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AN EMPIRICAL MODEL OF LABOR SUPPLY IN A LIFE CYCLE SETTING

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ABSTRACT

This paper formulates and estimates a structural life cycle model of labor supply. Using theoretical characterizations derived from an economic model of life cycle behavior, a two-stage empirical analysis yields estimates of intertemporal and uncompensated substitution effects which provides the information needed to predict the response of hours of work to life cycle wage growth and shifts in the lifetime wage path. The empirical model developed here provides a natural framework for interpreting estimates found in other work on this topic. It also indicates how cross section specifications of hours of work can be modified to estimate parameters relevant for describing labor supply behavior in a lifetime setting.

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Introduction

Over the past several years there has been considerable activity in formulating life cycle models of labor supply. The crucial implication of this work is that hours of work is a function of both current and future wages, as well as such variables as wealth and the worker's future prospects. Most of this work has gone unnoticed in the empirical literature. This study synthesizes existing knowledge into an estimable model of labor supply that incorporates life cycle factors, and relates it to empirical models typically found in the literature. A full life cycle model of hours of work is estimated using data on prime age males from the Michigan Panel on Income Dynamics.

Most empirical work on labor supply ignores life cycle theory, assuming instead a one-period model. Typically, annual hours of work are regressed on the current hourly wage rate and some measure of property income. A worker, however, determines his current labor supply in a life cycle setting. Unless credit markets are "perfectly imperfect" and there is no human capital accumulation, the supply of labor is a function of current and future discounted wage rates as well as wealth and constraints in other periods. Accordingly, regressions of hours of work on current hourly wage rates yield a wage coefficient that mixes the response of labor supply to wage changes of three types: those arising from movements along a given lifetime wage profile; those arising from shifts in the wage profile; and those arising from changes in the profile slope. As a result, the wage coefficient usually reported in empirical studies has no behavioral interpretation, in the context of a life cycle framework.

This study uses theoretical characterizations derived from an economic model of lifetime consumption and hours of work to formulate an empirical model of labor supply. These theoretical characterizations represent a natural extension of Friedman's (1957) permanent income theory to a situation in which the relative price of consumption and leisure varies over the life cycle. These characterizations separate those factors determining a consumer's dynamic behavior from those factors determining differences in consumption and labor supply across consumers. This separation leads to a manageable empirical model that allows one to disentangle and to estimate the separate responses of labor supply to wage changes due to movements along a lifetime wage profile and also due to parametric changes in this profile. In addition, one can estimate the effects of wealth and demographic characteristics on lifetime hours of work.

The empirical model also provides a natural framework for interpreting labor supply estimates found in the literature. It suggests how the typical cross sectional models of labor supply can be modified with the introduction of control variables so that wage coefficients represent economically meaningful parameters. It provides a new and simpler interpretation of work due to Ghez and Becker (1975). This model also provides a new interpretation of empirical work due to Lucas and Rapping (1969).

The organization of this paper is as follows. Section I outlines an economic model of life cycle behavior. Section II develops and discusses an empirical model of labor supply. Section III relates this empirical model to other empirical specifications found in the literature. Section IV contains the empirical analysis.

I. A Life Cycle Model of Consumption and Labor Supply

The consumer is assumed to choose consumption and hours of leisure at each age to maximize a lifetime preference function that is strongly separable over time, subject to a wealth constraint. Let utility at age t be given by the concave function $U(C(t),L(t))$ where $C(t)$ is the amount of market goods consumed and $L(t)$ is the number of hours spent in nonmarket activities at age t. Consumers are assumed to operate in an environment of perfect certainty. The consumer starts life with assets $A(0)$. At each age t he faces a real wage rate equal to $W(t)$ and this wage rate is assumed to be exogenously given. The consumer can freely borrow and lend at a real rate of interest equal to r, and his rate of time preference is ρ. A lifetime is assumed to consist of T+1 periods with τ being the total number of hours in each period.

Formally, the consumer's problem is to choose $C(t)$ and $L(t)$ at each age to maximize the lifetime preference function

$$(1) \qquad G\left[\sum_{t=0}^{T} \frac{1}{(1+\rho)^t} U(C(t),L(t))\right]$$

subject to the wealth constraint

$$(2) \qquad A(0) + \sum_{t=0}^{T} \frac{1}{(1+r)^t} N(t)W(t) = \sum_{t=0}^{T} \frac{1}{(1+r)^t} C(t)$$

where $G(\cdot)$ is a monotonically increasing function and $N(t) \equiv \tau - L(t)$ is hours of work at age t.[1]

Conditions for an optimum are satisfaction of the budget constraint and

$$(3) \qquad U_1(C(t),L(t)) = \left(\frac{1+\rho}{1+r}\right)^t \lambda \qquad t = 0,\cdots,T$$

$$(4) \qquad U_2(C(t),L(t)) \geq \left(\frac{1+\rho}{1+r}\right)^t \lambda, \ W(t) \qquad t = 0,\cdots,T$$

where subscripts denote partial derivatives and λ is defined by $\lambda = \lambda^*/G'$ where λ^* is the LaGrange multiplier associated with the budget constraint (i.e., λ^* is the marginal utility of wealth in period 0) and G' is the derivative of G. According to condition (3), consumption is chosen so that the marginal utility of consumption equals the marginal utility of wealth after adjusting for a discount factor which depends on the rate of time preference and the rate of interest. Condition (4) determines the consumer's choice of leisure. If it is an equality, then a positive amount of labor is supplied to the market. If it is a strict inequality, then all time is devoted to nonmarket activities.

Using the definition of labor supply (i.e., $N(t) = \tau - L(t)$) and the implicit function theorem, it is possible to jointly solve equations (3) and (4) for consumption and labor supply as functions of the form

$$(5) \qquad C(t) = C\left(\left(\frac{1+\rho}{1+r}\right)^t \lambda, \ W(t)\right) \qquad t = 0,\cdots,T$$

$$(6) \qquad N(t) = N\left(\left(\frac{1+\rho}{1+r}\right)^t \lambda, \ W(t)\right) \qquad t = 0,\cdots,T$$

The functions $C(\cdot,\cdot)$ and $N(\cdot,\cdot)$ depend only on the functional form of $U(\cdot,\cdot)$. As a consequence of concavity of U and the assumption that leisure is a normal good, they satisfy

(7) $\quad C_1 < 0 \qquad N_1 \geq 0 \qquad N_2 \geq 0.$[2]

These consumption and labor supply functions allow for corner solutions for hours of work either at age t or at any other age t'. No matter what the consumer's labor force participation pattern over his lifetime, consumption and labor supply decisions at any age (including the decision to set hours of work equal to zero) are completely determined by the functions $C(\cdot,\cdot)$ and $N(\cdot,\cdot)$ and the values of the variables $\left(\frac{1+\rho}{1+r}\right)^t \lambda$ and $W(t)$.[3,4]

The relationships given by (5) and (6) hereafter will be referred to as the "λ constant" consumption and labor supply functions. These relationships represent the marginal utility of wealth constant demand functions for consumption and leisure for a particular form of the lifetime preference function given by (1); namely, the one obtained when G is the identity transformation. For this particular choice of G, λ is the marginal utility of wealth in period 0. Given a choice of $U(\cdot,\cdot)$, it is theoretically possible to compute a unique value for λ using data on an individual's consumption, labor supply and wage rate at a point in time. This fact receives much attention in the formulation of the empirical model which is discussed in the next section.

Substituting the λ constant consumption and labor supply functions into the budget constraint given by (2) yields the equation

$$(8) \qquad A(0) = \sum_{t=0}^{T} \frac{1}{(1+r)^t}\left[C\left(\left(\frac{1+\rho}{1+r}\right)^t \lambda,\ W(t)\right) - W(t)N\left(\left(\frac{1+\rho}{1+r}\right)^t \lambda,\ W(t)\right)\right]$$

This equation implicitly determines the optimal value of λ. λ, then, can be expressed as a function of initial assets, lifetime wages, interest rates, rates of time preference and "consumer tastes." Concavity of preferences and the assumption that leisure at all ages is a normal good imply

$$(9) \qquad \frac{\partial \lambda}{\partial A(0)} < 0 \quad \text{and} \quad \frac{\partial \lambda}{\partial W(t)} \leq 0 \qquad t = 0,\cdots,T.$$ [5]

Inspection of the λ constant functions reveals that consumption and labor supply decisions at a point in time are related to variables outside the decision period only through λ. Thus, except for the value of the current wage rate, λ summarizes all information about lifetime wages and property income a consumer requires to determine his optimal current consumption and labor supply. At any age, any path of wages or property income over a consumer's lifetime that keeps λ and the current wage constant implies the same optimal current consumption and labor supply behavior.

The λ constant functions represent an extension of Friedman's (1957) permanent income theory to a situation in which the relative price of consumption and leisure varies over the life cycle. According to these functions, current consumption and labor supply decisions depend on a permanent component and the current wage rate. λ is like permanent income in the theory of the consumption function. At each point in time it is a sufficient statistic for all historic and future information about lifetime wages and property income that is relevant to the current choice of consumption and labor supply. The usual concept of permanent income or wealth does not qualify as a sufficient statistic for this retrospective and prospective information. Given permanent income, a consumer also requires information on future wages to determine his optimal current consumption and labor supply. Only if wages are constant over the life cycle, or labor supply is exogenously determined, can λ be written as a simple function of permanent income or wealth.

II. An Empirical Model

This section formulates an empirical model of labor supply that is based on the economic model described above. Specific functional forms are proposed for the λ constant labor supply function and the relationship between λ and such variables as lifetime wages and initial assets.

An Empirical Specification for the λ Constant Labor Supply Function

The λ constant consumption and labor supply functions fully characterize a consumer's dynamic behavior in a world of perfect certainty.[6] According to these functions, there are two reasons why a consumer might change his consumption or hours of work as he ages: (1) the real wage rate changes or (2) the rate of interest is not equal to the rate of time preference.[7] Estimating these relationships estimates all of the parameters one requires to predict how a given individual's consumption or labor supply changes over his lifetime.

This study uses panel data to directly estimate the parameters of a λ constant labor supply function. To construct such a function requires a specific form for the contemporaneous utility function.

Assume that consumer i at age t has utility given by

$$(10) \qquad U_i(t) = \gamma_i^*(t)(C_i(t))^{\omega^*} - \gamma_i(t)(N_i(t))^{\omega} \qquad t = 0,\cdots,T$$

where $0 < \omega^* < 1$ and $\omega > 1$ are time invariant parameters common across workers and $\gamma_i^*(t)$, $\gamma_i(t) > 0$ are age specific modifiers of "tastes."

$\gamma_i^*(t)$ and $\gamma_i(t)$ depend on all of consumer i's characteristics which plausibly affect his preferences at age t; these characteristics may include such variables as the number of children present at age t, the consumer's education and even age itself.

Assuming an interior optimum,[8] the implied λ constant labor supply function for consumer i at age t in natural logs is

$$(11) \qquad \ell n N_i(t) = \frac{1}{\omega - 1}\left[\ell n\lambda_i - \ell n\gamma_i(t) - \ell n\omega + t\ell n\left(\frac{1+\rho}{1+r}\right) + \ell n W_i(t)\right]$$

Assuming that "tastes" for work are randomly distributed over the population according to the equation $\ell n\gamma_i(t) = \sigma_i - u_i(t)$, the labor supply function can be written as

$$(12) \qquad \ell n N_i(t) = F_i + t \cdot b + \delta \ell n W_i(t) + u_i(t)$$

where $F_i = \frac{1}{\omega - 1}\ (\ell n\lambda_i - \sigma_i - \ell n\omega)$, $b = \frac{1}{\omega - 1}\ \ell n\left(\frac{1+\rho}{1+r}\right)$ and $\delta = \frac{1}{\omega - 1}$. σ_i and $u_i(t)$ are unobserved variables representing the unmeasured characteristics of consumer i; $u_i(t)$ is an error term with zero mean.[9]

The intercept term F_i in this equation represents a time invariant component that is unique to individual i. This study treats F_i as a fixed effect. Since F_i contains $\ell n\lambda_i$ as one of its components, one cannot assume that F_i is a "random factor" uncorrelated with exogenous variables of the model. Inspection of equation (8) reveals that λ depends on the values of variables and constraints in all periods. By construction, F_i is correlated with any exogenous variables used to predict a consumer's wages or wealth. Hence, treating F_i as part of the error term would result in biased parameter estimates of the labor supply function. Treating F_i as a fixed effect, on the other hand, avoids this bias.

