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An Empirical BVAR-DSGE Model of the Australian Economy

Sean Langcake and Tim Robinson

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Abstract

In this paper, we develop a multi-sector dynamic stochastic general equilibrium (DSGE) model with a simple commodity sector and assess whether forecasts from this model can be improved by using it as a prior for an empirical Bayesian vector autoregression (BVAR). We treat the world economy as being observed and exogenous to the small economy, rather than unobserved, as has been done in some previous studies, such as Hodge, Robinson and Stuart (2008) and Lees, Matheson and Smith (2011). We find that the forecasts from a BVAR that uses this DSGE model as a prior are generally more accurate than those from the DSGE model alone. Nevertheless, these forecasts do not outperform a small open economy VAR estimated using other standard priors or simple univariate benchmarks.

JEL Classification Numbers: C53, E13
Keywords: empirical Bayesian VAR, forecasting, small open economy

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1. Introduction

An important aspect of the Australian economy recently has been the divergence in growth between the commodity and non-commodity sectors, reflecting strong demand for commodities from abroad, particularly China (see Kearns and Lowe (2011) and Plumb, Kent and Bishop (2013)). In this paper we develop a multi-sector DSGE model with an explicit commodity sector that allows the price of commodities to differ from that of domestically produced consumer goods. This is important given the sizeable increase in the terms of trade for Australia since the mid 2000s. The domestic economy in our model is treated as 'small' in that it has no pricing power in world markets and external demand is treated as exogenous.

DSGE models place a large weight on economic theory; for example, they are derived from agents' utility and profit maximising behaviour. This enables economic interpretations to be given to the shocks, and hence DSGE models can be useful for scenario analysis. However, this comes at a cost: the model may be misspecified, and the high degree of structure in the model can place tight restrictions on its parameters. These tight restrictions reduce statistical uncertainty but may impede the forecasting performance.

At the other end of the spectrum, unrestricted VARs have many free parameters, and therefore can provide a better in-sample description of the data than a DSGE model. However, these parameters may be imprecisely estimated, particularly in small samples, which can reduce the VAR's forecasting performance. One possibility is to use a long run of data, but this is undesirable if the economy has experienced structural change in the period under investigation. Another possibility is to use Bayesian methods, which can limit over-fitting by introducing prior information or shrinking the parameter estimates towards some value.

In this paper we study the forecasting performance of a small open economy VAR model when information from the DSGE model developed earlier in the paper is used as a prior. We refer to this as an empirical BVAR-DSGE model.[1] The aim of this approach is to reach a compromise between theory and data that may be useful for forecasting.[2]

Many papers have estimated BVAR-DSGE models and evaluated their forecasting performance (see Del Negro and Schorfheide (2004) and Del Negro *et al* (2007) for the United States, Hodge *et al* (2008) for Australia and Lees *et al* (2011) for New Zealand). However, the methodology used in these papers, following Del Negro and Schorfheide (2004), does not impose the restrictions necessary to ensure that the small economy does not affect the large economy. To achieve this, we follow the estimation approach outlined in Robinson (2013).

We compare the point forecasting performance of our BVAR-DSGE model, which has exogenous foreign variables and error-correction terms arising from non-stationary technology, to those from the DSGE model alone, a small open economy variant of the Minnesota prior, and univariate autoregressive models. The main finding of this paper is that the BVAR-DSGE model generally improves the forecasts from the estimated DSGE model (as measured by their root mean squared error (RMSE)). However, we also find that this model does not perform better than the univariate benchmarks.

The rest of the paper is structured as follows. Section 2 discusses the DSGE model of the Australian economy and possible options for modelling the foreign sector. Section 3 describes the forecasting 'competition'. Section 4 briefly reviews the estimation results from each of the models. We then examine the results in Section 5 and conclude.

1 We refer to it as an 'empirical' BVAR-DSGE model as the data are used in the construction of the prior, as well as in the BVAR estimation.

2 BVAR-DSGE models are only one method of using information from DSGE and VAR models together (see Gerard and Nimark (2008) and Bache *et al* (2011) for alternative approaches in the context of central bank forecasting that have yielded promising results).

2. The DSGE Model

This section describes the DSGE model that we later use as a forecasting benchmark and as a prior for the empirical BVAR. We begin by describing the domestic economy, with particular emphasis on departures from the standard small open economy framework that allow us to introduce a separate commodity sector.

2.1 The Domestic Economy

The domestic economy in the DSGE model differs from the standard small open economy framework by the inclusion of a second source of production, which we add to represent the commodity sector. The result of this addition is that the terms of trade are exogenous, and there are sector-specific expressions for labour supply and investment. We describe each agent's problem, but focus primarily on the non-standard aspects. The log-linearised and steady state equations are in Appendices A and B. A subscript of 1 denotes the domestic good sector, a subscript of 2 the commodity sector, which produces for export.

2.1.1 Consumers

The representative consumer derives utility from consuming C_t and holding real money balances M_t, and disutility from working. One departure from the standard literature is that the hours worked are divided between the domestic and export good sectors, $L_{1,t}$ and $L_{2,t}$ respectively. We assume that these are imperfect substitutes, and therefore that wages can differ across the sectors. The consumer also derives income from renting capital to both sectors, $r_{1,t}$ and $r_{2,t}$, dividends from monopolistically competitive firms, $\Pi_{1,t}$ and $\Pi_{2,t}$, holdings of domestic and foreign one-period risk-free bonds, B_t and B_t^f, which earn R_t and R_t^f, and transfer payments, T_t. Utility is defined to be:

$$U(C_t, L_{1,t}, L_{2,t}, M_t) = \max_{C_t, L_{1,t}, L_{2,t}, M_t} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[e^{u_t^c} \ln(C_t - \upsilon C_{t-1}) + \ln M_t - \frac{(L_{1,t}^{1+\xi} + L_{2,t}^{1+\xi})^{\frac{1+\psi}{1+\xi}}}{1+\psi} \Bigg].$$

Consumers face a budget constraint, expressed in real terms as:

$$\begin{aligned} C_t + \frac{P_{1,t}}{P_t}\left(I_{1,t}+I_{2,t}\right) + B_t + S_t B_t^f + M_t = {} & \frac{R_{t-1}B_{t-1}}{\pi_t} + W_{1,t}L_{1,t} \\ & + W_{2,t}L_{2,t} + \frac{P_{1,t}}{P_t} r_{1,t} K_{1,t-1} + \frac{P_{2,t}}{P_t} r_{2,t} K_{2,t-1} \\ & + \Pi_{1,t} + \Pi_{2,t} + \frac{R_{t-1}^f S_t B_{t-1}^f}{\pi_t} \Phi(h_{t-1}, e^{u_{t-1}^{rp}}) \\ & + \frac{M_{t-1}}{\pi_t} + T_t. \end{aligned}$$

P_t is the price of the final consumption good and $I_{i,t}$ denotes investment. Foreign borrowing is done at a premium on domestic bond returns determined by a function Φ, which depends on the level of net foreign assets as a share of GDP, h_t, and a risk premium shock u_{t-1}^{rp}. π_t is consumer price inflation, S_t the nominal exchange rate (A$ per unit of foreign currency), $W_{i,t}$ are real wages (deflated by consumer prices) and $K_{i,t}$ the capital stock with gross rental rate $r_{i,t}$. u_t^c is a preference shock; β is the discount factor; υ is the strength of external habits; ψ is the inverse Frisch elasticity and ξ governs the substitutability of labour between the sectors.[3] An expression for the evolution of nominal net foreign assets as a share of GDP, h, can be obtained by manipulating this budget constraint.

The capital stock evolves in the same way in each sector, although depreciation rates differ. Investment adjustment costs are included, following Smets and Wouters (2007), and there is an investment-specific shock that is common across sectors.

The consumption good is a constant elasticity of substitution (CES) aggregate of domestically produced goods from sector 1 (C_t^d) and imports (C_t^m), where σ is the elasticity of substitution between goods and γ governs the relative importance of domestically produced goods in the consumption basket:

$$C_t = \left(\gamma^{\frac{1}{\sigma}} C_t^{d\ \frac{\sigma-1}{\sigma}} + (1-\gamma)^{\frac{1}{\sigma}} C_t^{m\ \frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

3 Note that the lag of consumption in the utility function, which appears due to external habit persistence, is based on aggregate consumption, not that of the individual consumer.

The first order conditions of the consumer's problem leads to a standard Euler equation and an uncovered interest rate parity condition. The non-standard features are the labour supply conditions. For example, for the domestic good sector, the labour supply equation is determined by

$$W_{1,t} = L_{1,t}^{\xi}(L_{1,t}^{1+\xi} + L_{2,t}^{1+\xi})^{\frac{\psi-\xi}{1+\xi}} \frac{1}{\kappa_t},$$

where κ_t is the Lagrange multiplier for the budget constraint. An analogous condition applies to the export sector. The allocation of hours worked across the sectors will depend on relative wages. The sensitivity of labour supply to wage differentials is determined by ξ. The marginal utility from investing in a further unit of capital relative to consumption, Tobin's Q, is defined for each sector as the ratio of the Lagrange multiplier on the capital accumulation equation for a sector to that on the budget constraint. We express Tobin's Q in terms of consumer prices for the domestic good sector as:

$$q_{1,t} = \beta E_t \left[\frac{\kappa_{t+1}}{\kappa_t} \frac{\pi_{1,t+1}}{\pi_{t+1}} \left(q_{1,t+1}(1-\delta_1) + r_{1,t+1} \right) \right],$$

where $\pi_{1,t}$ is inflation in the domestic good sector. The depreciation and rental rates of capital, δ_i and r_i, and the Euler equations are sector-specific.

2.1.2 Domestic good sector

Domestic good sector firms rent capital and hire labour from households, and use a Cobb-Douglas production function. α_1 governs how capital intensive domestic good production is, and it is assumed to be less than the corresponding parameter in the relatively capital intensive commodity sector. There is a unit continuum of firms; for the ith firm:

$$Y_{1,t}(i) = \left(Z_t L_{1,t}(i)\right)^{1-\alpha_1} K_{1,t-1}(i)^{\alpha_1},$$

where $Y_{1t}(i)$ is output and Z_t is labour-augmenting technology. We assume that Z_t follows a unit root without drift, and allow for autocorrelation in its growth rate.[4] A consequence of this is that most real variables will follow the same trend and

4 In all, there are 12 shocks in the model.

be cointegrated with output. If we had allowed for drift then the level of most real variables would also share a common linear trend; we relax this assumption in estimation by demeaning the observed variables (which for output, investment and exports will be achieved by using growth rates). The domestic good sector firm determines its demand for capital $(K_{1,t-1}(i))$ and labour $(L_{1,t}(i))$ by minimising real cost, deflated by the price of their output, taking W_{1t} and r_{1t} as given.

