# An Empirical Assessment of Endogeneity Issues in Demand Analysis for Differentiated Products

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This article explores the issue of price and expenditure endogeneity in empirical demand analysis. The analysis focuses on the U.S. carbonated soft drink market. We test the null hypothesis that price and expenditures are exogenous in the demand for carbonated soft drinks. Using an almost ideal demand system (AIDS) specification, we strongly reject exogeneity for both prices and expenditures. We find that accounting for price/expenditures endogeneity significantly impacts demand elasticity estimates. We also evaluate the implications of endogeneity issues for testing weak separability.

Key words: almost ideal demand system, carbonated soft drinks, endogeneity, separability.

Over the last few decades, strong linkages between economic theory and econometric methods have stimulated much empirical analysis of consumer behavior (e.g., Deaton and Muellbauer 1980b). The basic approach involves estimating Marshallian demand functions, expressing quantities consumed as functions of prices and household expenditures. The usual practice is to treat prices and expenditure as exogenous variables. In this article, we question the validity of these exogeneity assumptions, especially when focusing on the demand for differentiated products.<sup>1</sup> This is particularly important to the extent that food consumption typically involves differentiated products in market economies. We also examine the interactions between endogeneity of prices/expenditure in demand systems and the testing for weak separability of consumer preferences.

A price endogeneity problem can arise in the estimation of aggregate demand functions when the price determination process involves significant interplay of supply and demand. Such interaction may result in simultaneous equation bias. Econometrically this implies least square estimates of demand parameters are biased and inconsistent. Following Berry and Vilas-Boas and Winer, we argue that price endogeneity is particularly relevant in analyzing demand for differentiated products.

Economists focusing on consumer behavior often ignore this potential problem of price endogeneity (e.g., Teisl, Bockstael, and Levy; Nayaga and Capps).<sup>2</sup> A common justification for treating prices as exogenous in household demand analysis is that consumers are price takers and therefore have no impact on prices. However, having price-taking households is not a sufficient condition to treat prices as exogenous. Price-taking households can still make purchase decisions based on the actions of suppliers (e.g., merchandising and price-discounting efforts by the retailers and manufacturers).

Besides price endogeneity, the endogeneity of household expenditures can also be a problem. Most empirical demand analyses do not cover all products and services that a household purchases. Such analyses typically

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<sup>&</sup>lt;sup>1</sup> In a differentiated product market products are identified by their characteristics; for example, taste, packaging, advertisement, etc. (Tirole).

<sup>&</sup>lt;sup>2</sup> The issue of price endogeneity in demand models has been addressed by economists working on imperfect competition (e.g., Hausman, Leonard, and Zona; Cotterill, Putsis, and Dhar; Nevo 2000).

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represent the last stage of a multi-stage budgeting process justified on the assumption of weak separability of preferences (Deaton and Muellbouer 1980b). In this context, expenditure endogeneity issues may arise whenever the household expenditure allocation process across products or product groups is correlated with the demand behavior of the products being analyzed. Again, this would generate a situation where least square estimation leads to biased and inconsistent parameter estimates.

Market-level demand analyses have often ignored this problem of expenditure endogeneity (e.g., Hausman, Leonard, and Zona; Cotterill, Putsis, and Dhar). There are a few exceptions. For example, Blundell and Robin test for and reject the presence of expenditure endogeneity. However, they do not consider the issue of price endogeneity. Only LaFrance (1993) tests for the presence of both price and expenditure endogeneities in demand analysis. Using aggregate U.S. commercial disappearance data, he rejects expenditure exogeneity and finds that such endogeneity significantly impacts the demand parameter estimates.

Given the above-mentioned price and expenditure endogeneity issues, we undertake an analysis of the structure of soft drink demand using market-level sales data. Besides testing for the presence of price and expenditure endogeneity, we also explore the interaction between tests of weak separablility of preferences and endogeneity issues. Previous empirical tests of weak separability have treated prices and expenditure as exogenous (e.g., Nayaga and Capps; Eales and Unnvehr). As noted by LaFrance (1991, 1993), separability assumptions may be associated with endogeneity of right-hand side variables in demand specifications. This suggests the possibility of significant interactions between price/expenditure endogeneity and empirical testing of weak separability.

Our analysis is based on quarterly IRI (Information Resources Inc.)-Infoscan scanner data of supermarket sales of carbonated nondiet soft drinks (hereafter CSD) from 1988-Q1 to 1992-Q4.<sup>3</sup> This is the first study to use brandlevel data to test for both price and expenditure endogeneity and separability. This seems particularly relevant for two reasons. First, disaggregated analyses of the demand for differentiated products are becoming more common due to the increased availability of scanner data. Second, such investigations are useful in market structure and anti-trust policy analysis (e.g. Cotterill, Dhar, and Franklin; Nevo 2000).

The article is organized as follows. First, we discuss our demand system specification and our approach to endogeneity and separability tests. Second, we provide an overview of the data used in this analysis. Third, we present our empirical model, followed by the econometric results. We find strong evidence of endogeneity for both prices and expenditures. Also, the evidence against weak separability restrictions is found to remain strong even after taking into consideration price/expenditure endogeneity.

#### **Demand Model and Test Specification**

We specify a disaggregate nonlinear almost ideal demand system (AIDS) model. To control for price and expenditure endogeneities, we also specify reduced form equations for prices and expenditure.

## AIDS Demand Specification

The AIDS specification (Deaton and Muelbauer 1980a) can be stated as

(1) 
$$w_{ilt} = \alpha_i + \sum_{j=1}^N \gamma_{ij} \ln(p_{jlt}) + \beta_i \ln(M_{lt}/P_{lt})$$

where  $p = (p_1, ..., p_N)$  is a  $(N \times I)$  vector of prices for x, M denotes expenditure on the Ngoods,  $w_{ilt} = (p_{ilt} x_{ilt}/M_{lt})$  is the budget share for the *i*th commodity consumed in the *l*th city at time *t*. The term P can be interpreted as a price index defined by  $\ln(P_{lt}) = \delta + \sum_{m=1}^{N} \alpha_m$  $\ln(p_{mlt}) + 0.5 \sum_{m=1}^{N} \sum_{j=1}^{N} \gamma_{mj} \ln(p_{mlt}) \ln(p_{jlt})$ .

