An Energy Sharing Game in Prosumers based on Generalized Demand Bidding: Model and Properties

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***Abstract*—The advent of energy "prosumers" that not only consume but also produce energy, advocates a sharing market to encourage energy exchange. Motivated by the recent technology of online platforms, this paper proposes a simple but effective mechanism for energy sharing by generalizing demand bidding. Towards this end, a generic supply-demand function (SDF) is devised for individual prosumers to determine their role of buyer or seller in the sharing market, where the outcome is shown to be a Nash equilibrium (NE) among prosumers. The existence and uniqueness of NE are proved. Properties of the equilibrium price are uncovered. Compared with individual decision-making, the disutility of each prosumer can always be reduced via purchasing cheaper energy in the sharing market, leading to a Pareto improvement. It is revealed that the total cost of prosumers decreases with the price elasticity and the sharing market equilibrium can achieve social optimum when the number of prosumers becomes large enough. It is also found that introducing competition benefits social welfare. Case studies confirm the theoretical results with analyses on the impacts of several key factors. This work is expected to provide insights on understanding and designing future energy sharing markets.**

***Index Terms*—Prosumer, energy sharing, supply-demand function, game theory, Nash equilibrium**

NOMENCLATURE

A. Indices and Sets

i	Index of prosumers.
n	Index of resources.
k	Index of resource of prosumer.
$\mathscr{I}$	Set of prosumers.
$\mathscr{N}$	Set of resources.
$\mathscr{S}$	Set of sellers.
$\mathscr{D}$	Set of buyers.
$\mathscr{K}_i$	Set of resource of prosumer i.
$f_i(\cdot)$	Disutility function of prosumer i.
$s_i(\cdot)$	Sharing cost of prosumer i.
$md_i(\cdot)$	Marginal disutility of prosumer i.
$\Pi_i(\cdot)$	Total cost function of prosumer i.
X_i	Action set of player i, and $X = \prod_i X_i$.

B. Parameters

I	Number of prosumers.
N	Number of resources.
D_i^0	Fixed amount of energy prosumer i consumes.
p_i^0	The amount of energy i generates originally.
E_i^0	The amount of energy bought from grid by i.
D_i	Required load reduction of prosumer i.
$\underline{D},\overline{D}$	Lower/upper bound of D_i.
$\underline{c},\overline{c}$	Lower/upper bound of c_i.
$\underline{d},\overline{d}$	Lower/upper bound of d_i.
a	Price elasticity of prosumers
K_i	Number of resource of prosumer i.
c_i, d_i	Coefficients of the disutility function for prosumer i.
c_i^k, d_i^k	Cost coefficients of resource k of prosumer i. .

C. Decision Variables

p_i	Output adjustment of prosumer i.
λ_c	Sharing market clearing price.
b_i	Willingness to pay/buy of prosumer i.
$\bar{b}$	Average purchase desire of all prosumers.
q_i	Amount of energy bought/sell from/to the sharing market.
p_i^k	Output adjustment of resource k of prosumer i.
$\mu_i,\ \mu_i^{'}$	Dual variable of energy balance equation in the sharing problem of prosumer i.
$\eta_i,\ \eta_i^{'},\ \eta$	Dual variable of the market clearing condition.
$\xi, \xi^{'}$	Dual variable of the energy balance equation of the equivalent central decision-making problem.

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I. INTRODUCTION

THE proliferation of distributed wind, solar power and energy storage have been endowing traditional "pure" consumers with capability of generation, precipitating the advent of *prosumers* [1]. Different from traditional consumers, prosumers can not only consume, but also produce energy. Hence they can choose to either buy or sell energy when participating in an energy market, which provides an opportunity to flexibly exchange energy so as to enhance both the individual utility and social efficiency [2]. In this context, a well-designed market mechanism is desired to encourage individual prosumers to participate in energy sharing. This paper proposes a simple but effective mechanism based on generalized demand bidding, making an initial step to better understand the behavior of prosumers in energy sharing.

Nowadays, the advent of online platforms and applications have been enabling resource sharing in more and more sectors, such as ride-sharing (e.g., Uber, Lyft) [3], room-sharing (e.g., AirBnB) [4], workplace-sharing (e.g., Upwork, Amazon Mechanical Turk) [5]. These sharing platforms allow people to provide their idle goods for someone just in need and earn profit from doing so, resulting in a win-win game. These successes motivate a new paradigm of high-efficiency energy utilization in power systems, where energy prosumers could share their energy via online platforms in a similar way [6]. No surprising, such a paradigm has been gaining increasing attentions from both the academia and the public.

Existing studies in economics have investigated the operation of resource sharing. The benefits and drawbacks of sharing economy is discussed in [7]. The main difficulty of sharing platform construction is the design of an appropriate sharing mechanism, which means how the products be provided, how the market be cleared and how the revenue be allocated. Performance of typical sharing platforms is studied in [8]–[10] and also their impacts on the social welfare [11]. The influence of prices and subsidies is revealed in [12]. A review of sharing economy can be found in [13].

As for energy sharing, the potential of game-theoretic approaches was summarized in [14], including the applications in electric vehicles (EV), demand-side energy resource (DER) and storage managements. The economic efficiencies of autarky scheme, sharing scheme and aggregation scheme were quantitatively compared in [15], showing that energy sharing can achieve near-optimal efficiency without a central coordinator, which is a promising scheme for future energy market organization. An exchange article by article sharing paradigm was investigated. Random sharing clearing price in a storage investment problem is characterized in [16]. A simplified time-of-use (TOU) model with peak price and off-peak price was used. Above work initially explores the problem and opportunity of sharing in smart grid and the models are relatively abstract and simple. More detailed analytical studies related to resource sharing can be roughly cast into the following three categories.

Two-sided market with clearing price. It is assumed that there is a third-party platform. The sellers report the amount of products they are willing to share or their cost coefficients; the buyers report the amount of products they want or the money they are willing to pay. After receiving all the bids, the third-party sharing platform solves an optimization problem with the objective function of social welfare maximization or self-revenue maximization and clears the market. Ref. [17] provides interesting insights into the tradeoff between revenue maximization and social welfare maximization. The clearing price of sharing market is analyzed in [18]. Incentive design for electric vehicle-to-vehicle charge sharing is investigated in [19]. System constraints such as energy-flow limits can be taken into account in the two-sided market analysis. However, since the supply and demand statuses of participants are predetermined, it can not fully capture the behaviors of prosumers who can choose to purchase or sell changeably.

Single-sided market with set price. Different from the two-sided market, it assumes that the statuses of participants are symmetric, which means all of them can flexibly choose to purchase or sell. The benefits from sharing are distributed among prosumers via prices set by the sharing platform. An hour-ahead optimal pricing model of energy sharing management platform is proposed based under the framework of Stackelberg game in [20]. Energy sharing among photovoltaic (PV) prosumers is considered in [21], taking into account uncertainty of renewable energy generation. Two kinds of sharing schemes, the direct sharing (within one time period) and the buffered sharing (across different time periods), are discussed. A supply demand ratio based pricing algorithm is adopted in [22] for the energy sharing in PV prosumers. In the above studies, the sharing prices are set by the platform via solving a Stackelberg game, in which the upper level is the platform's pricing problem and the lower level prosumers' decision making problems. The impact of one prosumer's strategy on the other prosumers' decision is not fully captured.

Single-sided market with re-allocation. In this kind of sharing, the benefit distribution is achieved via re-allocation instead of price regulation. The main difficulty stems from the design of re-allocation scheme. The renowned Vickrey-Clarke-Groves (VCG) mechanism [23] could be regarded as an example. Under VCG, each agent not only gains its own value but also an additional payment based on an arbitrary function of the values of the other agents. Although the VCG re-allocation approach is ease to implement, it is not self-budget balancing as extra bonuses outside the sharing market is required. A cost re-allocation method for a group of electricity storages is presented in [24], resulting in a cooperative game. A coalitional game based algorithm was proposed in [25] for energy exchange among microgrids. A conceptual design for the DERs sharing is proposed in [26], where an aggregator coordinates all DERs in real-time operation and evaluates coordination surplus, which is split between aggregators and prosumers. However, the redistribution after a sharing transaction is difficult in practice, as it requires some private information of individual participants, e.g. the storage capacity and the cost coefficient.

This paper proposes a simple but transparent and effective energy sharing mechanism based on generalized demand bidding. A similar framework is known as the *supply function bidding* [27], [28] in demand response programs. Under this mechanism, each seller submits his supply function to the auctioneer, then the auctioneer sets a market clearing price according to the submitted supply functions and the expected total load shedding. Supply function bidding can fully capture the impact of seller's bid on his contracted quantity as well as the market clearing price, and is effective in competitive markets [29]. When it comes to the sharing market, the situation is more complex since the prosumers not only aim to minimize his cost but should also maintain power balancing. Besides, sellers and buyers coexist and can change their roles from time to time, and the equilibrium quantity is not known in advanced. In this regard, we generalize the supply function to a generic supply-demand function (SDF), based on which we build a sharing market mechanism for energy prosumers. This work possesses three salient features:

1) **Sharing mechanism design**. A generic supply-demand function is proposed, enabling a generalized demand biding based sharing mechanism. In contrast to the two-sided market based analysis, any participant can be a prosumer that aims

to minimize his own disutility. Moreover, different from the single-sided market with set price, the mutual impacts among prosumers are considered, which can better characterize the market behavior. The model encapsulating the decision making of prosumers turns out to be a generalized Nash game (GNG), which can be further reduced to a standard Nash game. The existence and uniqueness of Nash equilbrium are proved.

2) **Provable properties of the sharing mechanism**. Properties of the sharing equilibrium price are disclosed. It is proved that every prosumer's cost is no more than with individual decision-making, leading to a Pareto improvement and meaning that every prosumer has the motivation to participate in sharing. Moreover, the total cost of all prosumers decreases with the price elasticity and when the number of prosumers is large enough, the sharing market will lead to the same outcome as the social optimum. It is also revealed that the proposed generalized demand bidding based sharing mechanism is budget self-balancing and no private information is needed for re-allocation, which is easier to implement compared with the re-allocation based schemes.

3) **Impacts of competition on social efficiency**. The basic model is based on a perfectly competitive situation, in which every prosumer owns and controls only one resource. We further investigate a more realistic case, in which a prosumer could possess multiple resources. A special case provides a proof of concept that social cost can be reduced by spreading the resources among more prosumers which means more competition is introduced.