The output from the domestic good sector is sold to a domestic good aggregator, that aggregates it using CES technology, with a time-varying parameter λ_t^d.

Cost minimisation by the domestic aggregator leads to a demand curve for individual goods, which the producers take as given. The domestic good sector is imperfectly competitive, and there is stickiness in prices, generated by Calvo pricing, whereby there is a fixed probability that a firm can reset its price each period. When log-linearised, this gives rise to a standard New-Keynesian Phillips curve for $\pi_{1,t}$. We allow for rule-of-thumb pricing, following Galí and Gertler (1999), such that a fraction of firms that are able to reset their prices do not do so optimally, but use a simple rule based on past inflation. The variable elasticity of substitution λ_t^d introduces a mark-up shock to the Phillips curve.

2.1.3 Export sector

The model for the export sector draws on Dib (2008). It is perfectly competitive, and takes the world price for exports (commodity prices in foreign currency), $P^*_{2,t}$, as given, and in domestic currency $P_{2,t} = S_t P^*_{2,t}$. The world price is distinct from foreign consumer prices, P_t^*. The exporter's problem in real terms (deflated by consumer prices) is:

$$\max_{L_{2,t},\, K_{2,t-1}} \left(\frac{S_t P^*_{2,t}}{P_t} Y_{2t} - W_{2,t} L_{2,t} - \frac{S_t P^*_{2,t}}{P_t} r_{2,t} K_{2,t-1}\right)$$
$$s.t.\; Y_{2,t} = A_{2,t} \left(Z_t L_{2,t}\right)^{1-\alpha_2} \left(K_{2,t-1}\right)^{\alpha_2},$$

where $A_{2,t}$ is an export-specific technology shock. While we calibrate the export firms to match the Australian mining sector, for simplicity we have not allowed for decreasing returns to scale, even though this may be a better characterisation of the natural resource sector. Consequently, these effects may be captured by the productivity shock (or to some extent the investment shock), together with the investment adjustment costs. The assumption that the export sector is perfectly competitive, and hence a price taker, for Australia is also questionable, as it is a major exporter of both iron ore and coking coal and may have some pricing power in these markets, but we leave exploring alternative pricing assumptions to future research.

2.1.4 Importing firms

The importing sector is standard and is similar to that in Monacelli (2005). The law-of-one-price is assumed to hold at the docks, and the importer purchases a bundle of goods at price $S_t P_t^*$. They then costlessly differentiate these goods and, as the sector is monopolistically competitive, the retail price faced by the consumer is a mark-up on real marginal costs, namely $\frac{S_t P_t^*}{P_t^m}$. There is Calvo pricing in import prices, and consequently we obtain a New-Keynesian Phillips curve. Analogous to the domestic good aggregator, we allow the elasticity of substitution to vary, which introduces an exogenous mark-up shock into the Phillips curve. The aggregation of the import goods into the bundle C_t^m can be done by either a perfectly competitive firm using CES technology or the consumer. Note that the terms of trade, namely the ratio of export prices to import prices at the docks, is $\frac{S_t P_{2,t}^*}{S_t P_t^*} = \frac{P_{2,t}^*}{P_t^*}$, which corresponds to real commodity prices and is exogenous to the small economy.

2.1.5 Monetary policy

The central bank sets the policy rate using a Taylor rule, in which it reacts to the past interest rate, the log-linearised level (˜ denotes log-deviations from steady state) of inflation and value-added output, va_t, as well as growth in value-added output.[5] The rule is:

$$\widetilde{R}_t = \zeta^R \widetilde{R}_{t-1} + \zeta^R \left(\zeta^\pi \widetilde{\pi}_t + \zeta^{va} \widetilde{va}_t + \zeta^{gr} \left(\widetilde{va}_t - \widetilde{va}_{t-1} \right) \right) + \varepsilon_t.$$

5 Policy is implemented by the central bank making lump-sum transfers to consumers so as to achieve this interest rate. Our measure of value added is defined as $VA_t \equiv Y_{1,t} + \frac{P_{2,t}}{P_{1,t}} Y_{2,t}$.

2.1.6 Market-clearing conditions

Apart from factor markets, the market-clearing conditions for the *i*th domestic good firm is that its output equals that demanded by the aggregator. Identical conditions hold for importers. The market-clearing condition for the domestic good aggregator is:

$$Y_{1,t} = C_t^d + I_{1,t} + I_{2,t}.$$

Aggregated output from the domestic good producers can be consumed or invested. The assumptions that exports are comprised of commodities alone and all capital goods are produced domestically are strong ones. Capital goods are a substantial proportion of imports in the data, and the model will be unable to capture the downward relative price trend of capital imports relative to consumption goods, which is in part due to the falling prices of imported information technology goods. Expanding the model to capture this is left to future research. Domestic bonds are in zero net supply.

3. Estimating the Models

3.1 Estimating the DSGE Model

We estimate the DSGE model outlined above in two stages. The first stage is to estimate a BVAR model of the foreign sector with Minnesota priors using aggregates of foreign variables. This is a legitimate strategy if it is assumed that the foreign economy is predetermined and exogenous to the small economy (Jääskelä and Nimark 2011). The estimates may not be efficient if there are restrictions on the parameters across the economies included in the aggregates, but will be consistent. Second, we take the results from this BVAR model to calibrate the foreign sector in the DSGE model, and estimate the domestic parameters.

3.1.1 Estimating the large economy Minnesota VAR

We estimate a BVAR for the foreign sector closely following Kadiyala and Karlsson (1997), using the following variables:

- detrended level of output of Australia's trading partners
- trade-weighted inflation

- a G7 PPP-weighted interest rate
- detrended level of the terms of trade.

We use broad aggregates for output and inflation in an attempt to partially capture the increasing importance of China in the world economy, which is of particular relevance for Australia as a net exporter of commodities. Further details about the data are given in Appendix C.

We use univariate first-order autoregressive priors. These Minnesota priors are commonly used in forecasting. The parameters in each equation are assumed to follow a multivariate normal distribution. As we assume that these series are stationary, the Minnesota prior would usually set the means of the prior distributions to be zero, including those on the first lag. In an attempt to improve forecasting performance, we estimate first- and higher-order autoregressive processes for each series, over the sample 1982:Q3–1992:Q4. Based on these autoregressions, we set the prior on the own first lag parameter to be 0.7 and 0.4 for output and inflation, and 0.8 for the interest rate and the terms of trade. The mean of the prior for all other coefficients is set to zero, and we ensure that the prior is held more tightly for parameters on longer lags and on lags of non-dependent variables.

The variance of the prior is governed by two parameters, ϑ_1 and ϑ_2, for coefficients on own lags and other coefficients respectively. The variance of the prior for the coefficient on the *j*th variable in the *i*th equation of the large economy VAR, where underbars denote that the parameters are priors, is set following Kadiyala and Karlsson (1997) as:

$$\mathrm{Var}(\underline{\gamma_{ij}}) = \begin{cases} \frac{\vartheta_1}{k} & \text{for own lags} \\ \frac{\vartheta_2 \sigma_i^2}{k \sigma_j^2} & \text{for lags of variable } j \neq i, \end{cases}$$

where k is the lag length and σ_i is the standard error from an autoregression of the same order as the VAR for the *i*th variable. To select ϑ_1 and ϑ_2, we examine the forecasting performance of the model over the pre-sample period 1989:Q4–1992:Q4. We select $\vartheta_1 = 0.01$ and $\vartheta_2 = 0.003$; if more weight is placed on the one-quarter-ahead forecasts than year-ahead forecasts, a slightly tighter prior would be chosen. Subsequently, this VAR is used to generate our large economy forecasts.

3.1.2 DSGE calibration

In the second step we use the parameter estimates from the Minnesota VAR described above as the large economy parameters of the DSGE model and calibrate the parameters of the DSGE model that enter the steady state (with one exception).[6] The remaining parameters are estimated using Bayesian methods in Dynare. The observed variables are described in Section 3.4.

The calibrated parameter values are shown in Table 1. The discount factor, β, is chosen to imply a steady state annual interest rate of 3 per cent. The inverse Frisch elasticity of substitution is set to 1, approximately the median estimated by Justiniano and Preston (2010b). The value governing the sensitivity of domestic interest rates to foreign debt is also taken from Justiniano and Preston (2010b). The elasticity of substitution between individual domestically produced and imported goods is set to imply a steady state mark-up of 25 per cent, following Cagliarini, Robinson and Tran (2011).

Table 1: Calibrated Parameters

Coefficient	Description	Value
β	Discount factor	0.9925
ψ	Inverse Frisch elasticity	1
χ	Interest debt sensitivity	0.01
γ	Weight on domestic good in consumption	0.65
σ	Elasticity of substitution: domestic and imported goods	1.5
λ^m	Elasticity of substitution: individual imported goods	4
λ	Elasticity of substitution: individual domestic goods	4
ξ	Elasticity of substitution of labour between sectors	0.75
δ_1	Depreciation rate, domestic goods	0.018
δ_2	Depreciation rate, commodity sector	0.012
α_1	Weight on capital in domestic goods production	0.3
α_2	Weight on capital in commodity production	0.77

6 Pre-estimating the large economy does restrict the data that can be used to estimate the DSGE model. In particular, the (stationary) level of foreign output and terms of trade are of importance for the small economy. If the VAR is estimated in first differences, the DSGE with it imposed would not have a solution, and therefore we detrend these series.

There is little literature on the mobility of labour between sectors in response to real wage differentials in Australia, and as we estimate the model without using labour market data we set $\xi = 0.75$.[7] The depreciation rates are chosen with reference to annual national accounts data, which suggest that the depreciation rate in the mining sector is lower than in the rest of the economy at nearly 5 per cent per annum. Capital's share of mining sector income, α_2, is selected using the values reported in the experimental multifactor productivity estimates from the Australian Bureau of Statistics (ABS).

The one parameter entering the steady-state that we estimate is υ, the intensity of external habits. We do this as different papers have very different estimates; for example, Jääskelä and Nimark (2011) obtain a posterior mean of 0.76, whereas the median value from Justiniano and Preston (2010b) is 0.33. Consequently we set the mean of the prior to be 0.5.

Using the parameter values discussed above, and the mean of the prior for υ, we calibrate the remaining parameters γ, σ and α_1 so that the implied ratios of nominal consumption, investment, imports and compensation of employees to value-added output are approximately average of the sample 1993:Q1–2011:Q2 (Table 2). The consumption to value-added output ratio is particularly sensitive to σ, and a low value is necessary to match the data. The capital intensity of the domestic good sector influences the compensation of employees to value-added output ratio; setting α_1 to 0.3 means that it is considerably more labour intensive than the commodity sector, where α_2 is set to 0.77.