The above-mentioned AIDS specification can be modified to incorporate the effects of socio-demographic variables  $(Z_{1lt}, \ldots, Z_{klt})$  on consumption behavior, where  $Z_{klt}$  is the *k*th socio-demographic variable in the *l*th city at time  $t, k = 1, \ldots, K$ . Under demographic translating, assume that  $\alpha_i$  takes the form  $\alpha_{ilt} =$  $\alpha_{0i} + \sum_{k=1}^{K} \lambda_{ik} Z_{klt}$ ,  $i = 1, \ldots, N$ . Then, the

<sup>&</sup>lt;sup>3</sup> Information Resources Inc. collects data from supermarkets with more than \$2 million in sales from major U.S. cities. The size of supermarket accounts for 82% of grocery sales in the United States.

AIDS specification (1) becomes

(2) 
$$w_{ilt} = \alpha_{0i} + \sum_{k=1}^{K} \lambda_{ik} Z_{klt} + \sum_{j=1}^{N} \gamma_{ij} \ln(p_{ijlt}) + \beta_i \ln(M_{lt}) - \beta_i \left[ \delta + \sum_{m=1}^{N} \alpha_{0m} \times \ln(p_{mlt}) + \sum_{m=1}^{N} \sum_{k=1}^{K} \lambda_{mk} Z_{klt} \times \ln(p_{mlt}) + 0.5 \sum_{m=1}^{N} \sum_{j=1}^{N} \gamma_{mj} \times \ln(p_{mlt}) \ln(p_{jlt}) \right].$$

The theoretical restrictions are composed of symmetry restrictions:

(3a) 
$$\gamma_{ij} = \gamma_{ji}$$
 for all  $i \neq j$ 

and homogeneity restrictions:

(3b) 
$$\sum_{i=1}^{N} \alpha_{0i} = 1 \qquad \sum_{i=1}^{N} \lambda_{ik} = 0, \forall k$$
$$\sum_{i=1}^{N} \gamma_{ij} = 0, \quad \forall j \qquad \sum_{i=1}^{N} \beta_i = 0.$$

The system of share equations represented by (2) is nonlinear in the parameters. The parameter  $\delta$  can be difficult to estimate and is often set to some predetermined value (Deaton and Muellbauer 1980a). For the present analysis, we follow the approach suggested by Moschini, Moro, and Green and set  $\delta = 0.^4$ 

#### Price and Expenditure Endogeneity

As mentioned earlier, endogeneity problems arise as a result of explanatory variables being correlated with the residual error terms in the demand specification. In our AIDS specification, let  $u_{ilt}$  be the residual error of the *i*th demand equation in the *l*th city at time *t*. The price  $p_{ilt}$  would be endogenous if  $p_{ilt}$  and  $u_{ilt}$  are correlated. In this case, using least squares to estimate model parameters is subject to simultaneous equation bias and results in biased and inconsistent estimates. Any inference based on these least squares estimates would be invalid. Similar arguments apply to the endogeneity of expenditures  $(M_{lt})$ .

Under what scenarios are such endogeneity issues likely to arise? Whenever there are factors affecting consumer behavior which are not taken into account by the analyst and that are related to price determination and/or expenditure allocation to the commodities of interest. With respect to price endogeneity, this is a likely scenario for differentiated products. Retail prices for differentiated products are determined by strategic pricing rules of firms incorporating supply and demand characteristics for these products. Whenever some of the determinants of the pricing rules involve demand characteristics unobserved by the econometrician, treating prices as exogenous would lead to biased and inconsistent demand parameter estimates. Note that this argument applies even if the consumer behaves as a price taker. To the extent that product differentiation is extensive in retail sectors of a market economy, it suggests that the endogeneity of prices is likely to be a generic issue in demand analysis.

With respect to expenditure endogeneity, it also seems likely that demand behavior of consumers and expenditure allocation would be affected by common factors unobserved by the econometrician. Again, it would suggest that the endogeneity of total expenditures is likely to be a generic issue in demand analysis (LaFrance 1991).

Two questions arise. How does one control for price and/or expenditure endogeneity? And how does one test for such endogeneity? In empirical studies, two approaches have been used to control for price endogeneity. The first approach uses an instrumental variable estimation method after determining a set of instruments that are uncorrelated with the residual errors. For example, Hausman, Leonard, and Zona and Nevo (2001) use an instrumental variable approach first proposed by Hausman and Taylor for panel data. The second approach involves the explicit specification of price (supply) equations reflecting strategic firm behavior and the joint estimation of both the demand and price (supply) equations (e.g., Kadiyali, Vilcussim, and Chintagunta). The principal difference between the two approaches is the source of instruments. The first approach takes advantage of the panel nature of multi-city scanner data

<sup>&</sup>lt;sup>4</sup> The parameter  $\delta$  is difficult to estimate, as the likelihood function is extremely flat in  $\delta$  (Deaton and Muellbauer, 1980b; Moschini, Moro, and Green). As a result, setting  $\delta$  at zero does not affect the likelihood function or our likelihood ratio tests. And Moschini, More, and Green (p. 65) have argued that this has minimal effect on demand elasticity estimates.

and uses prices of neighboring cities as instruments. It assumes that neighboring cities have the same cost specification and that the demand idiosyncrasies (unobservable to the analyst) are independent. In the second approach, instruments for estimation are the demand and supply shifters within a city or region.

For the present analysis, we utilize the second approach to control for endogeneity using a nonlinear full information maximum likelihood (FIML) estimation procedure. This generates consistent and asymptotically efficient estimates based on the assumption that the errors are normally distributed. One major advantage of using FIML is that the asymptotic efficiency does not depend on the choice of instruments; this contrasts with instrumental variable estimators where the choice of instruments can be complex in nonlinear models (Hayashi, p. 482).

We specify reduced form price equations similar to that of Cotterill, Franklin, and Ma and Cotterill, Putsis, and Dhar to capture the supply side of the price formation mechanism. The price equation for the *i*th commodity in the *l*th city at time *t* is

(4) 
$$p_{ilt} = f(\text{supply/demand shifters}).$$

Similar to Blundell and Robin, we specify a reduced form expenditure equation where household expenditure in the *l*th city at time *t* is a function of median household income and a time trend:

## (5) $M_{lt} = f$ (time trend, income).

Given these reduced form specifications for the price and expenditure equations, we estimate jointly (2), (4), and (5) by FIML. The resulting parameter estimates have desirable asymptotic properties (Amemiya). Here it is important to note that the simultaneous equation bias issue arises because of the covariances in the error terms between equations (4), (5) and equation (2). Thus, in this study, FIML gives consistent parameter estimates, taking into account the effects of these covariances. Assuming the correct model specification, estimates are also asymptotically efficient and have the smallest possible asymptotic variance among all estimators of equations (2), (4)and (5).

