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Abstract

The phenomenon of sponsored search advertising—where advertisers pay a fee to Internet search engines to be displayed alongside organic (nonsponsored) Web search results—is gaining ground as the largest source of revenues for search engines. Using a unique six-month panel data set of several hundred keywords collected from a large nationwide retailer that advertises on Google, we empirically model the relationship between different sponsored search metrics such as click-through rates, conversion rates, cost per click, and ranking of advertisements. Our paper proposes a novel framework to better understand the factors that drive differences in these metrics. We use a hierarchical Bayesian modeling framework and estimate the model using Markov Chain Monte Carlo methods. Using a simultaneous equations model, we quantify the relationship between various keyword characteristics, position of the advertisement, and the landing page quality score on consumer search and purchase behavior as well as on advertiser's cost per click and the search engine's ranking decision. Specifically, we find that the monetary value of a click is not uniform across all positions because conversion rates are highest at the top and decrease with rank as one goes down the search engine results page. Though search engines take into account the current period's bid as well as prior click-through rates before deciding the final rank of an advertisement in the current period, the current bid has a larger effect than prior click-through rates. We also find that an increase in landing page quality scores is associated with an increase in conversion rates and a decrease in advertiser's cost per click. Furthermore, our analysis shows that keywords that have more prominent positions on the search engine results page, and thus experience higher click-through or conversion rates, are not necessarily the most profitable ones—profits are often higher at the middle positions than at the top or the bottom ones. Besides providing managerial insights into search engine advertising, these results shed light on some key assumptions made in the theoretical modeling literature in sponsored search.

Keywords

online advertising, search engines, hierarchical Bayesian modeling, paid search, click-through rates, conversion rates, keyword ranking, cost per click, electronic commerce, internet monetization

Disciplines

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An Empirical Analysis of Search Engine Advertising: Sponsored Search in Electronic Markets[1]

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Abstract

The phenomenon of sponsored search advertising – where advertisers pay a fee to Internet search engines to be displayed alongside organic (non-sponsored) web search results – is gaining ground as the largest source of revenues for search engines. Using a unique 6 month panel dataset of several hundred keywords collected from a large nationwide retailer that advertises on Google, we empirically model the relationship between different metrics such as click-through rates, conversion rates, cost-per-click, and ranks of these advertisements. Our paper proposes a novel framework and data to better understand what drives these differences. We use a Hierarchical Bayesian modeling framework and estimate the model using Markov Chain Monte Carlo (MCMC) methods. Using a simultaneous equations model, we quantify the impact of keyword type and length, position of the advertisement and the landing page quality on consumer search and purchase behavior as well as on advertiser's cost per click and the search engine's ranking decision for different ads. Our results provide descriptive and quantitative insights to advertisers about what attributes of sponsored keyword advertisements contribute to variation in advertiser value, and how much to invest in search engine optimization campaigns versus search engine marketing campaigns. Our analyses also lend quantitative insights into the relative economic impact of different kinds of advertisements such as retailer-specific ads, brand specific ads or generic ads. We also discuss how our empirical estimates shed light on some assumptions made by existing theoretical models in sponsored search advertising.

Keywords: *Online advertising, Search engines, Hierarchical Bayesian modeling, Paid search, Click-through rates, Conversion rates, Keyword ranking, Bid price, Electronic commerce.*

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1. Introduction

The Internet has brought about a fundamental change in the way users generate and obtain information, thereby facilitating a paradigm shift in consumer search and purchase patterns. In this regard, search engines are able to leverage their value as information location tools by selling advertising linked to user generated queries and referring them to the advertisers. Indeed, the phenomenon of sponsored search advertising – where advertisers pay a fee to Internet search engines to be displayed alongside organic (non-sponsored) web search results – is gaining ground as the largest source of revenues for search engines. The global paid search advertising market is predicted to have a 37% compound annual growth rate, to more than $33 billion in 2010 and has become a critical component of firm's marketing campaigns.

Search engines like Google, Yahoo and MSN have discovered that as intermediaries between users and firms, they are in a unique position to try new forms of advertisements without annoying consumers. In this regard, sponsored search advertising has gradually evolved to satisfy consumers' penchant for relevant search results and advertisers' desire for inviting high quality traffic to their websites. These advertisements are based on customers' own queries and are hence considered far less intrusive than online banner ads or pop-up ads. The specific 'keywords' in response to which the ads are displayed are often chosen by firms based on user-generated content in online product reviews, social networks and blogs where users have posted their opinions about firms' products, often highlighting the specific product features they value the most. In many ways, the increased ability of users to interact with firms in the online world has enabled a shift from 'mass' advertising to more 'targeted' advertising.

How does this mechanism work? In sponsored search, firms who wish to advertise their products or services on the Internet submit their product information in the form of specific 'keyword' listings to search engines. Bid values are assigned to each individual ad to determine the placement of each listing among search results when a user performs a search. When a consumer searches for that term on a search engine, the advertisers' web page appears as a sponsored link next to the organic search results that would otherwise be returned using the neutral criteria employed by the search engine. By allotting a specific value to each keyword, an advertiser only pays the assigned price for the people who click on their listing to visit its website. Because listings appear when a keyword is searched for, an advertiser can reach a more targeted audience on a much lower budget.

Despite the growth of search advertising, we have little understanding of how consumers respond to contextual and sponsored search advertising on the Internet. In this paper, we focus on previously unexplored issues: How does sponsored search advertising affect consumer search and purchasing behavior on the Internet? More specifically, what content attributes of a sponsored advertisement contribute to variation in advertiser value in terms of consumer click-through rates and conversions? How does keyword content influence the advertiser's actual bidding decisions, and the search engine's advertisement ranking decision? While an emerging stream of theoretical literature in sponsored search has looked at issues such as mechanism design in auctions, no prior work has empirically analyzed these kinds of questions. Given the shift in advertising from traditional banner advertising to search engine advertising, an understanding of the determinants of conversion rates and click-through rates in search advertising is essential for both traditional and Internet retailers.

Using a unique panel dataset of several hundred keywords collected from a large nationwide retailer that advertises on Google, we study the effect of sponsored search advertising on consumer and firm behavior. In particular, we propose a Hierarchical Bayesian modeling framework in which we build a simultaneous model to jointly estimate the impact of various attributes of sponsored keyword advertisements on consumer click-through and purchase propensities, on the advertiser's cost per click decision and on the search engine ad ranking decision. The presence of retailer-specific information in the keyword is associated with an increase in click-through and conversion rates by 14.72% and 50.6%, respectively, the presence of brand-specific information in the keyword is associated with a decrease in click-through and conversion rates by 56.6% and 44.2%, respectively, while the length of the keyword is associated with a decrease in click-through rates by 13.9%. Keyword rank is negatively associated with the click-through rates and conversion rates and this is increasing at a decreasing rate. Our findings show that an increase in the landing page quality of the advertiser can lead to an increase in conversion rates by as much as 22.5%. Further, we show that the advertiser's CPC is negatively affected by the landing page quality as well as by the presence of its own information but positively affected by the presence of brand-specific information in the keyword.

Our paper aims to make both methodological and substantive contributions to the literature. These can be summarized as follows. *First,* to the best of our knowledge, our paper is the first empirical study that simultaneously models and documents the impact of search engine advertising on all three entities involved in the process – consumers, advertisers and search engines. The proposed simultaneous model provides a natural way to account for endogenous relationships between decision variables, leading to a robust identification strategy and precise estimates. The model can be applied to similar data from other industries. Moreover, unlike previous work, we jointly study consumer click-through behavior and conversion behavior conditional on a click-through in studying consumer search behavior. Ignoring consumer click-through behavior can lead to selectivity bias if the error terms in the click-through probability and in the conditional conversion probability are correlated (Maddala 1983), and this is an additional contribution. The proposed Bayesian estimation algorithm provides a convenient way to estimate such model by using data augmentation.

Second, our model provides useful descriptive insights to advertisers about what content attributes of sponsored keyword advertisements contribute to variation in advertiser value. In particular, our study quantifies the relationship between branded/retailer/generic and shorter/longer keywords and demand side variables like click-through rates and conversion rates – a question of increasing interest to many firms. *Additionally,* advertisers are interested in determining how much to invest in search engine optimization campaigns (for example, by improving landing page qualities) versus search engine marketing campaigns (for example, by investing in higher bids in the auction process). By quantifying the impact of landing page quality on conversion rates and the cost per click of search advertising, and by comparing this to the relationship of keyword attributes with these variables, our study can also help managers make better decisions regarding investments in the online advertising domain.

Third, by showing a direct negative relationship between conversion rates and rank, we show that the value per click to an advertiser is not uniform across slots. This finding refutes a commonly held assumption in the industry is that the value of a click from a sponsored search campaign is independent of the position of the advertisement. Prior theoretical work (Aggarwal et al. 2006, Edelman et al. 2007, Varian 2007) also make a common assumption of uniform value per click across all ranks and show that under this condition, sponsored search auctions maximize social welfare. Our finding of non-uniformity in value per click paves the way for future theoretical models in this domain that could relax this assumption and design new mechanisms with more robust equilibrium properties. The recent work by Borgers et al. (2007) that incorporates non-uniform values for clicks in their theoretical model is a step in this direction. Further, by demonstrating that search engines are indeed taking into account both the bid price as well as prior click-through rates of the keywords before deciding the final rank of an advertisement, our findings contribute towards providing empirical evidence regarding other assumptions made in the theoretical work about sponsored keyword auction mechanisms in search engine advertising.