Table 2: Implied Nominal Ratios to Value Added

Nominal ratio to value added	Data	Model
Consumption	0.748	0.724
Non-residential investment	0.258	0.210
Compensation of employees	0.492	0.477
Imports	0.203	0.219

Note: 'Consumption' is all sectors
Sources: ABS; authors' calculations

The priors for the parameters that do not enter the steady state are selected as follows. For the Phillips curves, our prior is that values higher than those implied by microeconomic data for both the Calvo parameter and indexation will be

7 Drawing on the estimates of Iacoviello and Neri (2010) for the United States.

necessary to provide a realistic description of the behaviour of aggregate inflation (for further discussion on this see Cagliarini *et al* (2011)). The values selected for the Taylor rule are mostly standard and similar to those in Justiniano and Preston (2010b), although we place a slightly larger prior on growth in the Taylor rule since, in some DSGE models (including Justiniano and Preston (2010b)), this has been estimated to be more important than the response to the log-linearised level of output.[8]

We allow for autocorrelation in the growth rate of technology. However, as little persistence is evident in Australian output growth, we select a loose prior, to accommodate the possibility that it is low. For the other shocks, our prior on the autoregressive coefficients are similar to Justiniano and Preston (2010b). The adjustment cost parameter prior is based on Smets and Wouters (2007).

Turning to the standard deviations of the shocks, we set the mean of the prior for a quarterly monetary policy shock to be 25 basis points. Relatively large values are used for the standard deviation of technology shocks in the export sector and, in particular, the investment productivity shocks, reflecting the fact that growth in these expenditure aggregates are more volatile than aggregate GDP.

3.1.3 DSGE posterior

We used random-walk Metropolis-Hastings chains with 400 000 observations to find the posterior densities, dropping the first 80 per cent of these observations.[9] Table 3 below shows some characteristics of the posterior densities from the sample 1993:Q1–2011:Q2.

8 We do not allow a role for the exchange rate in the Taylor rule, reflecting the findings of Lubik and Schorfheide (2007). The exchange rate may still be of importance to policy, but through its impact on inflation and output, rather than directly.

9 Two chains were run in order to check for convergence. For the forecasting exercise, a single chain for each period was used.

Table 3: DSGE Estimation Results

Coefficient	Description	Prior			Posterior	
		Density	Mean	Std dev	Mean	90% HPD
θ_1	Calvo domestic prices	B	0.75	0.1	0.95	0.94–0.97
ω_1	Indexation, domestic	B	0.3	0.05	0.33	0.25–0.40
ζ_R	Taylor rule, smoothing	B	0.5	0.1	0.82	0.77–0.87
ζ^{π}	Taylor rule, inflation	N	1.5	0.1	1.48	1.31–1.64
ζ^{va}	Taylor rule, output	B	0.5	0.2	0.72	0.53–0.90
ζ_{dy}	Taylor rule, growth	B	0.5	0.2	0.16	0.02–0.29
θ_m	Calvo import	B	0.75	0.1	0.89	0.83–0.96
ω_m	Indexation, imports	B	0.3	0.05	0.31	0.23–0.39
ρ_a	Technology growth	B	0.5	0.2	0.33	0.12–0.55
ρ_{rp}	Risk premium	B	0.75	0.1	0.92	0.88–0.96
ρ_i	Investment technology	B	0.75	0.1	0.51	0.38–0.64
$\rho_{a,2}$	Export technology	B	0.75	0.1	0.79	0.68–0.90
ρ_g	Preferences	B	0.75	0.1	0.75	0.65–0.85
η''	Adjustment costs	N	4	1	1.48	0.72–2.27
υ	Habits	B	0.5	0.1	0.34	0.23–0.45
Standard deviations						
σ_{rp}	Risk premium	IG	1	1	0.46	0.31–0.59
σ_M	Monetary policy	IG	0.25	0.1	0.10	0.08–0.12
σ_a	Technology	IG	1	1	0.46	0.32–0.59
σ_2	Export technology	IG	2	1	2.21	1.91–2.51
σ_i	Investment technology	IG	5	1	6.52	3.79–9.37
σ_c	Preferences	IG	1	1	3.37	2.46–4.20
σ_1	Domestic mark-up	IG	0.5	1	0.16	0.13–0.18
σ_m	Imports mark-up	IG	0.5	1	0.33	0.13–0.54

Notes: Prior distributions are B – Beta, N – Normal, IG – inverse Gamma; HPD denotes highest probability density; foreign sector estimates are excluded from this table

There are several aspects of the posterior estimates to note. First, the mean Calvo parameter for domestically produced goods is very high (0.95), and the posterior is very tight. Taken literally, this implies price adjustment occurs very infrequently, and is at odds with microeconomic pricing data. It is, however, similar to estimates based on the generalised method of moments (GMM) (e.g. Kuttner and Robinson 2010), but much higher than the DSGE-based estimates from Robinson (2013), although the latter probably are artificially low due to the way that model is mapped to the data.[10] Such a high value implies a very flat Phillips curve, and consequently monetary policy shocks, while having substantial real effects, have only a small impact on inflation. We present analysis later that examines the sensitivity of the results to this parameter being calibrated to the mean of its prior. The estimates of the indexation parameters do not differ considerably from their priors.[11]

Turning to the Taylor rule, policy is estimated to be much less aggressive towards inflation than was found by Robinson (2013) and remains close to the prior. Policy is more responsive to the deviation of the level of output from its steady state than to output growth (considerably more than was found by Justiniano and Preston (2010b)).[12] A one standard deviation quarterly monetary policy shock is found to be around 10 basis points.

Considering investment, the mean adjustment cost parameter is lower than our prior. We experimented with allowing separate capital technology shocks and adjustment cost parameters for the export sector. It is plausible that these have separate effects, given that the investment boom currently underway in the mining sector has not been reflected in export volumes to a commensurate degree, reflecting the long time it takes for mining investment to become operational and production being disrupted as capacity is expanded. Unfortunately, neither could be identified, which may reflect the fact that aggregate, rather than sectoral, investment data is being used, and therefore both were excluded. Also, it may be difficult to disentangle a sectoral technology shock from a sectoral investment-specific shock.

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10 In particular, in Justiniano and Preston (2010a), import prices in the terms of trade are matched to retail prices, whereas in the Australian data import prices are measured ‘at the dock’, without any retail mark-up.

11 We experimented with allowing autocorrelation in the mark-up shocks. However, for domestic prices the autocorrelation coefficient tended to be very small and for import prices it appeared not to be identified. Consequently, autocorrelation is excluded for both.

12 To some extent this might reflect that we have allowed technology to be non-stationary, whereas they linearly detrend observed output.

3.2 Estimating the BVAR-DSGE

We now turn to the form of the BVAR-DSGE model that we wish to estimate and use for forecasting. The large economy variables are included as exogenous variables in the equations for the small open economy. We do this because central banks will typically have detailed information about their domestic economy, but fewer resources to monitor other countries. They might use forecasts from international organisations (e.g. the International Monetary Fund's (IMF) World Economic Outlook and the Organisation for Economic Co-operation and Development's (OECD) Economic Outlook) or Consensus Economics, which survey market economists, in addition to those compiled internally. Note that if these non-VAR forecasts for the large economy were used it is still necessary to incorporate a model for the large economy variables in the DSGE model.

The form of the BVAR with exogenous variables (BVARX) model we estimate is:

$$y_t = \sum_{i=1}^{p} \Phi_i y_{t-i} + \alpha\beta' y_{t-1}^{ns} + \sum_{j=0}^{q} \Phi_j^x x_{t-j} + u_t,\ u_t \sim N(0, \Sigma_u),$$

where y_t is a vector of n small economy variables that can be split into two components, namely Δy_t^{ns} which are the first difference of variables that according to the DSGE model are non-stationary, and y_t^s, which are stationary. Φ_i are coefficients on lags of y_t, Φ_j^x are the coefficients on the contemporaneous terms and lags of x_t, which is a vector of the large economy variables, and u_t are the shocks, which are normally distributed with variance-covariance matrix Σ_u.[13] Because the DSGE model contains a permanent technology shock, it predicts cointegration among the non-stationary observed variables. As in Del Negro *et al* (2007), we estimate a vector error correction model, where β is the cointegrating vector (we impose the values estimated from the DSGE model) and α are the loadings on the error-correction terms. We will refer to this model as a BVECMX model.[14] We also estimate a variant without these cointegrating terms, namely a BVARX model.

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13 For simplicity we set $p = q$.

14 In the DSGE model, variables such as exports, investment and output are stationary when normalised by technology, for example, $\frac{Y_{2,t}}{Z_t}$ and $\frac{VA_t}{Z_t}$. Consequently $\frac{Y_{2,t}}{VA_t}$ also is stationary, and therefore the log of exports and output are cointegrated with coefficients 1 and –1.

3.2.1 The BVECMX prior

The approach we take to construct the prior follows Robinson (2013), which itself draws on DeJong, Ingram and Whiteman (1993). In brief, by repeatedly drawing DSGE parameters from their posterior, simulating data using these parameters, and estimating a VAR on the simulated data, we map out what the DSGE posterior implies for the BVECMX parameters and the variance-covariance matrix of the shocks. We then use these estimates to inform our selection of the parameters of the prior. We assume that the parameters are distributed according to the independent Normal-Wishart prior described by Koop and Korobilis (2010), and restrict the parameter space to where the VAR is stable.[15] This prior allows different equations to include different variables and consequently accommodates block exogeneity. While this is not relevant when estimating a BVECMX model, the prior can accommodate the Minnesota prior and its small open economy variant, which we use as a benchmark. Finally, one way of relaxing the prior on the BVECMX parameters is to scale up its variance-covariance matrix by a factor of λ, which implies that the prior is given less weight in the estimation of the BVECMX.[16]

3.2.2 The BVECMX posterior

The joint posterior of the BVECMX parameters and the inverse of the variance-covariance matrix do not have a convenient form. However, the posterior distribution of the BVECMX parameters, conditional on the inverse of the variance-covariance matrix, and vice-versa, do have a convenient form. This enables us to use Gibbs sampling to produce realisations from the posterior.[17] The details of these distributions are given in Robinson (2013); the stability condition truncates these to the region where the VAR is stable.[18]

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15 This places a multivariate normal prior over the (vectorised) parameters of the VAR, and a Wishart prior over the inverse of the variance-covariance matrix of the reduced-form shocks. We set the degrees of freedom parameter in the latter to be $n+2$.

16 As the number of parameters in the BVECX is greater than that in the DSGE, to ensure that the prior for the variance-covariance matrix of the parameters is non-singular we add a small amount (0.01) to the standard deviation for all parameters. This is added after any scaling by λ.