Estimating the parameters of equation (12) only requires that data actually observed within the sample period. Regressions of current hours of work on individual specific intercepts and current wage rates produces a full set of parameter estimates.[10] Because F_i captures the effect of $\ln\lambda_i$, its estimated value summarizes all of the retrospective and prospective information relevant to consumer i's current choices. As there is no need to forecast any life cycle variables that are outside the sample period, the λ constant functions afford a considerable simplification of the empirical analysis.

Use of the λ constant functions allows one to estimate parameters needed to characterize dynamic behavior without introducing any assumptions regarding a consumer's behavior outside the sample period. To fully appreciate this point, consider the problem of predicting the additional hours of work a consumer will supply in response to observing a higher wage rate than he observed at a younger age. To obtain an estimate of this response using a traditional model of life cycle labor supply, one must formally incorporate the worker's future plans in the model. For example, if the worker anticipates an early retirement, future wages corresponding to the retirement years do not influence current labor supply. Thus, the researcher must not include these wages as explanatory variables. Such considerations lead to difficult data requirements and complicated estimation procedures. Using equation (12), on the other hand, it is possible to empirically analyze this problem without knowing anything about a worker's future plans; an individual constant term for each worker accounts for a worker's future plans in a parametrically simple way.

An Empirical Specification for Individual Effects

Estimation of the λ constant labor supply function given by (12) does not directly estimate all of the parameters required to characterize life cycle labor supply. The level of a consumer's hours of work also depends on the value of his individual component, F_i, and we know that F_i, through λ_i, is a function of the consumer's wealth and his lifetime wage path. To explain any aspect of labor supply other than dynamic behavior (e.g., how the hours of work of two individuals differ at a point in time), one must confront the problem of predicting individual effects.

From the above theoretical analysis, we know that the value of F, or more properly λ, is uniquely determined by the implicit equation given by (8). This equation does not admit an analytical solution for λ given the specific form of the utility function given by (9), even if it is known that this function applies to all ages and the consumer works in each period. λ is a complicated function of initial assets, lifetime wages, the interest rate, the rate of time preference, and parameters representing unobserved "taste" variables.[11] Using such a relationship as an empirical specification is not feasible.

This study assumes that equation (8) implies a solution for λ in which $\ell n\lambda$ can be approximated as a linear function of measured characteristics, the natural log of wages at each age, initial wealth, and an unobserved random variable representing unmeasured characteristics. With this assumption the implied equation for F_i is

$$(13) \qquad F_i = Z_i\phi + \sum_{t=0}^{T}\gamma(t)\ell nW_i(t) + A_i(0)\ \theta + a_i^*$$

where Z_i is a vector of observed variables (e.g., family background variables), a_i^* is an error term, and ϕ, $\gamma(t)$, and θ are parameters assumed to be constant across consumers.[12] This structural relationship for F_i implicitly assumes that each consumer has a working life of T+1 years. According to the theoretical restrictions given by (9), the $\gamma(t)$'s and θ should all be negative.

Unfortunately, to formulate an estimable version of an equation for F_i, we require additional assumptions concerning the forms of the lifetime wage and income paths. In contrast to the λ constant labor supply function, estimating the parameters of equation (13) requires data which normally is not available. Most variables appearing in this equation are not directly observed, including the dependent variable F_i, wages outside the sample period, and initial wealth. While estimates of F_i are obtained as a by-product from estimating equation (12), we still require a mechanism for predicting wages outside the sample period and initial permanent income. We do this by introducing lifetime profiles for wages and income.

This study assumes that the lifetime wage path is

$$(14) \qquad \ell n W_i(t) = \pi_{0i} + t\pi_{1i} + t^2\pi_{2i} + V_i(t)$$

where π_{0i}, π_{1i}, and π_{2i} are linear functions of the form

$$(15) \qquad \pi_{ji} = M_i g_j \qquad j = 0, 1, 2,$$

M_i is a vector of exogenous determinants of wages which are constant over the consumer's lifetime (e.g., education and background variables), g_j, $j = 0, 1, 2$ are vectors of parameters, and $V_i(t)$ is an error term. This path assumes that wages follow a quadratic equation in age with an intercept and slope coefficients that depend on age-invariant characteristics of the consumer.

To predict a consumer's initial wealth is complicated by the fact that most data sets do not contain extensive measures of even the consumer's current wealth. Some measure of the consumer's property or nonwage income during the sample period, however, is usually available.[13] Let $Y_i(t)$ and $A_i(t)$ denote the property income and assets of consumer i at age t. If $Y_i(t)$ is income flow generated by investing assets $A_i(t)$ at a rate of interest equal to r, we have the relationship $Y_i(t) = A_i(t)r$. Assume that the following quadratic equation in age approximates the lifetime path for property income

$$(16) \qquad Y_i(t) = \alpha_{0i} + t\alpha_{1i} + t^2\alpha_{2i} + \nu_i(t)$$

where α_{0i}, α_{1i}, and α_{2i} are linear functions of the form

$$(17) \qquad \alpha_{ji} = S_i q_j \qquad j = 0, 1, 2,$$

S_i is a vector of measured age-invariant characteristics of consumer i (e.g., education and background variables), q_j, j = 0, 1, 2 are parameter vectors and $\nu_i(t)$ is an error term.[14] The intercept α_{0i} can be thought of as a measure of consumer i's permanent income at age 0; that is, $\alpha_{0i} = A_i(0)r$.

Combining the lifetime paths for wages and income with equation (13) creates an equation for F_i that can be estimated using data observed within the sample period. Substituting the wage process given by (14) and the relationship $\alpha_{0i} = rA_i(0)$ into equation (13) yields

(18) $F_i = Z_i\phi + \pi_{0i}\bar{\gamma}_0 + \pi_{1i}\bar{\gamma}_1 + \pi_{2i}\bar{\gamma}_2 + \alpha_{0i}\bar{\theta} + a_i$

or, substituting relations (15) and (17) yields

(19) $F_i = K_i\psi + a_i$

where

(20) $\bar{\gamma}_j = \sum_{t=0}^{T} t^j \gamma(t) \qquad j = 0, 1, 2, \qquad \bar{\theta} = \theta/r,$

K_i is a vector including all age-invariant characteristics determining either wages, income or λ (i.e., all the elements of M_i, S_i, and Z_i), ψ is a vector of coefficients and $a_i = a_i^* + \sum_{t=0}^{T} \gamma(t)V_i(t)$ is a disturbance term which is randomly distributed across workers with zero mean. Equation (18) is a structural relationship between F and the characteristics of a consumer's wage and income profiles. The empirical analysis of this study focuses on estimating the parameters of this equation, ϕ, $\bar{\gamma}_0$, $\bar{\gamma}_1$, $\bar{\gamma}_2$, and $\bar{\theta}$. As we shall see shortly, these parameters have a sound economic interpretation. Equation (19) is essentially a reduced form equation for (18). By estimating the parameters of this equation, ψ, it is possible to predict how F varies across consumers using only age-invariant characteristics of the consumer as explanatory variables.

This section has developed an empirical model of lifetime labor supply. The λ constant hours of work function given by (12) and the structural equation for F given by (18) provide a manageable empirical model for analyzing labor supply behavior in a life cycle setting. This model naturally suggests a two-stage estimation procedure. In the first stage one estimates the parameters of the λ constant hours of work function. This first stage provides all the information a researcher requires to predict how the labor supply of a given consumer will differ at two points in time. A second-stage regression of estimated "individual effects" on the determinants of F_i permits estimation of the coefficients ϕ, $\bar{\theta}$ and the $\bar{\gamma}$'s. These coefficient estimates provide the additional information one requires to predict how labor supply will differ across consumers. Two-stage estimation of the λ constant specifications exploits the special characteristics of panel data to characterize life cycle behavior with a minimal amount of computational burden.

The empirical specifications for the λ constant system developed above can be extended to account for the presence of more than one form of market consumption or type of leisure. They also are consistent with a world in which the consumer is uncertain about his future lifetime path of wages and property income. Elsewhere, I show that much of the formal structure of the deterministic theory carries over with full force to the uncertainty model (MaCurdy, 1978). The empirical specification for the λ constant labor supply function and the equation for F proposed above are still applicable in the uncertainty case.

Interpretation of Parameters

In investigating the effect of changes in wages on labor supply, it is important to separate parametric change of the sort usually contemplated in comparative static exercises from evolutionary change due to movement along a life cycle wage path.[15] A parametric wage change refers to shifts in a life cycle wage profile (e.g., a shift from path II to path I in Figure 1), while an evolutionary wage change refers to movements along a given profile (i.e., along any path in Figure 1). Thus, parametric wage changes refer to differences in wages across

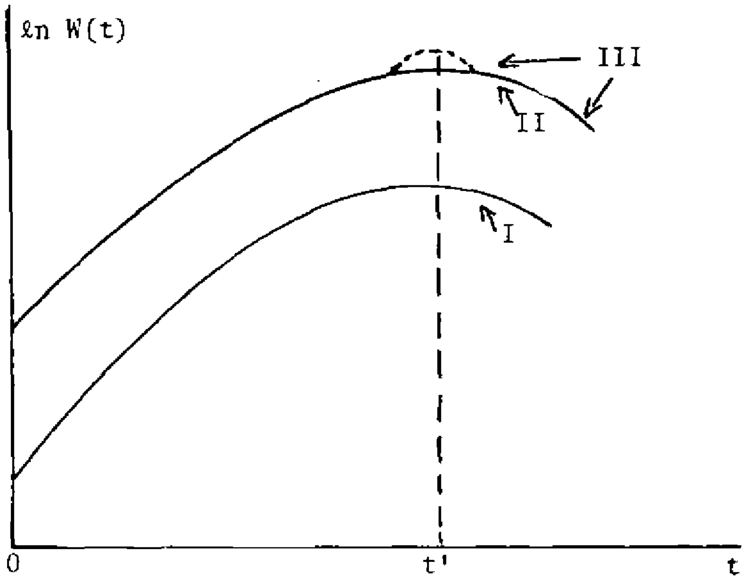

Figure 1

consumers while evolutionary wage changes refer to differences in wages across time for the same consumer.[16]

Consider the behavior of labor supply over the life cycle. As a consumer ages, he adjusts his hours of work in response to the different wage rates he observes at each point in his lifetime. These labor supply adjustments represent responses to evolutionary wage changes; they reflect the consumer's desire to supply more hours in those periods with highest wages. There is no wealth effect associated with this kind of wage variation since the wage profile is known to the consumer at the beginning of his lifetime and changes in wages are due only to movement along this given profile. It is apparent from the labor supply function given by (12) that the value of the parameter δ determines the hours of work response to evolutionary wage changes. Hereafter, I will refer to δ as the intertemporal substitution elasticity. The theoretical prediction for its sign is positive.[17] For the particular form of the utility function given by (10), δ is also the direct elasticity of substitution for hours of work in any two periods.[18]

Now compare the labor supply profiles of two consumers who face wage paths II and III respectively. As illustrated in Figure 1, the wage profiles for consumers II and III are the same except at age t' when consumer III's wage rate is higher than consumer II's. Let Δ denote the absolute value of this difference in period t' wages. This wage difference represents a parametric wage change because it involves a shift in the lifetime path of wages. It causes the labor supply profiles for consumers II and III to be different at all ages. Comparing these labor supply profiles is the sort of problem usually considered in comparative static exercises. In terms of the empirical model outlined above, this higher wage rate has two effects on consumer III's labor supply. The first effect is on the value of F. According to equation (13), consumer III will set a value for F which is lower than the value of F for consumer II by an amount equal to $\gamma(t') \cdot \Delta$. This decline in F implies that at all ages other than t' consumer III's labor supply will be less than consumer II's by a constant fraction. At age t' there is a second effect of the wage difference. Neglecting the decline in F, consumer III's labor supply at age t' will be higher by an amount equal to $\delta \cdot \Delta$. Thus, the total impact on consumer III's hours of work at age t' is $(\delta + \gamma(t')) \cdot \Delta$. Since $\delta > 0$ and $\gamma(t') < 0$, there is no sign prediction for $\delta + \gamma(t')$, so consumer III's hours of work at age t' may be greater than or less than consumer II's.

