INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2016, VOL. 62, NO. 2, PP. 159–165
Manuscript received January 12, 2016; revised June, 2016. DOI: 10.1515/eletel-2016-0022

An Energy Efficient QAM Modulation with Multidimensional Signal Constellation

Tomasz G. Markiewicz

***Abstract*—Packing constellations points in higher dimensions, the concept of multidimensional modulation exploits the idea drawn from geometry for searching dense sphere packings in a given dimension, utilising it to minimise the average energy of the underlying constellations. The following work analyses the impact of spherical shaping of the constellations bound instead of the traditional, hyper-cubical bound. Balanced constellation schemes are obtained with the N-dimensional simplex merging algorithm. The performance of constellations of dimensions 2, 4 and 6 is compared to the performance of QAM modulations of equivalent throughputs in the sense of bits transmitted per complex (two-dimensional) symbols. The considered constellations give an approximately $0.7\,\mathrm{dB}$ to $1\,\mathrm{dB}$ gain in terms of BER over a standard QAM modulation.**

Keywords*—communication system, QAM modulation, lattice, multidimensional constellations

I. Introduction

The use of multidimensional constellations was introduced by Gersho and Lawrence in [10]. Their work showed the method of generating four-dimensional signals based on the A_n lattice. They assumed a cubical topology of the underlying constellations (4D or 8D). However, constellations that are bounded by an N-dimensional sphere result in a much lower average energy due to a more cohesive packing [18]. A $1.2\,\mathrm{dB}$ gain with respect to the 16-QAM was achieved at $BER = 10^{-6}$ for a four-dimensional constellation in an experiment, and a gain of $2.4\,\mathrm{dB}$ was calculated mathematically for an eight-dimensional constellation [10]. In [4], Bourtos *et. al.* considered certain lattice packings that led to good constellations for both Rayleigh and Gaussian channels, which were used for a mathematical description of terrestrial and satellite links. They presented a thorough method of construction of multidimensional lattices from totally complex cyclotomic algebraic number fields, using a Q-homomorphism (rational number invariant homomorphism), in order to find a uniform class of constellations with a good performance for terrestrial and satellite links. Despite analysing a plethora of lattices, their work does not cover A_n lattices. Porath and Aulin described in [16] the method of constructing constellations from the behaviour of equally charged particles in free space. However, constellations constructed in such a manner suffer from the 'zero symbol' that is located in the beginning of the coordinate system. Unfortunately, removing the 'zero symbol' may cause a non-optimal distribution of points of a constellation, increase in its average energy and degradation of the overall performance as a final result. The method proposed by them results in a constellation that achieves approximately a $0.2\,\mathrm{dB}$ gain with respect to the 16-QAM at $BER = 10^{-4}$.

An attempt to generalise multidimensional modulations in the time or frequency domains was undertaken by Sari in [20]. It was done using Walsh-Hadamard sequences, therefore, the solution was limited only to the dimensions of integer powers of 2. The main emphasis of multidimensionality in his work was put on a multi-user communication instead of enhancing the performance of a communication system.

Various aspects of multidimensional communication methods have been studied so far, either from the perspective of multidimensional signal sets [11], [19], [21] together with TCM [17], or with STBC codes [2], as well as from the point of view of information theory, in order to generate optimum codes for band-limited channels [3]. The impact of forming constellations based on multidimensional lattices was exploited in [4], [10], [16] as well. Multidimensional constellation schemes were also applied with success in optical communications [6], [8], [9], [13], [14]. It led to the rise of many new modulations, e.g., RGI-CO-OFDM (Reduced Guard Interval Coherent Optical Orthogonal Frequency Division Multiplexing), DP-BPSK (Double Polarisation BPSK), (6PolSK)-QPSK (6-ary Polarisation Shift Keying QPSK) or PDM-QPSK (Polarisation Division Multiplexed QPSK).

This paper is organised as follows. Section II covers the basics of the lattice theory and the motivation and justification of using multidimensional constellations instead of complex ones. In Section III, a simulation system is described together with the applied methodology. That section also gives some details about the A_n lattice and the algorithm of constellation building. Section IV presents the results of the simulation experiment performed in order to compare the performance of the system based on typical QAM constellations and multidimensional constellations derived from the A_n lattice. The behaviour of the particular constellation of various dimensions is shown under different, but comparable, bit rates. A discussion on the results obtained from the computer simulation is presented in Section IV. Finally, in Section V, a brief summary and possible further research ideas are enclosed.

II. Background

A. Communication System

A traditional communication system can be roughly presented as in Fig. 1; it is composed of two entities at the transmitter side: symbol mapper and modulator (with an implicit pulse shaping), and two dual entities at the receiver side: symbol de-mapper and demodulator (Fig. 1). In such a system, bits are mapped into two-dimensional (complex) symbols according to a bijective mapping, which very often happens

Autor is PhD student at Poznań University of Technology, Poznań, Poland (e-mail: tomasz.g.markiewicz@gmail.com).