Besides price and expenditure endogeneities, there are two other possible sources of inconsistency and asymptotic bias in parameter estimates: errors in variables, and omission of relevant variables. The IRI-Infoscan data used in our empirical analysis are directly collected from supermarket scanners. Such scanner data are of high quality and reliability. So, we do not think that errors in variables are a serious problem in our analysis. Omitted variables, on the other hand, can be a potential source of problem in any econometric analysis. Given data limitations, we have specified our empirical model the best we could such that the problems of omitted variables are minimized.

#### *Price and Expenditure Endogeneities Test Procedure*

The primary objective of our analysis is to examine the endogeneity of price and expenditure within a scanner-data-based demand system for differentiated products. Two approaches have been used to test for endogeneity. Blundell and Robin and Vilas-Boas and Winer use an ad hoc but direct approach. The basic premise of their approach is that it is possible to estimate the bias in demand-side errors due to the presence of endogenous variables. Regression of an endogenous variable (price or expenditure) on a set of exogenous variables generates residual errors that uncover information related to such bias. They use the resulting residuals as an independent variable in the demand specification and test for the significance of the corresponding parameter. A significant parameter estimate means the unexplained variation of the endogenous variable also affects the variations in demand, implying endogeneity of the variable.

An alternative approach suggested by LaFrance (1993) is based on a test developed by Durbin, Wu, and Hausman (hereafter DWH). This approach can be used with multiple endogenous variables in a demand specification. The DWH tests for the consistency of parameter estimates. Under the DWH test, one first determines the potential endogenous variables in the demand system and control for such endogeneity. The test is based on the difference between parameter estimates with and without controlling for potential endogeneity. The null hypothesis is that parameters estimated without controlling for endogeneity are consistent. Rejecting the null hypothesis implies endogeneity of the explanatory variables. The DWH test statistic can be specified as

(6) 
$$H = (\Phi_{NLS} - \Phi_{\text{FIML}})[\operatorname{var}(\Phi_{NLS}) - \operatorname{var}(\Phi_{\text{FIML}})]^{-1}(\Phi_{NLS} - \Phi_{\text{FIML}})$$

where,  $\Phi_{NLS}$  is the vector of estimated parameters without controlling for endogeneity and  $\Phi_{\text{FIML}}$  is the vector of consistent parameter estimates using FIML (treating prices and expenditures as endogenous).<sup>5</sup> Under the null hypothesis, *H* is asymptotically distributed as  $\chi^2(g)$ , where *g* is the number of potentially endogenous variables. In this article, we use the DWH test procedure.

#### Test Specification for Separability

A secondary objective of our analysis is to investigate interactions between endogeneity and tests of separability. The separability test used here follows the approach proposed by Moschini, Moro, and Green.

Weak separability of a direct utility function implies that the Slutsky substitution terms between two goods in different groups are proportional to the expenditure effects of the two goods (Goldman and Uzawa). This condition is only valid in the case of symmetric separability. Blackorby, Davidson, and Schworm (hereafter BDS) develop a more general condition that holds both for symmetric and asymmetric separability. The Moschini, Moro, and Green test procedure is based on the BDS condition. They show that if  $I_g$  and  $I_s$  are two mutually exclusive and exhaustive separable product groupings where products  $(i, j) \in I_{\sigma}$ and  $(k, m) \in I_s$ , then the following restrictions on elasticities from the separable group should hold

(7) 
$$\sigma_{ik}/\sigma_{jm} = \varepsilon_i \varepsilon_k/(\varepsilon_j \varepsilon_m)$$

where  $\sigma_{ik}$  is the Allen-Uzawa elasticity of substitution between commodities *i* and *k*, and  $\varepsilon_i$  is the expenditure elasticity for the *i*th commodity. Such restrictions can be imposed in a demand system and tested against an unrestricted model using a likelihood ratio test.

To impose the restrictions (7) locally (as suggested by Moschini, Moro, and Green, p. 65), we normalize our right-hand side variables of the AIDS by the mean of the respective variable. Then at sample mean, the parametric restrictions on the demand system (8) can be written as

(8) 
$$\frac{\gamma_{ik} + \alpha_i \alpha_k}{\gamma_{jm} + \alpha_j \alpha_m} = \frac{(\alpha_i + \beta_i)(\alpha_k + \beta_k)}{(\alpha_j + \beta_j)(\alpha_m + \beta_m)}.$$

Statistical tests based on large demand systems tend to be biased toward rejection in small samples (Laitinen, Meisner). So, following Italianer and Pudney, we correct our test statistics for the size of the demand system.

#### **Description of Data**

In 1998, carbonated soft drinks (CSD) accounted for 49%, in terms of volume, of total U.S. beverage sales, generating over \$54 billion in revenues with 56.1 gallons per capita consumption. In contrast, the second largest beverage category: beer, accounted for only 19.4% of sales volume, with 22.1 gallons per capita being consumed.<sup>6</sup> CSD demand provides an excellent example of differentiated product category where the products are differentiated by taste, packaging and brand-based advertisement to influence consumers' perception of different brands.

IRI-Infoscan data used in this analysis contain detailed brand-level information of supermarket CSD sales; merchandising and price discount information from 46 major metropolitan marketing areas within the continental United States. A total of 920 quarterly observations (46 cities with 20 quarters) by brands (i.e., nine brands) are used in this analysis.

The following CSD brands are included in the data set: Coke, Pepsi, 7-Up, Mountain Dew, Sprite, RC Cola, Dr. Pepper, Private label, and an aggregate All-Other brand.<sup>7</sup> Detailed descriptive statistics of the brand and metropolitan area (city) level variables used in this study are presented in table 1. In terms of prices, Dr. Pepper is the most expensive (\$3.97/gal) and Private label the least expensive (\$2.34/gal). In terms of share of consumer expenditures, Coke has the highest share (25.7%) and RC Cola the lowest share (1.8%). More detailed descriptions of other variables are presented in the empirical section of the article.

