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AN ENGINEERING THEORY OF CREEP OF FROZEN SOILS

by

Branko Ladanyi

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An Engineering Theory of Creep of Frozen Soils

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Most of the existing theories of creep have been developed from two different viewpoints: micromechanistic and macroanalytical. The former deal with events occurring at the atomic level and provide knowledge of the processes that control creep. The latter are based on certain macroscopic experimental findings and represent, in fact, an extension of the theory of plasticity to include time and temperature effects. Both the micromechanistic and macroanalytical approaches lead to fruitful resuts and each can benefit from the other. However, although the former has the advantage of being derived from physical concepts, the use of the latter is often preferred in practice if it provides basic relations that are broad in scope and can lead to improved procedures for designing structures.

In this paper, a macroanalytical view of the problem of creep of frozen soils is presented. The proposed theory of creep has been developed mainly with the purpose of being used as a basis for solving a specific soil engineering problem, i.e., the bearing capacity of buried footings and anchors.

Since the problem is itself rather complex, it was endeavoured to present the creep information in a relatively simple mathematical form. The theory, while using certain concepts and data from the frozen soils literature, follows more closely, nevertheless, the methods usual in certain engineering theories of creep of metals.

La majorité des théories de fluage existantes ont été développées soit à partir d'un point de vue micro-mécanistique, soit à partir d'un point de vue macroanalytique. Les premières traitent des phénomènes se produisant à l'échelle atomique et tâchent d'expliquer des processus qui dirigent le fluage. Les dernières, par contre, sont basées sur certaines constatations expérimentales macroscopiques et représentent, en effet, une extension de la théorie de la plasticité dans laquelle on tient compte des effets du temps et de la température. Les deux types de théories de fluage sont utiles en pratique et bénéficient l'une de l'autre. Cependant, malgré que les premières ont l'avantage d'être déduites des conceptions physiques, en pratique on préfère souvent utiliser les deuxièmes si les relations qu'elles fournissent sont suffisamment générales et en même temps assez simples pour permettre une amélioration des méthodes de calcul des éléments de construction.

La présente étude montre un aspect macroanalytique du problème du fluage des sols gelés. La théorie proposée a été développée avec le but principal de servir de base pour la solution d'un problème géotechnique particulier, notamment celui de la capacité portante des fondations enterrées et des ancrages.

Le problème étant d'une complexité considérable en soi-même, on a fait tout effort pour présenter l'information de fluage dans une forme mathématique aussi simple que possible. La théorie utilise certaines conceptions et données tirées de la littérature sur les sols gelés. Néanmoins, dans sa conception générale, elle suit de plus près les méthodes usuelles dans certaines théories du fluage des métaux à haute température.

Introduction

As in metals, research efforts in the field of the creep of frozen soils have taken two different paths over the years: one aiming at an engineering theory of creep to be used in design work, the other aiming at a physical theory capable of describing the creep phenomena in terms of already established concepts of physics.

An engineering or macroanalytical theory of creep can simply be considered as a collection of laws that are found, by experience, to describe adequately the observed manifestations of creep. The criterion of a sound engineering theory is that it can describe a number of different creep manifestations in simple mathematical terms keeping the number of material parameters as small as possible. Typical examples of such theories are Odquist and Hult's theory of creep of metals (Hult 1966; Odquist and Hult 1962), and Vialov's theory of creep of frozen soils (Vialov 1959, 1962, 1963). On the other hand, the aim of a physical or micromechanistic theory of creep is to establish a set of laws that would be able to describe the observed phenomena of creep in terms of previously established quantities and laws of physics. An example is the theory of soil

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creep proposed by Mitchell *et al.* (1968) which is based on the concept of rate processes developed in statistical mechanics. The same physical concept has been used by Andersland and Akili (1967) for describing the creep behavior of frozen soils.