The parameters $\gamma(t')$ and $\delta + \gamma(t')$, then, determine the difference in consumer II's and consumer III's labor supply profiles which is due to the discrepancy in their wage rates at age t'. $\gamma(t')$ and $\delta + \gamma(t')$ correspond to the usual concepts of cross- and own-uncompensated substitution elasticities. These elasticities describe the response of labor supply to parametric wage changes. They can be used to predict differences in labor supply across consumers. These elasticities do not directly provide information on the response of labor supply to evolutionary wage changes; so, they cannot be used to predict differences in a given consumer's labor supply over time. Since the intertemporal substitution elasticity exceeds the own uncompensated substitution elasticities (i.e., $\delta > \delta + \gamma(t')$), one expects an evolutionary wage change to induce a larger labor supply response than a comparable parametric wage change. The wealth effect associated with a parametric wage change accounts for the smaller labor supply response.

Comparing the labor supply profiles for consumers I and II also involves a parametric wage change. As illustrated in Figure 1, consumer II's wage profile exceeds consumer I's by a constant fraction over the entire life cycle. The parametric wage change associated with moving from consumer I's to consumer II's wage profile is analogous to increasing the value of the intercept of the lifetime wage path, π_0. This has two effects on labor supply in each period. First, a consumer adjusts his value of F in response to the profile shift. According to equation (19), F declines by an amount equal to $\bar{\gamma}_0 = \sum_{t=0}^{T} \gamma(t)$ times the increase in the value of π_0. This decline in F implies a fall in hours of work at each age. Second, there is a direct impact on each period's labor supply. Holding the value of F constant, a consumer increases his hours of work by an amount equal to δ times the increase in π_0. The implied total impact on each period's labor supply, therefore, is $\bar{\gamma}_0 + \delta$ times the change in π_0. Because $\bar{\gamma}_0$ is unambiguously negative, there is no sign prediction for this total impact. Since $\bar{\gamma}_0 + \delta$ is less than $\gamma(t') + \delta$, however, the response of labor supply to a shift in π_0 should be less in algebraic value than the response to a shift in the wage profile only at age t'. The wealth effect associated with a shift in π_0 is greater. The labor supply profile for consumer II, then, can lie above or below consumer I's labor supply profile. It will lie above consumer I's if $\bar{\gamma}_0 + \delta$ is positive.

The empirical specification of life cycle supply given by equations (12) and (18) provides a convenient framework for estimating the response of labor supply to the different kinds of wage changes described above. Estimation of the λ constant labor supply equation produces an estimate of the intertemporal substitution elasticity δ. This elasticity can be used to predict the response of labor supply to evolutionary wage changes; and, so, it provides the information one needs to describe a consumer's dynamic behavior. Estimating the equation for F given by (18) produces estimates of the parameters $\bar{\gamma}_0$, $\bar{\gamma}_1$, $\bar{\gamma}_2$, and $\bar{\theta}$. These estimates provide the additional information one requires to predict the response of labor supply to parametric wage and wealth changes; and, so, they can be used to explain labor supply differences across consumers. Combining the estimates of $\bar{\gamma}_0$, $\bar{\gamma}_1$, and $\bar{\gamma}_2$ with the estimate of δ allows one to predict how labor supply profiles adjust to changes in the wage path coefficients π_0, π_1, and π_2. This includes both shifts and slope changes of the wage profiles. The estimate of $\bar{\theta}$ provides the information one needs to predict the response of labor supply profiles to changes in a consumer's initial permanent income. Estimating the empirical model proposed in this paper, then, fully characterizes a consumer's lifetime labor supply behavior.

III. Relating Different Models of Labor Supply

This section relates the empirical model developed above to three empirical specifications of labor supply commonly found in the literature. The three specifications considered are: traditional models of lifetime labor supply, the usual cross sectional specifications, and particular models proposed by Ghez-Becker (1975) and Lucas-Rapping (1969).

Alternative Specifications of Lifetime Labor Supply

To formulate an empirical model of hours of work over the life cycle, one naturally uses some form of the labor supply function as a guide for his empirical specifications. There are several alternative, but theoretically equivalent, representations of labor supply behavior in a life cycle setting. The model proposed in the previous section is unfamiliar to many economists.[19] The following analysis relates this model to more traditional empirical specifications of lifetime labor supply.

There are primarily two theoretical characterizations of life cycle labor supply behavior that underlie most empirical models in the literature. The first characterization writes hours of work at any point in time as a function of initial wealth and the lifetime wage path. The empirical model outlined above is obviously consistent with such a characterization. Substituting the expression for F given by (13) into the λ constant labor supply function given by (12) yields

$$(21) \qquad \ell n N_i(t) = Z_i\phi + bt + (\delta + \gamma(t))\ell n W_i(t) + \sum_{\substack{j=1 \\ j \neq t}}^{T} \gamma(j)\ell n W_i(j) + \theta A_i(0) + \nu_i(t)$$

where $\nu_i(t)$ is an error term.

This equation shows how labor supply at any age depends on the consumer's initial assets and the relative wage rates he faces at all ages. This equation verifies results obtained in the previous section; namely, that θ measures the effect of a pure wealth change on labor supply; $\delta + \gamma(t)$, $t = 1,\cdots,T$ are own compensated substitution elasticities; and $\gamma(j)$, $j = 1,\cdots,T$ are cross uncompensated substitution elasticities.

The second theoretical characterization of labor supply behavior, and probably the most familiar to economists, is one that treats current hours of work as a function of current wealth and current and future wages. To derive such a function, a consumer is viewed as resolving his lifetime optimization problem at each age t. Using the model and notation of Section I, it is easy to show that reoptimizing at each age allows one to write labor supply as a function of the form $N(t) = N(\lambda(t), W(t))$ where $\lambda(t)$ is the Lagrange multiplier associated with the wealth constraint at age t. Noting that λ in Section I corresponds to $\lambda(0)$, we see that $\lambda(t) = \left(\frac{1+\rho}{1+r}\right)^t \lambda(0)$. Whereas, $\lambda(0)$ is a function of wages at ages 0 thru T and $A(0)$($\equiv$ assets at age 0), $\lambda(t)$ is a function of wages at ages t thru T and $A(t)$($\equiv$ assets at age t); thus, $N(t)$ can be written as a function of $A(t)$ and $W(t), W(t+1),\ldots,W(T)$.

The empirical model of life cycle labor supply presented in the previous section implies a specific functional form for this second theoretical characterization of labor supply behavior. The empirical specification for the λ constant hours of work function given by (12) is unchanged, except that $\lambda(t)$ replaces $\left(\frac{1+\rho}{1+r}\right)^t \lambda(0)$ as an argument; we account for this in (12) by deleting the linear trend, bt, and replacing $F_i(\equiv F_i(0))$ by $F_i(t) = \frac{1}{\omega-1}(\ln \lambda_i(t) - \sigma_i - \ln \omega)$. Our problem, then, is to determine what the specification for $F_i(0)$ given by (13) implies about an empirical specification for $F_i(t)$. The following observations aid in determining this specification: (1) According to equation (13), $\frac{\partial \ln\lambda(0)}{\partial A(0)} = (1-\omega)\theta$ and $\frac{\partial \ln\lambda(0)}{\partial \ln W(t)} = (1-\omega)\gamma(t)$, $t = 0,\ldots,T$. (2) Wealth at age t, $A(t)$, is endogenously determined by the consumer, and it depends both on initial wealth, $A(0)$, and the entire lifetime path of wages. Defining $m(t) = \frac{\partial A(t)}{\partial A(0)}$, we have $0 < m(t) < 1$ which reflects the fact that a consumer spends part of a dollar increase in $A(0)$ at ages prior to age t. (3) Using the above relation for $\lambda(t)$, we see that $\frac{\partial \ell n\lambda(0)}{\partial A(0)} = \frac{\partial \ell n\lambda(t)}{\partial A(t)} \frac{\partial A(t)}{\partial A(0)}$. Differentiating Roys identity for N(k) and rearranging terms yields $\frac{\partial \ell n\lambda(t)}{\partial \ell nW(k)} = \left(\frac{\partial \ell nN(k)}{\partial \ell n\lambda(0)} + 1\right) \frac{\partial \ell n\lambda(t)}{\partial A(t)} \frac{E(k)}{(1+r)^{k-t}}$, $k = t,\ldots,T$, where $E(k) \equiv N(k)W(k)$ is earnings in period k.[20] Combining these three observations we conclude: $\frac{\partial \ell n\lambda(t)}{\partial A(t)} = \frac{(1-\omega)}{m(t)}$; and $\frac{\partial \ell n\lambda(t)}{\partial \ell nW(k)} = (1-\omega)\gamma(k) \frac{(1+r)^t}{m(t)}$, $k = t,\ldots,T$. Therefore, $F_i(t)$ has the form $F_i(t) = Z_i\phi + \frac{(1+r)^t}{m(t)} \sum_{j=1}^{T} \gamma(j)\ell n\ W_i(j) + A_i(t) \frac{\theta}{m(t)} + a_i^*(t)$. Substituting this expression for $F_i(t)$ into the λ constant hours of work function yields

$$(22) \quad \ell n\ N_i(t) = Z_i\ \phi + \left(\delta + \frac{(1+r)^t}{m(t)} \lambda(t)\right) \ell n\ W_i(t)$$

$$+ \sum_{j=t+1}^{T} \frac{1+r^t}{m(t)} \gamma(j)\ell n\ W_i(j) + A_i(t) \frac{\theta}{m(t)} + \nu_i^*(t)$$

where $\nu_i^*(t)$ is an error term. This equation, then, shows the implied empirical relationship between current hours of work and current wealth and current and future wages.

There are two important points to observe about this empirical specification. First, linearity in log wages and assets requires additional assumptions. In general, $m(t) \left(\equiv \frac{A(t)}{A(0)}\right)$ is a function of both A(t) and W(t),...,W(T); thus, we must assume that m(t) is a constant which depends only on t if we are to treat $\delta + \frac{(1+r)^t}{m(t)} \gamma(t)$, $\frac{(1+r)^t}{m(t)} \gamma(k)$, and $\frac{\theta}{m(t)}$ as parameters.

Second, we see that own and cross uncompensated substitution and wealth coefficients appearing in specification (22) are different than ones we have seen so far. Whereas $\delta + \gamma(t)$ and the $\gamma(j)$'s are own and cross uncompensated elasticities that hold <u>initial</u> wealth and the <u>entire</u> path of wages constant, $\delta + \frac{(1+r)^t}{m(t)} \gamma(t)$ and the $\frac{(1+r)^t}{m(t)} \gamma(j)$'s are these elasticities when one holds wealth <u>at age t</u> and only wages <u>after age t-1</u> constant. Since $\frac{(1+r)^t}{m(t)} > 1$, we see that these latter elasticities are algebraically smaller than the elasticities relative to age 0. This reflects the fact that the total wealth effect arising from a change in a wage rate at any age is spread over fewer periods; so, there is a larger wealth effect in each period. This factor also accounts for the larger absolute value of the coefficient on assets at time t, $\frac{\theta}{m(t)}$, than the coefficient on initial assets, θ. Notice further, since m(t) declines as a function of t, that uncompensated substitution and wealth coefficients fall monotonically as one advances the lifetime optimization problem to a later age. For this reason, use of specifications like (22) for an empirical analysis of life cycle labor supply is not as attractive as using a specification like (21).

<u>Cross Section Specifications</u>

Most cross sectional work on labor supply in the literature to date ignores life cycle theory. Typically, annual hours of work are regressed on the current hourly wage rate and some measure of property income. The estimated wage and income coefficients from such a regression are interpreted as measuring uncompensated substitution and income effects.

Recognition of life cycle considerations suggests two questions: (1) What behavioral parameters are being estimated in the typical cross sectional model of labor supply? (2) Is it possible to modify cross sectional models so that they account for life cycle factors? The following analysis answers these two questions.