## **Empirical Model Specification**

## Demographic Translating of the AIDS Model

As noted above, we modify the traditional AIDS specification with demographic

<sup>&</sup>lt;sup>5</sup> We estimate our model under the null hypothesis of exogeneity using Zellner's iterated SUR, which is equivalent to maximum likelihood estimation (Malinvaud).

<sup>&</sup>lt;sup>6</sup> Beverage World, 15 May 1999.

<sup>&</sup>lt;sup>7</sup> The All-Other brand is an aggregate of all residual brands. Most of these brands have less than 1% market share. Aggregating them into a single brand had little impact in our analysis.

		Mean Pure	chase Characteristic	S	
Brands	Price $(\$/gal) [p_i]$	Expenditures Share [w <sub>i</sub> ]	Unit Per Volume [UPV <sub>i</sub> ]	Price (%) Reduction [PRD <sub>i</sub> ]	Merchandising (%) [MCH <sub>i</sub> ]
7-Up	3.74 (0.40)	0.05 (0.02)	2.5 (0.3)	25.9 (7.0)	69.2 (13.6)
Coke	3.71 (0.31)	0.26 (0.08)	2.2 (0.4)	27.5 (6.8)	83.3 (7.6)
Dr. Pepper	3.97 (0.47)	0.04 (0.04)	2.3 (0.3)	24.7 (7.1)	63.5 (18.3)
Mt. Dew	3.88 (0.41)	0.03 (0.02)	2.2 (0.4)	25.7 (6.6)	71.5 (13.3)
Pepsi	3.65 (0.37)	0.24 (0.07)	2.2 (0.3)	27.1 (6.7)	83.8 (8.0)
RC Cola	3.33 (0.45)	0.02 (0.01)	2.5 (0.4)	22.2 (7.5)	63.8 (21.4)
Sprite	3.63 (0.33)	0.04 (0.01)	2.3 (0.3)	27.5 (7.0)	79.5 (9.7)
Private Label	2.34 (0.27)	0.08 (0.05)	5.6 (2.2)	21.3 (6.9)	50.4 (20.5)
All-Other	3.56 (0.40)	0.24 (0.07)	3.6 (0.9)	23.6 (5.0)	54.4 (11.2)
		Mean Values of C	Other Explanatory V	Variables	
Variables				Units	Mean
Median Age (E	Demand Shift V	Variable – $[Z_{lt}]$		Years	33.2 (2.4)
Median HH Siz	e (Demand Sh	ift Variable – $[Z_i]$	[t])	#	2.6 (0.1)
% of HH less th	han \$10k Incon	%	15.0 (3.3)		
% of HH more than \$50k Income (Demand Shift Variable $-[Z_{ll}]$ )				%	24.2 (6.5)
Supermarket to Grocery Sales ratio (Demand Shift Variable $-[Z_{ll}]$ )				]) %	75.8 (5.7)
Percentage of H	Hispanic Popula	ation (Demand S	hift Variable – $[Z_{lt}]$	) %	7.2 (9.6)
Concentration	Ratio (Price Fu	unction: $CR_{t}^{4}$ )		%	64.7 (13.1)
Per Capita Exp	enditure $(M_{lt})$			\$	5.91 (1.22)
Median Income	e (Expenditure	<i>Function:</i> $INC_{lt}$ )		\$	32,353 (7130)

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Note: Numbers in parenthesis are the standard deviations.

translating. As a result, our AIDS model incorporates a set of regional dummy variables along with selected socio-demographic variables. In previous studies using multi-market scanner data, Cotterill, Franklin, and Ma and Hausman, Leonard, and Zona use city specific dummy variables to control for city specific fixed effects for each brand. Here we control for regional differences by including nine regional dummy variables.<sup>8</sup>

Our AIDS specification incorporates six demand shifters, Z, capturing the effects of demographics across marketing areas. These variables include: percentage of Hispanic population, median household size, median household age, percent of household earning less than \$10,000, percentage of household earning more the \$50,000. To capture the effect of any city specific variation in outlet types used to purchase soft drinks, we also use data on the ratio of supermarket sales to total grocery sales as a demand shifter in the share equation.<sup>9</sup> AIDS model, the following restrictions based on (3b) are applied to demographic translating parameter  $\alpha_{0i}$ :

(9) 
$$\alpha_{0i} = \sum_{r=1}^{9} d_{ir} D_r$$
  $\sum_{r=1}^{9} d_{ir} = 1$   
 $i = 1, \dots, N$ 

where  $d_{ir}$  is the parameter for the *i*th brand associated with the regional dummy variable  $D_r$  for the *r*th region. Note that as a result, our demand equations do not have intercept terms.

## Specifications of the Reduced Form Price and Expenditure Functions

For products like CSD, raw material cost is only a small fraction of retail price. Conversely, merchandising and packaging costs tend to be a larger portion of the retail price. As a result most recent studies of differentiated products modeled price as a function of supply and demand shifters, assuming these shifters are exogenous to the price formation mechanism (e.g., Cotterill, Franklin, and Ma; Cotterill, Putsis, and Dhar; Kadiyali, Vilcassim, and Chintagunta). Our specification is similar in spirit and we specify the price

<sup>&</sup>lt;sup>8</sup> A list of the cities and definitions of the nine regions used in our analysis can be obtained from the authors upon request. Our region definitions are based on census definition of divisions. <sup>9</sup> For example, in a city with more supermarkets than any other

<sup>&</sup>lt;sup>9</sup> For example, in a city with more supermarkets than any other store format, consumers will be able to take advantage of larger package size and shorter trip time.

functions in (4) with marketing and other product characteristics as explanatory variables:

(10) 
$$p_{ilt} = \theta_{i0} + \theta_{i1} UPV_{ilt} + \theta_{i2} MCH_{ilt} + \theta_{i3} PRD_{ilt} + \theta_{i4} CR_{lt}^4$$

where  $UPV_{ilt}$  in is the unit volume of the *i*th product in the *l*th city at time *t* and represents the average size of the purchase. For example, if a consumer purchases only 1 gallon bottles of a brand, then unit volume for that brand will be just 1. Conversely, if this consumer buys a half-gallon bottle then the unit volume will be 2. This variable is used to capture packagingrelated cost variations, as smaller package size per volume implies higher costs to produce, to distribute and to shelve. The variable  $MCH_{ilt}$ measures percentage of a CSD brand i sold in a city *l* through any types of merchandising (e.g., buy one get one free, cross promotions with other products, etc.). This variable captures merchandising costs of selling a brand. For example, if a brand is sold through promotion such as: 'buy one get one free', then the cost of providing the second unit will be reflected in this variable. The variable *PRD<sub>ilt</sub>* is the percent price reduction of brand *i* and is used to capture any costs associated with specific price reductions (e.g., aisle end displays, freestanding newspaper inserts). Simply lowering the shelf price with no aisle end display or local newspaper advertisement telling consumer the brand is 'on special' does not effectively communicate the price change to consumers. Finally, the variable  $CR_{lt}^4$  measures the four firm concentration ratios of supermarkets in city *l*. This variable captures any market power effect on price formation. In earlier studies, it is found that supermarket concentration is a significant variable in explaining retail price variations across regions. Regions with higher supermarket concentration tend to have higher price (Cotterill, Dhar, and Franklin).

The reduced form expenditure function in (5) is specified as

(11) 
$$M_{lt} = \eta T R_t + \sum_{r=1}^{9} \delta_r D_r + \phi_1 I N C_{lt} + \phi_2 I N C_{lt}^2$$
  $t = 1, \dots, 20,$ 

where  $TR_t$  in (11) is a linear trend, capturing any time specific unobservable effect on consumer soft-drink expenditure. The  $D_r$ 's are the regional dummy variables defined above and capture region specific variations in per capita expenditure. The variable  $INC_{lt}$  is the median household income in city l and is used to capture the effect of income differences on CSD purchases.

We assume the demand shifters and the variables in the reduced form price and expenditure specification are exogenous. In general, the reduced form specifications (i.e., equations (4) and (5)) are always identified. The issue of parameter identification is rather complex in nonlinear structural model.<sup>10</sup> We checked the order condition for identification that would apply to a linearized version of the demand equations (2) and found them to be satisfied. Finally, we did not uncover numerical difficulties in implementing the FIML estimation. As pointed out by Mittelhammer, Judge, and Miller (pp. 474–75), we interpret this as evidence that each of the demand equations is identified.11

#### Utility Trees for Testing Separability

A secondary objective of this analysis is to explore the interactions between endogeneity and the hypothesis tests for alternative separability assumptions. The assumption of weak separability implies multi-stage budgeting in household purchases. In this article, we consider several two-stage budgeting processes in the CSD market based on earlier studies, brand, and market characteristics. Under multi-stage budgeting, we consider alternative structures for the household decision to purchase soft drinks. Table 2 presents four such household budgeting structures. Model [1] represents the base model and does not impose any separability assumptions. Model [2] is based on earlier studies of Cotterill, Franklin, and Ma, where the consumer first chooses between different segments of the CSD market: Private Label, All-Other, and Branded CSD, and in the second stage from Branded segment chooses any specific brand from the Cola (Coke, Pepsi, RC Cola and Dr. Pepper) or Clear (Sprite, 7-Up and Mt. Dew) subsegment. In Model [3], Branded Cola and Clear sub-segments are merged and create a single-branded segment (seven soft drinks) and are assumed to be separable from the other two segments of soft drinks (Private Label and All-Other). This implies that consumer

<sup>&</sup>lt;sup>10</sup> For a detailed discussion please refer to Mittelhammer, Judge, and Miller (pp. 474–75).

<sup>&</sup>lt;sup>11</sup> Due to space limitations, we report only related econometric results. More complete reports of the results are available from the authors on request.

				Assumed	Budgeting	Structure	
Segment	Sub-Segment	Brand	[1]	[2]	[3]	[4]	[5]
Branded	Cola	Coke Pepsi RC Cola Dr. Pepper	I II III IV	I I I I			I II III III
Branded	Clear	7-Up Sprite Mt. Dew	V VI VII	Ш Ш Ш		II III IV	$\frac{\mathbf{V}}{\mathbf{V}}$
Private Label All-Other			VIII IX	III IV	II III	V VI	VI VII

Table 2.	Structure of Se	parable Demand	Models Based on	Multi-Stage	Budgeting

Note: The same Roman numeral implies that products are in the same stage of consumer's decision process. A highlighted numeral implies decision to purchase is not in the first stage of budgeting. For example: a consumer to purchase Coke in model [2], first chooses to buy branded soda, then within branded soda s/he chooses to purchase Cola and in the last stage s/he purchases Coke from the Cola sub-segment.

first chooses between branded, private label, and All-Other, and then in the second stage within the Branded, the consumer may choose any specific brand. Model [4] is based on the assumption that only the branded Cola segment is separable from the rest of the brands. This implies consumers first choose from All-Other, Private Label, three clear brands, and the Cola-brand segment. In the second stage, given the choice of Cola-brand segment the consumer then chooses a specific Cola brand. Finally, model [5] is similar to model [4], only in this case there is a branded Clear segment but not a branded Cola segment.

Given the above alternative assumptions concerning the CSD purchase process, we impose the corresponding separability restrictions (8) on the AIDS specification and estimate the restricted models with and without controlling for endogeneity. We then test these restricted models (models [2]–[5]) against our unrestricted model (model [1]) using likelihood ratio test statistics.

## **Empirical Results**

We use the GAUSSX<sup>®</sup> programming module of the GAUSS software system to estimate model parameters. Our base nonlinear AIDS model without controlling for endogeneity consists of nine share equations (2). We drop one share equation due to the addingup constraints of the AIDS specification. The model specification that controls for only price endogeneity is based on the same eight share equations and nine reduced form price equations (10). Similarly the model that only controls for expenditure endogeneity has nine equations: eight budget share equations (2) with the addition of one expenditure equation (11). Finally, the model specification controlling for both price and expenditure endogeneities is composed of 18 equations: eight budget share (2), nine price (10), and one expenditure equation (11). Given the large number of parameters estimated with different model specifications, we do not present a detailed report of all the estimated parameters.