In analytical terms, the main difference between the two approaches is that, for expressing the stress dependence of creep rate, the former uses mainly a power form while the latter uses either a hyperbolic sine or an exponential form. As far as experimental evidence is concerned the power creep law has been found to fit experimental steady state creep data, at least at low and intermediate stress levels, for a large range of materials such as metals at high temperature (Dorn 1954, Laks *et al.* 1957), plastics (Marin *et al.* 1951), ice (Glen 1955; Gold 1970), and frozen soils (Vialov 1959, 1962, 1963). For the latter, the exponential form has also been found to fit experimental data at intermediate and high stress levels (Akili 1970; Andersland and Akili 1967; Andersland and AlNouri 1970).

In more general terms, as stated by Scott (1969), "each approach has advantages which depend on the problem to be studied: one may be of value in interpreting material properties from a test, another may be used in the calculation of a time-dependent stress or displacement field in the same material."

The theory shown in this report has been developed with the main purpose of being used as a basis for solving a bearing capacity problem. As the problem to be solved is itself rather complex, an engineering type theory, based on a single power term and leading to relatively simple mathematical expressions, has been found preferable to a physical theory. The proposed theory is similar in concept to Vialov's but has the advantage of being derived from a single concept and of using only normalized forms with a relatively small number of experimental parameters.

Type of Creep Information Required

It is well known that within the field of linear viscoelasticity, if the response to a step input is known (from stress relaxation and creep experiments) then the response to any arbitrary input can be calculated from the superposition integral, a procedure often known as Boltzmann's superposition principle. Solution to many creep problems can be obtained from the corresponding elastic solutions using the Laplace transform method.

It is now well recognized, however, that frozen soils show nonlinear viscoelastic behavior to a degree that precludes the adoption of linear approximations in most practical problems. Unfortunately, for such materials the superposition integrals cannot be applied without considerable modification which renders them rather intractable. Among alternative methods that have been proposed for non-linear viscoelastic materials, the following three have found increasing application in practice.

(1) Hoff's elastic analogue, which enables a non-linear viscoelasticity problem to be solved, under certain conditions, as a non-linear elasticity problem (Finnie and Heller 1959; Hoff 1954; Odquist 1966; Odquist and Hult 1962). It is most suitable for predicting primary and steady state creep behavior of structures subjected to constant or proportionally varying loads.

(2) Method of isochronous stress-strain curves, which transforms a non-linear visco-elastic-plastic problem into a non-linear elastic-plastic problem (Smith and Sidebottom 1965; Vialov 1959, 1962). It furnishes a direct answer to step inputs of stress only, but can also be used for a succession of step inputs with some modification.

(3) Method of time-dependent strength (Vialov 1959, 1962) or limiting strain (Turner 1966), which enables approximate solutions of creep failure problems to be found from the corresponding solutions in the theory of plasticity.

It is the purpose of this paper to show how the creep information necessary for the aforementioned three methods of solution can be deduced from a set of constant stress creep tests.

Uniaxial State of Stress

Stress – Strain – Strain Rate Relations

The type of creep curve shown in Fig. 1(a), obtained by step loading under uniaxial stress condition and at a constant temperature,

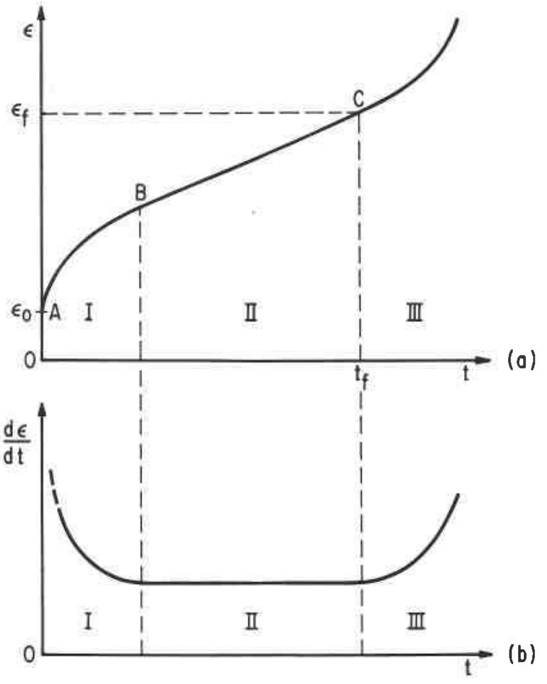

FIG. 1. Stress and strain rate in constant stress creep test.