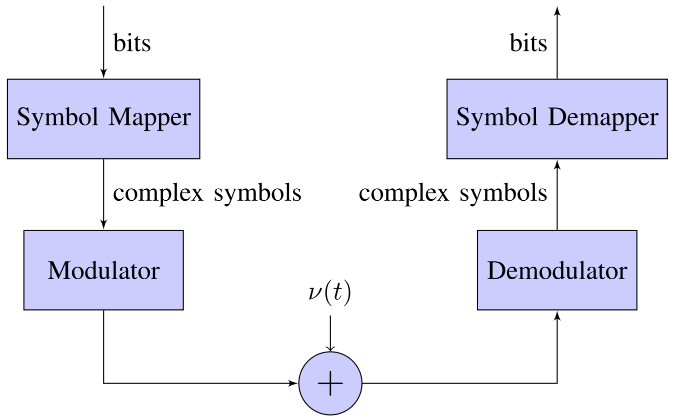

Fig. 1. Traditional two-dimensional system, $\nu(t)$ – Gaussian noise with $\mathcal{N}(0, N_0/2)$ distribution.

to be a Gray-based one. The carrier signal is modulated by complex symbols and the band-pass signal is produced as a result according to:

$$s(t) = \sum_i \mathfrak{Re}\left\{d_i \cdot e^{j2\pi(t-i\cdot T_s)}\right\}, \tag{1}$$

where T_s is the duration of a single symbol, and d_i is a complex data symbol in the i-th interval. However, an implicit assumption of using complex symbols to modulate a carrier signal limits modulations to two-dimensional constellations. Such a constraint can be overcome by distributing the coordinates of a multidimensional symbol over several complex symbols. Multidimensional symbols are chosen from an arbitrarily selected number of dimensions of the A_n lattice [5]. The fundamental block (the inseparable set of points that unambiguously define a lattice in terms of its basic vectors) of a given A_n lattice is an N-dimensional simplex. In order to construct an energy-balanced constellation (i.e., with symbols distributed uniformly in an N-dimensional hyperspace), the simplex merging algorithm was applied to N-dimensional simplexes with the initial simplex placed in the centre of the coordinate system.

B. Brief Theory of Lattices

The most naïve definition of a lattice would be *a set of dots*. Although, not just an ordinary set of some randomly picked dots, but rather a structured, infinite, countable and self-repetitive pattern of points in some N-dimensional hyperspace. An example of a two-dimensional square lattice is shown in Figure 2.

From the mathematical point of view, a lattice Λ is an algebra of a discrete subset of $\mathbb{R}^N$ that displays the following properties:

- closure

$$\bigwedge_{\Lambda\ni a,b} : a + b \in \Lambda \tag{2}$$

- associativity

$$\bigwedge_{\Lambda\ni a,b,c} : (a + b) + c = a + (b + c) \tag{3}$$

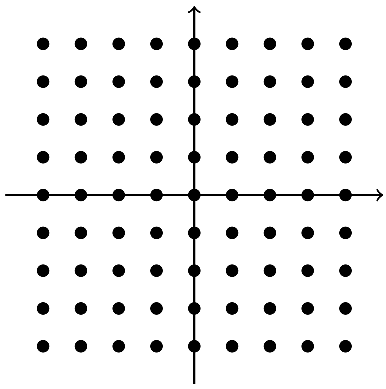

Fig. 2. Square lattice.

- commutativity

$$\bigwedge_{\Lambda\ni a,b,c} : a + b = b + a \tag{4}$$

- identity

$$\bigwedge_{\Lambda\ni a} \bigvee_{\Lambda\ni e} : a + e = e + a = a \tag{5}$$

- invertibility

$$\bigwedge_{\Lambda\ni a} \bigvee_{\Lambda\ni a^{-1}} : a + a^{-1} = e \tag{6}$$

In general, lattice Λ can be sufficiently described by its generation matrix. A generation matrix is used to compute the coordinates of lattice points. In order to create an N-dimensional lattice, a square generation matrix of size N is required, e.g.:

$$\mathbf{G}_\Lambda = \begin{bmatrix} \mathbf{v}_1{}^T \\ \mathbf{v}_2{}^T \\ \vdots \\ \mathbf{v}_N{}^T \end{bmatrix}^T . \tag{7}$$

