damage evaluation (TDE), and weapon cost evaluation (WCE). In ADE, the proportion of the total value of surviving assets in that of all assets (**PTVSA**) is evaluated. In TDE, the ratio of destroyed targets to all targets [i.e., interception ratio (**IR**)] is analyzed. In WCE, we measure the defense cost (DC) of generated WTA decisions by the average engagement times for destroying one target.

In the simulation, we fixed the number of stages (S) at four. The setting of the other three primary parameters W, T, and K includes the following cases:

$$W10T10K10(\mathbf{No.1}),\quad W10T10K5(\mathbf{No.2}),$$
$$W50T50K50(\mathbf{No.3}),\quad W50T50K20(\mathbf{No.4}),$$
$$W100T50K50(\mathbf{No.5}),\quad W100T50K20(\mathbf{No.6}),$$
$$W50T100K100(\mathbf{No.7}),\quad W50T100K50(\mathbf{No.8}),$$
$$W100T100K100(\mathbf{No.9}),\quad W100T100K50(\mathbf{No.10}),$$
$$W100T200K200(\mathbf{No.11}), W100T200K100(\mathbf{No.12}),$$
$$W200T200K200(\mathbf{No.13}).$$

The setting of the first generator's parameters is shown as follows:

$$q_L = 0.6, q_H = 0.99 \quad a_L = 0.6,\ a_H = 0.99$$
$$p_L = 0.4,\ p_H = 0.9 \quad f_L = 0.1,\ f_H = 0.9.$$

For comparison, we also employed a MCM to solve each DWTA instance. The technique of constraint handling in MCM is the same as that used in the proposed heuristic algorithm. In MCM, 10 000 random samplings were implemented at each stage, and the best solution among these samplings was selected as the global decision at corresponding DWTA stage. At the first stage, we recorded the expected total value of assets surviving after the final stage (**ETVSA**), that is, the objective value of the global decision made at the first stage. Its proportion in the total value of all assets (**PETVSA**) is used as another index for performance comparisons. In each case, both MCM and the heuristic algorithm run 30 times, and corresponding results are statistically analyzed. All programs were compiled by the well-known mathematical software—MATLAB (version 6.5.0). Particularly, we employed, for convenience, the built-in sorting algorithm in MATLAB [31]. All experiments described in this paper were performed on a PC with Pentium M 1.5-GHz CPU and 512-MB internal memory.

C. Results and Analysis

Experimental results are presented in Figs. 2 and 3 and Table II with the mean and standard deviation (s.t.d.) of the results of 30 runs provided. Figs. 2 and 3 show that the proposed heuristic can produce better global decisions in almost all test cases. From case 1 to 4, the PETVSA index of the rule-based heuristic is on average 34.0%, 24.6%, 5.7%, and 8.6% better than that of MCM, respectively. Besides, the results produced by the heuristic are stable since it is in nature a deterministic algorithm. Regarding the PTVSA index, the advantages of the heuristic over MCM in four cases are 18.9%, 12.2%, 0.8%, and 1.4%, respectively. On the whole, the results indicated by PETVSA are consistent with those by PTVSA. However, the values of PTVSA in most cases are a little larger than those of PETVSA. This is because the global decision made at the first stage, corresponding to PETVSA, has to consider the threats of all targets. In other words, all targets have potential damage to assets at this time. In contrast, the destruction of targets prior to the final stage eliminates the corresponding threats to assets and thus reduces defense pressure. In fact, as it is in the process of DWTA simulations earlier, the global decision *w.r.t.* PETVSA will be discarded on the occasion of the destruction of targets, and a new one will be produced at the subsequent stage. Therefore, it is not unusual to see that more assets will survive through all stages than expected.

As shown in Fig. 3, the heuristic also results in higher IR in almost all cases. The use of the proposed heuristic, compared with MCM, increases the IR from Case 1 to 4 by 20.7%, 11.3%, 0.9%, and 1.5%, respectively. Besides, the DCs of the DWTA decisions made by the heuristic in all cases are less than those corresponding to MCM (see Fig. 3). The heuristic reduces the DCs in the four cases by 20.1%, 20.7%, 18.9%, and 20.1%, respectively.

It should be noted that the aforementioned better results are achieved with less computation cost, as shown in Table II. The computation time of the heuristic averaged on all cases is about 114 times less than that of MCM. Even when solving the time-consuming DWTA instance No.13Case3, the heuristic runs about 24 times faster than MCM. In addition, the heuristic can solve small-scale DWTA instances almost instantly. Even for DWTA problems with larger scale, e.g., those in the cases of No.11 and No.12 $(W = 100, T = 200, S = 4)$, it can achieve efficient weapon assignment within only a few seconds. It can be expected that, with the support of advanced computing systems, the actual computation time of the proposed heuristic will be further reduced.