Consider cross sectional estimates of a labor supply equation of the form

$$\ell nN_i(t) = \beta_1 + \beta_2 \, \ell n\hat{W}_i(t) + \beta_3 \hat{Y}_i(t) + \beta_4 Q_i + e_i \tag{23}$$

where $\ell n\hat{W}_i(t)$ and $\hat{Y}_i(t)$ are the predicted values of the consumer's current wage rate and property income, Q_i is a vector of exogenous variables that control for demographic characteristics, the β's are parameters, and e_i is an error term. Equation (23) assumes that a researcher estimates the parameters of the labor supply function using simultaneous equation techniques. Such techniques recognize that a consumer's wage rate and property income are endogenous variables. There are three sources of endogeneity for these variables: reporting error; the presence of "transitory" components of wages and income;[21] and mutual dependence due to past investment decisions. Accounting for the endogeneity of $\ell nW_i(t)$ and $Y_i(t)$ has been shown to have a significant effect on cross sectional estimates.[22]

Equation (23) provides a useful framework for restating the two questions posed above. The typical cross sectional analysis assumes that $\beta_4 = 0$; it excludes all control variables Q_i. Our first question amounts to asking whether the parameters β_2 and β_3 have an economic interpretation in such an analysis. The second question asks whether one can introduce a set of control variables Q_i so that β_2 and β_3 represent parameters relevant to describing lifetime labor supply behavior.

To answer these questions we need to develop cross section specifications that are consistent with empirical models of lifetime labor supply. Using equations (12), (18), and (19) and the lifetime wage and income paths given by (14) and (16), it can be shown that the following two specifications apply:

$$\ell nN_i(t) = K_i \psi + bt + \delta \, \ell n\hat{W}_i(t) + f_i \tag{24}$$

$$\ell nN_i(t) = \beta_1 + (\delta + \bar{\gamma}_0) \ell n\hat{W}_i(t) + \bar{\theta} \hat{Y}_i(t) + \beta_4 t + \beta_5 t^2 + f_i^* \tag{25}$$

where $\beta_1 = Z_i\phi + \pi_{1i}\bar{\gamma}_1 + \pi_{2i}\bar{\gamma}_2$, $\beta_4 = b - \bar{\gamma}_0\pi_{1i} - \bar{\theta}\alpha_{1i}$, $\beta_5 = \bar{\gamma}_0\pi_{2i} - \bar{\theta}\alpha_{2i}$, and f_i and f_i^* are error terms. The derivation of equation (25) assumes that the coefficients on age and age squared for the lifetime wage and income paths (i.e., π_1, π_2, α_1, and α_2) are constant across consumers. Referring to the definitions of the π's and the α's given by (15) and (17), we see that the implied predicted values for wages and income are $\ell n\hat{W}_i(t) = M_i g_0 + \pi_1 t + \pi_2 t^2$ and $\hat{Y}_i(t) = S_i q_0 + \alpha_1 t + \alpha_2 t^2$ where the vectors M_i and S_i contain such variables as education and family background variables. Prediction equations such as these are commonly found in cross sectional analysis.

Let us start by answering our second question first. According to equation (24), if a researcher excludes property income (i.e., sets $\beta_3 = 0$), and he chooses age and all the age-invariance characteristics determining either the lifetime wage path or initial permanent income as his control variables, then the coefficient on the current wage rate is the intertemporal substitution elasticity (i.e., $\beta_2 = \delta$). Intuitively, this set of control variables accounts for individual effects and controls for differences in the value of F across consumers.

If, on the other hand, a researcher chooses age and age-squared as his control variables, we see from equation (25) that the wage coefficient measures the response of labor supply to a parallel shift in the wage profile (i.e., $\beta_2 = \delta + \delta_0$), and the income coefficient measures the response of labor supply to changes in initial permanent income (i.e., $\beta_3 = \bar{\theta}$). Intuitively, this set of control variables adjusts for age effects and the only remaining difference in wages and income across consumers is due to profile shifts. Notice that this latter choice of control variables crucially depends on the assumption that the lifetime wage path coefficients on age and age-squared are not functions of a consumer's characteristics. Violation of this assumption implies that age-invariant characteristics as well as interactions between these characteristics and t and t^2 must also be included as control variables in equation (25). Inclusion of such variables obviously leads to identification problems.

Using different sets of control variables, then, allows one to estimate different behavioral parameters. If the rate of time preference equals the rate of interest (i.e., $b = 0$), then including education and background variables in a cross sectional specification of labor supply yields an estimate of the hours of work response to an evolutionary wage change. If, instead, one includes polynomials in age, the estimated wage coefficient measures the response of labor supply to a change in the lifetime average wage rate.

To answer our first question concerning the interpretation of parameters in the usual cross sectional models of labor supply, consider equation (25). This equation relates hours of work to measures of average lifetime wages and income which is the interpretation that most analysts would give to cross section models of labor supply. Typically, however, these cross section specifications fail to include age and age-squared as explanatory variables; they assume that the parameters β_4 and β_5 in equation (25) are zero. Hence, the usual cross section analysis suffers from a left-out variables problem. As a result, wage and income coefficients are inconsistently estimated to the extent that the predicted values of wages and income are correlated with age. The bias of these estimates may be either positive or negative. The degree of bias depends on the age distribution of the population. Changing this age distribution, in general, will change the estimated values of the wage and income coefficients. This is certainly a disturbing feature of typical cross sectional estimates.

The standard cross sectional analysis of labor supply, then, generally estimates wage and income coefficients that do not have an economic interpretation. Because this analysis fails to include age variables as controls, estimates confuse shifts of profiles with movements along profiles for both wages and income. The resulting wage and income coefficients are, in general, complicated functions of wage effects, income effects and the distribution of consumer characteristics. The situation even worsens if the slopes of the lifetime paths for wages and income are allowed to depend on a consumer's characteristics. Not only will estimates in this case confuse profile shifts from movements along profiles, they will also confuse tilts in profiles from parallel shifts. In the light of this argument, it is hard to see why anyone would be interested in estimates produced by a typical cross sectional analysis, especially if their ultimate interest is policy recommendations.

Other Life Cycle Models of Labor Supply in the Literature

Two of the major published empirical investigations of life cycle labor supply are by Becker in Ghez-Becker (1975) and Lucas-Rapping (1969).[23] Becker's objective is to estimate intertemporal substitution effects. Lucas-Rapping attempt to estimate hours of work responses to "permanent" and "transitory" wage changes. The following discussion relates the work in these studies to the empirical model proposed in this paper. It is argued that both Becker and Lucas-Rapping estimate parameters of the λ constant labor supply function, and neither of these authors appears to recognize this possibility.

In his theoretical work Becker investigates the effect of life cycle wage growth on the timing of labor supply. He analyzes only the effects of evolutionary wage changes; there is no analysis of parametric wage change.

To test his theory, Becker uses data on wage rates and hours worked by age from the 1960 U.S. Census to form synthetic cohorts. He divides his cross sectional sample of workers into three education groups, and then groups them by age. A synthetic profile is constructed by computing means of the natural log of annual hours worked and hourly wages for each age group. This synthetic cohort is assumed to represent the life cycle of a typical individual.

Becker then uses this synthetic cohort data to estimate the parameters of what is effectively a λ constant labor supply function. Averaging observations over individuals in the same age group, equation (12) becomes

$$(26) \qquad \overline{\ell nN(t)} = \bar{F} + bt + \delta\, \overline{\ell nW(t)} + \overline{u(t)} \qquad t = 0,\cdots,T$$

where the bars denote means. This equation closely resembles the one actually estimated by Becker using least squares. The crucial assumption underlying this approach is that the value of $\bar{F}$ is the same for all age groups. Averaging observations for each age group produces a consistent estimate of the group intercept $\bar{F}$. In contrast to using observations on individuals, least-squares estimation of equation (30) produces consistent parameter estimates using the averaged data if there are no cohort effects. One eliminates endogeneity of wages by using group means as instruments. The issue of treating F as a random variable does not arise since it is assumed to be the same for all age groups.

To allow for smooth vintage effects, Becker treats $\bar{F}$ as a linear function of age, average property income, and average family size.[24] A measure of the permanent wage rate is the obvious variable missing from this relationship. If one cannot delete this variable, however, it is impossible to identify the intertemporal substitution effect, δ, using the synthetic cohort approach. After controlling for age, income and family size, this approach requires all vintage effects to be uncorrelated with wages. This seems to be a strong and implausible assumption which suggests that cohort effects may contaminate estimates produced by the synthetic cohort method.

Becker's study represents the first attempt to estimate a pure intertemporal substitution effect. Although he does not interpret his empirical specification as a λ constant labor supply function, he is clearly using such a relationship to estimate the response of labor supply to evolutionary wage changes.

Lucas-Rapping formulate an empirical model of labor supply along more traditional lines. They write log hours of work as a log linear function of the current real wage rate, the anticipated future real wage rate, and the level of wealth.[25] This specification is equivalent to the one given by (27). They write this equation as

$$\text{(27)} \qquad \ln N(t) = \phi + \delta(\ln W(t) - \overline{\ln W}) + (\delta + \gamma^*)\overline{\ln W} + \theta A(t) + \nu^*(t) \qquad t = 0,\ldots$$

where $\gamma(t)$ is assumed to be a constant for all t, $\overline{\ln W}$ is the average future wage rate defined by $\frac{1}{T-t+1}\sum_{j=t}^{T} \ln W(j)$, and $\gamma^* = (T-t+1)\gamma$.[26] They call $\overline{\ln W}$ the permanent or normal real wage rate, and they refer to its coefficient γ^* as the long-run wage elasticity. Indeed, given their assumptions, γ^* is equivalent to the coefficient $\bar{\gamma}_0$ in the empirical model described above, and $\delta + \gamma^*$ determines the response of labor supply to a permanent wage change. The coefficient on the current wage rate, δ, is the intertemporal wage elasticity. Lucas-Rapping call this the short-run wage elasticity.

Lucas-Rapping estimate the parameters of their model using an aggregate U.S. time series covering the years 1930-65. They convert most of their variables to per-household terms. Assuming their data refers to a representative individual, they estimate their model using two-stage least squares, and they report a long-run wage elasticity equal to .03, or essentially zero. Their estimate of the short-run elasticity is 1.4.[27] Thus, they conclude that the deviation of the current wage rate from its normal level is the major determinant of labor supply.

There is a serious difficulty with Lucas'-Rapping's interpretation of their results. How are they able to estimate labor supply responses to parametric wage changes? Presumably, their aggregate time series data depicts the life cycle of a representative individual. Their data, then, effectively consists of observations on a single worker's labor supply and wages at various points in his lifetime. Using such data, all one can hope to isolate is the effect of an evolutionary wage change on labor supply since all wage changes arise from movements along a given lifetime wage path.

The λ constant labor supply function, like the one given by (12), offers a more reasonable description of the empirical relation actually estimated by Lucas-Rapping. It is this specification of the hours of work function that determines responses to evolutionary wage changes. It explicitly recognizes that the value of λ fully summarizes the effects of the permanent wage rate and wealth, and that λ is naturally absorbed into the constant term of the labor supply equation. Since the constant term summarizes all information on wealth and future wages relevant to the current labor supply decision, measures of either the permanent wage rate or wealth should not directly enter the labor supply equation. In the light of this argument, it is not surprising that Lucas-Rapping find a long-run wage elasticity near zero in value and no effect of wealth on hours of work.

In fairness to Lucas-Rapping, they assume an uncertain environment in which it is theoretically possible to estimate the effects of parametric wage and income changes on hours of work. In their model, a consumer is uncertain about future wages and prices. As the consumer ages, he acquires new information concerning his future prospects which causes him to adjust his forecasts of wage and income profiles. Thus, the consumer experiences and responds to shifts in his expected lifetime wage and income paths. To actually estimate labor supply responses to such parametric changes, however, one requires instruments for the unanticipated components of wages and income. Such instruments are not easily found in any analysis. It is hard to argue that Lucas-Rapping identify responses to unanticipated changes in wages because they use a two-stage least-squares procedure to estimate their relationships. They predict wages using primarily predetermined variables. Given any sort of rational expectations scheme, these variables will not be correlated with unanticipated components. Even in this uncertainty environment, then, it is difficult to see how Lucas-Rapping are able to estimate the impact of a permanent wage change or change in wealth on hours of work.

IV. Empirical Analysis

This portion of the paper reports empirical estimates for the structural model of lifetime labor supply developed in Section II. The discussion begins with a description of the data. Next, estimates obtained from a two-stage estimation of the full λ constant hours of work model are presented and discussed. The section concludes with a discussion of cross sectional models of labor supply and the possibility of using these models to estimate parameters relevant to describing hours of work behavior in a life cycle setting.