The remainder of this paper is organized as follows. Section 2 gives an overview of the different streams of literature from marketing and computer science related to our paper. Section 3 describes the data and gives a brief background into some different aspects of sponsored search advertising that could be useful before we proceed to the empirical models and analyses. In Section 4, we present a model to study the click-through rate, conversion rate and keyword ranking simultaneously, and discuss our identification strategy. In Section 5 we discuss our empirical findings. In Section 6, we discuss some implications of our findings and then conclude the paper.

2. Literature and Theoretical Background

Our paper is related to several streams of research. A number of approaches have been build to modeling the effects of advertising based on aggregate data (Tellis 2004). Much of the existing academic (e.g., Gallagher et al. 2001, Dreze & Hussherr 2003) on advertising in online world has focused on measuring changes in brand awareness, brand attitudes, and purchase intentions as a function of exposure. This is usually done via field surveys or laboratory experiments using individual (or cookie) level data. Sherman & Deighton (2001) and Ilfeld & Winer (2002), show using aggregate data that increased online advertising leads to more site visits. In contrast to other studies which measure (individual) exposure to advertising via aggregate advertising dollars (e.g., Mela et al. 1998, Ilfeld & Winer 2002), we use data on individual search keyword advertising exposure. Manchanda et al. (2006) look at online banner advertising. Because banner ads have been perceived by many consumers as being annoying, traditionally they have had a negative connotation associated with it. Moreover, it was argued that since there is considerably evidence that only a small proportion of visits translate into final purchase (Sherman & Deighton 2001, Moe & Fader 2003, Chatterjee et al. 2003), click-through rates may be too imprecise for measuring the effectiveness of banners served to the mass market. Interestingly however, Manchanda et al. (2006), found that banner advertising actually increases purchasing behavior, in contrast to conventional wisdom. These studies therefore highlight the importance of investigating the impact of other kinds of online advertising such as search keyword advertising on actual purchase behavior, since the success of keyword advertising is also based on consumer click-through rates. Our study is also related to other forms of paid placements available to retailers on the internet available such as sponsored listings on shopping bots (e.g., Baye and Morgan

2001, Baye et al. 2008) who have studied the role of shopping bots as information gate keepers and estimated the impact of retailers' rank during placement on click-through rates.

There is also an emerging theoretical stream of literature exemplified by Aggarwal et al. (2006), Edelman et al. (2007), Feng et al. (2007), Varian (2007), and Liu et al. (2008) who analyze mechanism design and equilibria in search engine auctions. Chen and He (2006), and Athey & Ellison (2008) build models that integrate consumer behavior with advertiser decisions, and the latter paper theoretically analyzes several possible scenarios in the design of sponsored keyword auctions. Katona &Sarvary (2007) build a model of competition in sponsored search and find that the interaction between search listings and paid links determine equilibrium bidding behavior.

Despite the emerging theory work, very little empirical work exists in online search advertising. This is primarily because of difficulty for researchers to obtain such advertiser-level data. Existing work has so far focused on search engine performance (Telang et al. 2004, Bradlow & Schmittlein 2000). Moreover, the handful of studies that exist in search engine marketing have typically analyzed publicly available data from search engines. Animesh et al. (2008) look at the presence of quality uncertainty and adverse selection in paid search advertising on search engines. Goldfarb and Tucker (2007) examine the factors that drive variation in prices for advertising legal services on Google. Agarwal et al. (2008) provide quantitative insights into the profitability of advertisements associated with differences in keyword position. Ghose and Yang (2008) build a model to map consumers' search-purchase relationship in sponsored search advertising. They provide evidence of horizontal spillover effects from search advertising resulting in purchases across other product categories. Rutz & Bucklin (2007b) showed that there are spillovers between search advertising on branded and generic keywords, as some customers may start with a generic search to gather information, but later use a branded search to complete their transaction. In an interesting paper related to our work, Rutz & Bucklin (2007a) studied hotel marketing keywords to analyze the profitability of different campaign management strategies. However, our paper differs from theirs and extends their work in several important ways. Rutz & Bucklin (2007a) only model the conversion probability conditional on positive number of click-throughs. However, our paper models click-through and conversion rates simultaneously in order to alleviate potential selectivity biases. In addition, we also model the search engine's ranking decision and the advertiser's decision on cost-per-click (CPC), both of which are absent in their paper. Our analysis reveals that it is important to model the advertiser and the search engine's decisions simultaneously with clicks and conversion since both ‘CPC’ and ‘Rank’ have been found to be endogenous. These issues are not addressed in their paper.

To summarize, our research is distinct from extant online advertising research as it has largely been limited to the influence of banner advertisements on attitudes and behavior. We extend the literature by empirically comparing the impact of different keyword characteristics on the performance of online search advertising in paid search towards understanding the larger question of analyzing how keyword characteristics drive consumers’ search and purchase behavior, as well as firms’ optimal bid prices and ranking decisions.

3. Data

We first describe the data generation process for paid keyword advertisement since it differs on many dimensions from traditional offline advertisement. Advertisers bid on keywords during the auction process. *A keyword may consist of one or more ‘words’.* Once the advertiser gets a rank allotted for its keyword ad, these sponsored ads are displayed on the top left, and right of the computer screen in response to a query that a consumer types on the search engine. The ad typically consists of headline, a word or a limited number of words describing the product or service, and a hyperlink that refers the consumer to the advertiser’s website after a click. The serving of the ad in response to a query for a certain keyword is denoted as an impression. If the consumer clicks on the ad, he is led to the landing page of the advertiser’s website. This is recorded as a click, and advertisers usually pay on a per click basis. In the event that the consumer ends up purchasing a product from the advertiser, this is recorded as a conversion.

Our data contains weekly information on paid search advertising from a large nationwide retail chain, which advertises on Google.[2] The data span all keyword advertisements by the company during a period of six months in 2007, specifically for the 24 calendar weeks from January to June. Each keyword in our data has a unique advertisement ID. The data is for a given keyword for a given week. It consists of the number of impressions, number of clicks, the average cost per click (CPC) which represents the bid price, the rank of the keyword, the number of conversions, and the total revenues from a conversion. While an impression often leads to a click, it may not lead to an actual purchase (defined as a conversion). Based on these data, we compute the *Click-through Rate* (clicks/impressions) and *Conversion Rate* (conversions/clicks) variables. The product of *CPC* and number of clicks gives the total costs to the firm for sponsoring a particular advertisement. Based on the contribution margin and the revenues from each conversion through a paid search advertisement, we are able to compute the gross profit per keyword from a conversion. The difference between gross profits and keyword advertising costs (the number of clicks times the cost-per-click) gives the net profits accruing to the retailer from a sponsored keyword conversion. This is the *Profit* variable.

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[2] The firm is a large Fortune-500 retail store chain with several hundred retail stores in the US but due to the nature of the data sharing agreement between the firm and us, we are unable to reveal the name of the firm.

Finally, while we have data on the URLs of the landing page corresponding to a given keyword, we do not have data on landing page quality scores or content, since the exact algorithm used by Google to impute the landing page quality is not disclosed to the public.[3] Hence, we use a semi-automated approach with content analysis to impute the landing page quality based on the three known metrics used by Google. Google uses a weighted average of Relevancy, Transparency and Navigability to impute the landing page quality of a given weblink. We hired two independent annotators to rate each landing page based on each of these metrics and then computed the weighted average of the scores. The inter-rater reliability score was 0.73, indicating a very high level of reliability.

Our final dataset includes 9664 observations from a total of 1878 unique keywords. Note that our main interest in this empirical investigation is to examine various factors that drive differences in click-throughs and conversions. Hence, we analyze click-through rates, conversion rates, cost-per-click, and rank by jointly modeling the consumers' search and purchase behavior, the advertiser's decision on cost per click, and the search engine's keyword rank allocating behavior. Table 1 reports the summary statistics of our dataset. As shown, the average weekly number of impressions is 411 for one keyword, among which around 46 lead to a click-through, and 0.85 lead to a purchase. Our data suggest the average cost per-click for a given keyword is about 25 cents, and the average rank (position) of these keywords is about 6.92. Finally, we have information on three important keyword characteristics, which we next briefly discuss with a focus on the rationale of analyzing them. As Table 1 shows, there is a substantial amount of variation in clicks, conversion, rank and CPC of each keyword over time.

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[3] Google computes a quality score for each landing page as a function of the site's navigability as well as the relevance and transparency of information on that page in order to provide higher user experience after a click-through to the site. Besides these relevancy factors, the quality score is also based on click-through rates. However, the exact algorithm for computing this score is not publicly available. The quality score is then used in determining the minimum bid price, which in turn affects the rank of the ad, given the typical advertiser budget constraints. Further information on these aspects is available at www.adwords.google.com.

We enhanced the dataset by introducing keyword-specific characteristics such as *Brand*, *Retailer* and *Length*. For each keyword, we constructed two dummy variables, based on whether they were (i) branded keywords or not (for example, "Sealy mattress", "Nautica bedsheets"), and (ii) retailer-specific advertisements (for example, "Wal-Mart", "walmart.com") or not. To be precise, for creating the variable in (i) we looked for the presence of a brand name (either a product-specific or a company specific) in the keyword, and labeled the dummy as 1 or 0, with 1 indicating the presence of a brand name. For (ii), we looked for the presence of the specific advertiser's (retailer) name in the keyword, and then labeled the dummy as 1 or 0, with 1 indicating the presence of the retailer's name.