The rest of this paper is organized as follows. The mathematical formulations of energy prosumers and description of the energy sharing mechanism are presented in Section II; some basic properties of the sharing game are given in Section III; The impact of competition on social welfare is studied in Section IV; Illustrative examples are provided in Section V. Finally, conclusions are summarized in Section VI.

II. Game Model of Energy Sharing

A. Energy Prosumers

In this paper, we consider the decision-making problems of a set of prosumers $\mathscr{I}$, indexed by $i \in \mathscr{I} = \{1,2...,I\}$. There are N kinds of resources, indexed by $n \in \mathscr{N} = \{1,2,...,N\}$, which can be a distributed generator (DG), virtual power plant (VPP) and etc. First, we consider the case under perfectly competitive market, where each prosumer i owns one kind of resource, and we have $I = N$. To distinguish from the case under imperfect competitive market, we use N to represent the number of prosumers here. The fixed amount of energy prosumer i consumes is D_i^0, and is satisfied by the amount of energy it generates p_i^0 as well as the energy bought from the grid E_i^0. These prosumers take part in a demand response program, and the required amount of load reduction for prosumer i is a given value D_i, which means the amount of energy it bought from the grid needs to be reduced by D_i ($\underline{D} \le D_i \le \overline{D}$). Each prosumer changes its resource output to meet the load adjustment. For example, to reduce its load by $D_i > 0$, prosumer i needs to increase p_i by D_i. Any deviation from the original operating point will cause disutility. The disutility function of prosumer i is a quadratic function $f_i(p_i) = c_i p_i^2 + d_i p_i$, where p_i is the output adjustment of resource and $0 < \underline{c} \le c_i \le \overline{c}$, $0 < \underline{d} \le d_i \le \overline{d}$ are the cost coefficients. When a prosumer $i \in \mathscr{I}$ takes part in a demand response program individually, there is no room for optimization since $p_i = D_i$ is clearly the solution. The corresponding cost is $f_i(D_i)$.

However, the result under individual decision-making may not be the most efficient if prosumers with different marginal disutilities are allowed to trade with others. In such a circumstance, the design of an effective profit allocation scheme, from which all prosumers take part in sharing can benefit, is desired. The traditional supply function bidding in demand response program cannot be applied because of the simultaneous non-deterministic clearing quantity and clearing price as a prosumer can changeably acts as either a producer or a consumer. Hence a more general bidding mechanism, which can reflect prosumers' willingness to buy or sell energy while determining both the clearing quantity and price, are necessary.

B. Generic Supply-Demand Function

In this subsection, we propose a generic supply-demand function by generalizing the conventional supply function, so as to consider the situation where the participant can flexibly change his role between a seller and a buyer.

In the sharing market, the demand (or supply) function of each prosumer can be expressed by

$$q_i = a_i \lambda_c + b_i \tag{1}$$

where λ_c is the market clearing price, q_i is the amount of energy ($q_i > 0$ means he is a buyer and gets energy from the sharing market, $q_i < 0$ means he is a seller and sells energy to the sharing market). $a_i < 0$ represents price elasticity and b_i shows his willingness to buy. For simplification, we assume all prosumer have the same price sensitivity, i.e. $a_i = a < 0, i \in \mathscr{N}$. The average purchase desire is defined as $\bar{b} = (\sum_i b_i)/N$. The market clears when the net quantity $\sum_i q_i = 0$ and the obtained sharing price is

$$\lambda_c = -\sum\nolimits_i b_i/Na = -\bar{b}/a \tag{2}$$

here $b_i \ge \bar{b}$ implies prosumer i is more willing to buy than the average. We have $q_i = a\lambda_c + b_i \ge 0$, and the prosumer appears to be a buyer. Similarly, a prosumer who has less willingness to buy than the average ($b_i \le \bar{b}$) turns to be a seller ($q_i \le 0$). In consequence, the statuses of prosumers are determined spontaneously by their purchase desires, which enable a simple but effective sharing mechanism, as we explain.

C. Energy Sharing Mechanism

The sharing mechanism follows these three steps.

Step 1: Estimate the value of price elasticity a via historical data. Each prosumer i bids b_i to the sharing platform. The average purchase desire is $\bar{b} = \sum_i b_i/N$

Step 2: Clear the sharing market by setting price to $\lambda_c(b) = -\sum_i b_i/Na$, which is called the equilibrium price. The amount of energy prosumer i gets is $q_i(b) = a\lambda_c(b) + b_i$

Step 3: If $b_i \geq \bar{b}$, the amount of energy $q_i(b) \geq 0$, which means prosumer i will buy $q_i(b)$ from the sharing market and his payment is $\lambda_c(b)q_i(b)$. Otherwise, if $b_i \leq \bar{b}$, the amount of energy $q_i(b) \leq 0$, which means prosumer i will sell $-q_i(b)$ to the sharing market and he will get $-\lambda_c(b)q_i(b)$.

Under this setting, the sharing market clears when

$$\sum\nolimits_{i\in\mathscr{S}}(-q_i) = \sum\nolimits_{i\in\mathscr{D}} q_i \tag{3}$$

$\mathscr{S}$ is the set of sellers, $\mathscr{D}$ is the set of buyers and each prosumer i belongs to either $\mathscr{S}$ or $\mathscr{D}$, which means $\mathscr{I} = \mathscr{S} \cup \mathscr{D}$. Hence equation (3) also implies

$$\sum\nolimits_{i\in\mathscr{I}}(a\lambda_c + b_i) = 0 \tag{4}$$

D. Energy Sharing as A Generalized Nash Game

It is easy to verify that the setting price $\lambda_c(b)$ clears the market. When participating the sharing market, the optimization problem of each prosumer $i \in \mathscr{I}$ becomes

$$\min_{p_i,b_i} \quad \Pi_i := c_i p_i^2 + d_i p_i + (a\lambda_c(b) + b_i)\lambda_c(b) \tag{5a}$$

$$\text{s.t.} \quad p_i + a\lambda_c(b) + b_i = D_i : \mu_i \tag{5b}$$

$$\sum\nolimits_i (a\lambda_c(b) + b_i) = Na\lambda_c + \sum\nolimits_i b_i = 0 : \eta_i \tag{5c}$$

Here, $\Pi_i(p_i, b_i, b_{-i})$ is the cost function, which can be divided into two parts: the disutility in terms of money $f_i(p_i) := c_i p_i^2 + d_i p_i$ and the sharing cost $s_i(b) := (a\lambda_c + b_i)\lambda_c$. (5b) represents the energy balancing. (5c) is the sharing market clearing condition, which appears in every prosumer's problem. μ_i and η_i are corresponding Lagrangian multipliers. Due to the common constraint (5c), problem (5) constitutes a generalized Nash game (GNG), where players' payoffs and strategy sets depend on each other.

In summary, the sharing game consist of the following elements: 1) the set of prosumers $\mathscr{I} = \{1, 2, ..., I\}$; 2) action sets $X_i(b_{-i})$[1],$\forall i$, and strategy space $X = \prod_i X_i$; 3) cost functions $\Pi_i(p_i, b_i, b_{-i}), \forall i$. For simplicity, we use $\mathscr{G} = \{\mathscr{I}, X, \Pi\}$ to denote the sharing game (5) in an abstract form.

III. Properties of the Sharing Game

A. Existence and Uniqueness of Equilibrium

In this subsection, we show that the GNG model of energy sharing problems can be reduced into a standard Nash game, based on which we prove the existence and uniqueness of its equilibrium.

Denote by b_j the bids of other prosumer j $(j \neq i)$. From (5c), we have

$$\lambda_c(b) = -\frac{b_i}{Na} - \frac{\sum_{j\neq i} b_j}{Na} \tag{6}$$

Substituting into (5b) yields

$$p_i = D_i - \frac{N-1}{N} b_i + \frac{\sum_{j\neq i} b_j}{N} \tag{7}$$

Using b_i to represent p_i and $\lambda_c(b)$, the GNG (5) degenerates into a equivalent standard Nash game (8).

$$\min_{b_i} \quad c_i\left(D_i - \frac{N-1}{N} b_i + \frac{\sum_{j\neq i} b_j}{N}\right)^2 + d_i\left(D_i - \frac{N-1}{N} b_i + \frac{\sum_{j\neq i} b_j}{N}\right) + \left(-\frac{b_i}{N} - \frac{\sum_{j\neq i} b_j}{N} + b_i\right)\left(-\frac{b_i}{Na} - \frac{\sum_{j\neq i} b_j}{Na}\right) \tag{8}$$

Direct computation shows that, the second derivative of the objective function is $2\left[c_i\left(\frac{N-1}{N}\right)^2 - \frac{N-1}{N^2 a}\right] > 0$, implying each prosumer solves a strictly convex optimization.

Definition 1. (Nash Equilibrium) A strategy profile $(p^*, b^*) \in X$ is a Nash Equilibrium (NE) of the sharing game $\mathscr{G} = \{\mathscr{I}, X, \Pi\}$ defined by (5) [2], if $\forall i \in \mathscr{I}$

$$\Pi_i(p_i^*, b_i^*, b_{-i}^*) \leq \Pi_i(p_i, b_i, b_{-i}^*), \forall (p_i, b_i) \in X_i(b_{-i^*})$$

Given p, define $\tilde{\lambda}(p) := \frac{1}{N}\sum_i(2c_i p_i + d_i)$ and $\tilde{b}_i(p) := D_i - p_i - a\tilde{\lambda}(p)$. We have the following proposition.

Proposition 1. There exists a unique NE for the sharing game (5). Moreover, a strategy profile (p^*, b^*) is the unique NE if and only if, $\forall i \in \mathscr{I}$, p_i^* is the unique solution of:

$$\min_{p_i,\forall i} \quad \sum_i \left(c_i - \frac{1}{2(N-1)a}\right) p_i^2 + \left(d_i + \frac{D_i}{(N-1)a}\right) p_i \tag{9a}$$

$$\text{s.t.} \quad \sum_i p_i = \sum_i D_i : \xi \tag{9b}$$

and $b_i^* = \tilde{b}_i(p^*)$.

The proof of Proposition 1 can be found in Appendix A. Proposition 1 is fundamental since it ensures that the proposed sharing game is well defined. Furthermore, it implies that the NE computation can be greatly simplified into solving a simpler optimization problem (9), which is strictly convex.