Data

The sample employed in this study is from the Michigan Panel Study of Income Dynamics. It consists of observations on 561 prime-age, white, married males for the years 1967-75. To be included in the sample, these males had to be continuously married to the same spouse during the period 1968-76, and they had to be between the ages of 25 and 48 in 1967 (or 33-56 in 1975). Other sample selection criteria are presented in Appendix A.

The labor supply variable used in the empirical analysis is annual hours of work. The wage variable is average hourly earnings deflated by the Consumer Price Index. Table A-1 in Appendix A contains summary statistics for these variables. This table indicates a much wider variability in hours of work and wages for males than many economists would predict.

Estimation Procedures

Estimation of the model of labor supply outlined in Section II is carried out in two stages. In stage one, the λ constant labor supply equation given by (12) is estimated using constrained two- and three-stage least squares. In stage two, the structural equation for F given by (18) is also estimated using simultaneous equation methods. This two-stage procedure allows one to fit separately economic relationships describing a worker's dynamic behavior (the first stage) and his lifetime average labor supply (the second stage). It is efficient computationally. Also, to produce consistent parameter estimates, this two-stage procedure requires fewer assumptions concerning the presence of correlation between a consumer's measured and unmeasured characteristics determining his individual effect.[28]

Estimates for the Intertemporal Substitution Elasticity

Estimation of the λ constant labor supply equation is simplified by working with a first-differenced version of this equation. First differencing produces a new equation that sets the percentage change in hours of work equal to a constant term b, δ times the percentage change in wages, and a disturbance. Thus, it relates a consumer's labor supply changes to wage changes. The significant point is that differencing eliminates individual fixed effects, and it avoids the introduction of incidental parameters.

The differenced labor supply and wage equations can be formed into a simultaneous equations model. First differencing the labor supply function given by (12) and the lifetime path for wages given by (14) yields

$$(28) \quad \begin{pmatrix} 1 & -\delta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} D\ell nN_i(t) \\ D\ell nW_i(t) \end{pmatrix} = \begin{pmatrix} X_{1i}(t) & 0 \\ 0 & X_{2i}(t) \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_{1i}(t) \\ \varepsilon_{2i}(t) \end{pmatrix}$$

or

$$\Gamma y_i(t) = X_i(t)\beta + \varepsilon_i(t)$$

where D denotes the difference operator (i.e., $D\ell nN_i(t) = \ell nN_i(t) - \ell nN_i(t-1)$), $X_i(t)$ is a matrix of exogenous variables, β is a vector of parameters, and $\varepsilon_i(t)$ is a disturbance vector. In the absence of any year effects, $X_{1i}(t)$ is simply an intercept term and $\beta_1 = b$. $X_{2i}(t)$ contains determinants of evolutionary wage changes; without year effects it is defined by $X_{2i}(t)\beta_2 = \pi_{1i} - \pi_{2i} + 2t\pi_{2i}$, where π_{1i} and π_{2i} are the slope coefficients of the wage profile. In the following empirical analysis, the elements of $X_{1i}(t)$ include an intercept and year dummies and $X_{2i}(t)$ contains year dummies, education, squared education, age, and interactions between age and education and age and age-squared education.

The simultaneous equation system estimated in this paper combines all of the labor supply and wage equations for a single worker into one model. This model is formed by stacking the $y_i(t)$ and $\varepsilon_i(t)$ vectors and the $X_i(t)$ matrix according to the t index. This new model contains all of the observations for the i^{th} worker. There are linear constraints across equations in this complete structural equation system. To allow for the most general contemporaneous and temporal correlation schemes, no restrictions are imposed on the covariance matrix for an individual's error vector. To account for correlation across workers, year dummies are included in the X_i matrix.

This paper also estimates a simultaneous equations model that relates hours of work and earnings. Consider the set of structural equations given by (28). Adding $\delta D\ell nN_i(t)$ to both sides of the labor supply equation and solving this new equation for $D\ell nN_i(t)$, and adding $D\ell nN_i(t)$ to the left-hand side of the wage equation and its expectation conditional on $X_i(t)$ to the right-hand side produces a new system of structural equations that looks exactly like the one given by equation (28) except that the log difference in earnings, $D\ell nE_i(t)$, replaces $D\ell nW_i(t)$, $\frac{\delta}{1+\delta}$ replaces δ, and there is a new parameter vector β and disturbance vector.[29] Using the coefficient on earnings in the labor supply equation, $\frac{\delta}{1+\delta}$, it is possible to construct an estimate of δ. Thus, this new simultaneous equations model provides another econometric framework for estimating δ.

To estimate the parameters of the two structural equation models described above, this study employs constrained two- and three-stage least squares. These simultaneous equation procedures take advantage of the time series aspect of panel data to estimate the intertemporal substitution elasticity δ with a minimal amount of computational burden. They avoid biases arising from pure reporting error in earnings and labor supply, and they offer a flexible framework for testing and estimating alternative functional forms.

Table 1 presents estimates of the intertemporal substitution elasticity. Two specifications of the labor supply equation are considered. One includes only an intercept term in the equation and the other includes dummy variables for each year. All of the implied estimates of the intertemporal substitution elasticity are positive. According to the estimates of the wage coefficients, δ lies in the range .10 to .20. The earnings coefficients indicate a range of .18 to .33 for δ. The earnings coefficients indicate a higher estimate for δ in all cases, but these differences are small relative to their standard errors.[30]

Estimating the labor supply equation with and without year dummies provides a check as to whether the response of labor supply is sensitive to the source of wage variation. Two sets of variables are used to predict the change in wages and earnings: age and education variables and year dummies. Age and education variables capture in part variations in the wage rate arising from investment in human capital. The addition of year dummies reflects variations in wages due to cyclical or secular factors. Excluding year dummies from the labor supply equation implies the wage and earnings coefficients measure the response of labor supply to any variation in the wage, no matter what its source. Including year dummies in the labor supply equation implies that the wage and earnings coefficients measure the response of labor supply to variations in the wage solely due to past investment activities. Comparing the results of Table 1, we see there is little difference between the estimates of the intertemporal substitution elasticity with and without the inclusion of year dummies. Thus, there appears to be no evidence suggesting that the labor supply response depends on the source of the wage change.

To check on the robustness of the findings, the above structural relations were estimated using averages of an individual's observations over the sample period. Averaging the data amounts to a simple linear transformation of the complete structural equation system described above. Averaging the first differences of a variable is equivalent to taking the difference of the variable for a period greater than one year; for example, averaging over a sample period of eight years creates a new variable that represents an eighth difference.

Table 2 presents estimates of the intertemporal substitution elasticity based on the averaged data. The estimates obtained for the eight-year averages are .18 and .25. These results strongly support the estimates based on the unaveraged data.

These empirical results and the ones that follow require qualification. Their interpretation relies on assumptions concerning the absence of taxes and the existence of perfect capital markets. Violation of these assumptions is certain in the sample used here. Simple tax schemes can be admitted with no modifications to the above results; if an individual's tax rate is approximately proportional over the sample period, for example, then first differencing eliminates tax effects.[31] In general, however, the presence of taxes and imperfect capital markets leads to complicated biases in the parameter estimates reported here which are difficult even to sign.

There is a further qualification of the above empirical results. The previous discussion was careful to state that simultaneous equation procedures avoid biases due to reporting error. There is, however, a problem with measurement error in hours of work. Measured hours of work includes both time spent working and time spent in on-the-job training. Thus, to the extent there is investment in human capital, true hours of work are measured with error. Given that measured hours of work are used to construct average hourly earnings, it follows that wages are inflicted with this measurement problem also. Unfortunately, this measurement error is known to be correlated with variables that the above empirical analysis treats as instruments, namely, age and education. Economic theory predicts that investment time is related to both an individual's education and his age. First differencing obviously does not eliminate this problem. The result is inconsistent estimates of the wage and earnings coefficients. Formally, the direction of this inconsistency is ambiguous. It depends on whether the life cycle profile for the fraction of working time spent in training is a concave or convex function of age and education.[32]

Estimates of Responses to Parametric Wage Changes

Estimating the structural equation for F given by (18) provides the additional information we require to predict a consumer's labor supply response to parametric wage changes. Estimating this equation is not as difficult as it may first appear. It is true that all of the variables appearing in this equation (F_i, π_{0i}, π_{1i}, π_{2i}, and α_{0i}) are not directly observable. But they are observable with error, and it is possible to use standard two-stage least-squares procedures to estimate the structural parameters of interest.

Consider the coefficients of the lifetime path for wages given by (14) and the following definitions

$$(29)\quad \begin{cases} \text{(a)} & \tilde{\pi}_{2i}(t) = \tfrac{1}{2}(D\ell nW_i(t) - D\ell nW_i(t-1)) \\ \text{(b)} & \tilde{\pi}_{1i}(t) = D\ell nW_i(t) - \tilde{\pi}_{2i}(1-2t) \qquad t = 3,\cdots,T \\ \text{(c)} & \tilde{\pi}_{0i}(t) = \ell nW_i(t) - \tilde{\pi}_{1i}t - \pi_{2i}t^2 \end{cases}$$

It can be shown that $E(\tilde{\pi}_{ji}(t)) = \pi_{ji}$, $j = 0, 1, 2$. This is an important result because the $\tilde{\pi}_{ji}(t)$'s are observable variables, and it is possible to use the $\tilde{\pi}_{ji}(t)$'s as dependent variables in a simultaneous equation analysis to consistently estimate the π_{ji}'s. In the following empirical analysis, the averages of the $\tilde{\pi}_{0i}(t)$'s, $\tilde{\pi}_{1i}(t)$'s, and $\tilde{\pi}_{2i}(t)$'s over the sample period are used as the dependent variables.

Similarly, given observations on consumer i's income over the sample period, one can construct the variables $\tilde{\alpha}_{2i}(t)$, $\tilde{\alpha}_{1i}(t)$, and $\tilde{\alpha}_{0i}(t)$ using the definitions given by (29) with $Y_i(t)$ replacing $\ell nW_i(t)$. The average of the $\tilde{\alpha}_{0i}(t)$'s over the sample period, then, can be used in a simultaneous equations analysis to predict the intercept of the lifetime income path, α_{0i}, which is a measure of consumer i's initial permanent income.

An analogous strategy can be used to construct a measurable variable to serve as a proxy for F_i. Define $\tilde{F}_i(t) = \ln N_i(t) - bt - \delta \ln W_i(t)$. The expectation of $\tilde{F}_i(t)$ is F_i, but $\tilde{F}_i(t)$ cannot be directly observed since it depends on unknown parameters b and δ. We have estimates of these parameters, however, from the first stage of the empirical analysis. The first column of Table 1 indicates $\hat{b} = -.01$ and $\hat{\delta} = .20$.[33] A logical alternative to $\tilde{F}_i(t)$, then, is to form the variable $\hat{F}_i(t) = \ln N_i(t) - \hat{b}t - \hat{\delta} \ln W_i(t)$. Asymptotically, $\hat{F}_i(t)$ has an expectation equal to F_i. The following empirical analysis averages the $\hat{F}_i(t)$'s over the sample period and treats this average as a dependent variable that measures F_i with error.

Collecting the above results, we have a complete simultaneous equations model given by

$$\bar{\pi}_{ji} = M_i g_j + \eta_j \qquad j = 0, 1, 2 \tag{30}$$

$$\bar{\alpha}_{0i} = S_i q_0 + \eta_3 \tag{31}$$

$$\bar{F}_i = \phi + \bar{\pi}_{0i}\bar{\gamma}_0 + \bar{\pi}_{1i}\bar{\gamma}_1 + \bar{\pi}_{2i}\bar{\gamma}_2 + \bar{\alpha}_{0i}\bar{\theta} + \eta_4 \tag{32}$$

where $\bar{\pi}_{0i}$, $\bar{\pi}_{1i}$, $\bar{\pi}_{2i}$, $\bar{\alpha}_{0i}$, and $\bar{F}_i$ are averages of the variables $\tilde{\pi}_{0i}(t)$, $\tilde{\pi}_{1i}(t)$, $\tilde{\pi}_{2i}(t)$, $\tilde{\alpha}_{0i}(t)$, and $\hat{F}_i(t)$ over the sample period, the vectors of exogenous variables M_i and S_i and the coefficient vectors g_0, g_1, g_2, and q_0 are defined by (15) and (17), and the η_j's are disturbances. We have one set of equations for each consumer i. The endogenous variables in this model are $\bar{F}_i$, $\bar{\alpha}_{0i}$, and the $\bar{\pi}_{ji}$'s; the exogenous variables are the elements of M_i and S_i; and the structural parameters of interest are $\bar{\gamma}_0$,

$\bar{\gamma}_1$, $\bar{\gamma}_2$, and $\bar{\theta}$. M_i and S_i contain variables determining the coefficients of the lifetime wage and income paths. In the following empirical analysis, they include the consumer's education, his education squared, and three background variables: his mother's and father's education and a bracketed variable indicating whether the normal income of the consumer's parents was considered below average, average, or above average.