17 We simulate 31 000 observations, dropping the first 1 000.

18 This is implemented by dropping all unstable draws.

3.3 Estimating the Benchmark Models

Two types of benchmark model are estimated to produce comparison forecasts for the BVECMX in addition to the DSGE model, namely a small open economy (SOE) Minnesota VAR and separate univariate AR(2) models for each observed variable. The latter are estimated using maximum likelihood. We also combine the point forecasts from the DSGE model with those from the SOE Minnesota VAR using equal weights as another benchmark.[19]

3.3.1 The small open economy Minnesota VAR

In the SOE Minnesota VAR, the large economy variables are included in the vector of endogenous variables and block exogeneity is imposed. Unlike the large economy Minnesota VAR used as a prior for the BVECMX, we do not treat the variance-covariance matrix of the shocks as fixed; instead, we use the variant of the independent Normal-Wishart prior described above. Consequently, the SOE Minnesota VAR is a natural benchmark for the BVECMX model.

The main way the SOE Minnesota VAR differs from the BVECMX model is in the parameters of the prior. For the large economy equations, the priors are the same and block exogeneity is imposed. For the small economy equations we estimate first-order and higher-order autoregressive models, as was done for the large economy Minnesota VAR above, with the longest pre-sample used being 1982:Q3–1992:Q4.[20] Based on these, we set the mean of the prior for the coefficient on the first lags to be:

- exports: 0
- investment: 0
- output: 0.3
- inflation: 0.6

19 More sophisticated approaches could have been used, such as those in Bache *et al* (2011) or Geweke and Amisano (2011), although equal weights have been found in the past to perform well (for example, Clemen (1989)).

20 For investment, the sample was 1986:Q1 onwards due to data availability.

- interest rates: 0.7
- real exchange rate: 0.0.

The remaining parameters of the prior for the SOE Minnesota VAR are ϑ_1 and ϑ_2, which determine the standard deviations of the prior on the coefficients on lags of the own variable and other variables. As for the large economy Minnesota VAR, to choose these we examined the performance of a Minnesota VAR in forecasting the Australian variables over a pre-sample period 1989:Q4–1992:Q4.[21] We set $\vartheta_1 = 0.07$; there appeared to be a trade-off between the one-quarter-ahead (which would suggest a smaller value for ϑ_1) and the year-ahead forecasts. The forecasting exercise suggested a very small value of ϑ_2, which would effectively reduce the VAR to a collection of univariate autoregressive processes. Consequently we set $\vartheta_2 = 0.005$, which Kadiyala and Karlsson (1997) describe as a standard value. We construct the prior for the variance-covariance matrix of the shocks so that the mean coincides with that of the Minnesota prior, following Kadiyala and Karlsson.[22]

3.4 Data

The endogenous and exogenous variables we include in the BVECMX are also the variables used in the estimation of the DSGE model. The 10 observed variables are:

- detrended trade-weighted output
- trade-weighted inflation
- a G7 interest rate
- detrended terms of trade
- exports growth
- investment growth

21 We use the Minnesota prior to save computational time, and exclude business investment due to data availability.

22 As before, we set the degrees of freedom parameter in the Wishart distribution to be $n+2$.

- non-farm output growth
- underlying inflation
- the cash rate
- the change in the real trade-weighted exchange rate.[23]

Details of the data are given in Appendix C.[24]

The observed real exchange rate is defined such that an appreciation is an increase and the terms of trade is the ratio of export to import prices; both definitions are the norm for Australia. The former is the inverse of that in the model, and the measurement equation is adjusted accordingly. The longest estimation sample in the forecasting exercise is 1993:Q1–2011:Q1.[25]

4. Forecasting

4.1 Methodology

The forecasting process for the BVECMX and the Minnesota VAR is as follows. In each case we draw from the joint posterior of the parameters and the variance-covariance matrix of the shocks. Using these simulated parameter values, a vector of shocks is drawn and a realisation of y_{t+1} is found. We then iterate forward, using the same parameters and repeatedly draw new shocks. To create a forecast density this process is repeated 1 000 times, from which we calculate a mean forecast. As we have demeaned the variables prior to estimation, we add back the mean of the data to obtain the final forecasts.

The BVECMX model requires forecasts of the exogenous variables, namely those of the large economy. As discussed above, in practice these might be sourced from international organisations, such as the IMF, although here we simply

23 There are 10 observable variables in the DSGE model, but 12 structural shocks. If one were to identify the model, rather than focus on forecasting, then setting the number of observable variables equal to the number of structural shocks may be preferable.

24 All series are demeaned before estimation. The detrending was done with a Hodrick-Prescott filter with the smoothing parameter set equal to 1 600.

25 The variables are included in the order listed.

generate them using the large economy Minnesota VAR. For simplicity, we use the posterior mean of the VAR parameters, rather than taking into account parameter uncertainty.[26]

4.2 Evaluation

To evaluate the forecast performance of the various models we recursively construct out-of-sample forecasts for the period 2006:Q1–2011:Q2, that is, for each quarter, we re-estimate the models before forecasting. The mean of the data over the estimation period is added back into the forecasts, and this may vary over time because of the recursive estimation. This process does not fully recreate the real-time problem faced by forecasters given that we use final, rather than real-time data. We focus on the quarterly forecasts one and two quarters ahead, as well as the year-ended forecasts one and two years ahead, for all variables except the interest rate.[27] We compare the forecasting performance across models primarily by their RMSE and their bias.

5. Results

The forecasting results for the BVECMX model and the BVARX model that excludes the error-correction terms are shown in Table 4. The results suggest that including the error-correction terms improves the accuracy of the investment forecasts at all horizons, but leads to some deterioration in the export forecasts. This is not surprising because, in the data, exports and output do not appear to be cointegrated (at least with the parameters implied by the model; for further detail see Dungey and Pagan (2009)).

Table 5 shows the performance of the BVECMX relative to our benchmark models, namely the SOE Minnesota VAR, the univariate autoregressions, the DSGE model and the DSGE and Minnesota forecasts combined with equal weights.[28]

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26 Kadiyala and Karlsson (1997) describe this as the ‘customary’ approach.

27 Consequently we obtain 22 and 19 one-quarter- and one-year-ahead forecasts.

28 A comparable table for the model excluding the error-correction terms is Table D1.

Table 4: BVECMX and BVARX Forecast Results – RMSE

Series	Including error-correction terms		Excluding error-correction terms	
Quarterly	*1-quarter-ahead*	*2-quarters-ahead*	*1-quarter-ahead*	*2-quarters-ahead*
Δ Exports	2.47	2.45	2.31	2.34
Δ Investment	3.68	3.70	3.92	3.98
Growth	0.80	0.80	0.79	0.83
Inflation	0.25	0.27	0.24	0.26
Interest rates	0.40	0.79	0.40	0.77
Δ Real exchange rate	6.01	6.13	6.18	6.11
Year-ended	*1-year-ahead*	*2-years-ahead*	*1-year-ahead*	*2-years-ahead*
Δ Exports	4.71	4.78	3.54	4.04
Δ Investment	9.71	7.73	11.02	9.33
Growth	2.05	1.85	2.11	2.03
Inflation	0.94	1.20	0.89	1.17
Interest rates	1.39	1.82	1.34	1.75
Δ Real exchange rate	12.89	14.15	13.16	13.90

Notes: 1- and 2-years-ahead interest rate forecasts are for the level; VAR has 2 lags

Table 5: Relative RMSEs

Benchmark models – includes error-correction terms

Series	Minn	AR	DSGE	Combined	Minn	AR	DSGE	Combined
Quarterly		*1-quarter-ahead*				*2-quarters-ahead*		
Δ Exports	1.18	1.08	0.99	1.12	1.04	1.03	0.97	1.04
Δ Investment	0.97	1.00	0.98	1.00	1.00	1.11	0.93	0.97
Growth	1.05	1.08	0.95	1.03	1.15	1.12	0.93	1.06
Inflation	1.04	1.09	0.99	1.03	1.01	1.02	0.93	0.98
Cash rate	1.03	0.97	0.89	1.02	0.99	0.90	0.96	1.03
Δ Rer	1.18	1.02	0.95	1.10	1.03	0.98	0.97	1.03
Year-ended		*1-year-ahead*				*2-years-ahead*		
Δ Exports	1.43	1.31	0.94	1.30	1.51	1.25	0.95	1.22
Δ Investment	0.81	1.04	0.83	0.83	0.68	0.79	0.76	0.72
Growth	1.24	1.08	0.83	1.03	0.93	0.89	0.81	0.87
Inflation	1.01	1.03	0.94	0.99	0.98	1.02	1.01	1.00
Cash rate	0.97	0.92	1.10	1.07	1.03	1.10	1.19	1.11
Δ Rer	0.99	0.98	0.89	1.01	1.15	1.02	0.95	1.06

Notes: Values less than 1 indicate the RSME of the BVECMX is less than the benchmark and therefore has more accurate forecasts; 1- and 2-years-ahead interest rate forecasts are for the level; VARs have 2 lags; Minn denotes SOE Minnesota VAR; AR denotes autoregressive; Rer denotes real exchange rate

BVARs with Minnesota priors have been shown to forecast well (for example Litterman (1986)). In this case, for several variables, such as growth in exports or output, the SOE Minnesota VAR forecasts more accurately than the BVECMX. The relative accuracy of forecasts of growth in exports appears to be related to the inclusion of the error-correction terms in the BVECMX.

These results imply that the SOE Minnesota VAR outperforms the DSGE model at long horizons, except for interest rates and investment. For inflation, it is only when the structure of the model is relaxed to some extent, as in the BVECMX, that the forecasting performance becomes more comparable to that of the statistical models. There are several reasons why this may be the case. First, it could be particular to the DSGE model we have used. While we have attempted to tailor the baseline small open economy model to Australia to some extent, much could be done to further enrich it. Second, the fit and/or forecasting performance of DSGE models might generally be better for relatively closed economies, such as the United States, and it could be intrinsically more difficult to forecast small open economies or commodity exporters.

Another aspect to note is that the performance of the SOE Minnesota prior is very similar to that of the univariate autoregressive models. For some variables the autoregressive (AR) models themselves are probably similar to simply assuming the forecast is equal to the average growth rate; for example, over the inflation-targeting period there is little persistence evident in quarterly output growth in Australia, and consequently the autoregressive coefficients are insignificant.

The performance of the forecasts formed by equally-weighting the DSGE and Minnesota VAR forecasts is broadly equivalent to that of the BVECMX. Computationally, this is a much simpler way of combining information from the DSGE and the VAR.

Another metric for evaluating the forecasts is their bias, i.e. do they over- or under-predict on average? This test is shown in Table 6. The forecasts for growth in exports and output are biased. Over this period, there was an investment boom in the mining sector, but there was not a commensurate increase in export growth. Consequently, productivity in the mining sector declined and the model forecasts were for stronger export growth than eventuated.