= = Insert Table 1 = =

4. A Simultaneous Model of Click-through, Conversion, CPC and Keyword Rank

We cast our model in a hierarchical Bayesian framework and estimate it using Markov Chain Monte Carlo methods (see Rossi and Allenby 2003 for a detailed review of such models). We postulate that the decision of whether to click and purchase in a given week will be affected by the probability of advertising exposure (for example, through the rank of the keyword) and individual keyword-level differences (both observed and unobserved). We simultaneously model consumers' click-through and conversion behavior, the advertiser's keyword pricing behavior, and the search engine's keyword rank allocating behavior.

4.1 Theoretical setup

Assume for search keyword i at week j, there are n_{ij} click-throughs among N_{ij} impressions (the number of times an advertisement is displayed by the retailer), where $n_{ij} \leq N_{ij}$ *and* $N_{ij} > 0$. Suppose that among the n_{ij} click-throughs, there are m_{ij} click-throughs that lead to purchases, where $m_{ij} \leq n_{ij}$. Let us further assume that the probability of having a click-through is p_{ij} and the probability of having a purchase conditional on a click-through is q_{ij}. In our model, a consumer faces decisions at two levels – one, when she sees a keyword advertisement, she makes decision whether or not to click it; two, if she clicks on the advertisement, she can take any one of the following two actions – make a purchase or not make a purchase.

Thus, there are three types of observations. First, a person clicked through and made a purchase. The probability of such an event is $p_{ij}q_{ij}$. Second, a person clicked through but did not make a purchase. The probability of such an event is $p_{ij}(1- q_{ij})$. Third, an impression did not lead to a click-through or purchase. The probability of such an event is $1- p_{ij}$. Then, the probability of observing (n_{ij}, m_{ij}) is given by:

$$f(n_{ij}, m_{ij}, p_{ij}, q_{ij}) = \frac{N_{ij}!}{m_{ij}!(n_{ij} - m_{ij})!(N_{ij} - n_{ij})!}\{p_{ij}q_{ij}\}^{m_{ij}}\{p_{ij}(1-q_{ij})\}^{n_{ij}-m_{ij}}\{1-p_{ij}\}^{N_{ij}-n_{ij}} \qquad (4.1)$$

4.2 Modeling the Consumer's Decision: Click-through

Prior work (Broder 2002, Jansen and Spink 2007) has analyzed the goals for users' web searches and classified user queries in search engines into three categories of searches: *navigational* (for example, a search query consisting of a specific firm or retailer), *transactional* (for example, a search query consisting of a specific product) or *informational* (for example, a search query consisting of longer words). Being cognizant of such user behavior, search engines not only sell non-branded or generic keywords as advertisements, but also well-known product or manufacturer brand names as well as keywords indicating the specific advertiser in order for the firm to attract consumers to its website.[4] Moreover, advertisers also have the option of making the keyword advertisement either generic or specific by altering the number of words contained in the keyword. Finally, the length of the keyword is also an important determinant of search and purchase behavior but anecdotal evidence on this varies across trade press reports. Some studies have shown that the percentage of searchers who use a combination of keywords is 1.6 times the percentage of those who use single-keyword queries (Kilpatrick 2003). In contrast, another study found that single-keywords have on average the highest number of unique visitors (Oneupweb 2005). In our data, the average length of a keyword is about 2.6. In sum, the number of advertisers placing a bid, which can affect the number of clicks received by a given ad, will vary based on the kind of keyword that is advertised. Hence, we focus on the three important keyword-specific characteristics for the firm when it advertises on a search engine: *Brand*, *Retailer* and *Length*. The click-through probability is likely to be influenced by the position of the ad (*Rank*), how specific or broad the keyword is (*Length*), and whether is contains any retailer-specific (*Retailer*) or brand-specific information (*Brand*). Hence, in equation (4.1), p_{ij} the click-through probability is modeled as:

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[4] For example, a consumer seeking to purchase a digital camera is as likely to search for a popular manufacturer brand name such as NIKON, CANON or KODAK on a search engine as searching for the generic phrase "digital camera". Similarly, the same consumer may also search for a retailer such as "BEST BUY" or "CIRCUIT CITY" in order to buy the digital camera directly from the retailer

$$p_{ij} = \frac{exp(\beta_{i0} + \beta_{i1}Rank_{ij} + \alpha_1 Retailer_i + \alpha_2 Brand_i + \alpha_3 Length_i + \alpha_4 Time_{ij} + \varepsilon_{ij})}{1 + exp(\beta_{i0} + \beta_{i1}Rank_{ij} + \alpha_1 Retailer_i + \alpha_2 Brand_i + \alpha_3 Length_i + \alpha_4 Time_{ij} + \varepsilon_{ij})} \tag{4.2}$$

We capture the unobserved heterogeneity with a random coefficient on the intercept by allowing β_{i0} to vary along its population mean $\overline{\beta_0}$ as follows:

$$\beta_{i0} = \overline{\beta_0} + \varsigma_{i0}^{\beta} \tag{4.3}$$

We also allow the *Rank* coefficient of the *i*th keyword to vary along the population mean $\overline{\beta_1}$ and the keywords' characteristics as follows:

$$\beta_{i1} = \overline{\beta_1} + \gamma_1 Retailer_i + \gamma_2 Brand_i + \gamma_3 Length_i + \varsigma_{i1}^{\beta} \tag{4.4}$$

$$\begin{bmatrix} \varsigma_{i0}^{\beta} \\ \varsigma_{i1}^{\beta} \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{11}^{\beta} & \Sigma_{12}^{\beta} \\ \Sigma_{21}^{\beta} & \Sigma_{22}^{\beta} \end{bmatrix} \right) \tag{4.5}$$

4.3 Modeling the Consumer's Decision: Conversion

Next we model the conversion rates. Prior work (Brooks 2005) has shown that there is an intrinsic trust value associated with the rank of a firm's listing on a search engine, which could lead to the conversion rate dropping significantly with an increase in the rank (i.e., with a lower position on the screen). Another factor that can influence conversion rates is the quality of the landing page of the advertiser's website. Anecdotal evidence suggests that if online consumers use a search engine to direct them to a product but don't see it addressed adequately on the landing page, they are likely to abandon their search and purchase process. Different keywords from a given advertiser lead to different kinds of landing pages. Hence, it is important to incorporate the landing page quality as a covariate in the model. Furthermore, different keywords are associated with different products. It is possible that product-specific characteristics influence consumer conversion rates, and thus, it is important to control for the unobserved product characteristics that may influence conversion rates once the consumer is on the website of the advertiser. Hence, we include the three keyword characteristics to proxy for the unobserved keyword heterogeneity stemming from the different products sold by the advertiser. Thus, the conversion probability is likely to be influenced by the position of the ad on the screen, the three keyword specific characteristics, and the landing page quality score. These factors lead us to model the conversion probabilities as follows:

$$q_{ij} = \frac{exp(\theta_{i0} + \theta_{i1}Rank_{ij} + \delta_1\,Retailer_i + \delta_2 Brand_i + \delta_3 Length_i + \delta_4 Landing\ Page\ Quality_i + \delta_5 Time_{ij} + \eta_{ij}\,)}{1 + exp(\theta_{i0} + \theta_{i1}Rank_{ij} + \delta_1\,Retailer_i + \delta_2 Brand_i + \delta_3 Length_i + \delta_4 Landing\ Page\ Quality_i + \delta_5 Time_{ij} + \eta_{ij}\,)} \tag{4.6}$$

As before, we capture the unobserved heterogeneity with a random coefficient specified on both the intercept and the *Rank* coefficient, as follows:

$$\theta_{i0} = \overline{\theta_0} + \varsigma_{i0}^{\theta} \tag{4.7}$$

$$\theta_{i1} = \overline{\theta_1} + \kappa_1\,Retailer_i + \kappa_2 Brand_i + \kappa_3 Length_i + \kappa_4 Landing\ Page\ Quality_i + \varsigma_{i1}^{\theta} \tag{4.8}$$

$$\begin{bmatrix} \varsigma_{i0}^{\theta} \\ \varsigma_{i1}^{\theta} \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{11}^{\theta} & \Sigma_{12}^{\theta} \\ \Sigma_{21}^{\theta} & \Sigma_{22}^{\theta} \end{bmatrix} \right) \tag{4.9}$$

Thus, equations (4.1) - (4.9) model the demand for a keyword, i.e. consumer's decision.

4.4 Modeling the Advertiser's Decision – Cost Per Click

Next, we model the advertiser's (i.e., the firm's) strategic behavior. The advertiser has to decide on how much to bid for each keyword *i* in week j and thus the cost per click (CPC) that it is willing to incur.[5] The advertiser decides on it CPC by tracking the performance of a keyword over time such that the current CPC is dependent on past performance of that keyword.[6] Specifically, the keyword's current CPC is a function of the rank of the same keyword in the previous period. In keeping with the institutional practices of Google which decides the minimum bid price of any given keyword ad as a function of landing page quality associated with that keyword, we control for the landing page quality in the advertiser's CPC decision. Different keyword attributes determine the extent of competitiveness in the bidding process for that keyword as can be seen in the number of advertisers who place a bid. For example, a 'retailer' keyword is likely to be far less competitive since the specific advertiser is usually the only firm that will bid on such a keyword. On the other hand, 'branded' keywords are likely to be much more competitive since there are several advertisers (retailers who sell that brand) who will bid on that keyword. Similarly, smaller keywords typically tend to indicate more generic ads and are likely to be much more competitive whereas longer keywords typically tend to indicate more specific ads, and are likely to be less competitive. Hence, the advertiser's CPC for a given keyword also depends on the three keyword attributes. Thus, the CPC will be influenced by the rank of the ad in the previous time period, the three keyword specific characteristics, and the landing page quality. This leads to the following equation:

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[5] Since we do not have data on actual bids, we use the actual cost-per-click (CPC) as a proxy for the bid price. According to the firm whose data we use, they are very strongly correlated, and hence it's a very reasonable proxy.