Lattice points are generated by multiplying $\mathbf{G}_\Lambda$ by consecutive vectors of integer numbers, or more generally:

$$\Lambda \ni \lambda_\mathbf{v} = \mathbf{G}_\Lambda \cdot \mathbf{w}, \tag{8}$$

where $\mathbf{w} \in \mathbb{Z}^N$. The columns of generator matrix $\mathbf{G}_\Lambda$ must be linearly independent, i.e.:

$$\alpha_1 \cdot \mathbf{v}_1 + \alpha_2 \cdot \mathbf{v}_2 + \ldots \alpha_N \cdot \mathbf{v}_N = 0, \tag{9}$$

is satisfied only if $\alpha_1 = \alpha_2 = \cdots = \alpha_N = 0$ An equidistant lattice is a lattice that fulfils the following condition:

$$\|\mathbf{v}_1\| = \|\mathbf{v}_2\| = \ldots = \|\mathbf{v}_N\| > 0. \tag{10}$$

The minimum lattice distance is defined as:

$$d_{min}(\Lambda) = \min_{\Lambda\ni\lambda_{i,j}} (\|\lambda_i - \lambda_j\|) = \min_{\Upsilon\ni\mathbf{v}_i} (\|\mathbf{v}_i\|), \tag{11}$$

and $\Upsilon = \{\mathbf{v}_1, \mathbf{v}_2, \cdots \mathbf{v}_N\}$ is a set of the basic vectors over which a lattice is spanned. Of course, in the case of equidistant lattices $d_{min}(\Lambda) = \|\mathbf{v}_i\|$. Depending on its packing tightness, each lattice has its unique density which is defined in the following way:

$$\Delta(\Lambda) = \frac{V_n \rho^n}{\sqrt{V(\Lambda)}}, \quad (12)$$

where $V(\Lambda) = \|det(\mathbf{G}_\Lambda)\|$ is the volume of the lattice Voronoi region and V_n is the volume of the n-dimensional sphere of unit radius:

$$V_n = \frac{\pi^{n/2}}{n/2!} = \frac{2^n \pi^{\left(\frac{n-1}{2}\right)} \left(\frac{n-1}{2}\right)!}{n!}. \quad (13)$$

C. Motivation and Justification

In general, it is intuitively known that there is 'more space' in higher dimensions than in lower ones, e.g., the average energy ($\mathcal{E}_{ave}$) of a 16-QAM constellation, assuming its minimum distance $d_{min} = 2$, is $\mathcal{E}_{ave} = 9.75$, while for a 16-PAM it is $\mathcal{E}_{ave} = 28.125$, which results in an approximately $4.6\,\mathrm{dB}$ gain of the 16-QAM over the 16-PAM[1]. A simple analysis of extending constellations dimensionality gives a coarse view on the impact of the dimensionality on moving communication from PAM to QAM. However, unlike in the case of PAM and QAM, new constellations cannot be just Cartesian products of constellations of lower dimensions. Energy efficiency requires that constellation points be packed more compactly. It is due to the joint de-mapping of symbols from an N-dimensional QAM constellation which would not increase the energy efficiency of the underlying system, since it can be transmitted only on at most two available carriers – the in-phase and quadrature ones – therefore, such a system would differ from traditional a QAM system only at the receiver side by the joint decoding/de-mapping. It is why new constellation schemes are required in order to improve the energy efficiency.

Fortunately, there is no restriction in (1) to the dimension from which the data symbols d_i must be chosen. However, it is at the price of a longer transmission time and a greater complexity of the transmitter/receiver. The extended transmission time is a consequence of sending a multidimensional signal with a two-dimensional quadrature modulator. A higher dimensionality of constellation causes a longer transmission time of a single multidimensional symbol. An increased complexity is, on the other hand, caused by additional processing required for the proper transmission and reception of multidimensional symbols. Furthermore, in higher dimensions, a very high *kissing number* (i.e., the number of lattice points that are placed at the distance d_{min} from any other of its points) may decrease the constellation performance in terms of $BER(E_b/N_0)$ [5], [18].

[1]Neglecting error probabilities of these two modulations/mappings. In fact 16-QAM provides a definitely better performance in the term of BER than 16-PAM at the same E_b/N_0. It is slightly worse than performance of 4-PAM (due to the joint character of QAM modulation), while providing twice as good bit rate [18].

III. Methodology

A. Multidimensional QAM Signal

In the considered system, multidimensional symbols are carried by consecutive QAM signals that are constructed as follows:

$$s(t) = \sum_i \Re\mathfrak{e}\{\mathbf{E}_i(t) \cdot \mathbf{d}\}, \quad (14)$$

where $\mathbf{E}_i(t)$ is a modulation matrix, and $\mathbf{d}$ is a vector containing K consecutive complex symbols, hence the dimensionality of the the modulation is $2K$. Modulation matrix $\mathbf{E}_i(t)$ and multidimensional symbol $\mathbf{d}$ can be expressed in the following way:

$$\mathbf{E}_i(t) = \begin{bmatrix} E_{i,0} & 0 & \cdots & 0 \\ 0 & E_{i,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{i,K-1} \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ d_{K-1} \end{bmatrix} \quad (15)$$

where $E_{i,j} = e^{j2\pi(t-(Ki+j)\cdot T_s)}$.