Table 6: Bias – BVECMX Model
Includes error-correction terms

Variable	Quarterly	Year-ended
Δ Exports	1.21*	4.03*
Δ Investment	1.11	3.47
Growth	0.38*	1.23*
Inflation	–0.09	–0.55*
Interest rates	0.02	0.34
Δ Real exchange rate	–1.05	–3.90

Notes: * denotes significance at the 5 per cent level; forecast error is defined as the forecast minus the actual value; heteroskedasticity and autocorrelation consistent (HAC) standard errors for the year-ended forecasts were used; year-ahead interest rate forecasts are for the level

The rapid growth in investment was a response to a terms of trade boom from around 2003 onwards, and forecasters, including the RBA, under-predicted the extent of growth in the terms of trade (see Kearns and Lowe (2011)). Given these developments, and the fact that movements in the terms of trade and the real exchange rate are typically quite correlated, it is somewhat surprising that the bias in the forecasts for the change in the real exchange rate, while in the expected direction, is not significantly different from zero.

Output growth at both horizons is over-predicted on average. The demeaning of the data probably contributes to this bias. At the beginning of the sample there was strong growth as the economy emerged from the early 1990s recession. Also, since at least 2005, Australia's productivity performance has been relatively weak compared to the growth achieved over the 1990s.[29] As a result, the growth rate to which our forecasts are assumed to revert may be too high, even though we have allowed economy-wide technology to be non-stationary. Of course it is difficult to know whether the apparent change in the average rate of output growth will be permanent, especially given the considerable structural change occurring in the economy due to the terms of trade boom.

The bias in the inflation forecasts in the near-term is slight, but it is larger one-year-ahead. One period contributing to this result is immediately prior to the global financial crisis, when inflation in Australia increased noticeably. The BVECMX model initially fails to predict this increase, and its one-quarter-ahead inflation forecasts are progressively revised upwards throughout the period. The SOE

29 For one discussion of trends in Australian productivity over the 2000s see Eslake (2011).

Minnesota model also does not predict the initial increase. One possible reason for the sharp increase in inflation is that there might be a threshold level of capacity utilisation above which inflation increases strongly (Lowe 2011). As all of the models we have considered are linear, none can capture such dynamics.

5.1 Sensitivity Analysis

5.1.1 Prior sensitivity

The forecasting performance is relatively insensitive to the looseness of the prior, governed by the parameter λ (Table 7). For some variables, such as the interest rate and inflation, the forecasts made using different values of λ are highly correlated. This may in part reflect the strong persistence in these variables. The variations in the RMSEs that do occur as λ changes are not consistent across all of the variables or horizons; for example, for output growth one-quarter-ahead, tighter priors outperform looser priors, whereas one-year-ahead a higher λ performs better. For inflation, when the prior is tightly held ($\lambda = 1$) the forecasts are most accurate, although the impact of changing λ is small. Accordingly, focusing on the results for output and inflation, it is not clear what λ to choose; in the near-term a low λ is appropriate, whereas for longer horizons a higher value might be desirable. Alternatively, if λ was chosen with reference to the forecasting performance of all variables then $\lambda = 1$ would be selected.

Table 7: BVECMX Forecasts Results – RSME

	λ							
Series	1	10	50	100	1	10	50	100
	Including error-correction terms							
Quarterly	*1-quarter-ahead*				*2-quarters-ahead*			
Δ Exports	2.47	2.49	2.52	2.50	2.45	2.48	2.50	2.51
Δ Investment	3.68	3.98	3.81	3.81	3.70	3.70	3.46	3.33
Growth	0.80	0.85	0.87	0.87	0.80	0.80	0.78	0.76
Inflation	0.25	0.26	0.27	0.27	0.27	0.28	0.28	0.28
Interest rates	0.40	0.40	0.40	0.40	0.79	0.82	0.85	0.85
Δ Rer	6.01	5.96	5.92	5.92	6.13	6.11	6.13	6.15
Year-ended	*1-year-ahead*				*2-years-ahead*			
Δ Exports	4.71	4.77	4.80	4.79	4.78	4.69	4.64	4.62
Δ Investment	9.71	10.01	9.52	9.32	7.73	8.43	8.70	8.69
Growth	2.05	2.04	1.98	1.97	1.85	1.92	1.96	1.99
Inflation	0.94	0.97	0.99	1.00	1.20	1.21	1.23	1.24
Interest rates	1.39	1.44	1.55	1.58	1.82	1.80	1.83	1.87
Δ Rer	12.89	12.99	13.21	13.39	14.15	14.03	13.96	14.07

Notes: 1- and 2-years-ahead interest rate forecasts are for the level; VARs have 2 lags; Rer denotes real exchange rate

5.1.2 The Calvo parameter

As previously discussed, the estimated Calvo parameter for domestically produced goods is very high in this model. To examine the implications of this we re-estimated the model, calibrating the parameter at the mean of the prior previously used in estimation (0.75). The results, which are shown in Table 8, suggest that the main impact is that the near-term inflation forecasts become less accurate if the prior is tightly held (i.e. $\lambda = 1$). This suggests a looser prior may be appropriate.

Table 8: Relative RMSE Sensitivity to Calvo Parameter

Ratio to estimated Calvo parameter – includes error-correction terms

	$\lambda = 1$		$\lambda = 100$	
Series	Quarterly	Year-ended	Quarterly	Year-ended
Δ Exports	0.99	1.01	1.00	0.97
Δ Investment	1.06	1.06	1.14	1.09
Growth	0.96	0.98	0.99	1.01
Inflation	1.13	1.04	0.96	0.93
Interest rates	1.04	1.02	1.00	0.91
Δ Real exchange rate	1.01	1.00	1.05	1.03

Note: Year-ahead interest rate forecasts are for the level

5.1.3 Excluding the exports error-correction term

In the results presented in Table 4 it appears that including both error-correction terms leads to a deterioration in the performance of the forecasts for export growth, but improves those for investment growth. Re-estimating the model excluding the exports cointegrating term improves the export forecasts, but has little impact on the other variables (Table 9).

Table 9: RMSE Without the Export Error-correction Term

Ratio to baseline model

	$\lambda = 1$		$\lambda = 100$	
Series	Quarterly	Year-ended	Quarterly	Year-ended
Δ Exports	0.94	0.79	0.93	0.80
Δ Investment	0.99	0.98	0.94	0.97
Growth	1.00	1.02	1.00	1.01
Inflation	1.02	1.06	1.01	1.03
Interest rates	1.00	0.99	0.99	1.00
Δ Real exchange rate	1.00	1.00	1.01	1.02

Notes: Baseline model includes both cointegrating terms; year-ahead interest rate forecasts are for the level

5.1.4 Extending the estimation period

The estimation sample used was the inflation-targeting period (1993:Q1 onwards). Hence, the models for the early samples in the forecasting exercise were estimated with only a small amount of data, which may make the use of a relatively tight prior for the VAR preferable. To test the importance of sample size, we re-estimate the BVECMX over a longer sample (1986:Q2 onwards). To do this we regress all variables against a constant and a dummy for the inflation-targeting period, rather than demeaning the data, which is akin to allowing the intercept to shift for all equations in the VAR in 1993:Q1. Given that other countries, such as the United Kingdom and Canada, also deflated at around this time and are included in the aggregates we use for the foreign variables, we also allow for breaks in these series. The DSGE parameters obtained are similar to the short sample estimates, and are given in Table D1.

In general, the BVECMX estimated over the longer sample tends to forecast more accurately; for example, the ratio of its RMSE for the one-quarter-ahead inflation forecasts to that for the original sample is 0.85 (when $\lambda = 1$). However, the accuracy of the AR models, for example, also increases when estimated over the longer sample (and adjusting for possible breaks in the same way). Overall, it appears that in the short to medium term the BVECMX model forecasts inflation more accurately than an AR model when the long sample is used. In contrast, output growth is forecast less accurately by the BVECMX, but the relative performance of the BVECMX model generally improves (Table 10). It is also noticeable that a looser prior improves the accuracy of the output growth forecasts, but causes both the inflation and, particularly, the interest rate forecasts to deteriorate.

Table 10: RSME BVECMX Relative to AR Models – Long Sample

	λ							
Series	1	10	50	100	1	10	50	100
Quarterly		*1-quarter-ahead*				*2-quarters-ahead*		
Δ Exports	1.05	1.06	1.04	1.04	1.00	1.02	1.01	1.01
Δ Investment	0.98	0.98	1.02	1.02	1.08	1.04	1.04	0.99
Growth	1.08	1.07	1.06	1.07	1.09	1.09	1.04	1.07
Inflation	0.98	1.00	1.00	0.99	0.91	0.93	0.98	0.99
Cash rate	1.08	1.30	1.49	1.54	1.02	1.15	1.27	1.32
Δ Rer	1.06	1.06	1.07	1.07	1.01	1.01	1.04	1.04
Year-ended		*1-year-ahead*				*2-years-ahead*		
Δ Exports	1.20	1.18	1.09	1.05	1.17	1.19	1.14	1.08
Δ Investment	1.04	1.04	1.02	1.03	0.84	0.84	0.86	0.89
Growth	1.13	1.11	1.09	1.09	0.98	0.96	0.95	0.94
Inflation	0.88	0.93	0.99	1.00	1.06	1.10	1.08	1.07
Cash rate	0.99	1.04	1.15	1.18	1.04	1.05	1.11	1.16
Δ Rer	1.03	1.04	1.08	1.09	1.02	1.02	0.99	0.97

Notes: Includes the investment error-correction term only; values less than 1 indicate the RSME of the BVECMX is less than the AR model; 1- and 2-years-ahead interest rate forecasts are for the level; VARs have 2 lags; Rer denotes real exchange rate

5.2 The Global Financial Crisis

The major economic event during our forecast evaluation period is the global financial crisis. Consequently, it is interesting to consider the performance of the BVECMX around this time. Examining the forecasts, it is apparent that the early forecasts for the crisis period were for around trend growth. Throughout the first half of 2008, the mean one-quarter-ahead forecasts were generally revised lower, reflecting the soft growth outcomes which occurred. The BVECMX does not forecast the sharp drop in output that occurred in the December quarter of 2008. The SOE Minnesota VAR similarly predicted moderate growth during this period.

One possible explanation as to why the mean forecasts from these models missed the extent of the impact of the global financial crisis is that they do not contain enough financial variables, for example, credit aggregates and spreads are not included in the model. An obvious shortfall of the BVAR-DSGE methodology is that it is necessary to have a DSGE model that incorporates these variables. Developing this aspect of these models is currently a very active area of research.[30] The Minnesota VAR we have used to generate the large economy forecasts is also very simple and could be expanded or generalised. Edge and Gürkaynak (2010) review the forecasting performance for the United States economy of the Smets and Wouters (2007) DSGE model, a Minnesota VAR in the observable variables of the DSGE and Blue Chip (a monthly survey of business economists). They found that '[a]lthough all the forecasts clearly first miss the [United States'] recession, and then miss its severity, the Blue Chip forecasts in general fare better as the quarter to be forecasted gets closer, and especially when nowcasting' (p 235). This improved performance is likely to be due to the fact that the analysts included in the Blue Chip forecasts can utilise additional timely information in constructing their forecasts, which are typically not observed or even included in these models.