It was also observed that the decisions produced by the heuristic have obviously higher quality in severe defense scenarios where weapon resources are insufficient. For example, the scenarios corresponding to No.7Case1, No.7Case2, No.8Case1, No.8Case2, No.11Case1, No.11Case2, No.12Case1, and No.12Case2 are severe for the defender, since the number of weapons in these instances is only half of the number of targets, and weapons cannot be adequately used. Although the IRs in these scenarios seem to be relatively low, the generated decisions under high defense pressure have saved as many assets as possible.

D. Discussion

One of the most prominent advantages of the proposed heuristic is its saving of decision-making time with guaranteed solution quality, which is the primary demand of real-time military decision making. The statistical results of the running time of random sampling (MCM) demonstrate that the function evaluation in DWTA with the satisfaction of all constraints is very time consuming. In contrast, the constructive heuristic proposed in this paper can solve larger scale problems within a few seconds. Therefore, the heuristic provides a viable and efficient way to solve DWTA problems.

In the aforementioned simulation, we only consider, for simplicity, a special defense scenario in which all information contained in DWTA model is reliable. In practice, information accuracy has a great impact

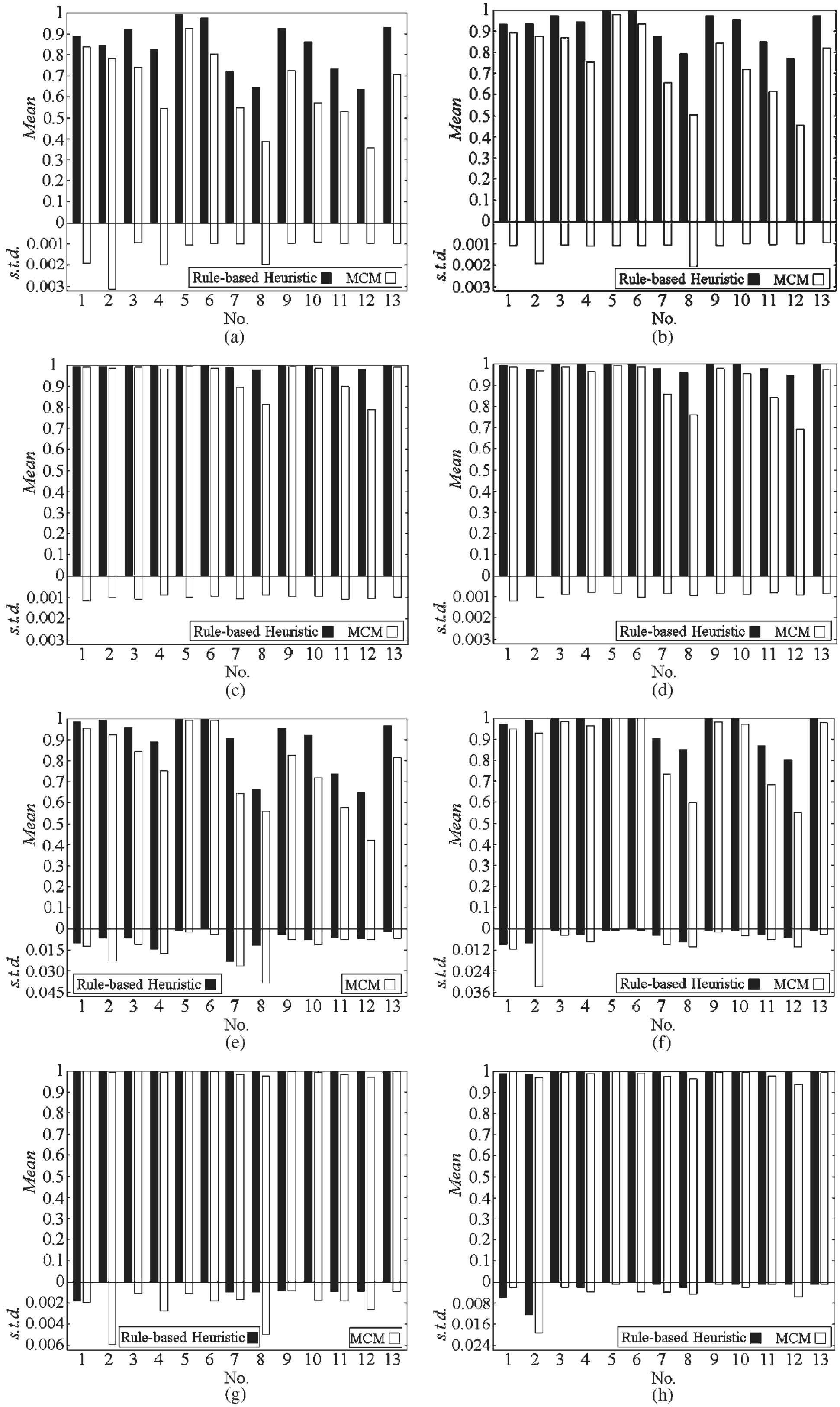

Fig. 2. Comparative results of rule-based heuristic and MCM w.r.t. the mean and s.t.d. of PETVSA and PTVSA. (a) PETVSA (Case 1). (b) PETVSA (Case 2). (c) PETVSA (Case 3). (d) PETVSA (Case 4). (e) PTVSA (Case 1). (f) PTVSA (Case 2). (g) PTVSA (Case 3). (h) PTVSA (Case 4).

on the effectiveness of DWTA decisions. If the aims of targets, for example, are not clear, a notable adjustment is usually essential in the use of domain knowledge. In this case, all potential aims of a target may be considered, which will result in a more complicated DWTA model and further a different decision scheme. Besides, the prediction of targets' movement also plays an important role in DWTA as its precision affects the analysis of weapons' combat effects and even engagement feasibility. An elaborate model should also have a careful