[6] This information about current bids being based on past performance (lagged rank) was given to us by the advertiser. The qualitative nature of all our results are robust to the use of both lagged rank and lagged profits from a given keyword ad.

$$
\begin{aligned}
ln(CPC_{ij}) = \omega_{i0} + \omega_{i1} Rank_{i,j-1} + \lambda_1 Retailer_i + \lambda_2 Brand_i + \lambda_3 Length_i + \\
\lambda_4 Landing\ Page\ Quality_i + \lambda_5 Time_{ij} + \mu_{ij}
\end{aligned}
\tag{4.10}
$$

$$\omega_{i0} = \overline{\omega_0} + \varsigma_{i0}^{\omega} \tag{4.11}$$

$$\omega_{i1} = \overline{\omega_1} + \rho_{11} Retailer_i + \rho_{12} Brand_i + \rho_{13} Length_i + \rho_{14} Landing\ Page\ Quality_i + \varsigma_{i1}^{\omega} \tag{4.12}$$

$$\omega_{i2} = \overline{\omega_2} + \rho_{21} Retailer_i + \rho_{22} Brand_i + \rho_{23} Length_i + \rho_{24} Landing\ Page\ Quality_i + \varsigma_{i2}^{\omega} \tag{4.13}$$

The error terms in equations (4.11) – (4.13) are distributed as follows:

$$
\begin{bmatrix} \varsigma_{i0}^{\omega} \\ \varsigma_{i1}^{\omega} \\ \varsigma_{i2}^{\omega} \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{11}^{\omega} & \Sigma_{12}^{\omega} & \Sigma_{13}^{\omega} \\ \Sigma_{21}^{\omega} & \Sigma_{22}^{\omega} & \Sigma_{23}^{\omega} \\ \Sigma_{31}^{\omega} & \Sigma_{32}^{\omega} & \Sigma_{33}^{\omega} \end{bmatrix} \right)
\tag{4.14}
$$

4.5 Modeling the Search Engine's Decision – Keyword Rank

Finally, we model the search engine's decision on assigning ranks for a sponsored keyword advertisement. During the auction, search engines like Google, MSN and Yahoo decide on the keyword rank by taking into account both the current CPC bid and a 'Quality Score' that is determined by the prior click-through rate (CTR) of that keyword (Varian 2007, Athey and Ellison 2008) amongst other factors. Since more recent CTR is given higher weightage by the search engine in computing this score, we use the one period lagged value of CTR. The three keyword attributes are used to control for unobserved characteristics such as the extent of competition in the auction bidding process as before in the CPC decision. Hence the rank is modeled as being dependent on these three keyword attributes. This leads to the following equation for the Rank of a keyword in sponsored search:

$$ln(Rank_{ij}) = \phi_{i0} + \phi_{i1} CPC_{i,j} + \overline{\phi_2} CTR_{i,j-1} + \tau_1 Retailer_i + \tau_2 Brand_i + \tau_3 Length_i + \tau_4 Time_{ij} + \nu_{ij} \tag{4.15}$$

$$\phi_{i0} = \overline{\phi_0} + \varsigma_{i0}^{\phi} \tag{4.16}$$

$$\phi_{i1} = \overline{\phi_1} + \pi_1 Retailer_i + \pi_2 Brand_i + \pi_3 Length_i + \varsigma_{i1}^{\pi} \tag{4.17}$$

The error terms in equations (4.16) and (4.17) are distributed as follows:

$$\begin{bmatrix} \varsigma_{i0}^{\phi} \\ \varsigma_{i1}^{\phi} \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{11}^{\phi} & \Sigma_{12}^{\phi} \\ \Sigma_{21}^{\phi} & \Sigma_{22}^{\phi} \end{bmatrix} \right) \tag{4.18}$$

Finally, to model the unobserved co-variation among click-through, conversions, CPC bid and the keyword ranking, we let the four error terms to be correlated in the following manner:

$$\begin{bmatrix} \varepsilon_{ij} \\ \eta_{ij} \\ \mu_{ij} \\ \nu_{ij} \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} \end{bmatrix} \right) \tag{4.19}$$

A couple of clarifications are useful to note here. First, the three characteristics of a keyword (*Retailer*, *Brand*, *Length*) are all mean centered. This means that $\overline{\beta_1}$ is the average effect of $\overline{\beta_{i1}}$ in equation (4.4). A similar interpretation applies to the parameters $\theta_{i1}, \omega_{i1}, \omega_{i2}$ and ϕ_{i1}. Second, in equations (4.2), (4.6), (4.11) and (4.15), we have controlled for the temporal effects by estimating time-period effects that captures unobserved industry dynamics.

4.6 Identification

To ensure that the model is fully identified even with sparse data (data in which a large proportion of observations are zero), we conduct the following simulation. We picked a set of parameter values, and generated the number of click-throughs, the number of purchases, CPC bid, and ranking for each keyword, which mimicked their actual observed values in the data according to the model and the actual independent variables observed in our data. We then estimated the proposed model with the simulated dataset and found that we were able to recover the true parameter values. This relieves a potential concern on empirical identification of the model due to the sparseness of the data.

In order to show any endogeneity issues and the identification of the proposed system of simultaneous equation model, we provide a sketch of the model below. Note that our proposed model boils down to the following simultaneous equations:

$$p = f_1(Rank, X_1, \varepsilon_1) \tag{4.20}$$

$$q = f_2(Rank, X_2, \varepsilon_2) \text{ Conditional on the number of click-throughs} > 0 \tag{4.21}$$

$$CPC = f_3(X_3, \varepsilon_3) \tag{4.22}$$

$$Rank = f_4(CPC, X_4, \varepsilon_4) \tag{4.23}$$

Here *p* is the click-through probability, *q* is the conversion probability conditional on click-through, CPC is cost per click and Rank is the position of a keyword in the listing. $X_1 - X_4$ are the exogenous covariates corresponding to the four equations. $\varepsilon_1 - \varepsilon_4$ are the error terms associated with the four equations, respectively. These error terms are mainly capturing information that is observed by the decision makers (consumer, advertiser, and search engine) but not observed by the researcher. Further, if ε_1 or ε_2 is correlated with ε_4, "Rank" will be endogenous. If ε_3 is correlated with ε_4, "CPC" will be endogenous.

Our proposed simultaneous model closely resembles the triangular system in standard econometric textbooks (Lahiri and Schmidt 1978, Greene 1999). To see this more clearly, CPC is modeled as exogenously determined (modeled as the advertiser's decision and a function of the advertiser's past performance with the same keyword and other keyword related characteristics). CPC, in turn, affects the search engine's ranking decision, and finally Rank affects both click-through and the conversion probabilities. As shown in Lahiri and Schmidt (1978) and discussed in Greene (1999), a triangular system of simultaneous equations can be identified without any further identification constraint such as nonlinearity or correlation restriction. In particular, the identification of such a triangular system comes from the likelihood function. This is also noted by Hausman (1975) who observes that in a triangular system, the Jacobian term in the likelihood function vanishes so that the likelihood function is the same as for the usual seemingly unrelated regressions problem (Hausman 1975). Hence, a GLS (generalized least squares) or SURE (seemingly unrelated regression) based estimation leads to uniquely identified estimates in a triangular system with a full covariance on error terms Lahiri and Schmidt (1978).

We also provide the parameters produced by the estimation of this system under the assumption of diagonality (restricting covariance elements to be zero) in order to be able to compare them to the generalized results. These are given in the tables in Appendix B. These estimates show that it is important to control for endogeneity since the parameter estimates are attenuated when we restrict the covariance elements to be zero, and thus biased. For example, in the case of estimating *CTR* and *conversion rates*, the parameter estimates on *Rank* are much closer to zero under the assumption of diagonality than otherwise. Similarly, in the case of estimating *Rank*, the parameter estimates on *Lag_CTR* and *CPC* are significantly closer to zero under the assumption of diagonality than otherwise.

Note that the conversion probability q is only defined when the number of click-throughs is greater than zero. In this case, if ε_1 and ε_2 are correlated as in our data, then the conditional mean of ε_2 conditional on a positive click-through probability is not going to be zero. Then, a model in which one only looks at the conversion conditional on positive number of click-throughs (i.e. does not model the click-through behavior simultaneously) is going to suffer from the selection bias. By jointly modeling click-through and conversion behavior, our proposed model accounts for such selectivity issues. The proposed Bayesian estimation approach also offers a computationally convenient way to deal with the selectivity problem by augmenting the unobserved click-through intention when there are no clicks.

5. Empirical Analysis

Next, we discuss our empirical findings. We first discuss the effects of various keyword characteristics and keyword ranking on click-through rates of the sponsored search advertisements.

5.1 Results

The coefficient of *Retailer* in Table 2a is positive and significant indicating that keyword advertisements that contain retailer-specific information lead to a significant increase in click-through rates. Specifically, this corresponds to a 14.72% increase in click-through rates with the presence of retailer information. Further, the coefficient of *Brand* in Table 2a is negative and significant indicating that keyword advertisements that contain brand-specific information can lead to a 56.6 % decrease in click-through rates. These results are useful for managers because they imply that keyword advertisements that explicitly contain information identifying the advertiser lead to higher click-through rates while those that explicitly contain information identifying the brand lead to lower click-through rates than keywords which lack such information. On the other hand, the coefficient of *Length* in Table 2a is negative suggesting that longer keywords typically tend to experience lower click-through rates. Specifically, we find that all else equal an increase in the length of the keyword by one word is associated with a decrease in the click-through rates by 13.9%.