A second explanation of why the BVECMX model's mean output growth forecasts fail to capture the crisis is that they are not very persistent, and quickly revert to the average growth rate. While this lack of persistence is a characteristic of the data, it makes it difficult for the model to explain fluctuations in growth.[31] None of the models that were considered forecast output growth well over this period.

6. Conclusions

In this paper we have extended a baseline small open economy DSGE model to include a second production sector which produces commodities. This allows the price of exports to differ from other domestically produced goods in the model. This addition is motivated by the sizable increase in Australia's terms of trade since the mid 2000s. The commodity sector we have introduced is very simple, and could be developed further; for example, by creating an explicit role

30 The financial accelerator model links credit spreads to the net worth of the borrower, and consequently potentially amplifies the impact of shocks (see, for example, Bernanke, Gertler and Gilchrist (1999)). However, Pagan and Robinson (2012) review the ability of several models including financial frictions to predict recessions and find that it is poor.

31 In the pre-sample period for the SOE Minnesota VAR, the first-order autocorrelation coefficient was approximately 0.3. However, over the estimation period this drops to 0.1 and is no longer statistically significantly different from zero. Tulip (2009) finds that for the United States, using Greenbook forecasts, during the Great Moderation period '... the predictable component of fluctuations in output and inflation has virtually disappeared' (p 1217).

for natural resources, or to consider alternative pricing assumptions. To better capture the changes in the terms of trade which have occurred, allowing permanent productivity shocks in the commodity sector, so as to capture the sluggish response of supply to the increased demand, could be examined.

We examine the forecasting performance of a BVECMX where this DSGE model is used as a prior, and the large economy variables remain exogenous, an approach which is potentially useful for forecasting for small open economies. We have concentrated on evaluating the point forecasting performance, and further research could consider the properties of the forecast distribution. We find that when estimated over the inflation-targeting period, the forecasts from the BVECMX generally outperformed those from the DSGE model. Simply combining the DSGE forecasts with those from a SOE Minnesota VAR yielded forecasts of a comparable accuracy, and forecasts from the BVECMX did not outperform the SOE Minnesota VAR alone or simple univariate models. When estimated over a longer sample, the performance of the BVECMX model forecasts improved relative to those of an AR model, particularly for inflation in the short to medium term.

In all, the Australian economy is in the midst of a once-in-a-lifetime terms of trade and investment boom. While the model in this paper is a first step, creating a DSGE model which can capture these developments more effectively is likely to be beneficial, but quite challenging.

Appendix A: Log-linearised Equations of the Model

This appendix lists the equations in the model. ˜ denotes that a variable is a log deviation from the steady-state. Recall that 1 denotes the domestic good sector, 2 the export sector, and * foreign variables. Parameters which are functions of the steady state needed for the log-linearised equations are in the subsequent appendix. We have eliminated the nominal exchange rate from the model; all relationships are expressed in terms of the real exchange rate. C_t^d, namely consumption of domestically produced goods, has also been substituted out. As the economy-wide technology, Z_t, is non-stationary, many of the real variables have been normalised by the level of technology, for example $c_t \equiv \frac{C_t}{Z_t}$. These are (not exclusively) denoted with lower case letters. f is introduced so as the model can be log-linearised around $h = 0$, and uses the approximation that $\ln(1+x) \approx x$. It effectively linearises terms involving h. v_m is similar. See Table A1 for a description of all variables.

Table A1: Variables

Variable	Description	Variable	Description
Variables which have been detrended			
c	Consumption	i	Investment
w	Wages	va	Value added
mc	Marginal costs, domestic goods	h	Net foreign assets share of value added
y	Output	f	$f \equiv h+1$
m	Real money balances	t_m	Real transfers
k	Capital	v_m	$v_m \equiv t_m + 1$
Other variables			
R	Nominal interest rate	π	Domestic good inflation
π	Consumer inflation	r	Gross rental rate on capital
Ψ	Imports marginal costs	s	Consumer terms of trade $\left(\frac{P^m}{P^d}\right)$
L	Labour	P_2^*	Nominal commodity prices in foreign currency
q	Tobin's Q	p_2^*	Real commodity prices in foreign currency $\left(\frac{P_2^*}{P^*}\right)$
g	Growth in nominal money	Rer	Real exchange rate
		Ω	$\Omega = \kappa Z$ Lagrange multiplier on the budget constraint
Shocks			
u^c	Preference	u^m	Import mark-up
u^i	Investment	a_2	Export sector technology
u^{rp}	Risk premium	a	Technology growth
ε	Monetary policy	u^d	Domestic mark-up
The foreign VAR contains 4 reduced-form shocks			
Observed variables			
Δy^{obs}	Output growth	π^{obs}	Inflation
Δi^{obs}	Investment growth	s^{*obs}	Terms of trade
y^{*obs}	Foreign output gap	Δrer^{obs}	Change in the real exchange rate
r^{obs}	Interest rate	Δx^{obs}	Change in exports

A.1 Consumer

Lagrange multiplier on budget constraint

$$\widetilde{\Omega}_t = u_t^c - \frac{1}{1-H}[\widetilde{c}_t - H(\widetilde{c}_{t-1} - \widetilde{a}_t)].$$

Euler equation

$$\widetilde{\Omega}_t = E_t[\widetilde{\Omega}_{t+1} + (\widetilde{R}_t - \widetilde{\pi}_{t+1}) - a_{t+1}].$$

Labour supply, domestic goods

$$\widetilde{w}_{1,t} = -\widetilde{\Omega}_t + \xi \widetilde{L}_{1,t} + (\psi - \xi)\left(\widetilde{L}_{1,t}\left(\frac{\bar{L}_1}{\bar{L}}\right)^{1+\xi} + \widetilde{L}_{2,t}\left(\frac{\bar{L}_1}{\bar{L}}\right)^{1+\xi}\right).$$

Labour supply, exports

$$\widetilde{w}_{2,t} = -\widetilde{\Omega}_t + \xi \widetilde{L}_{2,t} + (\psi - \xi)\left(\widetilde{L}_{1,t}\left(\frac{\bar{L}_1}{\bar{L}}\right)^{1+\xi} + \widetilde{L}_{2,t}\left(\frac{\bar{L}_2}{\bar{L}}\right)^{1+\xi}\right).$$

Tobin's Q, domestic goods

$$\widetilde{q}_{1,t} = \beta E_t[\widetilde{q}_{1,t+1}(1-\delta) + \widetilde{r}_{1,t+1}\bar{r}_1] - (\widetilde{R}_t - E_t[\widetilde{\pi}_{1,t+1}]).$$

Tobin's Q, exports

$$\begin{aligned}\widetilde{q}_{2,t} &= (1-\beta(1-\delta_2))(1-\gamma')\widetilde{s}_t + \widetilde{r}_{2,t+1} + \widetilde{rer}_{t+1} + \widetilde{p}^*_{2,t+1} + \widetilde{\pi}_{t+1}) \\ &\quad + \beta(1-\delta_2)E_t[\widetilde{q}_{2,t+1} + \widetilde{\pi}_{1,t+1}] - \widetilde{R}_t,\end{aligned}$$

where $\gamma' \equiv \gamma\left(\frac{\bar{P}_1}{\bar{P}}\right)^{1-\sigma}$.

Investment, domestic goods

$$\widetilde{i}_{1,t} = \frac{1}{(1+\beta)}\left(\frac{1}{\eta''(1)}\left(\widetilde{q}_{1,t} + u_t^i\right) + \beta E_t[\widetilde{i}_{1,t+1}] + \widetilde{i}_{1,t-1}\right).$$

Investment exports

$$\widetilde{i}_{2,t} = \frac{1}{(1+\beta)}\left(\frac{1}{S''(1)}(\widetilde{q}_{2,t} + u_t^i) + \beta E_t[\widetilde{i}_{2,t+1}] + \widetilde{i}_{2,t-1}\right).$$

Capital accumulation, domestic goods

$$\widetilde{k}_{1,t} = (1-\delta_1)(\widetilde{k}_{1,t-1} - \widetilde{a}_t) + \delta_1(u_t^i + \widetilde{i}_{1,t}).$$

Capital accumulation, exports

$$\widetilde{k}_{2,t} = (1-\delta_2)(\widetilde{k}_{2,t-1} - \widetilde{a}_t) + \delta_2(u_t^i + \widetilde{i}_{2,t}).$$

Money demand

$$\widetilde{m}_t = -\widetilde{\Omega}_t - \widetilde{R}_t \frac{\beta}{1-\beta}.$$

A.2 Domestic Good Firm

Phillips Curve

$$\widetilde{\pi}_{1,t} = \mu_{1,f}\widetilde{\pi}_{1,t+1} + \mu_{1,b}\widetilde{\pi}_{1,t-1} + \mu_1^{mc}\widetilde{mc}_t + u_t^d,$$

where

$$\mu_{1,f} = \frac{\beta\theta_1}{\theta_1 + \omega_1(1-\theta_1(1-\beta))}$$

$$\mu_{1,b} = \frac{\omega_1}{\theta_1 + \omega + 1(1-\theta_1(1-\beta))}$$

$$\mu_1^{mc} = \frac{(1-\beta)(1-\theta_1)(1-\theta_1\beta)}{\theta_1 + \omega_1(1-\theta_1(1-\beta))}.$$

Marginal costs

$$\widetilde{mc}_t = \alpha_1\widetilde{r}_{1,t} + (1-\alpha_1)\widetilde{w}_{1,t} + (1-\alpha_1)(1-\gamma')\widetilde{s}_t.$$

Demand for capital

$$\widetilde{k}_{1,t-1} = \widetilde{mc}_t - \widetilde{r}_{1,t} + \widetilde{y}_{1,t} + \widetilde{a}_t.$$

Demand for labour

$$\widetilde{L}_{1,t} = \widetilde{mc}_t + \widetilde{y}_t - (1-\gamma')\widetilde{s}_t - \widetilde{w}_{1,t}.$$

A.3 Market Clearing

$$\widetilde{y}_{1,t} = \left(\sigma(1-\gamma')\widetilde{s}_t + \widetilde{c}_t^d\right)\frac{\bar{C}^d}{\bar{Y}_1} + \widetilde{i}_{1,t}\frac{\bar{I}_1}{\bar{Y}_1} + \widetilde{i}_{2,t}\frac{\bar{I}_2}{\bar{Y}_1}.$$

A.4 Importer

$$\widetilde{\pi}_t^m = \mu_f^m \widetilde{\pi}_{t+1}^m + \mu_b^m \widetilde{\pi}_{t-1}^m + \mu^{m\ mc} \widetilde{\Psi}_t + \widetilde{u}_t^m,$$

where the parameters are defined as per the domestic good.