B. Individual Rationality of Prosumers

The next proposition shows that all the prosumers are incentivized to share by comparing the costs of the individual decision-making and the sharing game (5) at equilibrium.

Let $\Pi_i(p_i^*, b^*)$ be the cost of prosumer i at the NE of sharing game $\mathscr{G} = \{\mathscr{I}, X, \Pi\}$ defined by (5), and $f_i(D_i)$ the cost of prosumer i with his individual optimal decision.

Proposition 2. We have

$$\Pi_i(p_i^*, b^*) \leq f_i(D_i), \forall i \in \mathscr{I} \tag{10}$$

moreover, (10) holds with strictly inequality for at least one i unless the unique optimal solution of (9) is $p_i^* = D_i, \forall i$.

The proof of Proposition 2 can be found in Appendix B. It says that with the proposed sharing mechanism, a Pareto improvement can be achieved for all prosumers, since the cost of each prosumer is no worse than making decisions individually. Hence, the sharing mechanism provides positive incentives for prosumers to participate in the sharing market, which is crucial for the market design.

[1]The subscribe $-i$ means all players in $\mathscr{I}$ except i

[2]Given a collection of x_i for i in a certain set A, x denotes the vector $x := (x_i; i \in A)$ of a proper dimension with x_i as its components

C. Sharing Price and Prosumers' Behavior

In this subsection, we clarify the relationship between the sharing price λ_c that clears the market and prosumer's marginal disutility, as well as the resulting prosumers' behavior.

Definition 2. (Marginal Disutility) The marginal disutility of prosumer i, denoted by md_i, is defined as

$$md_i(p_i) := \frac{\partial f_i(p_i)}{\partial p_i} = 2c_i p_i + d_i$$

Proposition 3. Assume (p^*, b^*) is the NE of the sharing game (5). Then

1) the sharing price at equilibrium is given by

$$\lambda_c^* = \frac{1}{N}\sum_i md_i(p_i^*);$$

2) $md_i(p_i^*) > \lambda_c(b^*)$ if and only if $q_i(b^*) > 0$.

The proof of Proposition 3 can be found in Appendix C. Proposition 3 says that, the clearing price at the NE is simply the average marginal disutility of all prosumers participating in the sharing market. Moreover, the prosumers whose marginal disutility is larger than the average (which equals λ_c) have $q_i(b^*) > 0$ and hence will buy energy, while whose marginal cost is lower than the average have $q_i(b^*) < 0$ and hence will sell energy. Under the proposed sharing mechanism, a prosumer with higher/lower marginal disutility produces less/more and purchases/sells in the sharing market.

D. Social Efficiency

To investigate the social efficiency of the proposed sharing mechanism, consider the social planner's problem:

$$\min_{p_i, \forall i \in \mathscr{I}} \quad \textstyle\sum_i (c_i p_i^2 + d_i p_i) \tag{11a}$$

$$\text{s.t.} \quad \textstyle\sum_i p_i = \sum_i D_i \tag{11b}$$

Definition 3. (Socially Optimal) $\bar{p}$ is socially optimal if $\bar{p}$ is the unique optimal solution of (11).

Optimal solution of (11) is different from the case under individual decision-making, except for the case in which $p_i = D_i \ \forall i \in \mathscr{I}$ happens to be the optimal solution to problem (11). The difference in their optimal values interprets the loss of social welfare. Next we reveal that the proposed energy sharing mechanism can effectively reduce the loss of social welfare.

Invoking Proposition 1, it is easy to see, as the number of prosumers N in model (9) approaches infinity, the NE of the sharing problem (5) would turn to be identical to the solution to social optimization problem (11). Next we show the asymptotic convergence as $N \to \infty$.

Proposition 4. Let $(p^*(N), b^*(N))$ be the unique NE of (5) and $\bar{p}(N)$ be the socially optimal solution of (11). Then, we have

$$\textstyle\sum_{i\in\mathscr{I}} f_i(p_i^*(N)) \geq \sum_{i\in\mathscr{I}} f_i(\bar{p}_i(N))$$

and the average cost difference

$$\lim_{N\to\infty} \frac{1}{N}\left[\textstyle\sum_{i\in\mathscr{I}} f_i(p_i^*(N)) - \sum_{i\in\mathscr{I}} f_i(\bar{p}_i(N))\right] = 0$$

The proof of Proposition 4 can be found in Appendix D. Proposition 4 says that the proposed sharing mechanism asymptotically converges to the social optimum when there is an large enough number of prosumers in the sharing market.

Similarly, the impact of price elasticity can be analyzed by the following proposition.

Proposition 5. Let $(p^*(a), b^*(a))$ be the unique NE of (5) with price elasticity equals to $a < 0$. Then, we have $\sum_{i\in\mathscr{I}} f_i(p_i^*(a))$ is decreasing in $|a|$.

The proof of proposition 5 can be found in Appendix E. It reveals that when $|a|$ becomes larger, which means the prosumers are more sensitive to the change of price, the total social cost under sharing decreases and becomes closer to the social optimal cost. It is worthy nothing that, because $\sum_{i\in\mathscr{I}}(a\lambda_c + b_i)\lambda_c = 0$ holds, the group of prosumers are budget self-balancing. It implies that no extra bonus is needed to motivate the sharing market, which is a main superiority compared with the renown Vickrey-Clarke-Groves (VCG) mechanism.

IV. Impacts of Competition

Above analysis assumes a simplified competitive market, where each prosumer owns only one resource and there exists no monopoly power. Next we analyze a more complicated situation, in which prosumers could own multiple resources.

Assume that there are I multi-resource prosumers (MRP) indexed by $i \in \mathscr{I} = \{1, 2, ..., I\}$. Each prosumer i owns K_i kinds of resources labeled by $k \in \mathscr{K}_i = \{1, 2, ..., K_i\}$. The corresponding energy productions are $p_i^1, p_i^2, \cdots, p_i^{K_i}$. However, we assume there are still N resources in total, which means $\sum_{i\in\mathscr{I}} K_i = N$. The multi-resource prosumer (MRP) $i \in \mathscr{I}$ can either carry out the demand response command individually by solving the following problem

$$\min_{p_i^k, \forall k \in \mathscr{K}_i} \quad \sum_k \left[c_i^k \left(p_i^k\right)^2 + d_i^k \left(p_i^k\right)\right] \tag{12a}$$

$$\text{s.t.} \quad \sum_k p_i^k = D_i \tag{12b}$$

or take part in the sharing market by solving

$$\min_{p_i^k, \forall k \in \mathscr{K}_i, b_i} \quad \sum_k \left[c_i^k \left(p_i^k\right)^2 + d_i^k p_i^k\right] + (a\lambda_c + b_i)\lambda_c \tag{13a}$$

$$\text{s.t.} \quad \sum_k p_i^k + a\lambda_c + b_i = D_i : \mu_i' \tag{13b}$$

$$\sum_i (a\lambda_c) + \sum_i b_i = 0 : \eta_i' \tag{13c}$$

Following the similar process as in Section III, we can easily prove again that the proposed sharing mechanism can benefit all MRPs and the equilibrium sharing price reflects the average marginal disutility.

Definition 4. (Social Optimal for MRP) $\bar{p}$ is socially optimal (or the most efficient) if $\bar{p}$ solves

$$\min_{p_i^k, \forall i \in \mathscr{I}, k \in \mathscr{K}_i} \quad \sum_i \sum_k \left[c_i^k \left(p_i^k\right)^2 + d_i^k p_i^k\right] \tag{14a}$$

$$\text{s.t.} \quad \sum_i \sum_k p_i^k = \sum_i D_i \tag{14b}$$

If $I = 1$, the sharing problem for MRP (13) becomes the social optimal problem (14). If $I = N$, the sharing problem for MRP (13) degenerates to the sharing problem in the previous sections. If $1 < I < N$, the following propositions hold.

Proposition 6. The marginal disutilities of the resources for a prosumer are equal, which means[3]

$$md_i(p_i) := md_i^1\left(p_i^1\right) = \cdots = md_i^{K_i}\left(p_i^{K_i}\right), \quad \forall i \in \mathscr{I} \tag{15}$$

Proposition 6 can be directly deduced from the KKT condition. So we omit the proof here.

Given $p := (p_i, \forall i)$, we define $\tilde{\lambda}(p) := \frac{1}{I}\sum_i md_i(p_i)$ and $\tilde{b}_i(p) := D_i - \sum_k p_i^k - a\tilde{\lambda}(p)$. Then we have the following proposition.

Proposition 7. There exists a unique NE for the sharing problem with MRP (13). Moreover, a strategy profile (p^*, b^*) is the unique NE if and only if, $\forall i \in \mathscr{I}$, p_i^* is the unique solution of (16).

$$\min_{p_i^k, \forall i, k \in \mathscr{K}_i} \quad \sum_i \sum_k \left[\left(c_i^k - \frac{1}{2(I-1)a}\right)\left(p_i^k\right)^2 + \left(d_i^k + \frac{D_i}{(I-1)a}\right)p_i^k\right] - \frac{\sum_i \sum_{j>k \in \mathscr{K}_i} p_i^k p_i^j}{(I-1)a} \tag{16a}$$

$$\text{s.t.} \quad \sum_i \sum_k p_i^k = \sum_i D_i : \xi' \tag{16b}$$

and $b_i^* = \tilde{b}_i(p^*)$.

The proof of Proposition 7 can be found in Appendix F. Proposition 7 extends the result of existence and uniqueness of the NE in the sharing game from the single-resource case to the multi-resource one, and again provides an effective way to simplify the computation of NE.

Then we analyze the change in efficiency based on model (16). Generally speaking, as I varies from 1 to N, the change in the total socially optimal cost may not be monotonous. So we only consider a special case in which all $c_i^k = c, \forall k, \forall i$ and $K_i = K_I, \forall i$ and gives the following proposition.

Let (I, K_I, D) denotes a scenario that there are I prosumers, each has K_I resources and the required load adjustment for prosumer i is D_i. Then, the scenario $(I', K_{I'}, D')$ is an ***equal partition*** of (I, K_I, D) when there exists an $Z \in \mathbb{Z}^+$, such that $I' = ZI$ and $K_I = ZK_{I'}$, the resources one prosumer possesses and required load adjustment is distributed equally to Z prosumers and satisfies, $\forall i, \forall z_1, z_2 \in \{1, ..., Z\}$

$$\begin{aligned}(D'_{Z(i-1)+z_1} - \sum_{k=1}^{K_{I'}} p_{Z(i-1)+z_1}^{k*})(D'_{Z(i-1)+z_2} - \sum_{k=1}^{K^{I'}} p_{Z(i-1)+z_2}^{k*}) &\geq 0 \\ \sum_{z=1}^{Z} D'_{Z(i-1)+z} &= D_i\end{aligned}$$

where p^* is the NE under scenario (I, K_I, D).