To consistently estimate the parameters of the structural equation for F given by (32) one can employ a standard two-stage least-squares procedure. The standard errors reported by this procedure are valid if the number of time series observations for each consumer is sufficiently large. If one does not have a sufficiently large number of these time series observations, however, the usual standard errors are invalid. The problem lies in the fact that we use estimated values for b and δ to form the dependent variable $\bar{F}_i$. In cases where the number of time series observations is small, one must adjust the usual standard errors to account for the errors in estimating b and δ. The precise form of this adjustment is given in Appendix B. Although this adjustment is not complicated, it does require the use of matrix operations. Since the data set used here has only nine observations per person, the standard errors reported in this paper have been adjusted for estimation error. This adjustment was very minor in every instance for which it was used.

Table 3 presents two sets of estimates for the structural parameters of the individual effects equation given by (32). The first row presents unconstrained two-stage least-squares estimates. The second row presents estimates with the restriction that $\gamma(t)$ (i.e., the cross-substitution elasticity) is the same for all t. This restriction assumes a working life of 40 years.[34] The estimates of $\bar{\gamma}_0$, $\bar{\gamma}_1$, and $\bar{\gamma}_2$ are all negative, as theory predicts. Although one accepts the hypothesis that $\gamma(t)$ is constant at conventional levels of significance, there are large discrepancies between the constrained and the unconstrained estimates.

Combining estimates of $\bar{\gamma}_0$ and δ allows one to form estimates of cross- and own-uncompensated substitution elasticities. Dividing the unconstrained estimates of $\bar{\gamma}_0$ by the length of the working life produces an estimate of the average cross uncompensated elasticity; assuming a working life of 40 years implies an estimate for this elasticity equal to -.0042. An increase in a future wage rate, then, essentially has no effect on current hours of work. Adding an estimate for δ to this cross elasticity creates an estimate for the average own elasticity. Results from the first stage of the empirical analysis suggest a value for δ equal to about .2. This implies an estimate of .2 for the average own elasticity. Increasing a consumer's period t wage rate by 10%, then, leads to a 2% increase in his hours of working in period t.

Combining the estimates of $\bar{\gamma}_0$, $\bar{\gamma}_1$, and $\bar{\gamma}_2$ with an estimate of δ provides the information needed to predict a consumer's labor supply response to shifts in his wage profile. In response to a uniform 100% increase in wages at all ages (i.e., a parallel shift in the log wage profile), the unconstrained estimates of Table 3 predict that a consumer will adjust his hours of work by an amount equal to $\bar{\gamma}_0 + \delta = -.17 + .20 = .03$% at all ages. There is, then, essentially no response in a consumer's labor supply to parallel shifts in his log wage profile. If the slope of a consumer's wage profile is altered by changing the coefficient on the linear term (i.e., π_{1i}) by an amount equal to Δ, the unconstrained estimates predict that his hours of work at age t change by $(\bar{\gamma}_1 + \hat{\delta}t)\Delta$ = $(-4.69 + .2t)\Delta$ percent. t, here, is measured in decades and takes a value of 0 when the consumer is 18 years old. Hence, hours of work decline at all ages in response to this sort of increase in the slope of the wage profile with larger decreases occurring at earlier ages. The same is true when the slope of the wage profile is altered by changing the coefficient on the quadratic term (i.e., π_{2i}). Hours of work at age t decline by an amount equal to $(-18.45 + .2t^2)\Delta$ percent. The unconstrained estimates, then, are consistent with the popular notion that the lifetime labor supply curve of prime-age males is backward-bending or vertical.

The initial permanent income coefficient $\bar{\theta}$ has the wrong sign, but it is not statistically significant. The measure of property income used in the empirical analysis is total family income minus the husband's earnings. There are many problems with this measure of property income. It includes wife's earnings, and it does not include any income derived from consumer durables and the like which generally constitute the major components of a consumer's nonwage income.[35]

Cross Sectional Estimates

Theoretically, it is possible to estimate parameters relevant to describing a consumer's lifetime labor supply behavior using cross sectional models. The following analysis examines five such models and compares their empirical results to estimates obtained from the two-stage estimation procedure described above.

The first two approaches considered use synthetic cohort data to obtain an estimate of the intertemporal substitution elasticity. The first specification estimated is the one given by (26); namely, average of log annual hours of work is regressed on age and average log wages.[36] In this specification, the wage coefficient theoretically represents the intertemporal substitution elasticity δ. The first row of Table 4 presents the least-squares estimates of this wage coefficient. Results are given for each year of the panel, and a final column presents a pooled estimate obtained by full information estimation methods and imposing an equality restriction across years. This first row shows that the coefficients on wages are negative for each year. Clearly, they do not represent estimates of δ as they should. Given the relatively small number of observations per age cell in this sample, there is reason to believe that these negative estimates of δ are due to reporting errors in hours of work and earnings.[37]

The second use of synthetic cohort data employs an econometric specification that relies on a more appropriate set of instruments to estimate δ. It is possible to respecify the labor supply equation given by (26) by adding the quantity $\delta \, \overline{\ell n N(t)}$ to both sides of the equation and dividing each side by $1 + \delta$. This creates a new equation with average log hours of work as the dependent, age, and average log earnings as explanatory variables. Using an estimate of the earnings coefficient, $\frac{\delta}{1+\delta}$, one can compute an estimate for δ. Becker (1975) suggests that it is possible to reduce the effect of reporting error by specifying the labor supply equation in terms of earnings rather than wages. His reasoning is based on the assumption that reporting errors for earnings and hours of work are uncorrelated. Given this assumption, a regression of hours of work on earnings produces an earnings coefficient that is biased toward zero. One can show that regressing hours of work on average hourly earnings yields an estimate of the wage coefficient that is biased toward negative values rather than zero. This result suggests that better estimates of δ can be obtained using earnings data rather than wage data to estimate the parameters of the labor supply equation.

The second row of Table 4 presents the implied estimates of δ obtained by least-squares estimation of the hours of work-earnings specification using synthetic cohort data. Most of these estimates are positive, as economic theory predicts. The results are consistent with the above contention that there exists a less serious error in the variables problem for earnings than for wages. These estimates of δ are generally lower than those reported in Tables 1 and 2. The estimates based on averaged earnings data, however, may possess a downward bias as a consequence of reporting error in earnings and residual correlation due to unobserved individual effects.

The remaining three cross sectional models of labor supply are estimated using data on individuals and simultaneous equation techniques. These models include: the specification typically found in cross sectional analysis and the equations given by (24) and (25). The derivation of equation (25) requires slopes of lifetime wage and income paths to depend only on a consumer's age; only parallel shifts in these paths are admissible. The following analysis accounts for this restriction by using reduced form equations for log wages and income that include education, education squared, age, age squared, and background variables as explanatory variables.[38]

The third row of Table 4 presents estimates of the wage coefficient based on an empirical specification of labor supply typically found in cross sectional analysis; hours are regressed on predicted wages and income. As discussed in the previous section, this wage coefficient confuses wage effects, income effects, and properties of the population age distribution. It does not have a clean economic interpretation. The estimates of this coefficient are always positive and appear to increase over time as the population ages. The estimates of the income coefficients are not presented; they were all negative.[39] Thus, according to conventional labor supply theory, these estimates have the correct sign and show men's hours of work to be positively related to the wages. Viewed in a life cycle context, it is not clear what these coefficients tell us about labor supply behavior.

The fourth row of Table 4 reports cross sectional estimates of equation (24). Theoretically, the wage coefficient in this specification represents the intertemporal substitution effect, δ. The estimates of this coefficient for the years 1967-72 and 1974 range between .07 and .28 and are consistent with the estimates of δ reported in Tables 1 and 2. Estimates for the years 1973 and 1975 and the pooled estimate are less satisfactory because they are negative. All of these estimates are imprecisely estimated, reflecting the fact that only the age squared variable identifies the wage coefficient.[40]

The last row of Table 4 presents cross sectional estimates of equation (25). If wage and income profiles have slopes that are the same across consumers of a given age, the wage coefficient in this specification theoretically determines the response of labor supply to a 100% increase in wages over the entire lifetime. The pooled estimate of .04 differs only slightly from the .03 estimate obtained in stage two of the estimation procedure discussed above. There is, however, a fair amount of variation in the estimates of this coefficient from one year to the next; so much so that for two of the years (1974 and 1975) one would reject the hypothesis that the coefficients are zero using conventional levels of significance.

Conclusion

This study formulates a manageable empirical model of labor supply that fully incorporates life cycle considerations. This model naturally splits the empirical analysis into two stages. In stage one, the analysis concentrates on describing a consumer's dynamic behavior. The response of labor supply to evolutionary wage changes is estimated. In stage two, the analysis focuses on explaining differences in labor supply across consumers. This stage estimates the impact of changes in wealth and parametric changes in wages on hours of work over the life cycle. This two-stage analysis offers a very tractable estimation procedure and minimal data requirements.

A full set of estimates required to describe the lifetime labor supply behavior of prime-age males is presented. The effect of a change in the wage rate on hours of work is shown to depend on the source of the wage change. Estimates of the intertemporal substitution elasticities indicate that a 10% increase in the real wage rate which is due to life cycle wage growth induces a 1% to 3% increase in hours worked. The estimates of own period uncompensated substitution effects range between .1 and .3, and cross uncompensated effects are approximately zero. Yet, the evidence in this study is also consistent with the popular notion that an increase in men's lifetime average wage rate leads to no change or a decline in their hours of work.

This study argues that Ghez-Becker and Lucas-Rapping estimate the same structural labor supply parameter which determines the hours of work response to evolutionary wage changes. Given their empirical specifications, neither set of authors is able to estimate the effect of a parametric wage change on labor supply.

This study shows how cross sectional specifications of hours of work can be modified to estimate parameters relevant for describing lifetime labor supply behavior. Including such demographic variables as education and background variables in a cross sectional specification yields an estimate of the intertemporal substitution elasticity which determines the effect of wage growth on hours of work; for simple forms of the lifetime wage path including polynomials in age, on the other hand, yields an estimate of the response of hours of work to a change in the lifetime average wage rate. Unfortunately, these modified cross sectional models and the synthetic cohort models of Ghez-Becker met with mixed success in the empirical analysis of this study.

The important point for an analyst to extract from this study is the following. Recognizing that individuals make their decisions in a life cycle setting is crucial if one's objective is to estimate economically meaningful parameters. Creating an empirical model that accounts for such a setting need not complicate the analysis, and it generally leads to a more complete understanding of consumer behavior.

APPENDIX A

This appendix reports the exclusion criteria used to generate the samples used in the empirical analysis. The primary sample is the Panel Study of Income Dynamics for 1976.

The following criteria were used in order to construct the sample of nine years of data on 561 white males who were between the ages 25-48 in 1967 (or 33-56 in 1976). Households included in the sample had to satisfy the following criteria: (1) A stable family composition prevailed over the period in the sense that the husband and wife remained together. (2) A worker must be classified as employed or unemployed (i.e., permanently disabled and retired were deleted). (3) Wage and labor supply data must be available for all years. A worker's years of schooling must also be available. (4) A worker must report less than 4680 hours worked per year. The absolute value of the difference in his real average hourly earnings in adjacent years cannot exceed $16 or a change of 200 percent. The absolute value of the difference in the number of hours he works in adjacent years cannot exceed 2900 hours or a change of 190 percent. The purpose of this last criterion is to minimize difficulties arising from the presence of outliers. (Twenty-four workers were deleted by this outlier criterion.) (5) Households whose property income (defined as total family income minus husband's earnings deflated by the consumer price index) was less than $5,000 in any year were deleted. (Three observations were lost.)

Table A-1 presents summary statistics for annual hours of work and real average hourly earnings.

APPENDIX B

This appendix develops the correct standard errors for the parameter estimates of the individual effects equation given by (32).