B. Lattice Selection

The transmitted multidimensional symbols were chosen from n-dimensional constellations based on an A_n lattice [5]. The two-dimensional realisation of such lattice, namely A_2, is shown in Fig. 3.

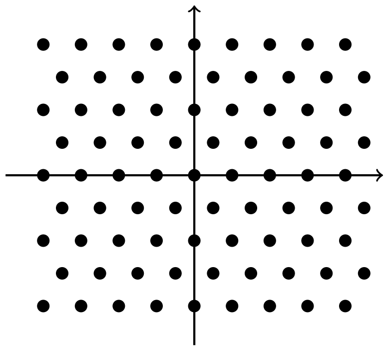

Fig. 3. Hexagonal (two-dimensional simplex, triangle) lattice, a two-dimensional example of an A_n lattice.

C. Fundamental Block

A fundamental block of the considered N-dimensional lattice is an N-dimensional simplex. Its construction was performed iteratively, exploiting the fact that an N-dimensional simplex is a polytope composed of exactly $N+1$ equally-distant vertices. Therefore, having the coordinates of a simplex of an arbitrary dimension N, the coordinates of a simplex of dimension $N+1$ were computed as follows. Assuming that an N-dimensional simplex S_N is composed of the set of points: $\{s_1, s_2, \ldots, s_{N+1}\}$, where every point s_i for $i = 1, \ldots, N+1$

is taken from an N-dimensional hyperspace, then the $N+1$-dimensional simplex is uniquely defined[2] by adding the point s_{N+2} to the set of points of simplex S_N:

$$s_{N+2}(k) = \begin{cases} \frac{1}{N+1} \sum_{i=1}^{N+1} s_i(k) & \text{for } k \leq N \\ \sqrt{1 - \sum_{i=1}^{N+1} s_{N+2}^2(i)} & \text{if } k = N+1, \end{cases} \tag{16}$$

where $s_i(k)$ is the k-th coordinate of the s_i point. An extension of the N-dimensional simplex onto the $N+1$-dimensional hyperspace is accomplished by ensuring that the last coordinate of s_{N+2}, i.e., $s_{N+2}(N+1)$, is non-zero, while the last coordinates of the remaining vertices are equal to zero.

D. Constellation Construction

The algorithm presented in this work overcomes the 'zero signal' problem [16] by assuring that no point is given in the centre of the constructed constellation. Therefore, having initially an N-dimensional simplex, which is equivalent to one surface of $N+1$ points, the next surfaces were constructed by gluing new simplexes to the primary one with $(N-1)$-dimensional cells. (The N-dimensional simplex is composed of $N+1$ $(N-1)$-dimensional cells, which are actually $(N-1)$-dimensional simplexes, e.g., an equilateral triangle is composed of three line segments, which are in fact one-dimensional simplexes. Similarly, a tetrahedron is built of four equilateral triangles, which are simplexes of the dimension of two.) The main advantage of building constellations by gluing is assuring a low average energy and keeping constellations balanced (the sum of its symbols was equal to zero). An example of building a constellation using the gluing algorithm is shown in Figure 4. The first step is to start with a triangle in the centre of the coordinate system which results in a base simplex that constitutes the first layer (i.e., the unpatterned triangle in Figure 4). The next three triangles are 'glued' to the edges of the first triangle (or in general to the walls of the base simplex) and, as a result, a second layer is built (the three triangles patterned with left-diagonal lines in Figure 4). After that the third layer of triangles is glued to the edges of the triangles of the second layer, and finally, the fourth layer is constructed. The constellation is composed of vertices of triangles (simplexes) from all built layers. Having built the constellation in such a way, any undesired points that could have come from the numerical noise of floating point operations [12] were removed by filtering with an N-dimensional hypersphere of radius $r = \Delta \cdot d_{min}$, where d_{min} is the minimum distance of the constellation, and the number $\Delta = 0.001$ was chosen arbitrarily and was linearly increased if a single filtering iteration did not remove all numerical noise. An example of such filtration is shown in Figure 5, where two distinct constellation points are initially constructed, but since both of them lie inside the filtration sphere, only one is finally chosen to be a valid constellation point. The decision which of the points should be chosen can be based upon a random choice due to the sufficiently small radius r of the filtration sphere, thus any errors are negligible. After filtering, if necessary, some constellation points are removed (typically those of the highest energy) in order to assure that the constellation is composed of the number of points that is a power of 2.