= = Insert Tables 2a and 2b = =

Intuitively, this result has an interesting implication if one were to tie this result with those in the literature on consideration sets in marketing. A longer keyword typically tends to suggest a more 'directed' or 'specific' search whereas a shorter keyword typically suggests a more generic search. That is, the shorter the keyword is, the less information it likely carries and the larger context should be supplied to focus the search (Finkelstein et al. 2001). This implies that the consideration set for the consumer is likely to shrink as the search term becomes 'narrower' in scope. Danaher & Mullarkey (2003) show that user involvement during search (whether the use is in a purchasing or surfing mode) plays a crucial role in the effectiveness of online banner ads. Since the consumers in our data get to see the ads displayed by all the retailers who are bidding for that keyword at the time of the search, the probability of a goal-directed consumer clicking on the retailer's advertisement decreases unless the retailer carries the specific product that the consumer is searching for. In contrast, a consumer who does not have a goal-directed search (has a wider consideration set) and is in the surfing mode, is likely to click on several advertising links before she finds a product that induces a purchase.

Some additional substantive results are as expected. *Rank* has an overall negative relationship with *CTR* in Table 2a. This implies that lower the rank of the advertisement (i.e., higher the location of the sponsored ad on the computer screen), higher is the click-through rate. The position of the advertisement link on the search engine page clearly plays an important role in influencing click-through rates. This kind of primacy effect is consistent with other empirical studies of the online world. Ansari & Mela (2003) suggested a positive relationship between the serial position of a link in an email and recipients' clicks on that link. Similarly, Drèze & Zufryden (2004) implied a positive relationship between a link's serial position and site visibility. Brooks (2004) showed that the higher the link's placement in the results listing, the more likely a searcher is to select it. In the context of shopping search engines, Baye at al. (2008) find that there is a 17.5% drop in click-through rates when a retailer is move down one position on the screen. Thus, *ceteris paribus*, website designers and online advertising managers would place their most desirable links toward the top of a web page or email and their least desirable links toward the bottom of the web page or email. A robustness test wherein we include a quadratic term for *Rank* highlight that the negative relationship between *CTR* and *Rank* increases at a decreasing rate. This finding has useful implications for managers interested in quantifying the impact of *Rank* on *CTR*.

When we consider the interaction effect of these variables on the relationship of *Rank* with click-through rates, we find that keywords that contain retailer-specific or brand-specific information lead to an increase in the negative relationship between *Rank* and *CTR*. That is, for keywords that contain retailer-specific or brand-specific information, a lower rank (better placement) leads to even higher click-through rates. On the other hand, we find that the coefficient of *Length* is statistically insignificant suggesting that longer keywords do not seem to affect the negative relationship between click-through rates and ranks. As shown in Table 2b, the estimated unobserved heterogeneity covariance is significant including all of its elements. This suggests that the baseline click-through rates and the way that keyword ranking predicts the click-through rates are different across keywords, driven by unobserved factors beyond the three observed keyword characteristics.

Next consider Tables 3a and 3b with findings on conversion rates. Our analysis reveals that the coefficient of *Brand*, δ_2, is negative and significant indicating that keywords that contain information specific to a brand (either product-specific or manufacturer-specific) experience lower conversion rates on an average. Specifically, the presence of brand information in the keyword decreases conversion rates by 44.2%. Similarly, the presence of retailer information in the keyword increases conversion rates by 50.6%. In contrast, *Length* is not statistically significant in its overall effect on conversion rates. We find a significant relationship between *Rank* and *conversion rates* such that lower the *Rank* (i.e., higher the sponsored keyword on the screen), higher is the *Conversion Rate*. A decrease in the rank from the maximum possible position or worst case scenario (which is 131 in our data) to the minimum position or best case scenario (which is 1 in our data) increases conversion rates by 92.5%. This finding can have an important implication for existing theoretical models in the domain of sponsored search advertising which have typically assumed that the value per click to an advertiser is uniform across all ranks. Our estimates suggest that the value per click is not uniform and motivates future theoretical models that modify the common assumption of uniformity in click values and re-examine the social welfare maximizing properties of generalized second price keyword auctions like those in Google.

The inclusion of a quadratic term for *Rank* highlights that the negative relationship between *Conversion Rates* and *Rank* increases at a decreasing rate. This finding is relatively new in the literature on online advertising. As speculated in trade press reports, our analysis empirically confirms that *Landing Page Quality* has a positive relationship with conversation rates. To be precise, an increase in landing page quality score from the lowest possible score (equal to 1) to the highest possible score (equal to 10) is associated with an increase in the conversion rates by 22.5%. These analyses suggest that in terms of magnitude, the rank of a keyword on the search engine has a larger impact on conversion rates than the quality of the landing pages.

When we consider the effect of these keyword characteristics on the relationship of *Rank* with *Conversion Rates*, we find that none of the keyword attributes have a statistically significant effect on the relationship between rank and conversion rates. As shown in Table 3b, the estimated unobserved heterogeneity covariance is significant including all of its elements. This suggests that the baseline conversion rates and the way that keyword ranking predicts the conversion rates are different across keywords, driven by unobserved factors.

= = Insert Tables 3a and 3b = =

Next, we turn to firms' behavior. Interestingly, the analysis of cost-per-click reveals that there is a negative relationship between *CPC* and *Retailer*, but a positive relationship between *CPC* and *Brand*. This implies that the firm incurs a lower cost per click for advertisements that contain retailer information and higher cost per click for those advertisements that contain brand information. This is consistent with theoretical predictions because *Retailer* keywords are far less competitive than *Brand* keywords, on an average. While *Length* does not have a direct statistically significant effect on *CPC*, it indirectly affects CPC through the interaction with *Rank*. There is a negative and statistically significant relationship between *CPC* and *Landing Page Quality,* implying that advertisers tend to place lower bid prices on keywords that lead to landing pages with higher quality. Further, there is a negative relationship between *CPC* and *Lag Rank*. These results are indicative of the fact that while there is some learning exhibited by the firm, it may not necessarily be bidding optimally.

= = Insert Tables 4a and 4b = =

Finally, on the analysis of *Rank*, we find that all three covariates-*Retailer, Brand* and *Length* have a statistically significant and negative relationship with *Rank*, suggesting that the search keywords that have retailer-specific information or brand-specific information or are more specific in their scope generally tend to have lower ranks (i.e., they are listed higher up on the search engine results screen).

How do search engines decide on the final rank? Anecdotal evidence and public disclosures by Google suggest that it incorporates a performance criterion along with bid price when determining the ranking of the advertisers. The advertiser in the top position might be willing to pay a higher price per click than the advertiser in the second position, but there is no guarantee that its ad will be displayed in the first slot. This is because past performance such as prior click-through rates are factored in by Google before the final ranks are published. The coefficients of *CPC* and *Lag CTR* are negative and statistically significant in our data. Thus, our results from the estimation of the *Rank* equation confirm that the search engine is indeed incorporating both the current CPC bid and the previous click-through rates in determining the final rank of a keyword. Note from Table 5a that the coefficient of *CPC* is almost twice the coefficient of *Lag CTR,* suggesting that the cost-per-click factor has a much larger role to play in determining the final rank.

= = Insert Tables 5a and 5b = =

Finally, it is worth noting in Table 6 that the unobserved covariance between (i) click-through propensity and keyword rank, (ii) between conversion propensity and keyword rank, and (iii) between CPC and keyword rank all turn out to be statistically significant. This suggests the endogenous nature of CPC and Rank. Therefore, it is important to simultaneously model the consumer's click-through and purchase behavior, and the advertiser's and search engine's decisions.

= = Insert Table 6 = =

As mentioned before, we provide the parameter estimates produced by the estimation of this system under the assumption of diagonality (restricting covariance elements to be zero) to the generalized results. Refer tables in Appendix B. These estimates further demonstrate that it is important to control for endogeneity since the parameter estimates are attenuated when we restrict the covariance elements to be zero, and thus biased.

5.2 Robustness Tests

Our results are robust to several different checks. We provide below a selected list of tests we have carried out. None of these make any change in the qualitative nature of our results. Results are omitted for brevity but are available from the authors on request.

(i) We have collected data on the average and maximum bid prices of the potential competitors of this firm from Google Keywords and Microsoft Ad Center who placed bids on the same set of keywords, and used them as controls in the CPC and Rank equations.

(ii) We have used the data on actual bid prices instead of the CPC for this advertiser for the same sample of keywords. Given the very high correlation of 0.95 between bid prices and CPC, our results are robust to this.

(iii) We have data on CPC of this advertiser on Yahoo and Microsoft, and we use them as additional controls. This helps controls for correlated keyword strategies across search engines for a given advertiser.

(iv) In addition to the linear term for Rank in the CTR and conversion rate equations, we have used a quadratic term as well in order to account for the fact that changes in CTR and conversion with rank may sometimes be non-linear.

(v) We have use the lagged values of profits in the CPC equation to account for the fact that the advertiser may use previous profits as an additional heuristic to place its bids in lieu of or in addition to using the lagged value of the CTR as the main heuristic to decide on its CPC.

(vi) We have used the product price as a control variable in the conversion rate equation in the event that it might influence the propensity to buy after clicking on an ad.