Definition 5. (Variance of marginal disutility) The variance of marginal utilities $md_i^{k*}, \forall k \in \mathscr{K}_I, \forall i \in \mathscr{I}$ is defined as

$$Var(md_i^{k*}, I) := \frac{1}{N}\sum_{i=1}^{I}\sum_{k=1}^{K_I}(md_i^{k*} - \frac{1}{N}\sum_{i=1}^{I}\sum_{k=1}^{K_I} md_i^{k*})^2$$

[3] p_i denotes the vector $p_i := (p_i^k, \forall k)$

Proposition 8. Suppose $c_i^k = c, \forall i \in \mathscr{I}, \forall k \in \mathscr{K}_i$ and $(p^*(I), b^*(I))$ is the unique NE of the sharing problem for MRP (13) with $I > 1$. For any I prosumers with the same number of resources, i.e. $K_i = K_I, \forall i \in \mathscr{I}$, there always exists an equal partition of (I, K_I, D) to $(I', K_{I'}, D')$, such that

$$\sum_{i=1}^{I'}\sum_{k=1}^{K_{I'}} f_{ik}(p_i^{k*}(I')) \leq \sum_{i=1}^{I}\sum_{k=1}^{K_I} f_{ik}(p_i^{k*}(I))$$

Moreover, if $-2ac \leq N$, then

$$Var(md_i^{k*}, I') \leq Var(md_i^{k*}, I)$$

The proof can be found in Appendix G. It shows that the system under $I = 1$ is the most efficient; otherwise, introducing competition by spreading resources benefits social welfare. Proposition 8 only considers a very special case. In Section IV, we provide empirical results of numerical experiments to further confirm this property.

V. Illustrative Examples

In this section, numerical experiments are presented to illustrate theoretical results. First, a simple case is used to illustrate the basic setup. Then, the impacts of several factors are analyzed, including the number of prosumers, their price elasticities as well as the impact of competition.

A. Benchmark Case

The simplest scenario with two prosumers is taken as an illustrative example. p_1 and p_2 are the output adjustment of prosumer 1 and 2. We assume the price elasticity $a = -1$, the cost coefficients $c_1 = 2$, $d_1 = 3$ and $c_2 = 4$, $d_2 = 5$. The required demand reduction are $D_1 = 1$ and $D_2 = 2$. The optimal output adjustments and the corresponding costs when making decisions individually (IDL), taking part in the sharing market (SMK) and the social optimal (SCO) are shown in Table I. The best response curves of two prosumers are shown in Fig.1.

TABLE I
Optimal solution under IDL, SMK and SCO

Scheme	IDL	SMK	SCO
Optimal output adjustment p_1	1.00	2.00	2.17
Optimal output adjustment p_2	2.00	1.00	0.83
Cost of prosumer 1	5.00	2.00	15.89
Cost of prosumer 2	26.00	21.00	6.95
Social total cost	31.00	23.00	22.83
Relative cost difference	35.76%	0.73%	–

From Table I, we can find that when the prosumers take part in sharing, their individual costs all decrease (prosumer 1 from 5.00 to 2.00, and prosumer 2 from 26.00 to 21.00), so does the social total cost, confirming Proposition 2. The relative social cost difference [4] between IDL and SCO is 35.75% while the relative social cost difference between SMK and SCO is

[4] Relative social cost difference(IDL,SCO)=$\frac{\sum_{i\in\mathscr{I}} f_i(D_i) - \sum_{i\in\mathscr{I}} f_i(\bar{p}_i)}{\sum_{i\in\mathscr{I}} f_i(\bar{p}_i)}$, Relative social cost difference(SMK,SCO)=$\frac{\sum_{i\in\mathscr{I}} f_i(p_i^*) - \sum_{i\in\mathscr{I}} f_i(\bar{p}_i)}{\sum_{i\in\mathscr{I}} f_i(\bar{p}_i)}$

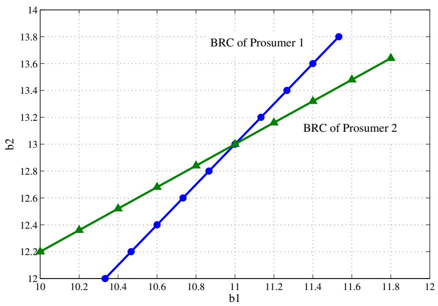

Fig. 1. Best response curves of two prosumers.

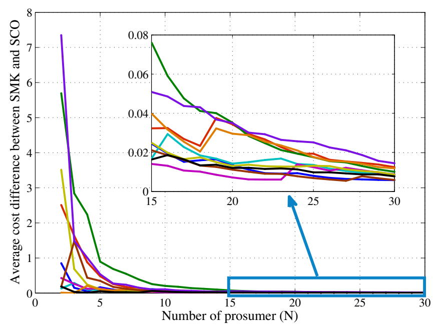

Fig. 2. Gap of average costs under different N.

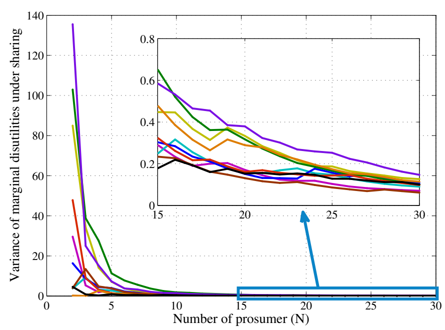

Fig. 3. Variance of marginal disutilities under different N.

0.73%, showing that the sharing mechanism can greatly reduce the social total cost. The intersection of best response curves in Fig.1 gives the sharing market equilibrium, which is $(b_1, b_2) = (11, 13)$ and the corresponding equilibrium output adjustment is $(p_1, p_2) = (2.0, 1.0)$, which is the same as the results in Table I offered by the proposed equivalent model (9), verifying Proposition 1.

B. Impact of the Number of Prosumers

We change the number N from 2 to 30. We assume that $\underline{c} = 1$, $\overline{c} = 10$, $\underline{d} = 2$, $\overline{d} = 12$, $\underline{D} = 0$, $\overline{D} = 10$. The parameters c_i, d_i, D_i are randomly chosen within the upper and lower bounds and 10 scenarios are tested. For each of the 10 random scenario, the average cost difference in Proposition 4 is plotted in Fig.2 and the variance of marginal disutilities in definition 5 (with $K_I = 1$) is plotted in Fig. 3, both as functions of N.

In Fig.2, the average cost with sharing is always larger than the average optimal social cost but the gap shrinks sharply with the increase of N in all scenarios, validating Proposition

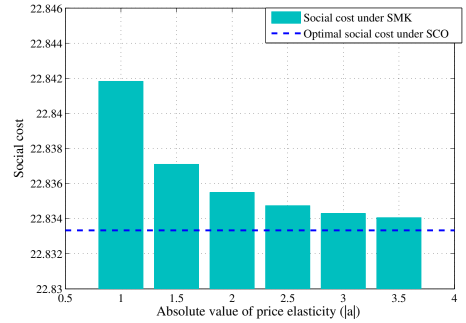

Fig. 4. Average costs under different a.

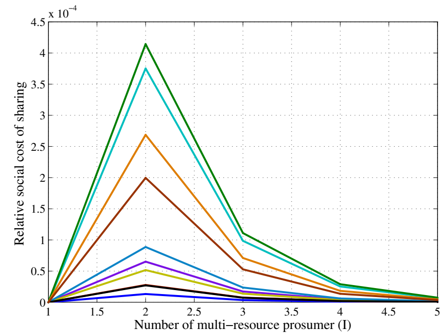

Fig. 5. Relative cost of sharing different numbers of MRPs with different c_i^k and the same K_i.

4. When N increases, the variance of marginal disutility also drops sharply, implying that the marginal disutilities of all prosumers under sharing become closer and all prosumers converge to the social optimum, as shown in Fig. 3.

C. Impact of Price Elasticity

When price elasticity coefficient a varies in $[-3.5, -1]$, the social costs under NE are shown in Fig. 4. The optimal social cost is marked by a dash line. From the figure, when the absolute value of a increases, the social cost under sharing is decreasing and gets closer to the optimal social cost. This is in accordance with Proposition 5.

D. Impact of Competition

A special case, in which all $c_i^k = c, \forall k, \forall i$ and $K_i = K_I, \forall i$, is analyzed in Section IV, showing that introducing competition improves social welfare. However, the general case is difficult to prove. Here, we first test cases with different c_i^k but the same K_i; and cases with different c_i^k and K_i. We assume that $\underline{c} = 1$, $\overline{c} = 10$, $\underline{d} = 2$, $\overline{d} = 12$, $\underline{D} = 0$, $\overline{D} = 10$. The parameters c_i^k, d_i^k and D_i are randomly chosen within the ranges and 10 scenarios are tested. The resources are allocated according to the condition (17) and the way in [30] (for same/different K_i). To eliminate the impact of scale, relative social cost, which equals to the ratio of costs with $I > 1$ and $I = 1$ minus 1, is used for comparison and its change under different scenarios with different c_i^k and same/different K_i are given in Fig.5 and Fig. 6, respectively.

It can be observed from Fig.5 that the relative cost is the smallest when $I = 1$, which means all the resources are

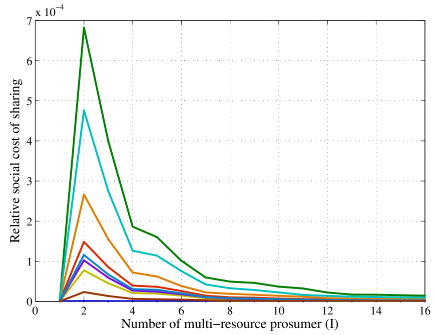

Fig. 6. Relative cost of sharing different numbers of MRPs with different c_i^k and K_i.

owned by one prosumer and the social optimal are achieved. When $I \geq 2$, the relative cost decreases with I, demonstrating that competition improves economic efficiency as stated in Proposition 8. This property extends to the case when K_i are different as shown in Fig. 6.