Consider the following set of equations

$$\text{(B-1)} \quad Y_{1i} = X_i\beta + U_i$$

$$i = 1, 2, \ldots, N$$

$$\text{(B-2)} \quad Y_{2i} - Z_i\beta = W_i\gamma + \varepsilon_i$$

where Y_1 and Y_2 are vectors of endogenous variables, Z_i is a matrix of either endogenous or exogenous variables, X_i and W_i are matrices of exogenous variables, β and γ are parameter vectors, and U_i and ε_i are correlated error vectors which are independently distributed over the index i. Suppose that one estimates β by least squares using equation (B-1), replaces β in equation (B-2) by its estimate, and estimates the resulting equation by least squares. The standard errors for the estimate of γ reported by this least squares procedure are incorrect.

Computing the correct standard errors is straightforward. Let $\hat{\beta} = (X'X)^{-1} \sum_{j=1}^{N} X_j'U_j = \beta + (X'X)^{-1} \sum_{j=1}^{N} X_j'U_j$ denote the estimate for β. Substituting $\hat{\beta}$ into equation (B-2) yields

$$\text{(B-3)} \quad Y_{2i} - Z_i\hat{\beta} = W_i\gamma + \varepsilon_i + Z_i(\beta - \hat{\beta}).$$

The least squares estimate for γ is

$$\text{(B-4)} \quad \hat{\gamma} = \gamma + (W'W)^{-1} \sum_{i=1}^{N} W_i'(\varepsilon_i + Z_i(\beta - \hat{\beta}))$$

$$= \gamma + (W'W)^{-1}\left[\sum_{i=1}^{N} W_i'\varepsilon_i - (\sum_{i=1}^{N} W_i'Z_i)(X'X)^{-1} \sum_{j=1}^{N} X_j'U_j\right]$$

$$= \gamma + \sum_{i=1}^{N} ((W'W)^{-1}W_i'\varepsilon_i - A(X'X)^{-1} X_i'U_i)$$

$$= \gamma + \sum_{i=1}^{N} \eta_i$$

where

$$A = (W'W)^{-1} \sum_{i=1}^{N} W_i'Z_i$$

$$\eta_i = (W'W)^{-1} W_i'\varepsilon_i - A(X'X)^{-1} X_i'U_i \ .$$

In large samples, it can be shown that

$$\text{(B-5)} \quad \hat{\gamma} \mathrel{\dot{\sim}} N(\gamma, (\sum_{i=1}^{N} \hat{\eta}_i\hat{\eta}_i'))$$

where

$$\hat{\eta}_i = (W'W)^{-1} W_i'\hat{\varepsilon}_i - A(X'X)^{-1} X_i'\hat{U}_i$$

$$\text{(B-6)} \quad \hat{\varepsilon}_i = Y_{2i} - Z_i\hat{\beta} - W_i\hat{\gamma}$$

$$\hat{U}_i = Y_{1i} - X_i\hat{\beta}.$$

A standard application of least squares reports standard errors neglects the second component of $\hat{\eta}_i$.

The procedure used to estimate the parameters of equation (32) is a simultaneous equation version of the estimation procedure described above. The standard errors reported by two stage least squares are incorrect. The correct standard errors have the same form as those given by (B-5) except that all the endogenous variables included in the matrices W, W_i, X and X_i are replaced by their predicted values in all expressions but the definitions of $\hat{\varepsilon}_i$ and $\hat{U}_i$.

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Footnotes

1. Adding a bequest function to the preference function does not alter any of the following analysis. Similarly, one could introduce time dependence in the period utility functions, $U(\cdot,\cdot)$, without any significant consequences.

2. See Heckman (1974,1976) for proofs of these inequalities. Heckman develops and uses functions equivalent to those given by equations (5) and (6) in his analysis of the behavior of consumption and labor supply over the life cycle.

3. If it is optimal for the consumer to work at age t, then condition (4) is an equality and the functions $C(\cdot,\cdot)$ and $N(\cdot,\cdot)$ represent the solutions of equations (3) and (4) for the variables $C(t)$ and $N(t) = \tau - L(t)$, respectively. If, on the other hand, the necessary condition given by (4) is inequality (i.e., $U_2(C(t),\tau) = \left(\frac{1+\rho}{1+r}\right)^t W(t)$), then the consumer chooses not to work. In this case, $N(\cdot,\cdot) = 0$ and the function $C(\cdot,\cdot)$ is the solution of the equation $U_1(C(t),\tau) = \left(\frac{1+\rho}{1+r}\right)^t \lambda$ for $C(t)$. In either case, $C(\cdot,\cdot)$ and $N(\cdot,\cdot)$ only contain the variables $\left(\frac{1+\rho}{1+r}\right)^t \lambda$ and $W(t)$ as arguments and their functional form depends only on the form of the period t utility function $U(\cdot,\cdot)$. For a further discussion on this issue see Heckman and MaCurdy (1979).

4. Introducing age dependence into the utility function does not change any of this analysis. If the utility function at age t is given by $U(C(t), L(t), X(t))$, where $X(t)$ is a vector of time-varying determinants of "consumer tastes," then a third argument $X(t)$ enters the consumption and labor supply functions given by (5) and (6). These functions satisfy the restrictions given by (7) and they also allow for corner solutions.

5. See Heckman (1974,1976) for proof of these propositions.

6. See MaCurdy (1978) for a discussion of the uncertainty case.

7. Changes in a consumer's tastes can also be a reason for adjustments in consumption and labor supply over the life cycle. As discussed in footnote 4, if the period utility function is age dependent, the λ constant consumption and labor supply functions will also be age dependent. It is still true, however, that the λ constant functions fully characterize a consumer's dynamic behavior.

8. Since this study's empirical objective is to examine the labor supply behavior of prime-age males, this assumption is not unreasonable.

9. There are other forms of the utility function that imply convenient empirical specifications for the λ constant consumption and leisure demand functions. Two such functions are

(a) $$U_i(t) = \kappa_i(t)(C_i(t)+\mu^*)^{\omega^*}(L_i(t)+\mu)^{\omega} \qquad \kappa_i(t),\ \omega^*,\ \omega > 0 \text{ or } \omega^* + \omega < 1$$
$$\kappa_i(t),\ \omega^*,\ \omega < 0$$

(b) $$U_i(t) = \kappa_i^*(t)\frac{(C_i(t)+\mu^*)^{\omega^*} - 1}{\omega^*} + \kappa_i(t)\frac{(L_i(t)+\mu)^{\omega} - 1}{\omega} \qquad \kappa_i(t),\ \kappa_i^*(t) > 0$$

where $\kappa_i^*(t)$, $\kappa_i(t)$, μ^*, μ, ω^*, and ω are all parameters. Both of these functions are concave. They include Cobb-Douglas, addilog, and Stone-Geary as special cases. The λ constant functions for consumption and leisure are log linear in λ, wages, and the coefficients $\kappa_i^*(t)$ and $\kappa_i(t)$ which represent specific modifiers of tastes.

This study uses the utility function given by (10) to formulate an empirical model because it implies a form for the labor supply function which can be readily compared to labor supply equations found in existing empirical work.

10. If utility at age t depends on measured characteristics of the consumer that vary over the sample period, then current values of these "taste shifter" variables would also be included as regressors. A natural way to introduce such "taste shifter" variables is to model the taste coefficient $\gamma_i(t)$ as a function of the form $\ell n \gamma_i(t) = \sigma_i + X_i(t)\beta - u_i(t)$ where $X_i(t)$ is a vector of variables influencing tastes and β is a parameter. For this case, $X_i(t)\beta$ enters as an additional linear term in the λ constant labor supply equation given by (12).

11. The equation determining λ_i is

$$A_i(0) = \sum_{t=0}^{T} \left[\frac{1}{(1+r)^t} \left(\frac{1}{\gamma_i^*(t)\omega^*} \left(\frac{1+\rho}{1+r} \right)^t \lambda_i \right)^{\frac{1}{\omega^*-1}} - W_i(t) \left(\frac{1}{\gamma_i(t)\omega} \left(\frac{1+\rho}{1+r} \right)^t \lambda_i W_i(t) \right)^{\frac{1}{\omega-1}} \right]$$

12. The effects of the interest rate and time preference are assumed to be absorbed into the intercept.

13. These income measures seldom include such imputed income as that generated by consumer durables which is a major source of property income for most consumers.

14. In contrast to W(t), Y(t) is determined endogenously in this model. Equation (16) can be viewed as an approximation to the optimal lifetime path for Y(t) expressed as a function of the exogenous variables of the model.

15. This distinction goes back to Ghez and Becker (1975).

16. This statement is true only in an environment of perfect certainty. If there is uncertainty about the future, a consumer can

experience parametric wage changes as he acquires new information about his lifetime wage path. For a discussion of this uncertainty case see MaCurdy (1978).

17. In the literature on consumer demand, δ is often referred to as the specific substitution effect. The specific substitution effect determines the effect of a price change holding the marginal utility of income constant (see Theil, 1976, Ch. 1).

18. See McFadden (1963).

19. Houthakker and Taylor (1970, Ch. 5) and Phlips (1974, pp. 190-93, and 250-60) have estimated marginal utility of wealth constant demand functions as an intermediate computational step toward estimating a system of ordinary demand functions. In contrast to their work, I directly estimate the λ constant demand functions as a means of characterizing a consumer's dynamic behavior. I treat λ as a fixed effect which summarizes the effect of historic and future information on current decisions. I thank Orley Ashenfelter for the Houthakker and Taylor reference.

20. The relation $\frac{\partial \ln\lambda(0)}{\partial A(0)} = \frac{\partial \ln\lambda(t)}{\partial A(t)} \frac{\partial A(t)}{\partial A(0)}$ follows immediately from the equation $\ln \lambda(t) = t \ln\left(\frac{1+\rho}{1+r}\right) + \ln \lambda(0)$ and the assumption that wages are exogenous. To verify the second relation, let V denote the value function defined by $V(W^*(t),\ldots,W^*(T),A(t)) \equiv \max\{ \sum_{j=1}^{T} U(C(j), L(j))\}$ subject to a wealth constraint where $W^*(j) \equiv \frac{1}{(1+r)^{j-t}} W(j)$ is the discounted wage. Roy's identity implies $\frac{\partial V}{\partial W^*(k)} = \lambda(t)N(k)$ where $\lambda(t) = \frac{\partial V}{\partial A(t)}$. Differentiating this identity with respect to A(t) yields $\frac{\partial\lambda(t)}{\partial W^*(k)} = \lambda(t) \frac{\partial N(k)}{\partial A(t)} + \frac{\partial\lambda(t)}{\partial A(t)} N(t)$. Multiplying both sides of this equation by $\frac{W^*(k)}{\lambda(t)}$, treating N(t) as a function of the form $N(k) = N\left(\left(\frac{1+\rho}{1+r}\right)^{k-t} \lambda(t),\ W(k)\right) = N\left(\left(\frac{1+\rho}{1+r}\right)^{k} \lambda(0),\ W(k)\right)$ $\left[\right.$which implies $\frac{\partial \ln N(k)}{\partial \ln\lambda(t)} =$

$\frac{\partial \ell n N(k)}{\partial \ell n \lambda(0)}\Bigg]$, and regrouping terms yields $\frac{\partial \ell n \lambda(t)}{\partial \ell n W(k)} = \left(\frac{\partial \ell n N(k)}{\partial \ell n \lambda(0)} + 1\right) \frac{\partial \ell n \lambda(t)}{\partial A(t)} \cdot$

$\frac{E(t)}{(1+r)^{k-t}}$ which is the desired result.

21. The implicit interpretation of cross sectional estimates held by most researchers relates to some sort of "lifetime average" relationship between hours, wages, and income. To estimate such a relationship, observations of $\ell n W_i(t)$ and $Y_i(t)$ must be purged of their transitory components so that they measure their "lifetime average." Predicting $\ell n W_i(t)$ and $Y_i(t)$ is one way of accomplishing this task.

22. See DaVanzo, Detray and Greenberg (1973).

23. The comments that follow on the work of Becker also apply to the work of Smith (1977).

24. The family size variable or age may enter as a regressor for another reason. If period-specific utilities are a function of family size, then the λ constant functions will also depend on family size directly.

25. They also allow for the price of market goods to vary over the life cycle. In terms of the model presented in this paper, this translates into a nonconstant real interest rate.