6. Managerial Implications and Conclusion

The phenomenon of sponsored search advertising is gaining ground as the largest source of revenues for search engines. In this research, we focus on analyzing the relationship between different keyword level covariates and different metrics of sponsored search advertisement performance taking both consumer and firm behavior into account. Our data reveals that there is a considerable amount of heterogeneity in terms of the profitability of various keywords accruing from significant differences in the decision metrics of the various players – consumers, advertisers and search engines.

Arguably, the mix of retailer-specific and brand-specific keywords in an online advertiser's portfolio has some analogies to other kinds of marketing mix decisions faced by firms in many markets. For instance, typically it is the retailer who engages in 'retail store' advertising that has a relatively 'monopolistic' market. In contrast, typically it is the manufacturer who engages in advertising 'national-brands'. From the retailer's perspective, these brand-specific advertisements are likely to be relatively more 'competitive' since national brands are likely to be stocked by its competitors too. Retailer-name searches are navigational searches, and are analogous to a user finding the retailer's or address in the White Pages. These searches are driven by brand awareness generated by catalog mailings, TV ads, etc, and are likely to have come from more 'loyal' consumers. Even though the referral to the retailer's website came through a search engine, the search engine had very little to do with generating the demand in the first place. On the other hand, searches on product or manufacturer specific brand names are analogous to consumers going to the Yellow Pages—they know they need a branded product, but don't yet know where to buy it (Kaufman 2007). These are likely to be "competitive" searches. If the advertiser wins the click and the order, that implies they have taken market share away from a competitor. Thus, retailer-specific keywords are likely to be searched for and clicked by 'loyal' consumers who are inclined towards buying from that retailer whereas brand-specific keywords are likely to be searched for and clicked by the 'shoppers' who can easily switch to competition. This would suggest that advertisers experience higher conversion rates on retailer-specific keywords and relatively lower conversion rates on brand-specific keywords, a feature that we also observe in our data.

Our results provide some managerial insights for an advertiser of sponsoring such retail store keywords (retailer-specific keywords) with national-brand keywords (brand-specific keywords). Most firms who sponsor online keyword advertisements set a daily budget, select a set of keywords, determine a bid price for each keyword, and designate an ad associated with each selected keyword. If the company's spending has exceeded its daily budget, however, its ads will not be displayed. With millions of available keywords and a highly uncertain click-through rate associated with each keyword, identifying the most profitable set of keywords given the daily budget constraint becomes challenging for firms (Rusmevichientong & Williamson 2006). The analyses of keyword content on conversion rates also provide insights into the cost per conversion and the value per click of different keywords. Such specific knowledge of acquisition costs at the keyword can also help advertisers re-optimize their keyword advertisement portfolio in the event that search engines enable a pay-per-conversion model in addition to a pay-per-click model as Google has recently adopted.

By quantifying the impact of landing page quality on product conversions and cost per click of advertising, and comparing it to the impact of keyword attributes on these variables and rank, our study can help managers make better decisions regarding investments in the online advertising domain. Appropriate investments in landing page quality in order to improve the ranking in paid search can also boost the organic rankings of that retailer for a given set of keywords. This is because organic rankings of advertisers' websites are based on a complex and proprietary indexing algorithm devised by the search engine involving the quality of the landing page and the website's "relative importance" with respect to other links. This can be important because of claims in the trade press that more people will visit an advertiser's website if it is listed in both paid and organic listings because there is a "second opinion effect" (iCrossing 2007). This happens because searchers are encouraged by the fact that a website is listed in both the organic listings and paid ads leading to higher click-through rates.

Our results have some similarities with the findings in the context of traditional media advertising in offline markets. Koschat and Putsis (2002) attempt to estimate the effect of unbundling in magazine advertising. They find that in terms of the pricing of magazine advertising space targeting specific reader segments is generally preferable to offering advertisers all its readers. This is consistent with our finding that advertisers have to incur a higher cost per click for brand specific keywords (that are relatively more targeted) compared to generic keywords that do not highlight the manufacturer or product brand. Wilbur (2007) empirically examines the determinants of television advertising pricing to estimate viewer demand for programs, and advertiser demand for audiences. His results suggest that advertiser preferences influence network choices more strongly than viewer preferences. This has an interesting parallel to our finding that search engines place a higher weightage on advertisers' bid prices relative to consumer click-through rates in deciding their choice of rank for a given ad. Using circulation data for US daily newspapers, Chandra (2008) shows that newspapers facing more competition have lower circulation prices but higher advertising prices than similar newspapers facing little or no competition. This corresponds well with our finding that advertisers tend to incur a higher cost-per-click on longer keywords (narrower searches) and is consistent with a story of targeted advertising. In the context of selling medical services, Tellis et al. (2001) find that effective TV ads that generate referrals may not necessarily be profitable too. This is very consistent with our data that suggests that keyword ads that generate higher click-through rates may also have lower profits (due to a higher cost per click and lower revenues) in comparison to other ads. It is also important for sponsored search advertisers to keep in mind that even if keyword clicks do not lead to immediate conversions in the short run, the mere act of repetitive exposure of a stimulus can increase the familiarity with the brand name and lead to a preference (Tellis 2004), which in turn enhances the effectiveness of future advertising. In other words, sponsored advertising can contribute to building equity for the brand or for the retailer, and thus generate a longer term business value.

Our estimates from the conversion rate equation show that an advertiser's relative value-per-click for each slot is not uniform. Instead, they are decreasing across slots, meaning that clicks from lower ranked slots are more valuable than clicks from higher ranked slots. Prior work (Edelman et al. 2006, Varian 2007) showed that in a model where all clicks on an ad gain the advertiser the same value, generalized second price (GSP) keyword auction maximizes the social welfare in equilibrium. Given that the probability that a click will convert to a sale for the advertiser depends on the position (rank) of the ad, the equilibrium results of (Aggarwal et al. 2006, Edelman et al. 2006, Varian 2007) do not hold. A conclusion can be made for pay-per-click online advertising that the value generated by clicks may vary due to various reasons and that this should be taken into account in the design of the advertising mechanism. There have been many theoretical papers advocating alternative auction formats for slot auctions (Aggarwal et al. 2006, Athey and Ellison 2008) and focusing on new mechanisms with good equilibrium properties. Our empirical results can thus pave the way for future theoretical models in this domain that could relax assumptions to design newer mechanisms with more robust equilibrium properties. The recent theoretical work by Borgers et al. (2007) is a step in this direction.

Our paper has several limitations. These limitations arise primarily from the lack of information in our data. For example, we do not have precise data on competition since our data is limited to one firm. That is, we do not know the keyword ranks or other performance metrics of keyword advertisements of the competitors of the firm whose data we have used in this paper. Further, we do not have any knowledge of other information that was mentioned in the textual description in the space following a paid advertisement during consumers' search queries. Future work could integrate that information with our modeling approach to have more precise estimates. In addition, future work could examine product specific characteristics to see how different kinds of products affect the click-through and conversion rates in different ways. This will help firms analyze which brands or products have higher conversions and lower costs per conversion. Another area for future work is to study whether keyword advertising acts like a coupon by always inducing an immediate purchase or more like a regular ad that can induce a delayed purchase as shown by prior work in traditional media (Weiss and Tellis 1995). This analysis requires access to consumer level data that captures whether exposure to a sponsored ad in one time period resulted in a conversion in a later time period. Finally another area of future research could be to examine the strategic interactions of search engines in allocating ranks for different firms who vary in their experience level in the auction process. We hope that this study will generate further interest in exploring this important and emerging inter-disciplinary area.

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Table 1: Summary Statistics of the Paid Search Data (N=9664)

Variable	Mean	Std. Dev.	Min	Max
Impressions	411.694	2441.488	1	97424
Clicks	46.266	716.812	0	38465
Orders	0.860	11.891	0	644
Click-through Rate (CTR)	0.156	0.262	0	1
Conversion Rate	0.023	0.132	0	1
Cost-per-Click (CPC)	0.245	0.181	0.001	1.46
Lag Rank	6.473	9.139	1	131
Log(Lag Profit)	0.026	1.726	-5.210	11.280
Rank	6.926	10.027	1	131
Lag CTR	0.154	0.250	0	1
Retailer	0.076	0.265	0	1
Brand	0.427	0.494	0	1
Length	2.632	0.755	1	6
Landing Page Quality	8.556	1.434	4	10

Table 2a: Coefficient Estimates on Click-through Rate

	Intercept	Retailer	Brand	Length
Intercept	$\overline{\beta_0}$ **-1.654** **(0.063)**	α_1 **1.290** **(0.124)**	α_2 **-0.299** **(0.065)**	α_3 **-0.106** **(0.045)**
Rank	$\overline{\beta_1}$ **-0.264** **(0.017)**	γ_1 **-0.205** **(0.031)**	γ_2 **-0.049** **(0.018)**	γ_3 -0.004 (0.010)
Time	α_4 **0.051** **(0.003)**			

Table 2b: Unobserved Heterogeneity Estimates in the Click-through Model (Σ^{β})

	β_{i0} (Intercept)	β_{i1} (Rank)
β_{i0} (Intercept)	**1.053** **(0.078)**	**-0.095** **(0.014)**
β_{i1} (Rank)	**-0.095** **(0.014)**	**0.035** **(0.004)**

Note: Posterior means and posterior standard deviations (in the parenthesis) are reported, and estimates that are significant at 95% are bolded in Tables 2a - 7.