is common to a large number of materials including frozen soils, plastics, and many metals at high temperature. Fig. 1(b) presents the corresponding creep rate, $d\varepsilon/dt$ (note that definition of symbols is given in the appendix), versus time. Three periods of time are observed during which the creep rate is in order (I) decreasing, (II) remaining essentially constant, and (III) increasing. These are often called the periods of primary, secondary and tertiary creep.

Figure 2 shows a set of such creep curves as obtained in a series of constant temperature creep tests, step loaded to different uniaxial stress levels $\sigma_1 < \sigma_2 < \sigma_3 < \sigma_4$.

If the type of creep curves shown in Fig. 2 are to be used as a basis for establishing the constitutive equation of the material, a convenient method is that described by Hult (1966). The method consists in approximating the creep curves by straight lines, as indicated in Fig. 2, and in establishing a law that describes these straight lines rather than the actual creep curves. It is evident that the predictions to be derived from such a law will be in error during the first phase of a

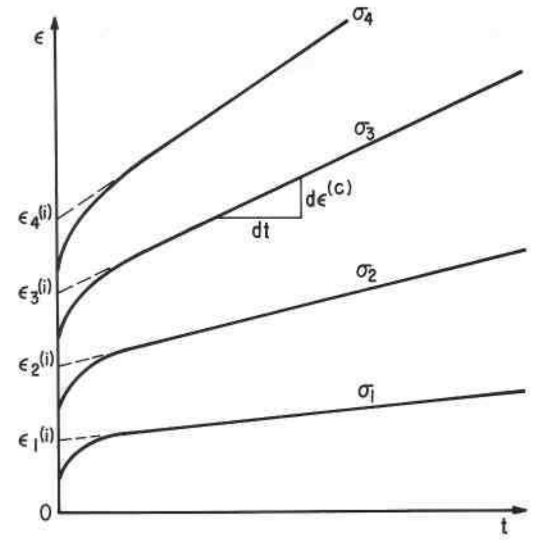

FIG. 2. Linearized creep curves according to Hult (1966).

creep process, but the error will decrease steadily during continued creep. In frozen soils, according to Vialov (1959), for time intervals longer than about 24 h, the amount of strain developed during the secondary creep period is large compared with the strain developed during primary creep, so that the proposed straight-line approximation seems acceptable for most practical long term problems. The validity of this method of time dependent strain prediction for long periods of time has been checked for various materials (Marin *et al.* 1951).

In the straight line approximation method, the strain in the secondary creep period is given by

[1] $$\varepsilon = \varepsilon^{(i)} + \varepsilon^{(c)}$$

where the pseudo-instantaneous strain $\varepsilon^{(i)}$, defined as indicated in Fig. 2, is governed by

[2] $$\varepsilon^{(i)} = F(\sigma, T)$$

and the creep strain $\varepsilon^{(c)}$ by the creep law

[3] $$d\varepsilon^{(c)}/dt = G(\sigma, T).$$

The form of the functions $F(\sigma, T)$ and $G(\sigma, T)$ is determined by plotting the intercepts $\varepsilon^{(i)}$ and the slopes $d\varepsilon^{(c)}/dt$ against the applied stresses, with the temperature as a parameter, as in Fig. 3, and by fitting suitable