A.5 Exporter

Labour demand

$$L_{2,t} = \widetilde{p}_t^* + \widetilde{rer}_t + \widetilde{y}_{2,t} - \widetilde{w}_{2,t}.$$

Capital demand

$$\widetilde{k}_{2,t-1} = \widetilde{y}_{2,t} - \widetilde{r}_{2,t} + \widetilde{a}_t.$$

Production function

$$\widetilde{y}_{2,t} = (1-\alpha_2)\widetilde{L}_{2,t} + \alpha_2(\widetilde{k}_{2,t-1} - \widetilde{a}_t) + \widetilde{a}_{2,t}.$$

A.6 Open Economy

Uncovered interest parity

$$\left(\widetilde{R}_t - \pi_{t+1}\right) - \left(\widetilde{R}_t^f - \pi_{t+1}^f\right) = E_t[\widetilde{rer}_{t+1}] - \widetilde{rer}_t - \chi \widetilde{f}_t + u_t^{rp}.$$

Net foreign assets

$$\widetilde{f}_t - \frac{1}{\beta}\widetilde{f}_{t-1} = \bar{p}_2^* \frac{\bar{P}}{\bar{P}_1} \frac{\bar{y_2}}{\bar{va}} \left(\widetilde{p}_{2,t}^* + \sigma\gamma'\widetilde{s} + \widetilde{y}_2 - \widetilde{c}_t\right).$$

Real exchange rate and consumption terms of trade relationship

$$\widetilde{rer}_t = \widetilde{\Psi}_t + \gamma' \widetilde{s}_t.$$

Consumption terms of trade growth

$$\widetilde{s}_t - \widetilde{s}_{t-1} = \widetilde{\pi}_t^m - \widetilde{\pi}_t^d.$$

A.7 Autoregressive Processes

Technology growth

$$\widetilde{a}_t = \rho_a \widetilde{a}_{t-1} + \varepsilon_{a,t}.$$

Risk premium

$$u_t^{rp} = \rho_{rp} u_{t-1}^{rp} + \varepsilon_t^{rp}.$$

Preference shock

$$u_t^c = \rho_c u_{t-1}^c + \varepsilon_{c,t}.$$

Export sector technology

$$\widetilde{a}_{2,t} = \rho_{a,2} \widetilde{a}_{2,t-1} + \varepsilon_{2,t}.$$

Investment technology shock

$$u_t^i = \rho_i u_{t-1} + \varepsilon_{t^i}.$$

A.8 Aggregates

CPI inflation

$$\widetilde{\pi}_t = \widetilde{\pi}_{1,t} + (1-\gamma')(\widetilde{s}_t - \widetilde{s}_{t-1}).$$

Value added

$$\widetilde{va}_t = \frac{\bar{y}_1}{\bar{va}} \widetilde{y}_{1,t} + \frac{\bar{y}_2}{\bar{va}} \frac{\bar{P}_2}{\bar{P}_1} \left(\widetilde{rer}_t + \widetilde{p}_{2,t}^* + (1-\gamma')\widetilde{s}_t + \widetilde{y}_{2,t} \right).$$

A.9 Monetary Policy

Taylor rule

$$\widetilde{R}_t = \zeta^R \widetilde{R}_{t-1} + \zeta^R \left(\zeta^\pi \widetilde{\pi}_t + \zeta^{va} \widetilde{va}_t + \zeta^{gr} \left(\widetilde{va}_t - \widetilde{va}_{t-1} \right) \right) + \varepsilon_t.$$

Growth in nominal money

$$\widetilde{g}_t = \widetilde{\pi}_t + \widetilde{a}_t - \widetilde{\Omega}_t - \widetilde{m}_{t-1} - \widetilde{R}_t \frac{\beta}{1-\beta}.$$

Real transfers

$$\widetilde{v}_t = \widetilde{g}_t \bar{m}.$$

A.10 Foreign Sector

The foreign sector is modelled as a BVAR in the foreign output gap, inflation, interest rates and the detrended Australian terms of trade.

A.11 Measurement Equations

As the observed data have been demeaned, constants are omitted from the measurement equations.

Output growth

$$\Delta y_t^{obs} = (\widetilde{va}_t - \widetilde{va}_{t-1}) + \widetilde{a}_t.$$

Investment growth

$$\Delta i_t^{obs} = \frac{\bar{I}_1}{I}((\widetilde{i}_{1,t} - \widetilde{i}_{1,t-1}) + \widetilde{a}_t) + \frac{\bar{I}_2}{I}((\widetilde{i}_{2,t} - \widetilde{i}_{2,t-1}) + \widetilde{a}_t).$$

Foreign output gap

$$y_t^{*obs} = \widetilde{y}_t^{*}.$$

Interest rate

$$r_t^{obs} = 4\widetilde{r}_t.$$

Foreign interest rates

$$r_t^{*obs} = 4\widetilde{r}_t^{*}.$$

Inflation

$$\pi_t^{obs} = \widetilde{\pi}_t.$$

Foreign inflation

$$\pi_t^{*obs} = \widetilde{\pi}_t^{*}.$$

Terms of trade

$$s_t^{*obs} = \widetilde{p}_{2,t}^{*}.$$

Change in the real exchange rate

$$\Delta rer_t^{obs} = \widetilde{rer}_t - \widetilde{rer}_{t-1}.$$

Change in exports

$$\Delta x_t^{obs} = \widetilde{y}_{2,t} - \widetilde{y}_{2,t-1} + \widetilde{a}_t.$$

Appendix B: Steady State

Superscript bars denote steady state values. We log-linearise around a deterministic steady state where the real exchange rate, $\frac{\bar{S}\bar{P}^*}{\bar{P}}$, is normalised to 1, and net foreign assets equal 0. Also, real commodity prices are exogenous to the small economy, so we set $\bar{p}_2^* = 1$. Steady state inflation is assumed to be 0. In this section we provide the details necessary to calculate the steady state quantities which appear in the log-linearised model or were used in calibration.

B.1 Cash Rate

In the steady state, from the consumer's first-order condition for bond holdings $\frac{1}{\beta} = \bar{R}$.

B.2 Prices

Import prices

In the steady state, retail import prices are a constant mark-up over nominal marginal costs (alternatively, real marginal costs are constant):

$$\frac{\bar{P^m}}{\bar{S}\bar{P^*}} = \frac{\lambda^m}{\lambda^m - 1} \equiv X^m.$$

Note also that

$$\frac{\bar{P^m}}{\bar{P}} = \frac{\bar{P^m}}{\bar{S}\bar{P^*}}\frac{\bar{S}\bar{P^*}}{\bar{P}} = \frac{\bar{P^m}}{\bar{S}\bar{P^*}}1 = X^m.$$

Domestic good prices

From the expression for the CPI we have:

$$1 = \left(\gamma\left(\frac{\bar{P}_1}{\bar{P}}\right)^{1-\sigma} + (1-\gamma)\left(\frac{\bar{P^m}}{\bar{P}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

Using the expression for real import prices above, we obtain:

$$\frac{\bar{P}_1}{\bar{P}} = \left(\frac{1-(1-\gamma)X^{m1-\sigma}}{\gamma} \right)^{\frac{1}{1-\sigma}}.$$

Real marginal costs in the domestic goods sector

Similar to import prices, we have

$$\bar{mc} = \frac{\lambda^d - 1}{\lambda^d}.$$

B.3 Investment

Domestic goods gross rental rate

Tobin's Q in steady state is 1. From the equation for Tobin's Q we can obtain:

$$\bar{r}_1 = \frac{1}{\beta} - (1-\delta_1).$$

Export sector gross rental rate

Similarly:

$$\bar{r}_2 = \frac{\bar{P}_1}{\bar{P}} \frac{1}{\bar{p}_2^*} \left(\frac{1}{\beta} - (1-\delta_2)\right).$$

Capital to output ratio, domestic good sector

From the factor demand equation

$$\frac{\bar{k}_1}{\bar{y}_1} = \alpha_1 \frac{\bar{mc}}{\bar{r}_1}.$$

Investment to output ratio, domestic good sector

From the capital accumulation equation

$$\frac{\bar{i}_1}{\bar{y}_1} = \delta_1 \alpha_1 \frac{\bar{mc}}{\bar{r}_1}.$$

Capital to output ratio, export sector

$$\frac{\bar{k}_2}{\bar{y}_2} = \alpha_2 \frac{1}{\bar{r}_2}.$$

Investment to output ratio, export sector

$$\frac{\bar{i}_2}{\bar{y}_2} = \delta_2 \alpha_2 \frac{1}{\bar{r}_2}.$$

Note for the measurement equation for investment $\bar{i} = \bar{i}_1 + \bar{i}_2$, and that $\frac{\bar{i}_2}{\bar{y}_1} = \frac{\bar{i}_2}{\bar{y}_2}\frac{\bar{y}_2}{\bar{y}_1}$. An expression for the $\frac{\bar{y}_2}{\bar{y}_1}$ is developed below. We can then express $\frac{\bar{i}_1}{\bar{i}} = \frac{\frac{\bar{i}_1}{\bar{y}_1}}{\frac{\bar{i}_1}{\bar{y}_1} + \frac{\bar{i}_2}{\bar{y}_1}}$ and $\frac{\bar{i}_2}{\bar{i}} = \frac{\frac{\bar{i}_2}{\bar{y}_1}}{\frac{\bar{i}_1}{\bar{y}_1} + \frac{\bar{i}_2}{\bar{y}_1}}$.