VI. Conclusion

Prosumers endowed with distributed generators are emerging nowadays, providing a great opportunity for energy sharing. By allowing prosumers to exchange energy with each other, energy sharing can greatly reduce the cost of prosumers while enhancing social efficiency. To promote energy sharing in smart grid, a simple but transparent and effective mechanism is proposed based on the generic supply-demand function. This paper establishes fundamental properties of such a sharing market by proving the existence and uniqueness of market equilibrium, disclosing the individual rationality of prosumers, characterizing the sharing price, comparing the social efficiency, as well as investigating the market impact of competition. Both theoretical analysis and case studies justify the effectiveness of the proposed sharing mechanism.

In contrast to the existing works, the proposed mechanism considers the choosability of prosumers to become a seller or buyer, the equilibrium price set by market sharing, and the fairness and operability of profit allocation. It is expected that this work provides a fundamental, though initial, framework for energy sharing problems. Future research directions include analyzing the behavior of such mechanism when coping with uncertainty due to the integration of renewables.

References

[1] P. G. D. Silva, D. Ilić, and S. Karnouskos, "The impact of smart grid prosumer grouping on forecasting accuracy and its benefits for local electricity market trading," *IEEE Trans. Smart Grid*, vol. 5, no. 1, pp. 402–410, 2014.

[2] A. D. Rathnayaka, V. M. Potdar, and S. J. Kuruppu, "An innovative approach to manage prosumers in smart grid," pp. 141–146, 2011.

[3] J. Hamari, M. Sjöklint, and A. Ukkonen, "The sharing economy: Why people participate in collaborative consumption," *Journal of the association for information science and technology*, vol. 67, no. 9, pp. 2047–2059, 2016.

[4] G. Zervas, D. Proserpio, and J. W. Byers, "The rise of the sharing economy: Estimating the impact of airbnb on the hotel industry," *Journal of marketing research*, vol. 54, no. 5, pp. 687–705, 2017.

[5] G. Paolacci and J. Chandler, "Inside the turk: Understanding mechanical turk as a participant pool," *Current Directions in Psychological Science*, vol. 23, no. 3, pp. 184–188, 2014.

[6] H. Heinrichs, "Sharing economy: A potential new pathway to sustainability," *GAIA - Ecological Perspectives for Science and Society*, vol. 22, no. 4, pp. 228–231, Jan. 2013.

[7] B. Fang, Q. Ye, and R. Law, "Effect of sharing economy on tourism industry employment," *Annals of Tourism Research*, vol. 57, pp. 264–267, 2016.

[8] Z. Li, Y. Hong, and Z. Zhang, "Do on-demand ride-sharing services affect traffic congestion? evidence from uber entry," *Available at SSRN:*, 2016, https://ssrn.com/abstract=2838043.

[9] G. Zervas, D. Proserpio, and J. Byers, "A first look at online reputation on airbnb, where every stay in above average," *Available at SSRN:*, Jan. 2015, https://ssrn.com/abstract=2554500.

[10] S. Zhang, D. Lee, P. V. Singh, and T. Mukhopadhyay, "Demand interactions in sharing economies: Evidence from a natural experiment involving airbnb and uber/lyft," *Available at SSRN:*, Jan. 2018, https://ssrn.com/abstract=3124712.

[11] C. Lam and M. Liu, "More than taxis with an app: How ride-hailing platforms promote market efficiency," *Available at SSRN:*, Jan. 2018, https://ssrn.com/abstract=2997190.

[12] L. Chen, A. Mislove, and C. Wilson, "Peeking beneath the hood of uber," *Proceedings of the 2015 Internet Measurement Conference. ACM*, pp. 495–508, 2015.

[13] M. Cheng, "Sharing economy: A review and agenda for future research," *International Journal of Hospitality Management*, vol. 57, pp. 60–70, 2016.

[14] W. Tushar, C. Yuen, H. Mohsenian-Rad, T. Saha, H. V. Poor, and K. L. Wood, "Transforming energy networks via peer to peer energy trading: Potential of game theoretic approaches," *arXiv preprint arXiv:1804.00962*, 2018.

[15] Y. Chen, W. Wei, F. Liu, Q. Wu, and S. Mei, "Analyzing and validating the economic efficiency of managing a cluster of energy hubs in multi-carrier energy systems," *Applied Energy*, vol. 230, pp. 403–416, 2018.

[16] D. Kalathil, C. Wu, K. Poolla, and P. Varaiya, "The sharing economy for the electricity storage," *IEEE Trans. Smart Grid*, 2017.

[17] Z. Fang, L. Huang, and A. Wierman, "Prices and subsidies in the sharing economy," *Proceedings of the 26th International Conference on World Wide Web*, pp. 53–62, 2017.

[18] T. Tanaka, N. Li, and K. Uchida, "On the relationship between the VCG mechanism and market clearing," *2018 Annual American Control Conference (ACC)*, pp. 4597–4603, 2018.

[19] P. Dutta and A. Boulanger, "Game theoretic approach to offering participation incentives for electric vehicle-to-vehicle charge sharing," *Transportation Electrification Conference and Expo (ITEC), 2014 IEEE*, pp. 1–5, 2014.

[20] N. Liu, X. Yu, C. Wang, and J. Wang, "Energy sharing management for microgrids with PV prosumers: A stackelberg game approach," *IEEE Trans. Ind. Inform*, vol. 13, pp. 1088–1098, 2017.

[21] N. Liu, M. Cheng, X. Yu, J. Zhong, and J. Lei, "Energy sharing provider for PV prosumer clusters: A hybrid approach using stochastic programming and stackelberg game," *IEEE Trans. Ind. Elec.*, vol. 65, no. 8, pp. 6740–6750, 2018.

[22] N. Liu, X. Yu, C. Wang, C. Li, L. Ma, and J. Lei, "Energy-sharing model with price-based demand response for microgrids of peer-to-peer prosumers," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3569–3583, 2017.

[23] A. Mas-Colell, M. D. Whinston, and J. R. Green, "Micoreconomic theory," *Oxford University Press*, 1995.

[24] P. Chakraborty, E. Baeyens, K. Poolla, P. P. Khargonekar, and P. Varaiya, "Sharing storage in a smart grid: A coalitional game approach," *IEEE Trans. Smart Grid, early access*, 2018.

[25] J. Mei, C. Chen, J. Wang, and J. L. Kirtley, "Coalitional game theory based local power exchange algorithm for networked microgrids," *Applied Energy*, vol. 239, pp. 133–141, 2019.

[26] W. Qi, B. Shen, H. Zhang, and Z. J. M. Shen, "Sharing demand-side energy resources-a conceptual design," *Energy*, vol. 135, pp. 455–465, 2017.

[27] P. D. Klemperer and M. A. Meyer, "Supply function equilibria in oligopoly under uncertainty," *Econometrica: Journal of the Econometric Society*, pp. 1243–1277, 1989.

[28] Y. Xu, N. Li, and S. H. Low, "Demand response with capacity constrained supply function bidding," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1377–1394, 2016.

[29] N. Li, L. Chen, and M. A. Dahleh, "Demand response using linear supply function bidding," *IEEE Trans. Smart Grid*, vol. 6, no. 4, pp. 1827–1838, 2015.

[30] *[Online] Available:*, https://sites.google.com/site/yuechenthu/appendix.

Appendix A
Proof of Proposition 1

Proof. Given the other prosumers' bids $b_j, j \neq i$, prosumer i solves a strictly convex optimization problem. Thus, the KKT condition below is the sufficient and necessary condition of the optimal solution.

$$\begin{aligned} 2c_i p_i + d_i + \mu_i &= 0 &\text{(A.1a)}\\ 2a\lambda_c + b_i + a\mu_i + Na\eta_i &= 0 &\text{(A.1b)}\\ \lambda_c + \mu_i + \eta_i &= 0 &\text{(A.1c)}\\ p_i + a\lambda_c + b_i &= D_i &\text{(A.1d)}\\ Na\lambda_c + b_i + \sum_{j\neq i} b_j^* &= 0 &\text{(A.1e)} \end{aligned}$$

where μ_i is dual variable of constraint (5b) and η_i is the dual variable of constraint (5c) for prosumer i.

Problem (9) is also a strictly convex optimization problem and the KKT condition is

$$2\left[c_i - \frac{1}{2(N-1)a}\right] p_i + d_i + \frac{D_i}{(N-1)a} + \xi = 0 \quad \text{(A.2a)}$$

$$\sum_i p_i = \sum_i D_i \quad \text{(A.2b)}$$

$\Rightarrow$: If (p^*, b^*) is the NE of the sharing game, it satisfies the KKT condition (A.1). If we sum up all (A.1d) for each i and together with (A.1e), constraint (A.2b) is obviously satisfied.

By $Na\times$ (A.1c) - (A.1b), we have

$$(N-2)a\lambda_c - b_i + (N-1)a\mu_i = 0 \quad \text{(A.3)}$$

Substitute (A.1d) into (A.3), we have

$$(N-1)a\lambda_c + p_i - D_i + (N-1)a\mu_i = 0 \quad \text{(A.4)}$$

For prosumer j, we also have

$$(N-1)a\lambda_c + p_j - D_j + (N-1)a\mu_j = 0 \quad \text{(A.5)}$$

With (A.4)-(A.5), we have

$$p_i - D_i + (N-1)a\mu_i = p_j - D_j + (N-1)a\mu_j \quad \text{(A.6)}$$

Then with (A.1a) we have

$$2c_i p_i + d_i + \mu_i = 2c_j p_j + d_j + \mu_j \quad \text{(A.7)}$$

(A.7)-(A.6)/$(N-1)a$ gets

$$\begin{aligned} &(2c_i - \frac{1}{(N-1)a})p_i + (d_i + \frac{D_i}{(N-1)a})\\ &= (2c_j - \frac{1}{(N-1)a})p_j + (d_j + \frac{D_j}{(N-1)a}) \quad \text{(A.8a)} \end{aligned}$$

Then we can always find a ξ such that (A.2a) is satisfied. As a result, p^* is also the optimal solution of (9).

Obviously, we have $b_i^* = D_i - p_i^* - a\lambda_c^*$. $\lambda_c^* = \frac{1}{N}\sum_i (2c_i p_i^* + d^i)$ will be proved latter in Appendix C.