Here we briefly discuss our main econometric results. Our estimated demand model without controlling for any endogeneity assumptions or imposing separability restrictions has 164 parameters. Of these parameters, 112 are significantly different from zero at 5% level of significance. We estimate 209 parameters in the model that controls for only price endogeneity with no separability restrictions. Of these, 159 parameters are significantly different from zero. Our model that controls for both price and expenditure endogeneity with no separability assumptions is based on 221 parameters with 164 estimated parameters found to be significantly different from zero.

The model specification where only the eight share equations are estimated represents the base model. In the following section, we implement the test of price and expenditure endogeneity relative to this base model using test statistic shown in (6).

## *Results of Price and Expenditure Endogeneity Tests*

We undertook a sequence of endogeneity tests. First we test for only price, then only expenditure, and lastly for joint price and expenditure endogeneity using the DWH test procedure outlined in (6). The estimated chisquare test statistic in the case of only price endogeneity is 1730.7, for only expenditure endogeneity is 3178.9, and for both price and expenditure endogeneity is 9056.6. In all three cases, we reject the null hypothesis (at 1% significance level with 164 df, the critical chisquare value is 194.9) that parameter estimates obtained without controlling for endogeneity are consistent. This provides strong evidence of endogeneity of prices as well as expenditure.

Our result on price endogeneity is similar to the results of Vilas-Boas and Zhao, and Vilas-Boas and Winer. Both studies using discrete choice demand system show that price is endogenous. Similar to our analysis, Vilas-Boas and Zhao use retail level scanner data, while Vilas-Boas and Winer rely on household purchase data. Our result with respect to expenditure endogeneity is consistent with those of LaFrance (1993) and Blundell and Robin.

Tables 3 and 4 present uncompensated price elasticity estimates before and after controlling for price and expenditure endogeneity, respectively. To compare elasticity estimates under different endogeneity assumptions and following LaFrance (1993), we define the absolute percentage difference (hereafter APD) between a pair of elasticity estimates,  $\varepsilon^*$  and  $\varepsilon^{**}$ , as

(12) 
$$APD = \{100|\varepsilon^* - \varepsilon^{**}|\}/\{0.5|\varepsilon^* + \varepsilon^{**}|\}.$$

The second to last column of table 4 presents average APD by brand. Our average estimated APD of elasticities (own and cross price elasticities) for all brands with and without controlling for endogeneity is 218%, suggesting significant differences due to endogeneity of variables. In terms of brands, the highest average APD is for 7-Up brand (399.6%) and lowest is for Sprite (74%). More specifically, our estimated own price elasticities after controlling for endogeneity suggest higher price sensitivity of the brands. On an average, own price elasticities increase 90% after controlling for endogeneity. In the case of cross price elasticities, in most cases similar claims can be made. This finding is consistent with the results of Vilas-Boas and Winer and Vilas-Boas and Zhao. In Vilas-Boas and Zhao, estimated own price elasticities increase by 50% after controlling for price endogeneity. Our estimated own price elasticities are higher than the estimates of Cotterill, Franklin, and Ma, who use the same data set but only controlled for price

endogeneity, using a linear approximation to the AIDS specification.

Estimated own and cross price elasticities after controlling for endogeneity are not only consistent but also asymptotically efficient (as discussed above). After controlling for both price and expenditure endogeneity, this is illustrated by noting that the estimated standard deviations decrease for 75 of the 81 elasticities.

In the last column of tables 3 and 4, we present estimated expenditure elasticities before and after controlling for endogeneity. In terms of APD, controlling for endogeneity generates a 64% average change in APD. For five of nine brands, estimated expenditure elasticities increase. And for all brands, standard deviations of the elasticities decrease.

## Results of the Separability Tests

The likelihood ratio test statistics obtained from the implementation of the Moschini, Moro, and Green are presented in table 5 with and without controlling for endogeneity. Our unrestricted model (model [1] in table 2) is estimated assuming no separability of the utility function. We test our unrestricted model against models [2]–[5] (see table 2). All the restricted models based on our separability assumptions are found to be significantly different from our unrestricted model [1] at the 5% level of significance. This is shown by the chi-square statistics in columns (3) and (4) of table 5. The rejection of weak separability holds with or without controlling for price and expenditure endogeneity.

Following the suggestion of Moschini, Moro, and Green, and Pudney, we also adjust our test statistics for model size. As a result, the test statistics decline marginally in magnitude, with Pudney's approach providing the lowest test statistics. After the size correction, we still reject the null hypothesis in all the four cases with or without controlling for endogeneity. Interestingly as we control for endogeneity, the estimated test statistics with Pudney correction also changes and in some cases quite significantly. For example, in the case of testing model [3] against our unrestricted model (model [1]), the test statistic without controlling for any endogeneity is 108.85, but it declines to 47.50 after we control for both price and expenditure endogeneities. This is the test statistic closest to the critical chi-square value that we found in our analysis of separable preferences. It suggests that the strength of the

<b>b</b>	•		a	<i>с</i> е						
	7-Up	Coke	Dr. Pepper	Mt. Dew	Pepsi	RC Cola	Sprite	<b>Private Label</b>	All-Other	Expenditures
7-Up	-1.761	0.420	-0.212	0.542	-0.152	0.045	-0.238	-0.210	0.642	0.925
	0.125	0.166	0.127	0.153	0.192	0.081	0.077	0.088	0.147	0.064
Coke	0.080	-1.952	0.151	0.272	0.074	0.097	-0.055	0.012	0.379	0.943
	0.032	0.135	0.062	0.061	0.102	0.034	0.030	0.063	0.092	0.052
Dr. Pepper	-0.247	0.960	-2.378	-1.301	2.060	-0.519	0.296	-0.083	0.382	0.829
	0.152	0.379	0.375	0.255	0.354	0.163	0.108	0.239	0.316	0.164
Mt. Dew	0.763	1.975	-1.572	-4.240	1.622	0.620	-1.049	-0.037	0.776	1.143
	0.220	0.450	0.307	0.586	0.550	0.215	0.186	0.204	0.348	0.132
Pepsi	-0.047	0.007	0.339	0.232	-1.803	0.019	0.190	0.069	-0.234	1.228
4	0.040	0.107	0.060	0.078	0.134	0.038	0.029	0.056	0.093	0.042
RC Cola	0.117	1.338	-1.177	1.171	0.312	-3.416	0.828	0.211	-0.369	0.985
	0.217	0.476	0.365	0.404	0.502	0.269	0.153	0.266	0.439	0.168
Sprite	-0.288	-0.360	0.289	-0.871	1.155	0.367	-1.948	0.336	0.313	1.005
4	0.092	0.187	0.108	0.156	0.172	0.068	0.122	0.079	0.126	0.061
<b>Private Label</b>	-0.159	-0.096	-0.071	-0.024	0.158	0.039	0.155	-1.952	0.477	1.473
	0.055	0.198	0.122	0.087	0.170	0.061	0.040	0.161	0.185	0.118
All-Other	0.145	0.469	0.072	0.126	-0.110	-0.023	0.067	0.225	-1.668	0.698
	0.145	0.469	0.072	0.126	-0.110	-0.023	0.067	0.225	0.109	0.045

