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An Empirical Mass Formula for Mesons. I

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One of the important problems in physics is to relate the masses and quantum numbers of elementary particles. There are many approaches to the problem from group theoretical[1] or composite models.[2] We expect that it may be possible to relate masses to quantum numbers by a (yet unknown) dynamical mechanism with constructive force different from interactive one.*) In this note we propose an empirical mass formula for mesons. The relevant formula is required to give a deep insight into dynamical origin of masses and should be suitable for overall fittings if possible. Taking into account this requirement, in our formula we will choose a small number of parameters and make coefficients of quantum numbers rational numbers. We can apply the formula to most of the meson particles listed in Ref. 4).*)

Our empirical mass formula for mesons is

$$M_{ei}^2(J^P, Y'=|Y|, T) = c_1+c_2[J(J+1)+Y'(Y'+1) -6T(T+1)]+\tfrac{1}{4}J(J+1), \qquad (1)$$

where M_{ei} denotes eigenvalues of mass operator and J is spin, P parity, Y hypercharge and T isospin of particles respectively, and c_1, c_2 are parameters. We will apply formula (1) to mesons which have $J^P=0^\pm, 1^\pm, 2^\pm$, $T=\frac{1}{2}, 1, 0$ and $Y=\pm 1, 0$. From (1), the mass relation for a multiplet with the same spin-parity $J^\pm$ (we call this multiplet a spin-multiplet) is

$$12M_{ei}^2(J^\pm, 1, \tfrac{1}{2}) - 2.5M_{ei}^2(J^\pm, 0, 1) = 9.5M_{ei}^2(J^\pm, 0, 0), \qquad (2)$$

c.f., the $SU(3)$ mass formula for mesons is

$$M^2=A+B[T(T+1)-\tfrac{1}{4}Y^2],$$

*) The constructive force means the one which works in internal spaces of particles and constructs masses of particles. The interactive force is an origin of interactions between particles.

*) We make some remarks: $\boldsymbol{E}$ meson has probably $J^P=0^-$ but an assignment $J^P=1^+$ is not excluded and $M_{A_2}^2=\frac{1}{2}(M_{A_2L}^2+M_{A_2H}^2)$.

Table I. Comparison of mass relation (2) with experimental data.[4] Here the fourth column gives outputs. Parentheses in the fourth column represent values from $SU(3)$ mass relations.

J^P \ M^2	$M_{\mathrm{ex}}^2(J^P,\ 1,\ \frac{1}{2})$	$M_{\mathrm{ex}}^2(J^P,\ 0,\ 1)$	$M_{\mathrm{ei}}^2(J^P,\ 0,\ 0)$	Experiments
0^-	$M_K^2=0.246(\mathrm{GeV})^2$	$M_\pi^2=0.019$	0.306 (0.322)	$M_\eta^2=0.301$ $M^2_{\eta'(X_0)}=0.917$
1^-	$M^2_{K^*}=0.796\pm0.045$	$M_\rho^2=0.585\pm0.095$	0.852 (0.866)	$M_\phi^2=1.039\pm0.004$ $M_\omega^2=0.614\pm0.009$ $\frac{1}{2}(M_\phi^2+M_\omega^2)=0.827$
2^+	$M^2_{K_N}=1.985\pm0.135$	$M^2_{A_2}=1.69$	2.063 (2.083)	$M^2_{f'}=2.29\pm0.11$ $M_f^2=1.60\pm0.19$ $\frac{1}{2}(M^2_{f'}+M_f^2)=1.945$
0^+	$M^2_{K_\pi}=1.166$	$M_\delta^2=0.933$	1.227 (1.244)	$M^2_{S^*}=1.13\pm0.09$ $M^2_{\sigma(\varepsilon)}=0.5$
1^+	$M^2_{K_A}=1.54\pm0.08$ $=1.77\pm0.09$	$M^2_{A_1}=1.14\pm0.10$ "	1.645 (1.673) 1.936 (1.980)	$M_D^2=1.66\pm0.04$ $M_E^2=2.02\pm0.10$
	$M^2_{K_A}=1.54\pm0.08$ $=1.77\pm0.09$	$M_B^2=1.53\pm0.13$ "	1.543 (1.543) 1.833 (1.850)	?
2^-	$M^2_{K_A(L)}=3.15\pm0.14$	$M^2_{\pi_A(A_3)}=2.67$ ±0.15	3.276 (3.310)	?

i.e.,

$$4M^2(J^\pm,1,\tfrac{1}{2})-M^2(J^\pm,0,1)=3M^2(J^\pm,0,0). \qquad (3)$$

Results from formula (2) are shown in Table I.

Formula (2) for respective spin-multiplet is in agreement with the experimental data.

Next we group mesons into two multiplets (we call it empirical super-multiplet) in which several spin-multiplets with different spin-parity are contained, and apply formula (1) to respective empirical super-multiplet. Here we choose $[0^-,1^-,2^+]$ and $[0^+,1^+,2^-]$ as two empirical super-multiplets. Results from formula (1) are shown in Tables II-(a) and II-(b).

Characteristic features of overall fittings of (1) are as follows:

(A) We can get overall fittings of two empirical supermultiplets by two parameters.

(B) The empirical super-multiplet $[0^-,1^-,2^+]$ which contains well-established spin-multiplets fits the formula better than $[0^+,1^+,2^-]$.