26. The exogenous variables Z_i and the index i have been suppressed. $\theta(t)$ is assumed to be constant.

27. See Lucas and Rapping (1969, p. 745).

28. An alternative estimation procedure would be to estimate either the specification given by (23) or (24) by a restricted simultaneous equations method. The two-stage estimation procedure described above has two advantages over this more familiar one-stage estimation scheme. The first is computational efficiency. The two-stage procedure significantly

diminishes the number of variables and parameters one must deal with at any one time. In the first stage one neglects all future and past variables; in the second stage, where all these variables enter, the problem is reduced to analyzing only one equation per worker. In the one-step analysis an analyst must include past, present, and future variables in every labor supply equation of which there are several for each worker. Thus, the one-step procedure forces one to consider the impact of all variables at once. This results in greater complexity and computational burden, and it can result in serious difficulties with multicollinearity.

The second advantage of the two-stage estimation procedure concerns the potential correlation of unobserved permanent "person effects" with exogenous variables. Such effects are captured by the disturbance term a^* in the relationship for λ given by equation (13). In the specification of labor supply given by either (21) or (22), this unobserved permanent component ends up in the error term. If these unobserved individual characteristics are correlated with the observed characteristics, the one-step estimation procedure produces inconsistent estimators for all parameters of the labor supply equation. This is not true in the case of the two-step estimation scheme. Only the second-stage estimators are inconsistent. Thus, using the two-step procedure one obtains "clean" estimates for at least the λ constant labor supply function.

29. Adding $\delta D\ell nN_i(t)$ to both sides of equation (12) yields

$$D\ell nN_i(t)(1+\delta) = b+\delta(D\ell nW_i(t)+D\ell nN_i(t))+\varepsilon_i(t)$$

or

$$D\ell nN_i(t) = \frac{b}{1+\delta}+\frac{\delta}{1+\delta}D\ell nE_i(t)+\varepsilon_i(t)$$

This new equation relates changes in hours to changes in earnings. Replacing the reduced form equation for $D\ell nW_i(t)$ by a reduced form equation for $D\ell nE_i(t)$ produces a new simultaneous equations model. Notice that these two reduced forms will have the same set of explanatory variables $X_{2i}(t)$.

30. All of these estimates are well within the -.068 to .44 range of estimates for δ obtained by Becker in Ghez-Becker (1975, pp. 112-13).

31. Such a situation arises in the case of progressive taxes when one approximates a consumer's budget constraint by a linear function that is evaluated at the same point each period.

32. I tried several remedies for this problem. Most of them consisted of entering polynomials of age and education directly into the labor supply equation to capture investment effects. The estimates of δ were, for the most part, unaffected. As one would predict, all parameters were estimated with very little precision.

33. The following empirical results are not sensitive to the particular estimate of b or δ picked from Table 1.

34. If $\gamma(t)$ is constant, we have: $\bar{\gamma}_0 = \gamma(T+1)$; $\bar{\gamma}_1 = \gamma \sum_{t=0}^{T} t$; and $\bar{\gamma}_2 = \gamma \sum_{t=0}^{T} t^2$. The restriction assumes that T+1 = 40. In the actual empirical analysis t is defined as (consumer's age - 18)/10.

35. If one includes dummy variables for house ownership in equation (32), these variables have coefficients that are typically negative. Such variables are better proxies for permanent nonwage income than the income measure reported in Table 3.

36. The synthetic cohort data used here includes all education groups. A small enough number of observations is present in each age group without further dividing the sample into education groups as well. The average number of observations per age group is about 25, with a minimum of seven and a maximum of 44.

37. From the summary statistics reported in Appendix A, we see wide variation in an individual's measured hours of work and wages in adjacent years. Such evidence leads to a strong suspicion of an error in the variables problem for wages. Given the small number of observations per age cell (as few as seven), it is likely that this problem carries over to group averages as well. Because this measurement error in wages is correlated with the dependent variable, estimates of δ are biased toward negative values rather than zero.

38. Background variables include the consumer's father's education, his mother's education, and a dummy variable representing the income level of his parents.

39. If one does not use predicted values for wages and income, the wage coefficient is usually negative and the income coefficient is usually positive, which is a common finding in the literature. See DaVanzo, Detray and Greenberg (1973).

40. Notice that stage one of the estimation procedure prescribed in this paper estimates a first-differenced version of equation (24). First differencing eliminates all time-invariant effects and reduces the number of maintained assumptions one requires to obtain consistent estimates of the intertemporal substitution elasticity. The fact that this first-stage estimation produced a more precise estimate of δ is primarily due to the use of an expanded set of explanatory variables and equality constraints across years on reduced form equations. Using an expanded set of explanatory variables to predict wages in estimating the parameters of equation (24) produces estimates of δ that are very near zero for all years and a pooled estimate of δ that is negative and significant at conventional levels of significance. There are good reasons for preferring the differenced equations.

TABLE 1

SIMULTANEOUS EQUATION ESTIMATION OF FIRST DIFFERENCED LABOR SUPPLY EQUATION ESTIMATES OF THE INTERTEMPORAL SUBSTITUTION ELASTICITY
(absolute values of t-statistics in parentheses)

Estimation Procedure	D(log wage)	D(log earning)	Intercept	Average of Year Dummies
2SLS	.20 (2.05)	. . .	-.0097 (4.31)	. . .
3SLS	.16 (2.01)	. . .	-.0089 (4.52)	. . .
2SLS	. . .	.33 (3.88)	-.009 (6.36)	. . .
3SLS	. . .	.20 (3.7)	-.0086 (6.57)	. . .
2SLS	.18 (1.14)	. . .	. . .	-.008
3SLS	.10 (.75)	. . .	. . .	-.008
2SLS	. . .	.25 (1.77)	. . .	-.008
3SLS	. . .	.18 (1.48)	. . .	-.007

[a]Reduced form equations for log wage and earnings differences include age, education, education squared, interactions between these variables and year effects. Reduced form coefficients are restricted to be the same across years.

[b]While the estimates in the second column refer to δ, the t-statistics are computed for the coefficients on earnings $\omega = \frac{\delta}{1 + \delta}$. To convert the t-statistics reported for ω to those for δ requires multiplication by the square of the quantity $\frac{d\omega}{d\delta} = \frac{1}{(1 + \delta)^2}$.

TABLE 2

SIMULTANEOUS EQUATION ESTIMATION OF AVERAGED FIRST DIFFERENCED LABOR SUPPLY EQUATIONS ESTIMATES OF INTERTEMPORAL SUBSTITUTION ELASTICITY
(absolute values of t-statistics in parentheses)

Estimation Procedure	Averages Computed over	Averaged D(log wage)	Averaged D(log earnings)	Intercept
2SLS	Entire 8 year period	.18 (1.10)		-.0095 (3.04)
2SLS	1968-1975		.25 (1.8)	-.0086 (5.06)

[a]The R^2 for the reduced form for averaged wage and earnings differences is .025 and .04 respectively.

[b]See Table 1, note b.

TABLE 3

SIMULTANEOUS EQUATION ESTIMATION OF FIXED EFFECTS EQUATIONS
(absolute values of t-statistics in parentheses)

$\bar{\gamma}_0$	$\bar{\gamma}_1$	$\bar{\gamma}_2$	$\bar{\theta}$	Intercept
-.17 (1.5)	-4.69 (1.3)	-18.45 (1.01)	.0246 (.3)	7.72 (56)
-.10 (1.6)	-.22 (1.6)	-.58 (1.6)	.008 (.25)	7.61 (150)

[a]t here is measured in decades and is 0 when the consumer is 18 years old (i.e., t = (consumer's age - 18)/10).

TABLE 4

CROSS SECTION ESTIMATION
(absolute value of t-statistics in parentheses)

Data and Estimation Procedure	Coefficient on $\ell n \hat{W}$										Other Included Variables[b, c]
	67	68	69	70	71	72	73	74	75	Pooled[a]	
Synthetic Cohorts LS	-.10 (1.15)	-.15 (1.69)	-.18 (1.91)	-.17 (1.82)	-.15 (1.51)	-.13 (1.87)	-.21 (2.84)	-.04 (.52)	-.04 (.6)	-.17 (3.17)	Age
Synthetic Cohorts LS	.07 (.8)	.04 (.4)	.04 (.4)	.06 (.5)	.08 (.7)	-.01 (.1)	-.06 (.6)	.11 (1.29)	.05 (.7)	.06 (2.09)	Age
Individual Data 2SLS	.04 (.89)	.05 (1.07)	.07 (1.66)	.12 (2.01)	.32 (1.65)	.11 (1.58)	.11 (1.42)	.23 (2.13)	.12 (1.55)	.02 (.8)	Predicted Income
Individual Data 2SLS	.13 (.5)	.25 (.6)	.27 (.9)	.24 (.6)	.20 (.7)	.07 (.3)	-.05 (.3)	.28 (.6)	-.07 (.3)	-.03 (.2)	Background Variables, Education, and Age
Individual Data 2SLS	.005 (.07)	-.06 (.51)	-.03 (.35)	.05 (.8)	.24 (1.32)	.07 (1.14)	.09 (1.3)	.18 (1.8)	.18 (1.9)	.04 (1.03)	Predicted Income, Age, and Age Squared

[a]Estimates in the pooled column were obtained by estimating equations for 1967-75 by seemingly unrelated regression techniques for the synthetic cohort models and by 3SLS imposed on the wage coefficients across years for the other models with an equality constraint.

[b]Intercept always included.

[c]Reduced form equations for log wages and earnings include education, education squared, age, age squared, and background variables.

TABLE A-1

SAMPLE SUMMARY STATISTICS

Variable Definitions	Variable	Mean	Standard Deviation	Minimum Value	Maximum Value	Skewness	Kurtosis
	W67	4.08	2.16	0.75	25.76	2.82	19.42
	W68	4.26	2.18	0.41	21.57	2.04	8.94
	W69	4.48	2.37	0.22	20.87	2.11	8.45
W(t) = average hourly earnings	W70	4.53	2.40	0.34	21.49	1.94	7.04
in year t	W71	4.60	2.61	0.52	21.49	2.68	13.81
	W72	4.80	2.56	0.68	26.17	1.80	5.71
	W73	4.96	2.83	0.68	19.24	2.52	10.55
	W74	4.94	2.90	0.71	24.61	2.48	10.08
	W75	4.82	2.92	0.31	25.42	2.91	15.22
	DW68	0.18	1.20	-5.09	9.12	1.66	13.50
	DW69	0.22	1.12	-6.24	8.20	0.80	11.07
	DW70	0.04	1.27	-10.24	10.34	-0.61	20.52
DW(t) = W(t) - W(t-1)	DW71	0.07	1.45	-7.05	15.54	3.43	41.03
	DW72	0.19	1.34	-14.06	5.33	-2.57	28.89
	DW73	0.15	1.41	-7.56	11.21	1.23	14.18
	DW74	-0.01	1.60	-7.67	10.52	1.50	12.26
	DW75	-0.12	1.48	-11.08	5.78	-2.04	13.49
	N67	2392.33	568.16	920.00	4368.00	0.97	1.56
	N68	2402.14	544.46	1031.99	4560.00	1.03	1.43
	N69	2363.22	543.38	400.00	4639.99	0.93	2.23
	N70	2315.52	522.58	520.00	4339.99	0.78	1.67
N(t) = annual hours worked	N71	2324.89	546.10	552.00	4455.00	0.79	1.48
in year t	N72	2351.55	523.09	800.00	4310.00	0.80	1.41
	N73	2340.83	520.65	600.00	4420.00	1.06	2.14
	N74	2289.95	529.57	400.00	4215.00	0.63	1.28
	N75	2268.25	506.19	879.99	4160.00	0.61	1.03
	DN68	9.81	447.82	-2397.99	1920.00	-0.10	3.83
	DN69	-38.91	421.21	-2382.00	2186.00	-0.15	4.54
	DN70	-47.69	431.45	-2366.99	1520.00	-0.41	4.59
DN(t) = N(t) - N(t-1)	DN71	9.36	418.12	-1937.99	2660.00	0.62	5.72
	DN72	26.66	451.03	-2030.00	2136.99	0.23	3.89
	DN73	-10.72	447.57	-1740.00	2150.00	0.11	3.79
	DN74	-50.88	431.80	-2508.00	1617.00	-0.51	4.32
	DN75	-21.70	393.13	-2324.99	1324.00	-0.52	3.96