Table 3. Elasticity Matrix Without Controlling for Endogeneity

Note: Standard deviations are in italic: Column represents 1% percentage price change and rows represents the percentage change in demand. For example: cross elasticity of 7-Up demand to a percentage change in price of Coke is 0.42. Last column presents the expenditure elasticities.

Table 4. El	Table 4. Elasticity Matrix After Controllin	After Co	ontrolling for	g for Price and Expenditure Endogeneity	xpenditur	e Endogene	ity				
	7-Up	Coke	Dr. Pepper	Mt. Dew	Pepsi	RC Cola	Sprite	Private Label	All-Other	MADS	Expenditures
7-Up	-2.941	0.766	-0.213	1.130	0.600	-0.090	0.041	0.258	-0.088	399.627	0.535
ı	0.120	0.154	0.123	0.141	0.181	0.074	0.068	0.090	0.134		0.061
Coke	0.118	-3.693	0.497	0.321	0.974	0.257	-0.210	0.535	0.070	103.431	1.130
	0.030	0.130	0.057	0.054	0.089	0.033	0.029	0.057	0.086		0.044
Dr. Pepper	-0.329	2.862	-3.693	-1.979	2.367	0.112	0.544	-0.511	-0.815	143.601	1.995
	0.147	0.353	0.342	0.232	0.336	0.141	0.109	0.216	0.313		0.161
Mt. Dew	1.634	2.593	-2.311	-8.400	5.651	0.885	-1.912	0.349	1.228	77.888	0.283
	0.202	0.395	0.279	0.538	0.505	0.187	0.178	0.198	0.326		0.117
Pepsi	0.084	0.989	0.434	0.779	-4.281	0.106	0.605	-0.238	0.188	395.038	1.333
4	0.038	0.095	0.057	0.072	0.122	0.034	0.030	0.054	0.084		0.039
RC Cola	-0.326	3.337	0.249	1.605	1.206	-9.845	0.835	0.126	0.654	203.654	2.159
	0.199	0.457	0.317	0.350	0.451	0.266	0.144	0.250	0.394		0.152
Sprite	0.032	-1.239	0.592	-1.617	3.620	0.395	-3.383	0.353	0.362	74.713	0.886
4	0.081	0.178	0.109	0.148	0.171	0.064	0.115	0.072	0.118		0.054
Private Label		1.444	-0.271	0.089	-0.914	0.028	0.125	-3.893	1.084	186.718	2.239
	0.055	0.182	0.110	0.085	0.163	0.057	0.037	0.151	0.166		0.102
All-Other	0.007	0.345	-0.061	0.179	0.487	0.090	0.097	0.546	-1.758	377.533	0.068
	0.028	0.091	0.054	0.048	0.087	0.031	0.020	0.057	0.119		0.044
Note: Numbers in	Note: Numbers in italic are standard deviation. Column represents 1%	tion. Column r		age price change and	l rows represents	s the percentage chi	ange in demand.	percentage price change and rows represents the percentage change in demand. For example: cross elasticity of 7-Up demand to a percentage change in price of Coke is	ty of 7-Up demand to	o a percentage cha	nge in price of Coke is

0.766.

 $MADS = Mean (by brand) of absolute percentage difference (APD); where APD = \{100 | \epsilon^* - \epsilon^{**} | \}/\{0.5 | \epsilon^* + \epsilon^{**} | \}.$ Last column presents the expenditure elasticities.

Test	Number of Restrictions	Test Statistic	Test Statistic After Size Adjustment	1% Critical Chi-Square Value
	Withou	t Controlling for Price	and Expenditure Endog	geneity
[1] versus [2]	23	313.6	303.4	41.6
[1] versus [3]	12	119.0	108.8	26.2
[1] versus [4]	15	186.4	176.2	30.6
[1] versus [5]	12	202.2	192.0	26.2
	Co	ontrolling for Price and	Expenditure Endogenei	ity
[1] versus [2]	23	199.4	189.2	41.6
[1] versus [3]	12	57.7	47.5	26.2
[1] versus [4]	15	217.7	207.5	30.6
[1] versus [5]	12	182.3	172.1	26.2

Table 5. Results of Alternative Separability	Hypothesis Tests	With and Without	<b>Controlling for</b>
Price and Expenditure Endogeneity			-

Note: Size adjustment factor is derived using Pudney's approach.

empirical evidence against weak separability declines after controlling for endogeneity.

#### Conclusion

Using retail scanner data, our empirical analysis suggests that both price and expenditure endogeneity significantly impacts the consistency of demand parameter estimates. In a differentiated product market such as in CSD's, price and expenditure endogeneity is likely due to the strategic nature of price formation and heterogeneity of consumers. This suggests that demand analysts who do not control for endogeneity may obtain inconsistent demand estimates and incorrect inferences. This is illustrated by the large impact of price/expenditure endogeneity on our estimated demand elasticities for CSD. The differences in the estimated price and expenditure elasticities can be quite large, with absolute percentage difference of 218% and 64%, respectively.

Our results are consistent with the existing literature concerned with the potential problem of price endogeneity in marketing science and industrial organization. These forms of endogeneity can significantly impact parameter and other statistical estimates of a demand system, and any empirical inferences thereof. Our results on expenditure endogeneity conform to those of LaFrance (1993), who showed that failure to control for expenditure endogeneity could severely affect applied welfare analysis.

In terms of separability, we find statistical evidence against multi-stage budgeting by consumers. But after we control for endogeneity, some of the test statistics do change significantly. This suggests that the presence of endogenous variables can affect tests of separability.