B.4 Trade

In steady state we have net foreign assets as a share of GDP equal to 0. This implies balanced trade. Further, we have

$$\frac{\bar{P}_2}{\bar{P}_1}\bar{x}_2 = \frac{\bar{S}\bar{P}^*}{\bar{P}_1} c^{\bar{m}}.$$

From which we can obtain:

$$\bar{x}_2 = \frac{1}{\bar{p}^*}\bar{c}^m.$$

Note that $\bar{x}_2 = \bar{y}_2$, so we have

$$\frac{\bar{y}_2}{\bar{y}_1} = \frac{1}{\bar{p}^*}\frac{\bar{c}^m}{\bar{c}}\frac{\bar{c}}{\bar{y}_1}.$$

B.5 Consumption

Lagrange multiplier on the budget constraint

$$\bar{\Omega} = \frac{1}{\bar{c}\,(1-H)}.$$

Imports

We have the following demand function:

$$\frac{\bar{c}^m}{\bar{c}} = (1-\gamma)\left(\frac{\bar{P^m}}{\bar{P}}\right)^{-\sigma}.$$

Domestic goods

Similarly, for domestic goods.

$$\frac{\bar{c}^d}{\bar{c}} = \gamma\left(\frac{\bar{P}_1}{\bar{P}}\right)^{-\sigma}.$$

Consumption share of domestic goods

From the market clearing for domestic goods we have:

$$1 = \frac{\bar{c}^d}{\bar{y}_1} + \frac{\bar{i}_1}{\bar{y}_1} + \frac{\bar{i}_2}{\bar{y}_2}\frac{\bar{y}_2}{\bar{y}_1}.$$

From this we can obtain

$$\frac{\bar{c}}{\bar{y}_1} = \frac{1-\frac{\bar{i}_1}{\bar{y}_1}}{\frac{\bar{c}^d}{\bar{c}} + \frac{\bar{i}_2}{\bar{y}_2}\frac{1}{\bar{p}^*}\frac{\bar{c}^m}{\bar{c}}}.$$

B.6 Labour

Labour, domestic goods

From the labour supply of consumers we can obtain an expression for the labour share:

$$(\bar{L}_1^{1+\xi} + \bar{L}_2^{1+\xi})^{\frac{\psi-\xi}{1+\xi}} \bar{L}_1^{1+\xi} \frac{1}{\bar{\Omega}\bar{y}_1} = \frac{\bar{w}_1\bar{L}_1}{\bar{y}_1}.$$

From the labour demand of firms we have

$$(1-\alpha_1)\bar{mc}\left(\frac{\bar{P}_1}{\bar{P}}\right) = \frac{\bar{w}_1\bar{L}_1}{\bar{y}_1}.$$

Equating these, and using results from above, we obtain:

$$\bar{L}_1 = \left(\frac{(1-\alpha_1)\bar{mc}\bar{\Omega}\bar{y}_1\frac{\bar{p}_1}{\bar{p}}}{\left(1+\frac{(1-\alpha_2)\bar{p}_2^*\frac{\bar{y}_2}{\bar{y}_1}}{(1-\alpha_1)\bar{mc}\frac{\bar{p}_1}{\bar{p}}}\right)^{\frac{\psi-\xi}{1+\xi}}} \right)^{\frac{1}{1+\psi}}.$$

Similarly,

$$\bar{L}_2 = \left(\frac{(1-\alpha_2)\bar{p}_2^*\bar{\Omega}\bar{y}_2}{\left(\frac{(1-\alpha_1)}{(1-\alpha_2)}\frac{\bar{mc}}{\bar{p}_2^*}\frac{\bar{p}_1}{\bar{p}}\frac{\bar{y}_1}{\bar{y}_2}+1\right)^{\frac{\psi-\xi}{1+\xi}}} \right)^{\frac{1}{1+\psi}}.$$

These can be substituted into

$$\bar{L} = \left(\bar{L}_1^{1+\xi} + \bar{L}_2^{1+\xi}\right)^{\frac{1}{1+\xi}}.$$

B.7 Real Money Balances

Note that from money demand we have

$$\bar{m} = \frac{1}{\bar{\Omega}}\left(1-\bar{R}^{-1}\right)^{-1}.$$

The $\bar{c}$ can be obtained by rearranging the production function of domestic good firms to be:

$$\bar{y}_1 = \bar{L}_1\left(\frac{\bar{k}}{\bar{y}}\right)^{\frac{\alpha_1}{1-\alpha_1}}.$$

As we have an expression for $\frac{\bar{c}}{\bar{y}_1}$ above we can solve for $\bar{c}$, and hence $\bar{m}$.

B.8 Value added

In steady state we have

$$\bar{va} = \bar{y}_1 + \frac{\bar{P}_2}{\bar{P}_1}\bar{y}_2.$$

The coefficients in the log-linearised expression for value-added are:

$$\frac{\bar{y}_1}{\bar{va}} = \frac{\bar{y}_1}{\bar{y}_1 + \frac{\bar{P}_2}{\bar{P}_1}\bar{y}_2} = \frac{1}{1 + \frac{\bar{P}_2}{\bar{P}_1}\frac{\bar{y}_2}{\bar{y}_1}},$$

and

$$\frac{\bar{P}_2\bar{y}_2}{\bar{P}_1\bar{va}} = \frac{\bar{P}_2\bar{y}_2}{\bar{P}_1\bar{y}_1 + \bar{P}_2\bar{y}_2} = \frac{1}{\frac{\bar{P}_1}{\bar{P}_2}\frac{\bar{y}_1}{\bar{y}_2} + 1}.$$

The ratios to value added, such as those in the net foreign assets equation, can easily be derived, e.g. $\frac{\bar{c}^m}{\bar{va}} = \frac{\bar{c}^m}{\bar{y}_1}\frac{\bar{y}_1}{\bar{va}}$. Similarly, $\frac{\bar{i}}{\bar{va}} = \frac{\bar{i}_1}{\bar{y}_1}\frac{\bar{y}_1}{\bar{va}} + \frac{\bar{i}_2}{\bar{y}_2}\frac{\bar{P}_2\bar{y}_2}{\bar{P}_1\bar{va}}\frac{\bar{P}_1}{\bar{P}_2}$. To determine the compensation of employees to nominal value added, i.e. $(\frac{\bar{w}_1\bar{L}_1+\bar{w}_2\bar{L}_2}{\bar{va}})\frac{\bar{P}}{\bar{P}_1}$, which were used in the calibration but not in the steady state of the model, the labour demand equations can be utilised, together with the previous expressions for the share of value added constituted by each sector's output and relative prices. In the domestic good sector $\frac{\bar{w}_1\bar{L}_1}{\bar{y}_1} = (1-\alpha_1)\bar{mc}_1\frac{\bar{P}_1}{\bar{P}}$ and $\frac{\bar{w}_1\bar{L}_1}{\bar{va}} = \frac{\bar{w}_1\bar{L}_1}{\bar{y}_1}\frac{\bar{y}_1}{\bar{va}}$. For the export sector we have $\frac{\bar{w}_2\bar{L}_2}{\bar{y}_2} = (1-\alpha_2)\bar{p}_2^*$.

Appendix C: Data Definitions

C.1 Foreign

- Output – merchandise export-weighted real gross domestic product, source: RBA; prior to 1991 back-cast with G7 output, source: RBA
- Inflation – trade-weighted, calculated from real and nominal trade-weighted exchange rate, source: RBA Statistical Table F15 Real Exchange Rate Measures
- Interest rates – G7-GDP PPP-weighted interest rates, source: RBA

C.2 Australia

- Exports – real exports, chain volume, seasonally adjusted, source: Australian Bureau of Statistics (ABS), Australian National Accounts: National Income, Expenditure and Product (ABS Cat No 5206.0).
- Investment – real business investment, chain volume, seasonally adjusted, source: ABS Cat No 5206.0
- Output – real non-farm gross domestic product, chain volume, seasonally adjusted, source: ABS Cat No 5206.0
- Inflation – trimmed mean inflation, excluding interest payments and tax changes, seasonally adjusted, source: RBA
- Interest rates – cash rate, average quarter, source: RBA Statistical Table F1 Interest Rates and Yields – Money Market
- Real exchange rate – real trade-weighted exchange rate, source: RBA Statistical Table F15 Real Exchange Rate Measures
- Terms of trade – source: ABS Cat No 5206.0

Appendix D: Further Results

Table D1: DSGE Estimation Results – Long Sample

Coefficient	Description	Prior			Posterior	
		Density	Mean	Std dev	Mean	90% HPD
θ_1	Calvo domestic prices	B	0.75	0.1	0.93	0.91–0.95
ω_1	Indexation, domestic	B	0.3	0.05	0.40	0.33–0.48
ζ_R	Taylor rule, smoothing	B	0.5	0.1	0.63	0.55–0.72
ζ^{π}	Taylor rule, inflation	N	1.5	0.1	1.51	1.35–1.66
ζ^{va}	Taylor rule, output	B	0.5	0.2	0.73	0.56–0.91
ζ_{dy}	Taylor rule, growth	B	0.5	0.2	0.17	0.02–0.33
θ_m	Calvo import	B	0.75	0.1	0.82	0.76–0.88
ω_m	Indexation, imports	B	0.3	0.05	0.31	0.23–0.39
ρ_a	Technology growth	B	0.5	0.2	0.29	0.43–0.53
ρ_{rp}	Risk premium	B	0.75	0.1	0.94	0.91–0.96
ρ_i	Investment technology	B	0.75	0.1	0.50	0.38–0.62
$\rho_{a,2}$	Export technology	B	0.75	0.1	0.78	0.68–0.88
ρ_g	Preferences	B	0.75	0.1	0.84	0.79–0.90
$\eta''(1)$	Adjustment costs	N	4	1	0.95	0.52–1.34
υ	Habits	B	0.5	0.1	0.26	0.16–0.36
Standard deviations						
σ_{rp}	Risk premium	IG	1	1	0.46	0.35–0.57
σ_M	Monetary policy	IG	0.25	0.1	0.21	0.17–0.26
σ_a	Technology	IG	1	1	0.57	0.37–0.76
σ_2	Export technology	IG	2	1	2.30	2.03–2.55
σ_i	Investment technology	IG	5	1	5.08	3.36–6.68
σ_c	Preferences	IG	1	1	3.26	2.51–3.97
σ_1	Domestic mark-up	IG	0.5	1	0.19	0.16–0.21
σ_m	Imports mark-up	IG	0.5	1	0.32	0.15–0.50

Notes: Prior distributions are B – Beta, N – Normal, IG – inverse Gamma; HPD denotes highest probability density

Table D2: RMSE BVARX Relative to Benchmark Models

Benchmark models – excludes error-correction terms

Series	Minn	AR	DSGE	Combined	Minn	AR	DSGE	Combined
Quarterly		*1-quarter-ahead*				*2-quarters-ahead*		
Δ Exports	1.10	1.01	0.93	0.98	0.99	0.99	0.93	0.96
Δ Investment	1.03	1.06	1.05	1.08	1.08	1.19	1.00	1.10
Growth	1.04	1.07	0.93	1.04	1.19	1.16	0.96	1.09
Inflation	1.01	1.06	0.96	0.99	0.96	0.98	0.89	0.93
Cash rate	1.02	0.96	0.88	0.98	0.97	0.89	0.94	0.98
Δ Rer	1.21	1.05	0.98	1.05	1.03	0.98	0.96	1.02
Year-ended		*1-year-ahead*				*2-years-ahead*		
Δ Exports	1.12	1.03	0.74	0.90	1.28	1.05	0.81	0.93
Δ Investment	0.88	1.13	0.90	0.98	0.82	0.96	0.92	0.93
Growth	1.25	1.09	0.84	1.00	1.02	0.97	0.89	0.94
Inflation	0.97	0.99	0.90	0.94	0.96	1.00	0.99	0.98
Cash rate	0.98	0.94	1.12	1.08	0.99	1.05	1.14	1.07
Δ Rer	1.03	1.02	0.92	1.05	1.13	1.00	0.93	0.98

Notes: Values less than 1 indicate the RSME of the BVARX is less than the benchmark; 1- and 2-years-ahead interest rate forecasts are for the level; VARs have 2 lags; Minn denotes SOE Minnesota VAR; AR denotes autoregressive; Rer denotes real exchange rate

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