(C) There is a common term, $\frac{1}{4}J(J+1)$, in formula (1).

(D) As experimental data for mesons having $J^P=1^+$, there seems to exist two groups One group includes lighter K_A, A_1 and D mesons and the other heavier K_A and B mesons. Our formula (1) for given values of c_1 and c_2 may represent the second group.

(E) A mean $(\mathrm{mass})^2$ difference between $[0^-,1^-,2^+]$ and $[0^+,1^+,2^-]$ is $0.921(\mathrm{GeV})^2$ because the respective values of c_2 for $[0^-,1^-,2^+]$ and $[0^+,1^+,2^-]$ are almost the same.

Two simple results (A) and (E) cannot be easily understood for the $SU(6)$ mass formula*)

*) This does not mean that $SU(6)$ symmetry is wrong, because in $SU(6)$ symmetry the formula (4) is not the most general mass formula and there is no such classification as $[0^-,1^-,2^+]$ and $[0^+,1^+,2^-]$.

Table II. Comparison of formula (1) with experimental data.[4] Here M^2_{ei} means predicted mass values.

(a) $[0^-, 1^-, 2^+]$ $c_1=0.306(\text{GeV})^2$ $c_2=0.024(\text{GeV})^2$

J^P	$Y'=\|Y\|$	T	$M^2_{ei}(\text{GeV})^2$	M^2 experiment
0^-	1	$\frac{1}{2}$	0.246 (input)	$M_K{}^2=0.246$
	0	1	0.019 (input)	$M_\pi{}^2=0.019$
	0	0	0.306	$M_\eta{}^2=0.301$
1^-	1	$\frac{1}{2}$	0.794	$M^2_{K^*}=0.796\pm0.045$
	0	1	0.567	$M_\rho{}^2=0.585\pm0.095$
	0	0	0.854	$\frac{1}{2}(M_\phi{}^2+M_\omega{}^2)=0.827$
2^+	1	$\frac{1}{2}$	1.889	$M^2_{K_N}=1.985\pm0.135$
	0	1	1.662	$M^2_{A_2}=1.69$
	0	0	1.949	$\frac{1}{2}(M^2_{f'}+M_f{}^2)=1.945$

(b) $[0^+, 1^+, 2^-]$ $c_1=1.227(\text{GeV})^2$ $c_2=0.025(\text{GeV})^2$

J^P	$Y'=\|Y\|$	T	$M^2_{ei}(\text{GeV})^2$	M^2 experiment
0^+	1	$\frac{1}{2}$	1.166 (input)	$M^2_{K_\pi}=1.166$
	0	1	0.933 (input)	$M_\delta{}^2=0.933$
	0	0	1.227	$M^2_{S^*}=1.13\pm0.09$
1^+	1	$\frac{1}{2}$	1.715	$M^2_{K_A}=1.54\pm0.08$ or 1.77 ± 0.09
	0	1	1.482	$M^2_{A_1}=1.14\pm0.10$, $M_B{}^2=1.53\pm0.13$
	0	0	1.776	$M_D{}^2=1.66\pm0.04$
2^-	1	$\frac{1}{2}$	2.813	$M^2_{K_A(L)}=3.15\pm0.14$
	0	1	2.580	$M^2_{\pi_A(A_3)}=2.67\pm0.15$
	0	0	2.874	?

$$M^2=a+b[T(T+1)-\tfrac{1}{4}Y^2]+cJ(J+1). \tag{4}$$

Further

(F) Since we get $c_1([0^+,1^+,2^-])-c_1([0^-,1^-,2^+])=0.921(\text{GeV})^2$ and on the other hand an experimental value of mean (mass)^2 of $[0^-,1^-,2^+]$ is $0.919(\text{GeV})^2$, we think that these values $\sim0.9(\text{GeV})^2$ may represent a global structure of meson mass spectrum.*)

Though we applied formula (1) to $[0^-, 1^-, 2^+]$ and $[0^+, 1^+, 2^-]$, taking the same parameters within respective empirical super-multiplets, we do not know a theory which derives these classifications of mesons. It is not always necessary to insist on this classification. For example, it may be possible to classify such as $[0^-, 1^-]$, $[2^+, \cdots]$, $\cdots$ and $[0^+, 1^+]$, $[2^-, \cdots]$, $\cdots$. In this case, however, we cannot include **g** meson ($J^P=1^-, 3^-, \cdots$ with 3^- favored) in $[2^+, \cdots]$ or $[2^-, \cdots]$.

We do not yet derive the mass formula theoretically, but anticipate that formula (1) will be derived from an internal mechanism of particles and, moreover, the se-

*) On the other hand, fine structures of masses are already pointed out, for example, by Nambu and Takabayasi.[3]

cond and third terms of formula (1) correspond to an anharmonic oscillator and a rotator[5] respectively.

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1) See, for example, as to $SU(3)$ and $SU(6)$, A. Pais, Rev. Mod. Phys. **38** (1966), 215, and as to $SU(4)$, S. Nakamura, Prog. Theor. Phys. **32** (1964), 418.
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3) Y. Nambu, Prog. Theor. Phys. **7** (1952), 595; T. Takabayasi, Nuovo Cim. **30** (1963), 1500.
4) N. Barash-Schmidt et al., *Particle properties* UCRL-8030 (1970).
5) As to a harmonic oscillator, T. Takabayasi, Phys. Rev. **139** (1965), B1381.
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