Looking at future research directions, it would be useful to develop structural models of the pricing rules that contribute to price endogeneity. Such exercise would help generate more efficient estimates of demand parameters. However, difficulties arise in deriving analytical and estimable forms of price and expenditure equations using flexible demand specifications. The resulting models are highly nonlinear and difficult to work with either analytically or empirically. Utilizing recent developments in numerical methods could improve the econometric tractability of such approaches. In terms of separability tests, our specifications of utility trees for multi-stage budgeting are not exhaustive. Given the complexities and time requirements of estimating large-scale nonlinear demand system, it remains a significant challenge to investigate all conceivable utility trees. As such, there will always exist a trade-off between disaggregate demand specification and empirical tractability. One possible solution could be to rely on the concept of latent separability (Blundell and Robin). With latent separability, researchers need to define a unique product in each separable group and the estimation procedure can help determine the optimum groupings for the rest of the products. This might also help overcome one of the objections of Nevo (2001) regarding the arbitrariness of multi-stage budgeting.

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# References

- Amemiya, T. Advanced Econometrics. Cambridge, MA: Harvard University Press, 1985.
- Berry, S.T. "Estimating Discrete-Choice Models of Product Differentiation." *Rand Journal of Economics* 25(1994):242–62.
- Blackorby, C., R. Davidson, and W. Schworm. "Implicit Separability: Characterization and Implications for Consumer Demands." *Journal of Economic Theory* 55 (1991):364–99.
- Blundell, R., and J.M. Robin. "Latent Separability: Grouping Goods without Weak Separability." *Econometrica* 68(2000):53–84.
- Cotterill, R.W., A.W. Franklin, and L.Y. Ma. "Measuring Market Power Effects in Differentiated Product Industries: An Application to the Soft Drink Industry." Food Marketing Policy Center, University of Connecticut, Storrs, CT, Research Report, 1996.
- Cotterill, R.W., T.P. Dhar, and A.W. Franklin. "Post-Merger Price Conduct: A Case Study of Pricing in Connecticut Markets After the 1996 Royal Ahold-Stop & Shop Merger." Food Marketing Policy Center, University of Connecticut, Storrs, CT, Research Report, October 1999.
- Cotterill, R.W., W.P. Putsis Jr., and R. Dhar. "Assessing the Competitive Interaction between Private Labels and National Brands." *Journal* of Business 73 (January 2000):109–37.
- Deaton, A.S., and J. Muellbauer. "An Almost Ideal Demand System." *American Economic Review* 70(1980a):312–26.
- —. *Economics and Consumer Behavior*. New York: Cambridge University Press, 1980b.
- Durbin, J. "Errors in Variables." *Review of the International Statistical Institute* 22(1954):23–32.
- Eales, J., and L. Unnevehr. "Demand for Beef and Chicken Products: Separability, and Structural Change." *American Journal of Agricultural Economics* 70(1988):521–32.
- Goldman, S., and H. Uzawa. "A Note on Separability in Demand Analysis." *Econometrica* 32(1964):387–98.
- Hausman, J.A. "Specification Tests in Econometrics." *Econometrica* 46(1978):1251–71.
- Hausman, J.A., and W.E. Taylor. "Panel Data and Unobservable Individual Effects." *Econometrica* 49(1981):1377–98.
- Hausman, J., G. Leonard, and J.D. Zona. "Competitive Analysis with Differentiated Products." Annales D'Economie et de Statistique 34(1994):159–80.
- Hayashi, F. *Econometrics*, 1st ed. Princeton: Princeton University Press, 2000.
- Italianer, A. "A Small Sample Correction for the Likelihood Ratio Test." *Economic Letters* 19 (1985):315–17.

- Kadiyali, V., N.J. Vilcassim, and P.K. Chintagunta. "Empirical Analysis of Competitive Product Line Pricing Decisions: Lead, Follow, or Move Together?" *Journal of Business* 69(1996):459– 87.
- LaFrance, J.T. "Weak Separability in Applied Welfare Analysis." *American Journal of Agricultural Economics* 75(1993):770–75.
- —. "When is Expenditure 'Exogenous' in Separable Demand Models?" Western Journal of Agricultural Economics 16(1991):49–62.
- Laitinen, K. "Why Demand Heterogeneity Often Rejected?" *Economics Letters* (1978):187– 91.
- Malinvaud, E. Statistical Methods in Econometrics, 2nd ed. New York: North-Holland, 1980.
- Meisner, J.F. "The Sad Fate of the Asymptotic Slutsky Symmetry Test for Large Systems." *Economic Letters* 2(1979):231–33.
- Mittelhammer, R.C., G.G. Judge, and D.J. Miller. *Econometric Foundations*, 1st ed. Cambridge, UK: Cambridge University Press, 2000.
- Moschini, G., D. Moro, and R.D. Green. "Maintaining and Testing Separability in Demand Systems." American Journal of Agricultural Economics 76(1994):61–73.
- Nayaga, R.M., and O. Capps Jr. "Tests of Weak Separability in Disaggregated Meat Products." *American Journal of Agricultural Economics* 76(1994):800–08.
- Nevo, A. "Measuring Market Power in the Ready-to-Eat Cereal Industry." *Econometrica* 69(2001):307–42.
- —. "Mergers with Differentiated Products: The Case of the Ready-to-Eat Cereal Industry." *RAND Journal of Economics* 31(2000):395– 421.
- Pudney, S.E. "An Empirical Method of Approximating the Separable Structure of Consumer Preferences." *The Review of Economic Studies* 48(1981):561–77.
- Teisl, M.F., N.E. Bockstael, and A. Levy. "Measuring The Welfare Effects of Nutrition Information." American Journal of Agricultural Economics 83(2001):133–49.
- Tirole, J. *The Theory of Industrial Organization*. Cambridge: The MIT Press, 1988.
- Villas-Boas, J.M., and Y. Zhao. "The Ketchup Marketplace: Retailer, Manufacturers, and Individual Consumers." Unpublished Mimeo, University of California, Berkeley, January 2001.
- Villas-Boas, J.M., and R. Winer. "Endogeneity in Brand Choice Models." *Management Science* 45(1999):1324–38.
- Wu, D. "Alternative Tests of Independence between Stochastic Regressors and Disturbances." *Econometrica* 41(1973):733–50.