Table 3a: Coefficient Estimates on Conversion Rate

	Intercept	Retailer	Brand	Length	Landing Page Quality
Intercept	$\overline{\theta}_0$ **-4.457** **(0.097)**	δ_1 **1.123** **(0.234)**	δ_2 **-0.879** **(0.136)**	δ_3 -0.041 (0.110)	δ_4 **0.152** **(0.066)**
Rank	$\overline{\theta}_1$ **-0.282** **(0.031)**	κ_1 -0.032 (0.089)	κ_2 0.014 (0.036)	κ_3 0.012 (0.023)	κ_4 0.013 (0.014)
Time	δ_5 **0.067** **(0.009)**				

Table 3b: Unobserved Heterogeneity Estimates in the Conversion Model (Σ^{θ})

	θ_{i0} (Intercept)	θ_{i1} (Rank)
θ_{i0} (Intercept)	**1.436** **(0.285)**	**-0.131** **(0.030)**
θ_{i1} (Rank)	**-0.131** **(0.030)**	**0.058** **(0.007)**

Table 4a: Coefficient Estimates on CPC

	Intercept	Retailer	Brand	Length	Landing Page Quality
Intercept	$\overline{\omega}_0$ **-1.660** **(0.024)**	λ_1 **-0.760** **(0.069)**	λ_2 **0.139** **(0.032)**	λ_3 -0.022 (0.023)	λ_4 **-0.036** **(0.016)**
LagRank	$\overline{\omega}_1$ **-0.041** **(0.005)**	ρ_{11} **0.036** **(0.010)**	ρ_{12} -0.008 (0.008)	ρ_{13} **0.018** **(0.005)**	ρ_{14} -0.001 (0.004)

Time	λ_6 **-0.020** **(0.001)**

Table 4b: Unobserved Heterogeneity Estimates in the CPC Model (Σ^{ω})

	ω_{i0} (Intercept)	ω_{i1} (LagRank)
ω_{i0} (Intercept)	**0.555** **(0.030)**	**-0.021** **(0.005)**
ω_{i1} (LagRank)	**-0.021** **(0.005)**	**0.011** **(0.001)**

Table 5a: Coefficient Estimates on Keyword Rank

	Intercept	Retailer	Brand	Length
Intercept	$\overline{\phi}_0$ **1.954** **(0.031)**	τ_1 **-0.213** **(0.075)**	τ_2 **-0.279** **(0.037)**	τ_3 **-0.172** **(0.030)**
CPC	$\overline{\phi}_1$ **-2.028** **(0.093)**	π_1 0.361 (0.306)	π_2 0.185 (0.108)	π_3 -0.003 (0.085)
Lag_CTR	$\overline{\phi}_2$ **-1.289** **(0.046)**			
Time	τ_5 **0.031** **(0.001)**			

Table 5b: Unobserved Heterogeneity Estimates in the Keyword Rank Model (Σ^{θ})

	$\overline{\phi}_0$ (Intercept)	$\overline{\phi}_1$ (CPC)
$\overline{\phi}_0$ (Intercept)	**1.020** **(0.048)**	**-1.677** **(0.108)**
$\overline{\phi}_1$ (CPC)	**-1.677** **(0.108)**	**4.073** **(0.294)**

Table 6: Estimated Covariance across Click-through, Conversion, CPC and Rank (Ω)

	Click-through	Conversion	CPC	Rank
Click-through	**0.956** **(0.055)**	**1.092** **(0.086)**	**-0.082** **(0.009)**	**0.472** **(0.022)**
Conversion	**1.092** **(0.086)**	**2.429** **(0.158)**	**-0.213** **(0.021)**	**0.528** **(0.043)**
CPC	**-0.082** **(0.009)**	**-0.213** **(0.021)**	**0.220** **(0.004)**	-0.003 (0.005)
Rank	**0.472** **(0.022)**	**0.528** **(0.043)**	-0.003 (0.005)	**0.319** **(0.007)**

Appendix A: The MCMC Algorithm

We ran the MCMC chain for 40,000 iterations, and used the last 20,000 iterations to compute the mean and standard deviation of the posterior distribution of the model parameters, in the application presented in the paper. We report below the MCMC algorithm for the simultaneous model of click-through rate, conversion rate, bid price and keyword rank.

1. Draw c_{ij}^{p} and c_{ij}^{q}

As specified, the likelihood function of the number of clicks (n_{ij}) and number of purchases (m_{ij}) is

$l(c_{ij}^{p}, c_{ij}^{q} \mid n_{ij}, m_{ij}) \propto \{p_{ij}q_{ij}\}^{m_{ij}}\{p_{ij}(1-q_{ij})\}^{n_{ij}-m_{ij}}\{1-p_{ij}\}^{N_{ij}-n_{ij}}$ where

$$p_{ij} = \frac{exp(c_{ij}^{p})}{1+exp(c_{ij}^{p})}, \qquad q_{ij} = \frac{exp(c_{ij}^{q})}{1+exp(c_{ij}^{q})}.$$

$$c_{ij}^{p} = \overline{m_{ij}^{p}} + \varepsilon_{ij},$$

$$\overline{m_{ij}^{p}} = \beta_{i0} + \beta_{i1}Rank_{ij} + \alpha_1 Retailer_i + \alpha_2 Brand_i + \alpha_3 Length_i + \alpha_4 Time_{ij}$$

$$c_{ij}^{q} = \overline{m_{ij}^{q}} + \eta_{ij},$$

$$\overline{m_{ij}^{q}} = \theta_{i0} + \theta_{i1}Rank_{ij} + \delta_1 Retailer_i + \delta_2 Brand_i + \delta_3 Length_i + \delta_4 LandingPageQuality_i + \delta_5 Time_{ij}.$$

We further define the following notations:

$$D = \Omega_{11}^{*} - \Omega_{12}^{*}\Omega_{22}^{*-1}\Omega_{21}^{*}$$

$$\Omega_{11}^{*} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}, \quad \Omega_{22}^{*} = \begin{bmatrix} \Omega_{33} & \Omega_{34} \\ \Omega_{43} & \Omega_{44} \end{bmatrix}, \quad \Omega_{12}^{*} = \Omega_{21}^{*}{'} = \begin{bmatrix} \Omega_{13} & \Omega_{14} \\ \Omega_{23} & \Omega_{24} \end{bmatrix}$$

$$u_{ij1} = ln(CPC_{ij}) - (\omega_{i0} + \omega_{i1}Rank_{i,j-1} + \lambda_1 Retailer_i + \lambda_2 Brand_i + \lambda_3 Length_i + \lambda_4 LandingPageQuality_i + \lambda_5 Time_{ij})$$

$$u_{ij2} = ln(Rank_{ij}) - (\phi_{i0} + \phi_{i1}CPC_{ij} + \overline{\phi_2}CTR_{i,j-1} + \tau_1 Retailer_i + \tau_2 Brand_i + \tau_3 Length_i + \tau_4 Time_{ij})$$

$$E_{ij} = \Omega_{12}^{*}\Omega_{22}^{*-1}u_{ij}$$

We use Metropolis-Hastings algorithm with a random walk chain to generate draws of $c_{ij} = (c_{ij}^{p}, c_{ij}^{q})$ (see Chib and Greenberg 1995, p330, method 1). Let $c_{ij}^{(p)}$ denote the previous draw, and then the next draw $c_{ij}^{(n)}$ is given by:

$$c_{ij}^{(n)} = c_{ij}^{(p)} + \Delta$$

with the accepting probability α given by:

$$\min\left[\frac{exp[-1/2(c_{ij}^{(n)} - \overline{m_{ij}} - E_{ij})' D^{-1}(c_{ij}^{(n)} - \overline{m_{ij}} - E_{ij})]l(c_{ij}^{(n)})}{exp[-1/2(c_{ij}^{(p)} - \overline{m_{ij}} - E_{ij})' D^{-1}(c_{ij}^{(p)} - \overline{m_{ij}} - E_{ij})]l(c_{ij}^{(p)})}, 1\right]$$

Δ is a draw from the density Normal(0, 0.015I) where I is the identity matrix.

2. Draw $b_i = [\beta_i', \theta_i', \omega_i', \phi_i']'$

$$y_{ij1} = c_{ij}^{p} - (\alpha_1 Retailer_i + \alpha_2 Brand_i + \alpha_3 Length_i + \alpha_4 Time_{ij})$$

$$y_{ij2} = c_{ij}^{q} - (\delta_1 Retailer_i + \delta_2 Brand_i + \delta_3 Length_i + \delta_4 LandingPageQuality_i + \delta_5 Time_{ij})$$

$$y_{ij3} = ln(CPC_{ij}) - (\lambda_1 Retailer_i + \lambda_2 Brand_i + \lambda_3 Length_i + \lambda_4 LandingPageQuality_i + \lambda_5 Time_{ij})$$

$$y_{ij4} = ln(Rank_{ij}) - (\overline{\phi_2} CTR_{i,j-1} + \tau_1 Retailer_i + \tau_2 Brand_i + \tau_3 Length_i + \tau_4 Time_{ij})$$

$$x_{ij} = \begin{bmatrix} x_{ij1}' & 0 & 0 & 0 \\ 0 & x_{ij2}' & 0 & 0 \\ 0 & 0 & x_{ij3}' & 0 \\ 0 & 0 & 0 & x_{ij4}' \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} \Sigma^{\beta} & 0 & 0 & 0 \\ 0 & \Sigma^{\theta} & 0 & 0 \\ 0 & 0 & \Sigma^{\omega} & 0 \\ 0 & 0 & 0 & \Sigma^{\phi} \end{bmatrix}$$

$$x_{ij1} = x_{ij2} = [1, Rank_{ij}],\ x_{ij3} = [1, Rank_{i,j-1}, Profit_{i,j-1}]',\ x_{ij4} = [1, CPC_{ij}]'$$

$$\overline{b_{i1}} = \overline{\beta_0}\ ,\ \overline{b_{i2}} = \overline{\beta_1} + \gamma_1 Retailer_i + \gamma_2 Brand_i + \gamma_3 Length_i$$

$$\overline{b_{i3}} = \overline{\theta_0}\ ,\ \overline{b_{i4}} = \overline{\theta_1} + \kappa_1 Retailer_i + \kappa_2 Brand_i + \kappa_3 Length_i + \kappa_4 LandingPageQuality_i$$

$$\overline{b_{i5}} = \overline{\omega_0}\ \overline{b_{i6}} = \overline{\omega_1} + \rho_{11} Retailer_i + \rho_{12} Brand_i + \rho_{13} Length_i + \rho_{14} LandingPageQuality_i$$

$$\overline{b_{i7}} = \overline{\omega_2} + \rho_{21} Retailer_i + \rho_{22} Brand_i + \rho_{23} Length_i + \rho_{24} LandingPageQuality_i\ \overline{b_{i8}} = \overline{\phi_0}\ ,$$

$$\overline{b_{i9}} = \overline{\phi_1} + \pi_1 Retailer_i + \pi_2 Brand_i + \pi_3 Length_i$$