[2]Note, that, in fact, in (16), the last coordinate can be defined either on the negative or on the positive half of the $N+1$-dimensional hyperspace – due to the square root. Since both simplexes defined in such a manner are congruent, the foregoing ambiguity was considered of no particular importance.

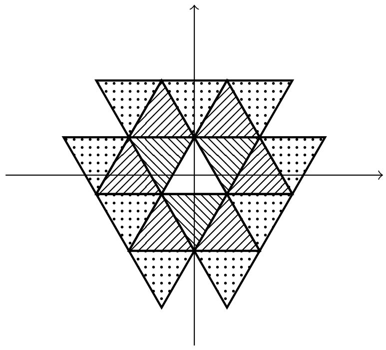

Fig. 4. Stages of building a 2-dimensional constellation composed of equilateral triangles.

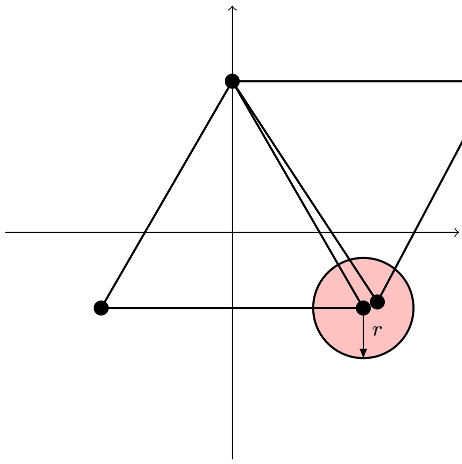

Fig. 5. Numerical noise filtration.

E. Energy Normalisation and Balancing

In order to assure a fair comparison between multidimensional constellations and two-dimensional ones, appropriate energy scaling has been performed. The average energy of multidimensional constellation was normalised to unity. Besides, multidimensional constellations were assured to be balanced, i.e.:

$$\sum_i \mathbf{s}_i = \mathbf{0}, \tag{17}$$

where $\mathbf{s}_i$ is a multidimensional symbol.

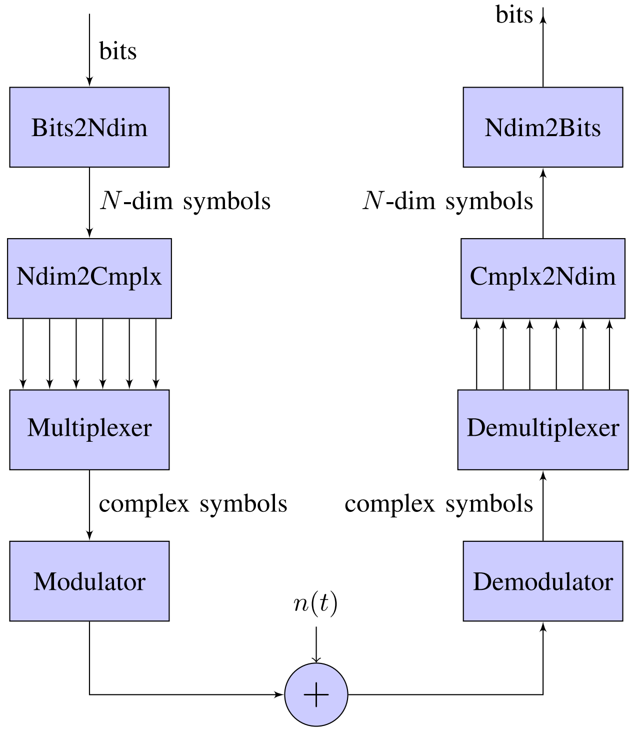

Fig. 6. N-dimensional system model

F. Bit Mapping

Although Gray-coding is widely used for various classes of real-world to digital-world source coding problems, it has no direct applicability to the multidimensional symbols set based on the simplex topology. It is due to a very complex (in the sense of binary labelling) nature of the neighbourhood of symbols of A_n-based constellations. Therefore, a simpler mapping was used. Natural binary mapping was applied to the lexicographical order of points.

G. System Model

A model of the considered system is shown in Fig. 6. While the modulator and demodulator are the same as in the system shown in Fig. 1, the mapper and demapper functionalities were divided into three distinct modules, namely: the bit-to-multidimensional symbol mapper – 'Bit2Ndim', the multidimensional-to-complex symbol mapper – 'Ndim2Cmplx' and the multiplexer (demultiplexer at the receiver side). Formally, the 'Bits2Ndim' component is a function $\phi : \mathbb{Z}_2{}^{K_{bit}} \to \mathbb{R}^N$, which maps a block of bits $b = (b_1, b_2, \ldots, b_{K_{bit}})$ to an N-dimensional symbol $p = (p_1, p_2, \ldots, p_N)$ (or equivalently to a point in an N-dimensional hyperspace), where $K_{bit} = \log_2 K$ and K is the number of N-dimensional constellation points. The 'Ndim2Cmpl' is responsible for mapping N-dimensional symbols onto complex ones. The Mapping is done in such a way that the pairs of coordinates from consecutive dimensions are composing the consecutive complex symbols. As a result, the $N/2$ complex symbols are multiplexed, modulated and transmitted through the AWGN channel. The parallel-to-serial conversion is done by the 'Multiplexer' element.