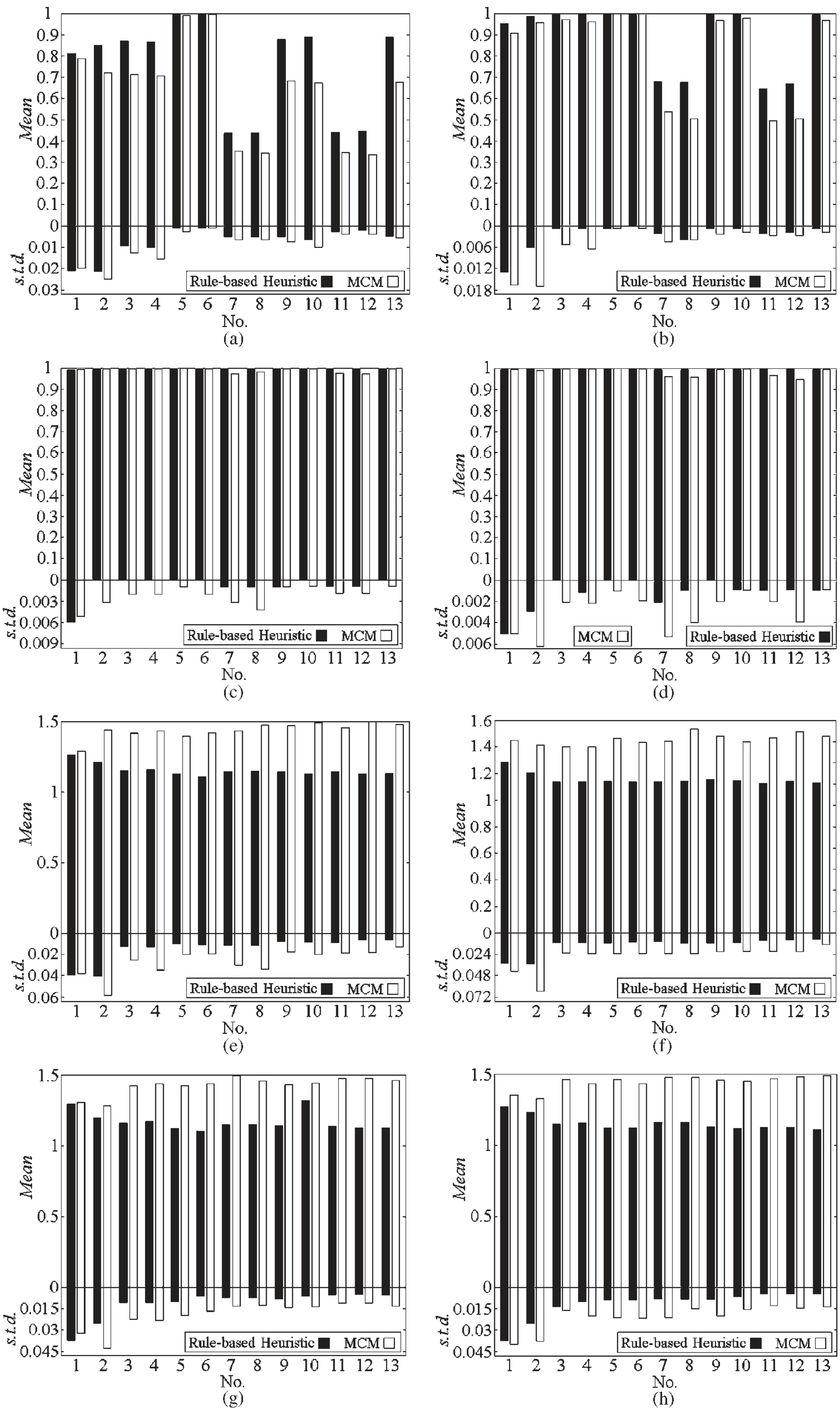

Fig. 3. Comparative results of rule-based heuristic and MCM w.r.t. the mean and s.t.d. of IR and DC. (a) IR (Case 1). (b) IR (Case 2). (c) IR (Case 3). (d) IR (Case 4). (e) DC (Case 1). (f) DC (Case 2). (g) DC (Case 3). (h) DC (Case 4).

evaluation of assets' values since an improper evaluation may lead to a bad decision. From an all-sided consideration of military operations, the aforementioned issues are also significant and worth devotion from researchers for reliable DWTA decision making.

V. Conclusion

The rule-based constructive heuristic we have proposed is an efficient straightforward solution to DWTA problems, and it benefits from the use of domain knowledge in the form of three crucial rules.

TABLE II
COMPARATIVE RESULTS W.R.T. COMPUTATION TIME (S)

No.	Case 1	Case 2	Case 3	Case 4
1h	**0.012±0.003**	**0.009±0.001**	**0.013±0.001**	**0.009±0.001**
1m	2.681±0.003	2.606±0.004	2.674±0.001	2.763±0.005
2h	**0.011±0.002**	**0.010±0.002**	**0.010±0.002**	**0.009±0.001**
2m	2.644±0.006	2.643±0.001	2.668±0.001	2.646±0.001
3h	**0.194±0.005**	**0.225±0.006**	**0.387±0.005**	**0.345±0.004**
3m	30.03±0.136	29.77±0.116	29.66±0.080	28.88±0.087
4h	**0.195±0.006**	**0.223±0.002**	**0.384±0.005**	**0.345±0.004**
4m	29.33±0.109	29.39±0.129	29.04±0.080	28.79±0.155
5h	**0.507±0.005**	**0.655±0.006**	**0.778±0.005**	**0.838±0.005**
5m	58.31±0.079	58.02±0.110	58.99±0.123	58.85±0.129
6h	**0.549±0.005**	**0.640±0.008**	**0.853±0.005**	**0.836±0.006**
6m	59.65±0.111	59.54±0.105	59.35±0.228	58.71±0.125
7h	**0.392±0.005**	**0.457±0.005**	**0.823±0.004**	**0.691±0.010**
7m	60.82±0.109	60.55±0.113	58.43±0.138	58.93±0.156
8h	**0.386±0.006**	**0.472±0.006**	**0.826±0.002**	**0.742±0.004**
8m	60.34±0.205	60.89±0.125	58.34±0.110	59.46±0.232