Then $b_i \sim MVN(A_i, B_i)$

$$B_i = [x_i'\Omega^{-1}x_i + \Sigma^{-1}]^{-1},\ A_i = B_i[x_i'\Omega^{-1}y_i + \Sigma^{-1}\overline{b_i}]$$

3. Draw $a = [\alpha', \delta', \lambda', \overline{\phi_2}, \tau']'$

$$y_{ij1} = c_{ij}^{p} - (\beta_{i0} + \beta_{i1} Rank_{ij})$$

$$y_{ij2} = c_{ij}^{q} - (\theta_{i0} + \theta_{i1} Rank_{ij})$$

$$y_{ij3} = ln(CPC_{ij}) - (\omega_{i0} + \omega_{i1} Rank_{i,j-1})$$

$$y_{ij4} = ln(Rank_{ij}) - (\phi_{i0} + \phi_{i1} CPC_{ij})$$

$$x_{ij} = \begin{bmatrix} x_{ij1}' & 0 & 0 & 0 \\ 0 & x_{ij2}' & 0 & 0 \\ 0 & 0 & x_{ij3}' & 0 \\ 0 & 0 & 0 & x_{ij4}' \end{bmatrix}$$

$$x_{ij1} = [Retailer_i, Brand_i, Length_i, Time_{ij}]$$

$$x_{ij2} = [Retailer_i, Brand_i, Length_i, LandingPageQuality_i, Time_{ij}]$$

$$x_{ij3} = [Retailer_i, Brand_i, Length_i, LandingPageQuality_i, Time_{ij}]$$

$$x_{ij4} = [CTR_{i,j-1}, Retailer_i, Brand_i, Length_i, Time_{ij}]$$

$$\overline{a} = 0_{21x1},\ \Sigma_0 = 100I$$

Then $a \sim MVN(A, B)$

$$B = [X'\Omega^{-1}X + \Sigma^{-1}]^{-1},\ A = B[X'\Omega^{-1}Y + \Sigma_0^{-1}\overline{a_0}]$$

4. Draw Ω

$$y_{ij1} = c_{ij}^{p} - (\beta_{i0} + \beta_{i1} Rank_{ij} + \alpha_1 Retailer_i + \alpha_2 Brand_i + \alpha_3 Length_i + \alpha_4 Time_{ij})$$

$$y_{ij2} = c_{ij}^{p} - (\theta_{i0} + \theta_{i1} Rank_{ij} + \delta_1 Retailer_i + \delta_2 Brand_i + \delta_3 Length_i + \delta_4 LandingPageQuality_i + \delta_5 Time_{ij})$$

$$y_{ij3} = ln(CPC_{ij}) - (\omega_{i0} + \omega_{i1} Rank_{i,j-1} + \lambda_1 Retailer_i + \lambda_2 Brand_i + \lambda_3 Length_i$$
$$+ \lambda_4 LandingPageQuality_i + \lambda_5 Time_{ij})$$

$$y_{ij4} = ln(Rank_{ij}) - (\phi_{i0} + \phi_{i1} CPC_{ij} + \overline{\phi_2} CTR_{i,j-1} + \tau_1 Retailer_i + \tau_2 Brand_i + \tau_3 Length_i + \tau_4 Time_{ij})$$

$$\Omega \sim IW\left(\sum_i \sum_j y_{ij}' y_{ij} + Q_0, N + q_0\right); \; Q_0 = 10I \text{ and } q_0 = 10; \text{N = \# of observations}$$

5. Draw Σ^{β}, Σ^{θ}, and Σ^{ω}

$$\Sigma^{\beta} \sim IW\left(\sum_i (\beta_i - \overline{\beta})'(\beta_i - \overline{\beta}) + Q_0, N + q_0\right); \; Q_0 = 10I \text{ and } q_0 = 10; \text{n = \# of keywords}$$

$$\Sigma^{\theta} \sim IW\left(\sum_i (\theta_i - \overline{\theta})'(\theta_i - \overline{\theta}) + Q_0, N + q_0\right); \; Q_0 = 10I \text{ and } q_0 = 10; \text{n = \# of keywords}$$

$$\Sigma^{\omega} \sim IW\left(\sum_i (\omega_i - \overline{\omega})'(\omega_i - \overline{\omega}) + Q_0, N + q_0\right); \; Q_0 = 10I \text{ and } q_0 = 10; \text{n = \# of keywords}$$

where IW stands for the Inverted Wishart Distribution.

6. Draw $f_1 = [\overline{\beta_0}, \overline{\beta_1}, \gamma_1, \gamma_2, \gamma_3]'$

$$x_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & Retailer_i & Brand_i & Length_i \end{bmatrix}$$

$$\overline{a} = 0_{5x1}, \; \Sigma_0 = 100I$$

Then $f_1 \sim MVN(A, B)$

$$B = [X'\Sigma^{\beta^{-1}} X + \Sigma_0^{-1}]^{-1}, \; A = B[X'\Sigma^{\beta^{-1}} \beta + \Sigma_0^{-1} \overline{a_0}]$$

7. Draw $f_2 = [\overline{\theta_0}, \overline{\theta_1}, \kappa_1, \kappa_2, \kappa_3, \kappa_4]'$ similar to step 6

8. Draw $f_3 = [\overline{\omega_0}, \overline{\omega_1}, \rho_{11}, \rho_{12}, \rho_{13}, \rho_{14}, \overline{\omega_2}, \rho_{21}, \rho_{22}, \rho_{23}, \rho_{24}]'$ similar to step 6

9. Draw $f_4 = [\overline{\phi_0}, \overline{\phi_1}, \pi_1, \pi_2, \pi_3]'$ similar to step 6

Appendix B: Estimates (Diagonal Ω)

Table B1: Coefficient Estimates on Click-through Rate

	Intercept	Retailer	Brand	Length
Intercept	$\overline{\beta_0}$ **-2.528** **(0.043)**	α_1 **1.505** **(0.117)**	α_2 **-0.178** **(0.058)**	α_3 -0.008 (0.048)
Rank	$\overline{\beta_1}$ **-0.079** **(0.008)**	γ_1 **-0.116** **(0.048)**	γ_2 -0.006 (0.014)	γ_3 **0.023** **(0.009)**
Time	α_4 **0.008** **(0.002)**			

Table B2: Coefficient Estimates on Conversion Rate

	Intercept	Retailer	Brand	Length	Landing Page Quality
Intercept	$\overline{\theta_0}$ **-5.461** **(0.098)**	δ_1 **1.623** **(0.213)**	δ_2 **-0.917** **(0.151)**	δ_3 -0.011 (0.106)	δ_4 **0.235** **(0.063)**
Rank	$\overline{\theta_1}$ **-0.114** **(0.016)**	κ_1 **0.178** **(0.080)**	κ_2 0.059 (0.040)	κ_3 0.004 (0.023)	κ_4 -0.002 (0.018)
Time	δ_5 **0.035** **(0.008)**				

Table B3: Coefficient Estimates on CPC

	Intercept	Retailer	Brand	Length	Landing Page Quality
	$\overline{\omega_0}$	λ_1	λ_2	λ_3	λ_4
Intercept	**-1.650**	**-0.732**	**0.165**	-0.027	**-0.038**
	(0.024)	**(0.072)**	**(0.032)**	(0.025)	**(0.016)**
	$\overline{\omega_1}$	ρ_{11}	ρ_{12}	ρ_{13}	ρ_{14}
LagRank	**-0.041**	**0.036**	-0.012	**0.019**	-0.001
	(0.004)	**(0.010)**	(0.008)	**(0.005)**	(0.004)
Time	λ_6				
	-0.020				
	(0.001)				

Table B4: Coefficient Estimates on Keyword Rank

	Intercept	Retailer	Brand	Length
	$\overline{\phi_0}$	τ_1	τ_2	τ_3
Intercept	**1.734**	**-0.530**	**-0.299**	**-0.236**
	(0.031)	**(0.085)**	**(0.040)**	**(0.033)**
	$\overline{\phi_1}$	π_1	π_2	π_3
CPC	**-1.881**	**0.673**	**0.322**	0.039
	(0.093)	**(0.325)**	**(0.128)**	(0.098)
	$\overline{\phi_2}$			
Lag_CTR	**-0.091**			
	(0.030)			
Time	τ_5			
	0.025			
	(0.001)			