On the receiver side, all operations are performed in reverse order with the except of 'Ndim2Bit' module, which is in fact a Maximum Likelihood (ML) detector that minimises the Euclidean distance between the received symbol and the candidate symbol selected from a constellation.

$$ML(\mathbf{r}, \mathbf{s}) = \min_{C \ni \mathbf{s}_i} \|\mathbf{r} - \mathbf{s}_i\|^2, \tag{18}$$

where C denotes the set composed of constellation points.

H. Computer Simulation

The performance of the considered system was measured by computer simulations written in C++ with the support of the IT++ library [15]. The comparison was made between constellations that yield to the same throughput, details are in provided Table I.

TABLE I. SIMULATION CASES

Name	Dim	Order	Throughput [b/cs][a]
QAM	2	4, 16, 64	2, 4, 6
An	4	16, 256, 4096	2, 4, 6
An	6	64, 4096, 262144	2, 4, 6

[a] bits per complex symbol

IV. RESULTS AND DISCUSSION

This section contains simulation results for evaluating the performance of the proposed multidimensional constellations based on A_n lattices (simplex lattices). For all simulations, the same number of bits were transmitted in order to provide both comparability and sufficient reliability of the obtained results. The system was simulated in the AWGN (Additive White Gaussian Noise) channel in order to allow a fair comparison of different constellations in additive noise environment without the impact of coding techniques which could obscure the results. The numbers in square brackets in the legend (Fig. 7 and Fig. 8) refer to the constellation's dimensionality and its order (number of points), e.g., An[4,256] describes a four-dimensional constellation with 256 points that is based on the A_4 lattice. Although dimensionality in the case of QAM is known *a priori* and is fixed, it is used to maintain the legend's consistency. The term 'NDIM-SER' refers to the multidimensional symbol error rate (error in the N-dimensional hyperspace) and is used in order to fairly compare symbol error rates of QAM and A_n constellations (neglecting the bit-to-symbol mapping impact). In the case of a QAM constellation, an NDIM-SER is exactly the same as an ordinary SER.

A. Results

Figure 7 depicts the performance in terms of bit error rates of the considered constellation schemes. Constellation An[4,256] gives an approximately $0.5\,\text{dB}$ gain over QAM[2,16] at the $BER = 10^{-6}$ and An[6,4096] gives an approximately $0.7\,\text{dB}$ gain at the level of $BER \approx 2 \cdot 10^{-6}$. An[4,4096] results

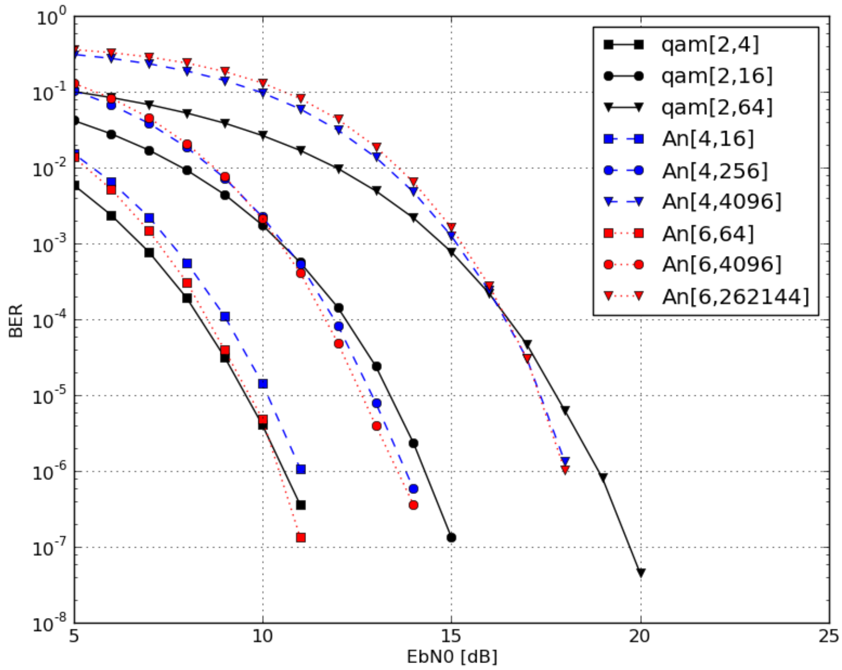

Fig. 7. Bit Error Rate of QAM and An constellations over AWGN channel.