$\Leftarrow$: If p^* is the optimal solution of problem (9), then by letting

$$\begin{aligned} \mu_i &= -2c_i p_i^* - d_i\\ \lambda_c &= \xi^* = \frac{1}{N}\sum_i (2c_i p_i^* + d_i)\\ \eta_i &= -\lambda_c - \mu_i\\ b_i &= D_i - p_i^* - a\lambda_c \end{aligned}$$

It is easy to prove that it satisfied the KKT condition (A.1). In consequence, the sharing problem (5) is equivalent to the central decision-making problem (9). This completes the proof. □

Appendix B
Proof of Proposition 2

Proof. Under SDF-based sharing mechanism, given other prosumers strategies $b_j, j \neq i$, by choosing

$$p_i = D_i,\ b_i = \frac{\sum_{j\neq i} b_j}{(N-1)}$$

with

$$\lambda_c = -\frac{\sum_{j\neq i} b_j}{(N-1)a}$$

We have $\Pi_i(p_i, b) = f_i(D_i)$, which means prosumer i can acheive the same cost as under individual decision-making. Because each prosumer solves a minimization problem, so that we always have $\Pi(p_i^*, b^*) \leq f_i(D_i)$. In consequence, a Pareto improvement is achieved for all prosumers.

If $p_i^* = D_i$ does not hold for all i, then as p^* is the unique optimal solution of problem (9), we always have

$$\sum_i (c_i(p_i^*)^2 + d_i p_i^*) - \sum_i \frac{(p_i^* - D_i)^2}{2(N-1)a} < \sum_i (c_i D_i^2 + d_i D_i) \quad \text{(B.1)}$$

and becasue $\sum_i \Pi(p_i^*, b^*) = \sum_i f_i(p_i^*)$, so we have

$$\begin{aligned} &\sum_i \Pi(p_i^*, b^*)\\ <\ &\sum_i f_i(D_i) + \sum_i \frac{(p_i^* - D_i)^2}{2(N-1)a}\\ <\ &\sum_i f_i(D_i) \quad \text{(B.2a)} \end{aligned}$$

so that at least one strict inequality of (10) holds. This completes the proof. □

Appendix C
Proof of Proposition 3

Proof. If (p^*, b^*) is the NE of the sharing game, then it satisfies the KKT conditions (A.1).

1) With (A.1b)-$a\times$(A.1c), we have

$$\eta_i^* = -\frac{a\lambda_c + b_i^*}{(N-1)a} \quad \text{(C.1)}$$

Sum up (C.1) for all i, we have $\sum_i \eta_i^* = 0$. Sum up (A.1c) for all i and then substitute (A.1a) into it, we have

$$\lambda_c^* = \frac{1}{N}\sum_i (2c_i p_i^* + d_i) = \frac{1}{N}\sum_i md_i(p_i^*) \quad \text{(C.2)}$$

2) Given any $b_j, j \neq i$, let $b_i(p_i) = \frac{-Np_i + \sum_{j\neq i} b_j + ND_i}{N-1}$ be the unique solution of constaints (5b), (5c). Then, for any fixed $b_j, j \neq i$, $\Pi_i(p_i, b_i, b_{-i}) = \Pi_i(p_i, b_i(p_i), b_{-i})$.

Derivatives of (5b) and (5c) with respect to p_i are

$$1+a\frac{\partial \lambda_c}{\partial p_i}+\frac{\partial b_i}{\partial p_i}=0 \tag{C.3}$$

$$Na\frac{\partial \lambda_c}{\partial p_i}+\frac{\partial b_i}{\partial p_i}=0 \tag{C.4}$$

Solving the equations we have

$$\begin{aligned}\frac{\partial \lambda_c}{\partial p_i} &= \frac{1}{(N-1)a}\\ \frac{\partial b_i}{\partial p_i} &= -\frac{N}{N-1}\end{aligned}$$

In consequence, the derivative of $\Pi_i(p_i, b_i(p_i), b_{-i})$ is

$$\begin{aligned}&\frac{\partial \Pi_i}{\partial p_i}(p_i, b_{-i})\\ =\ & (2c_ip_i+d_i)-\left[\lambda_c(b_i(p_i),b_{-i})-\frac{a\lambda_c(b_i(p_i),b_{-i})+b_i(p_i)}{(N-1)a}\right]\end{aligned} \tag{C.5}$$

The first term $2c_ip_i+d_i$ is the marginal disutility of prosumer i; the second term λ_c is the marginal cost he needs to pay when buying from the market regardless of the mutual impact of different prosumers; the third term $(a\lambda_c+b_i)/a(N-1)$ reflects the marginal profit deviation due to the interest conflicts among different prosumers.

If $md_i(p_i^*)>\lambda_c(b^*)$ and $q_i(b^*)<0$, then prosumer i can always choose $\Delta p_i<0$ to reduce his cost, and thus, it is not a stable situation. Similarly, if $md_i(p_i^*)<\lambda_c(b^*)$ and $q_i(b^*)>0$, the market is also unstable as prosumer i always has a better choice of $\Delta p_i>0$. As a result, $md_i(p_i^*)>\lambda_c(b^*)$ if and only if $q_i(b^*)>0$. This completes the proof. □

APPENDIX D
PROOF OF PROPOSITION 4

Proof. 1) According to Proposition 1, p_i^* is the unique solution of (9) and satisfies $\sum_i p_i^*=\sum_i D_i$. So p_i^* must be a feasible solution to the social optimal problem (11). As $\bar p_i$ is the optimal solution of (11), we always have

$$\sum_{i\in\mathscr{I}} f_i(p_i^*)\geq \sum_{i\in\mathscr{I}} f_i(\bar p_i)$$

2) With the KKT conditions, we can obtain the optimal solutions to problem (9) and (11) are

$$p_i^*=\left(\sum_j D_j+\sum_j\frac{d_j+\frac{D_j}{a(N-1)}}{2c_j-\frac{1}{a(N-1)}}\right)\frac{\frac{1}{2c_i-\frac{1}{a(N-1)}}}{\sum_j\frac{1}{2c_j-\frac{1}{a(N-1)}}}-\frac{d_i+\frac{D_i}{a(N-1)}}{2c_i-\frac{1}{a(N-1)}}$$

and

$$\bar p_i=\left(\sum_j D_j+\sum_j\frac{d_j}{2c_j}\right)\frac{\frac{1}{2c_i}}{\sum_j\frac{1}{2c_j}}-\frac{d_i}{2c_i}$$

respectively. Moreover, we have

$$\begin{aligned}\bar p_i &\leq \left(N\bar D+N\frac{\bar d}{2\underline{c}}\right)\frac{\bar c}{\underline{c}N}-\frac{\underline{d}}{2\bar c}\\ &= (\bar D+\frac{\bar d}{2\underline{c}})\frac{\bar c}{\underline{c}}-\frac{\underline{d}}{2\bar c} \qquad \text{(D.1a)}\\ \bar p_i &\geq (N\underline{D}+N\frac{\underline{d}}{2\bar c})\frac{\underline{c}}{\bar c N}-\frac{\bar d}{2\underline{c}}\\ &= (\underline{D}+\frac{\underline{d}}{2\bar c})\frac{\underline{c}}{\bar c}-\frac{\bar d}{2\underline{c}} \qquad \text{(D.1b)}\end{aligned}$$

Letting $\bar A:=\sum_j\frac{1}{2c_j}$ and $A^*:=\sum_j\frac{1}{2c_j-\frac{1}{a(N-1)}}$ gives

$$\begin{aligned}0\leq \bar A-A^* &= \sum_j\frac{-\frac{1}{a(N-1)}}{2c_j(2c_j-\frac{1}{a(N-1)})}\\ &= \sum_j\frac{1}{2c_j-4ac_j^2(N-1)}\\ &\leq \frac{N}{2\underline{c}-4a\underline{c}^2(N-1)}\\ &\leq -\frac{1}{2a\underline{c}^2}\end{aligned} \tag{D.2}$$

Consequently, we have

$$\frac{\bar A-A^*}{\bar A} \leq -\frac{\bar c}{a\underline{c}^2N} \tag{D.3a}$$

$$\frac{\bar A-A^*}{\bar A} \geq 0 \tag{D.3b}$$

Let $\bar B:=\sum_j\frac{d_j}{2c_j}$ and $B^*:=\sum_j\frac{d_j+\frac{D_j}{a(N-1)}}{2c_j-\frac{1}{a(N-1)}}$. Then there are

$$\begin{aligned}\bar B-B^* &= \sum_j\frac{d_j+2c_jD_j}{-4ac_j^2(N-1)+2c_j}\\ &\leq \sum_j\frac{|\bar d+2\bar c\bar D|}{-4a\underline{c}^2(N-1)+2\underline{c}}\\ &\leq \frac{|\bar d+2\bar c\bar D|}{-2a\underline{c}^2}\end{aligned} \tag{D.4a}$$

$$\begin{aligned}\bar B-B^* &= \sum_j\frac{d_j+2c_jD_j}{-4ac_j^2(N-1)+2c_j}\\ &\geq \frac{-N|\underline{d}+2\underline{cD}|}{-4a\underline{c}^2(N-1)+2\underline{c}}\\ &\geq \frac{-|\underline{d}+2\underline{cD}|}{-2a\underline{c}^2}\end{aligned} \tag{D.4b}$$

Furthermore, it is easy to see

$$\begin{aligned}|B^*| &\leq \sum_j\left|\frac{d_j+\frac{D_j}{a(N-1)}}{2c_i-\frac{1}{a(N-1)}}\right|\\ &\leq \sum_j\frac{|d_j|+|\frac{D_j}{a(N-1)}|}{2\underline{c}}\\ &\leq \sum_j\frac{\bar d-max\{|\bar D|,|\underline{D}|\}/a}{2\underline{c}}\\ &= \frac{\bar d-max\{|\bar D|,|\underline{D}|\}/a}{2\underline{c}}\cdot N\end{aligned} \tag{D.5}$$

Let $\bar C:=\frac{d_i}{2c_i}$ and $C^*:=\frac{d_i+\frac{D_i}{a(N-1)}}{2c_i-\frac{1}{a(N-1)}}$. Direct calculation gives

$$\bar C-C^* = \frac{d_i+2c_iD_i}{-4ac_i^2(N-1)+2c_i}$$

$$\leq \frac{|\bar{d}+2\overline{cD}|}{-2\underline{c}^2 aN} \tag{D.6a}$$

$$\begin{aligned}\bar{C}-C^* &= \frac{d_i+2c_iD_i}{-4ac_i^2(N-1)+2c_i}\\ &\geq \frac{-|\underline{d}+2\underline{cD}|}{-4a\underline{c}^2(N-1)+2\underline{c}}\\ &\geq \frac{-|\underline{d}+2\underline{cD}|}{-2\underline{c}^2 aN} \end{aligned}\tag{D.6b}$$