9h	**1.068±0.005**	**1.447±0.005**	**2.846±0.010**	**2.459±0.007**
9m	121.0±0.187	120.7±0.203	118.2e±0.157	117.5±0.208
10h	**1.135±0.005**	**1.487±0.005**	**2.962±0.010**	**2.481±0.005**
10m	123.9±0.288	123.7±0.313	117.2±0.193	118.3±0.235
11h	**2.221±0.007**	**2.783±0.004**	**5.847±0.016**	**5.177±0.010**
11m	273.6±3.003	267.5±0.183	255.9±0.211	261.0±0.196
12h	**2.226±0.007**	**2.899±0.006**	**5.918±0.009**	**4.913±0.012**
12m	266.7±0.174	269.8±0.483	260.8±0.326	264.8±0.203
13h	**7.637±0.005**	**10.48±0.015**	**24.65±0.023**	**20.61±0.057**
13m	627.4±0.294	624.7±0.281	602.1±0.257	610.8±0.295

The algorithm can directly obtain desirable solutions without function evaluations. In order to guarantee the feasibility of generated solutions, we achieve constraint satisfaction in the process of weapon assignment by validating the saturation states of different constraints dynamically. A DWTA simulation framework is built to investigate the performance of the heuristic. The framework can also serve as a test suite to compare the performances of different DWTA algorithms in future. Besides, the heuristic can also be taken as a reference for comparison. The experiments with DWTA instances under different scales validate the effectiveness of the heuristic including its decision-making quality and computation efficiency. It can efficiently solve DWTA instances under larger scales (e.g., W100T100S4) with guaranteed decision quality.

For the sake of enhancing the algorithm's scalability and solving DWTA problems more efficiently, a further research on the reduction of its time complexity is significant. Some advanced heuristics or metaheuristics are also worth investigation.

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