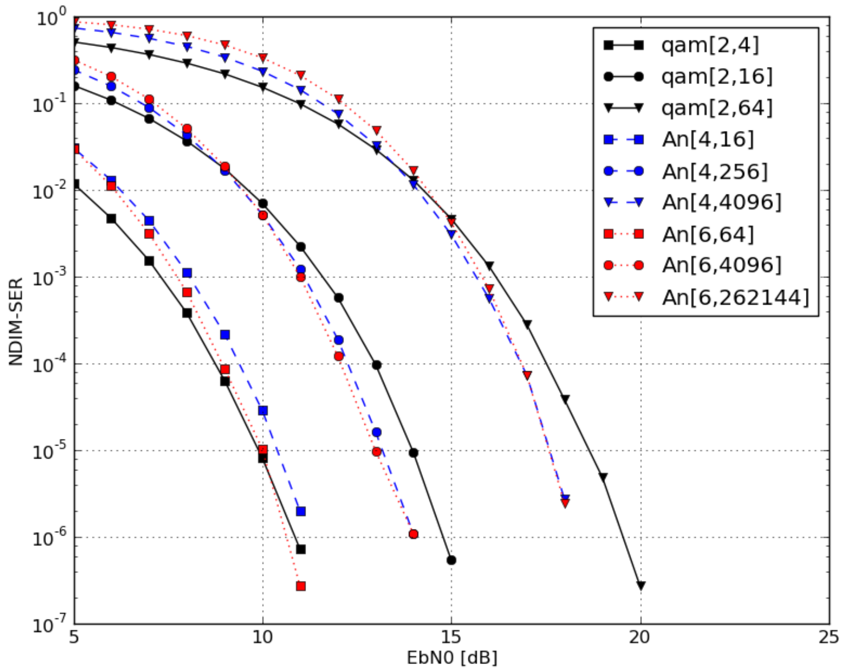

Fig. 8. Symbol Error Rate of QAM and An constellations over AWGN channel.

in *circa* $1\,\mathrm{dB}$ gain over QAM[2,64] at the $BER \approx 10^{-6}$. However, the performance of a four dimensional simplex-based constellation An[4,16] is worse by approximately $0.5\,\mathrm{dB}$ than QAM[2,4], An[6,64] behaves similar. All A_n-based constellations are worse than QAM-based constellations under some critical E_b/N_0 level; for QAM[2,16] it is $12\,\mathrm{dB}$ at the $BER \approx 10^{-3}$ and for QAM[2,64] it is $17\,\mathrm{dB}$ at the $BER \approx 3 \cdot 10^{-4}$.

Figure 8 reveals the performance of the considered constellations schemes in terms of symbol error rates. Basically, A_n-based constellations outperform QAM-based ones with a difference approximately from $0.1\,\mathrm{dB}$ to $0.2\,\mathrm{dB}$ higher than in the case of BER comparison. The only difference is in the case of critical points, where constellations based on A_n lattices start to outperform QAM constellations; simplex-based constellations become better than QAM faster in terms of NDIM-SER than in terms of BER. It is probably caused by (besides the better performance) the lack of Gray-based bit mappings suitable for A_n lattices.

B. Discussion

A very low gain of An[4,256] and An[6,4096] over QAM[2,16] (approximately $0.5\,\mathrm{dB}$) as well as a gain of An[4,4096] over QAM[2,64] (*circa* $1\,\mathrm{dB}$) is probably due to the lack of Gray-based mapping of bits onto multidimensional symbols, while at the same time, mapping used for QAM transmission was Gray-based. However, it is not a methodology deficiency, but rather a topological trait of simplex lattices; the optimisation of such a mapping is definitely not a trivial problem, due to the non-binary nature of A_n lattices. The justification of that assumption is confirmed by the results shown in Fig. 8; the NDIM-SER results definitely show that for high E_b/N_0 scenarios, A_n-based constellations outperform QAM-based. However, the gain of QAM over A_n constellations for low E_b/N_0 is probably caused by a high *kissing number* of multidimensional simplex lattices, leading to the higher error probability of the latter.

A solution proposed by Porath and Aulin in [16] gives an approximately $0.2\,\mathrm{dB}$ gain in terms of symbol error rate over 16-QAM. The one of solutions proposed in this paper (i.e., constellation composed of 4096 points that is based on A_6 lattice) is approximately $1\,\mathrm{dB}$ better than what Porath and Aulin considered, while still preserving throughput of the 16-QAM.

V. Conclusions

Simulation of the communication system based on multidimensional simplex constellations and its comparison between a typical communication system (based on two-dimensional QAM constellations) was made. The obtained results show the gain ($1\,\mathrm{dB}$ in terms of BERs and $1.2\,\mathrm{dB}$ in terms of multidimensional SERs – NDIM-SERs) of the proposed constellation over a typical QAM system, resulting in better overall energy efficiency in such scenario. However, further research is needed to confirm whether the performance of the proposed system can be increased; research focused on the possible bit mappings may lead to better results.