Let $\bar{E} := \frac{\frac{1}{2c_i}}{\bar{A}}$ and $E^* := \frac{\frac{1}{2c_i-\frac{1}{a(N-1)}}}{A^*}$, then we have

$$E^* \leq \frac{1}{2\underline{c}A^*} \leq \frac{2\bar{c}-1/a}{2\underline{c}}\cdot\frac{1}{N} \tag{D.7a}$$

$$E^* \geq \frac{\frac{1}{2\bar{c}-1/a}}{A^*} \geq \frac{2\underline{c}}{2\bar{c}-1/a}\cdot\frac{1}{N} \tag{D.7b}$$

and

$$\begin{aligned}\frac{\bar{E}}{E^*}-1 &= \frac{A^*}{\bar{A}}\frac{2c_i}{2c_i-\frac{1}{a(N-1)}}-1\\ &\leq (1-\frac{1}{(-2a\bar{c}+1)N})-1\\ &\leq -\frac{1}{(-2a\bar{c}+1)N}\end{aligned}\tag{D.8a}$$

$$\begin{aligned}\frac{\bar{E}}{E^*}-1 &= \frac{A^*}{\bar{A}}\frac{2c_i}{2c_i-\frac{1}{a(N-1)}}-1\\ &\geq \left(1+\frac{\bar{c}}{a\underline{c}^2N}\right)\left(1-\frac{1}{-a\underline{c}N}\right)-1\\ &\geq \frac{\bar{c}}{a\underline{c}^2N}+\frac{1}{a\underline{c}N}\end{aligned}\tag{D.8b}$$

Therefore, there must be

$$\bar{E}-E^* \leq \left(-\frac{1}{(-2a\bar{c}+1)N}\right)\frac{2\bar{c}-1/a}{2\underline{c}}\cdot\frac{1}{N} \tag{D.9a}$$

$$\bar{E}-E^* \geq \left(\frac{\bar{c}}{a\underline{c}^2N}+\frac{1}{a\underline{c}N}\right)\frac{2\underline{c}}{2\bar{c}-1/a}\cdot\frac{1}{N} \tag{D.9b}$$

The optimal solution of (9) and (11) can be represented as

$$\bar{p}_i = \left(\sum_j D_j+\bar{B}\right)\bar{E}-\bar{C}$$

$$p_i^* = \left(\sum_j D_j+B^*\right)E^*-C^*$$

First, we have

$$\begin{aligned}\left|\left(\sum_j D_j+B^*\right)(\bar{E}-E^*)\right| &\leq \left|\sum_j D_j+B^*\right||\bar{E}-E^*|\\ &\leq \left|\sum_j D_j+B^*\right|\frac{M_3}{N^2}\\ &\leq (\left|\sum_j D_j\right|+|B^*|)\frac{M_3}{N^2}\\ &\leq (M_1N+M_2N)\frac{M_3}{N^2}\\ &= \frac{M_3(M_1+M_2)}{N}\end{aligned}\tag{D.10}$$

where

$$\begin{aligned}M_1 &= max\{|\underline{D}|,|\bar{D}|\}\\ M_2 &= \frac{\bar{d}-max\{|\bar{D}|,|\underline{D}|\}/a}{2\underline{c}}\\ M_3 &= max\left\{\left|(\frac{1}{-2a\bar{c}+1})\frac{2\bar{c}-1/a)}{2\underline{c}}\right|,\right.\\ &\left.\left|(\frac{\bar{c}}{a\underline{c}^2}+\frac{1}{a\underline{c}})\frac{2\underline{c}}{2\bar{c}-1/a}\right|\right\}\end{aligned}\tag{D.11}$$

The difference between $\bar{p}_i$ and p_i^* is

$$\begin{aligned}&|\bar{p}_i-p_i^*|\\ =&\left|\left(\sum_j D_j+\bar{B}\right)\bar{E}-\bar{C}-\left(\sum_j D_j+B^*\right)E^*+C^*\right|\\ \leq&\left|\left(\sum_j D_j+\bar{B}\right)\bar{E}-\left(\sum_j D_j+B^*\right)\bar{E}\right|\\ &+\left|\left(\sum_j D_j+B^*\right)\bar{E}-\left(\sum_j D_j+B^*\right)\bar{E}\right|+\left|\bar{C}-C^*\right|\\ =&|\bar{E}(\bar{B}-B^*)|+\left|\left(\sum_j D_j+B^*\right)(\bar{E}-E^*)\right|+\left|\bar{C}-C^*\right|\\ \leq&\frac{\bar{c}}{N\underline{c}}max\left\{\frac{|\bar{d}+2\bar{c}\bar{D}|}{-2a\underline{c}^2},\frac{-|\underline{d}+2\underline{cD}|}{-2a\underline{c}^2}\right\}+M_3(M_1+M_2)\frac{1}{N}\\ &+max\left\{\frac{|\bar{d}+2\overline{cD}|}{-2\underline{c}^2a},\frac{|\underline{d}+2\underline{cD}|}{-2\underline{c}^2a}\right\}\frac{1}{N}\end{aligned}\tag{D.12}$$

It is easy to see that $|\bar{p}_i-p_i^*|\leq\frac{\alpha}{N}$ holds for a large enough positive number α. Because $\underline{P}\leq\bar{p}_i\leq\bar{P}$ and $-\frac{\alpha}{N}\leq p_i^*-\bar{p}_i\leq\frac{\alpha}{N}$, where

$$\underline{P}=\left(\underline{D}+\frac{\underline{d}}{2\bar{c}}\right)\frac{\underline{c}}{\bar{c}}-\frac{\bar{d}}{2\underline{c}}$$

$$\bar{P}=\left(\bar{D}+\frac{\bar{d}}{2\underline{c}}\right)\frac{\bar{c}}{\underline{c}}-\frac{\underline{d}}{2\bar{c}}$$

as a result, we have

$$2\underline{P}-\alpha\leq2\underline{P}-\frac{\alpha}{N}\leq p_i^*+\bar{p}_i\leq2\bar{P}+\frac{\alpha}{N}\leq2\bar{P}+\alpha \tag{D.13}$$

For a given $\varepsilon>0$, we choose a large enough number $N_0 := \frac{1}{(\bar{c}\alpha max\{|2\underline{P}-\alpha|,|2\bar{P}+\alpha|\}+\alpha\bar{d})\varepsilon}$. Then for arbitrary number $N>N_0$, there is

$$\begin{aligned}&\frac{1}{N}\left|\sum_{i\in\mathcal{N}}f_i(p_i^*)-\sum_{i\in\mathcal{N}}f_i(\bar{p}_i)\right|\\ \leq&\frac{1}{N}\sum_i\left|c_i(p_i^*)^2+d_ip_i-c_i(\bar{p}_i)^2-d_i\bar{p}_i\right|\\ \leq&\frac{\bar{c}}{N}\sum_i|p_i^*+\bar{p}_i||p_i^*-\bar{p}_i|+\frac{\bar{d}}{N}\sum_i|p_i^*-\bar{p}_i|\\ \leq&(\bar{c}\alpha max\{|2\underline{P}-\alpha|,|2\bar{P}+\alpha|\}+\alpha\bar{d})\frac{1}{N}\\ <&\ \varepsilon\end{aligned}\tag{D.14a}$$

This completes the proof. ∎

APPENDIX E
PROOF OF PROPOSITION 5

Proof. Suppose that $0<|a_1|<|a_2|$, and p^{1*} is the NE of (5) with $a=a_1$, and p^{2*} is the NE with $a=a_2$. According to Proposition 1, p^{1*} and p^{2*} are the unique optimal point of problem (9) under $a=a_1$ and $a=a_2$, respectively. Due to optimality,

$$\begin{aligned} &\sum_{i\in\mathscr{I}} f_i(p_i^{2*})-\frac{\sum_{i\in\mathscr{I}}(p_i^{2*}-D_i)^2}{2(N-1)a_1}\\ \geq\ &\sum_{i\in\mathscr{I}} f_i(p_i^{1*})-\frac{\sum_{i\in\mathscr{I}}(p_i^{1*}-D_i)^2}{2(N-1)a_1} \end{aligned} \tag{E.1}$$

which means

$$\begin{aligned} &2|a_1|(N-1)\left[\sum_{i\in\mathscr{I}} f_i(p_i^{2*})-\sum_{i\in\mathscr{I}} f_i(p_i^{1*})\right]\\ \geq\ &\left[\sum_{i\in\mathscr{I}}(p_i^{1*}-D_i)^2-\sum_{i\in\mathscr{I}}(p_i^{2*}-D_i)^2\right] \end{aligned} \tag{E.2}$$

If we have

$$\sum_{i\in\mathscr{I}} f_i(p_i^{1*})<\sum_{i\in\mathscr{I}} f_i(p_i^{2*})$$

then

$$\begin{aligned} &2|a_2|(N-1)\left[\sum_{i\in\mathscr{I}} f_i(p_i^{2*})-\sum_{i\in\mathscr{I}} f_i(p_i^{1*})\right]\\ \geq\ &2|a_1|(N-1)\left[\sum_{i\in\mathscr{I}} f_i(p_i^{2*})-\sum_{i\in\mathscr{I}} f_i(p_i^{1*})\right]\\ \geq\ &\left[\sum_{i\in\mathscr{I}}(p_i^{1*}-D_i)^2-\sum_{i\in\mathscr{I}}(p_i^{2*}-D_i)^2\right] \end{aligned} \tag{E.3}$$

which means

$$\begin{aligned} &\sum_{i\in\mathscr{I}} f_i(p_i^{2*})-\frac{\sum_{i\in\mathscr{I}}(p_i^{2*}-D_i)^2}{2(N-1)a_2}\\ \geq\ &\sum_{i\in\mathscr{I}} f_i(p_i^{1*})-\frac{\sum_{i\in\mathscr{I}}(p_i^{1*}-D_i)^2}{2(N-1)a_2} \end{aligned} \tag{E.4}$$

and is contradict to the assumption that p^{2*} is the NE under $a=a_2$ (also the optimal solution of problem (9) under $a=a_2$), which completes the proof. □