References

[1] Technical Radio Group 3GPP. 3GPP TS 36.211, Physical Channels and Modulations. Technical Report Release 8, 3rd Generation Partnership Project (3GPP), 2009.

[2] E. Ben Slimane, S. Jarboui, and A. Bouallegue. Concatenated Orthogonal Space-Time Block Coding with Four Dimensional 8-PSK Trellis Coded Modulation over Fading Channels in Absence of Channel State Information. In *Computing, Communications and Applications Conference (ComComAp), 2012*, pages 162–166. Ieee, jan 2012.

[3] E. Biglieri and M. Elia. Multidimensional Modulation and Coding for Band-Limited Digital Channels. *IEEE Transactions on Information Theory*, 34(4):803–809, 1988.

[4] J. Boutros, E. Viterbo, C. Rastello, and J. C. Belfiore. Good Lattice Constellations for Both Rayleigh Fading and Gaussian Channels. *IEEE Transactions on Information Theory*, 42(2):502–518, 1996.

[5] J. H. Conway and N. J. A. Sloane. *Sphere packings, lattices and groups*. Springer Science, New York, 2nd edition, 1993.

[6] A. Dochhan, B. Teipen, and M. Eiselt. A 3-Dimensional Simplex Modulation Format with Improved OSNR Performance Compared to DP-BPSK. In *Photonic Networks, 14. 2013 ITG Symposium. Proceedings*, pages 1–5, 2013.

[7] ETSI EN 302 755 Digital Video Broadcasting (DVB). Frame structure channel coding and modulation for a second generation digital terrestrial television broadcasting system (DVB-T2), 2012.

[8] J. K. Fischer, S. Alreesh, R. Elschner, F. Frey, N. Markus, C. Schmidt-Langhorst, and C. Schubert. Bandwidth-Variable Transceivers based on Four-Dimensional Modulation Formats. *Journal of Lightwave Technology*, 32(16):2886–2895, 2014.

[9] J. K. Fischer, S. Alreesh, T. Tanimura, R. Elschner, F. Frey, C. Meuer, and L. Molle. Four-Dimensional Coded Modulation : 6PolSK-QPSK. In *Photonic Networks, 14. 2013 ITG Symposium. Proceedings*, pages 5–10, 2013.

[10] A. Gersho and V. B. Lawrence. Multidimensional Signalling Constellations for Voiceband Data Transmission. *IEEE Journal on selected areas in communication*, SAC-2(5):687–702, 1984.

[11] J. S. Han and M. J. Kim. Offset Quadrature-Quadrature Phase Shift Keying with Half-Sine Pulse Shaping. In *2013 International Conference on ICT Convergence (ICTC)*, pages 931–935. Ieee, oct 2013.

[12] IEEE Task P754. IEEE 754-2008, Standard for Floating-Point Arithmetic, 2008.

[13] M. Karlsson and E. Agrell. Spectrally efficient four-dimensional modulation. In *Optical Fiber Communication Conference*, pages 1–3, Washington, D.C., 2012. Osa.

[14] J. Karout, X. Liu, C. Sethumadhavan, E. Agrell, M. Karlsson, and R. J. Essiambre. Experimental Demonstration of an Optimized 16-ary Four-Dimensional Modulation Format Using Optical OFDM. In *Optical Fiber Communication Conference/National Fiber Optic Engineers Conference 2013*, pages 1–3, Washington, D.C., 2013. Osa.

[15] NEWCOM. libitpp, 2007.

[16] J. Porath and T. Aulin. Design of multidimensional signal constellations. *Communications, IEE Proceedings-*, 150(5):317–323, 2003.

[17] K. H. Priya and M. Tamilarasi. A Trellis-Coded Modulation Scheme with 32-Dimensional Constant Envelope Q^2PSK Constellation. In *International Conference on Communication and Signal Processing*, pages 821–825, 2013.

[18] J. G. Proakis and M. Salehi. *Digital Communications*. McGraw-Hill Education, New York, 5 edition, 2008.

[19] D. Saha and T. Birdsal. Quadrature-Quadrature Phase-Shift Keying. *IEEE Transactions on Communications*, 37(5):473–448, 1989.

[20] H. Sari. A generalization of rnultidimensional modulation. In *IEEE International Conference on Communications*, pages 683–687, Seattle, 1995.

[21] M. Visintin, E. Biglieri, and V. Casteuani. Four-Dimensional Signaling for Bandlimited Channels. *IEEE Transactions on Communications*, 42(2):403–409, 1994.