APPENDIX F
PROOF OF PROPOSITION 7

Proof. The KKT conditions of the MRP sharing problem is

$$2c_i^k p_i^k+d_i^k+\mu_i^{'} = 0, \forall k\in K_i \tag{F.1a}$$
$$\lambda_c+\mu_i^{'}+\eta_i^{'} = 0 \tag{F.1b}$$
$$2a\lambda_c+b_i+a\mu_i^{'}+Ia\eta_i^{'} = 0 \tag{F.1c}$$
$$\sum_k p_i^k+a\lambda_c+b_i = D_i \tag{F.1d}$$
$$Ia\lambda_c+\sum_i b_i = 0 \tag{F.1e}$$

The KKT condition of problem (16) is

$$\begin{aligned} &\left[2c_i^k-\frac{1}{(I-1)a}\right]p_i^k+d_i^k\\ &+\frac{D_i}{(I-1)a}-\frac{\sum\limits_{j\in K_i,j\neq k} p_i^j}{(I-1)a} = -\xi^{'} \end{aligned} \tag{F.2a}$$

$$\sum_i\sum_k p_i^k = \sum_i D_i \tag{F.2b}$$

$\rightarrow$: If (p^*,b^*) is the NE of the sharing problem for MRP (13), then it satisfies the KKT conditions (F.1). Sum up the (F.1d) for all i and substitute (F.1e) into it, (F.2b) is met. With (F.1b)$\times Ia$-(F.1c) and (F.1d), we have

$$a(I-2)\lambda_c+\sum_k p_i^k+a\lambda_c-D_i+(I-1)a\mu_i^{'}=0 \tag{F.3}$$

For another MRP u, we also have

$$a(I-2)\lambda_c+\sum_k p_u^k+a\lambda_c-D_u+(I-1)a\mu_u^{'}=0 \tag{F.4}$$

(F.3)-(F.4) gives

$$\sum_k p_i^k-D_i+(I-1)a\mu_i^{'}=\sum_k p_u^k-D_u+(I-1)a\mu_u^{'} \tag{F.5}$$

Together with (F.1a), we have

$$\left[2c_i^k-\frac{1}{(I-1)a}\right]p_i^k+d_i^k+\frac{D_i}{(I-1)a}-\frac{\sum\limits_{j\in K_i,j\neq k} p_i^j}{(I-1)a}=\text{constant}$$

In consequence, (F.2a) is met and p^* is also the optimal solution of problem (16).

By (F.1c) $-\,a\times$ (F.1b), we have

$$\eta_i^{'}=-\frac{a\lambda_c+b_i}{(I-1)a}$$

so that $\sum\limits_i \eta_i^{'}=0$.

By (F.1a) and (F.1b), we have

$$\lambda_c+\eta_i^{'}=md_i(p_i) \tag{F.6}$$

Sum up (F.6) for all i tells that

$$\lambda_c^*(p^*)=\frac{1}{I}\sum_i md_i(p_i^*)$$

and

$$b_i^*(p^*)=D_i-\sum_k p_i^{k*}-a\lambda_c^*(p^*)$$

$\leftarrow$: if (p^*,b^*) is the optimal solution of problem (16), then it satisfies the KKT conditions (F.2). If we let

$$\begin{aligned} \mu_i^{'} &= -(2c_i^k(p_i^k)^*+d_i^k)\\ \lambda_c &= \frac{1}{I}\sum_i md_i(p_i^*)\\ \eta_i^{'} &= -(\mu_i^{'}+\lambda_c)\\ b_i &= -(a\lambda_c+\sum_k (p_i^k)^*) \end{aligned} \tag{F.7a}$$

It is easy to verify that (F.7) satisfies KKT condition (F.1), and thus, (p^*,b^*) is the NE of the sharing problem (13), which completes the proof. □

APPENDIX G
PROOF OF PROPOSITION 8

Proof. We consider a special situation, where all the $c_i, \forall i$ equal to a same value c, and each prosumer possess the same number of resource, which is K_I. IK_I is a fixed value equals to N.

With the KKT condition (F.2), we have

$$2cp_i^k + d_i^k + \frac{D_i - \sum_{k\in\mathscr{K}_i} p_i^k}{(I-1)a} = -\xi' \quad \text{(G.1a)}$$
$$\sum_i\sum_k p_i^k = \sum_i D_i \quad \text{(G.1b)}$$

Denote $2cp_i^k + d_i^k$ as md_i^k. It is obvious that for all $k \in \mathscr{K}_i$, md_i^k are equal, and so we use md_i to represent md_i^k for all k. Sum up (G.1a) for all i and k, we have

$$2c\sum_i\sum_k p_i^k + \sum_i\sum_k d_i^k + \frac{K_I(\sum_i D_i - \sum_i\sum_{k\in\mathscr{K}_i} p_i^k)}{(I-1)a} + N\xi' = 0 \quad \text{(G.2)}$$

Together with (G.1b), it is easy to find that ξ' is independent of I and $\sum_i md_i + I\xi' = 0$. Assume that the optimal marginal disutility is md_i^*, then according to (G.1a), we have

$$\frac{D_l - \sum_{k\in\mathscr{K}_i} p_i^{k*}}{(I-1)a} = -md_i^* - \xi'^* \quad \text{(G.3)}$$

The objective function (16a) can be rewritten as

$$\pi := \pi_1 + \pi_2$$

where

$$\pi_1 = \sum_i\sum_k [c(p_i^{k*})^2 + d_i^k p_i^{k*}] = \sum_i\sum_k \frac{(md_i^*)^2 - (d_i^k)^2}{4c}$$

$$\pi_2 = -\frac{\sum_i (D_i - \sum_{k\in\mathscr{K}_i} p_i^{k*})^2}{2(I-1)a} = -\frac{(I-1)a}{2}\sum_i (md_i^* + \xi'^*)^2$$

Then we have

$$\begin{aligned}
\pi_1 &= \sum_i\sum_k \frac{(md_i^*)^2 - (d_i^k)^2}{4c} \\
&= \frac{K_I}{4c}\sum_i (md_i^*)^2 - \sum_i\sum_k \frac{(d_i^k)^2}{4c} \\
&= \frac{K_I}{4c}\sum_i (md_i^* + \xi'^*)^2 + \frac{N(\xi'^*)^2 - \sum_i\sum_k (d_i^k)^2}{4c} \quad \text{(G.4)}
\end{aligned}$$

The first term of (G.4) is variational and the second term is a constant. So the change of π_1 is related with $\frac{K_I}{4c}\sum_i (md_i^* + \xi'^*)^2$, the change of π_2 is related with $-\frac{(I-1)a}{2}\sum_i (md_i^* + \xi'^*)^2$ and the change of π is related with $\frac{K_I - 2ac(I-1)}{4c}\sum_i (md_i^* + \xi'^*)^2$. Obviously, $\frac{K_I - 2ac(I-1)}{4c}$ is always positive for all $I > 1$.

Next, we define an equal partition such that π is decreasing. Assume that there is I multi-resource prosumers, and the optimal output is p_i^{k*}. Then we introduce competition and allocate the resources owned by one prosumer and its demand to $Z \in \mathbb{Z}^+$ prosumers such that $I' = ZI$ and $K_I = K_{I'}$, and satisfies $\forall i, \forall z_1, z_2 \in \{1, ..., Z\}$

$$\begin{aligned}
(D'_{Z(i-1)+z_1} - \sum_{k=1}^{K_{I'}} p_{Z(i-1)+z_1}^{k*})(D'_{Z(i-1)+z_2} - \sum_{k=1}^{K^{I'}} p_{Z(i-1)+z_2}^{k*}) &\geq 0 \\
\sum_{z=1}^{Z} D'_{Z(i-1)+z} &= D_i
\end{aligned}$$

Then

$$\begin{aligned}
\pi^{I*} &= \pi_1^{I*} - \sum_i \frac{(D_i - \sum_{k=1}^{K_I} p_i^{k*})^2}{(I-1)a} \\
&= \pi_1^{I*} - \sum_i \frac{(\sum_{z=1}^{Z} D'_{Z(i-1)+z} - \sum_{z=1}^{Z}\sum_{k=1}^{K_{I'}} p_{Z(i-1)+z}^{k*})^2}{(I-1)a} \\
&\geq \pi_1^{I*} - \sum_i \frac{\sum_{z=1}^{Z} (D'_{Z(i-1)+z} - \sum_{k=1}^{K_{I'}} p_{Z(i-1)+z}^{k*})^2}{(I-1)a} \\
&\geq \pi_1^{I*} - \sum_i \frac{\sum_{z=1}^{Z} (D'_{Z(i-1)+z} - \sum_{k=1}^{K_{I'}} p_{Z(i-1)+z}^{k*})^2}{(I'-1)a} \\
&\geq \pi^{I'*} \quad \text{(G.5a)}
\end{aligned}$$

(G.5) tells us that the objective function π decreases with I. When I grows, the coefficient of π_1 (which is $\frac{K_I}{4c}$) decreases while the coefficient of π_2 (which is $\frac{-(I-1)a}{2}$) increases. If $\sum_i (md_i + \xi')^2$ decreases with I, then obviously π_1 decreases with I. Otherwise, if $\sum_i (md_i + \xi')^2$ increases with I, π_2 increases with I. If π_1 also increases, then $\pi^{I*} < \pi^{I'*}$, which is contradict to (G.5). In conclusion, we always have the social total cost π_1 reduces with I, which means introducing effective competition can improve social welfare.

Moreover, as $\frac{K-2ac(I-1)}{4c}$ is always positive and reaches the minimum when $I = \sqrt{-2ac/N}$. If $\sqrt{-2ac/N} \leq 1$, then $\frac{K-2ac(I-1)}{4c}$ increases with $I > 1$, which tells that $\sum_i (md_i + \xi')^2$ is decreasing in I, so that

$$\begin{aligned}
Var(md_i^{k*}, I) &= \frac{1}{N}\sum_{i=1}^{I}\sum_{k=1}^{K_I} (md_i^{k*} + \xi')^2 \\
&= \frac{K_I}{N}\sum_{i=1}^{I} (md_i + \xi')^2 \\
&\geq \frac{K_{I'}}{N}\sum_{i=1}^{I'} (md_i + \xi')^2 = Var(md_i^{k*}, I')
\end{aligned}$$

This completes the proof. ∎