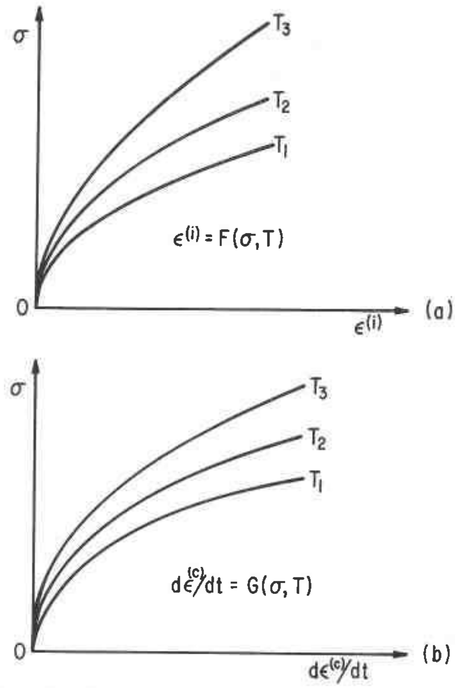

FIG. 3. Constant temperature curves of pseudo-instantaneous strain versus stress (a), and creep rate versus stress (b).

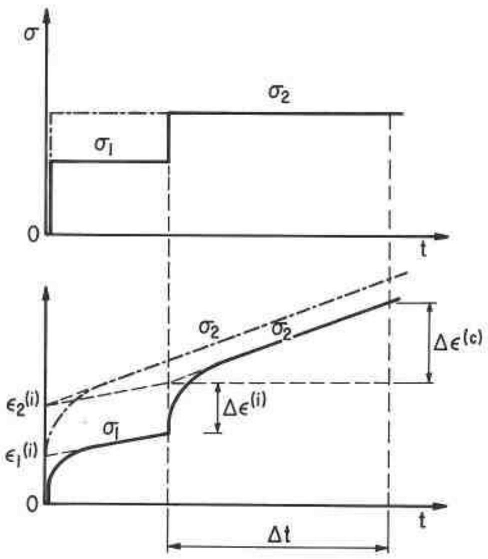

FIG. 4. Creep curve for step loading.

mathematical expressions to the experimental curves.

Once the mathematical form of the functions F and G has been determined, the total strain after the time t in a constant temperature creep test step loaded to a stress σ, is from Eqs. [1] to [3] given by

$$\varepsilon = F(\sigma, T) + G(\sigma, T)\, t. \quad [4]$$

The strain in a creep process in which the load is increased in steps, σ and T being constant for each step, can be obtained by a summation procedure, as shown schematically in Fig. 4. At any time t, the total strain is then a function of the complete loading and temperature history of the process.

If a relation similar to Eq. [4] is assumed to hold also when σ and T vary continuously with time, the total strain is given by

$$\varepsilon = F(\sigma, T) + \int_0^t G(\sigma, T)\, dt \quad [5]$$

from which the total rate of strain

$$d\varepsilon/dt = (d/dt) \,.\, F(\sigma, T) + G(\sigma, T). \quad [6]$$

The pseudo-instantaneous strain $\varepsilon^{(i)}$ as defined above is, in fact, composed of an elastic (reversible) portion, $\varepsilon^{(ie)}$, and a plastic (irreversible) portion $\varepsilon^{(ip)}$. Hence,

$$\varepsilon^{(i)} = \varepsilon^{(ie)} + \varepsilon^{(ip)}. \quad [7]$$

The elastic portion of $\varepsilon^{(i)}$ can be written as

$$\varepsilon^{(ie)} = \sigma / E(T) \quad [8]$$

where $E(T)$ is a fictitious Young's modulus, smaller than the instantaneous elastic modulus, because $\varepsilon^{(ie)}$ contains also the delayed elasticity effect. The plastic portion of $\varepsilon^{(i)}$ may often be written as a pure power expression (Marin *et al.* 1951; Odquist and Hult 1962)

$$\varepsilon^{(ip)} = \varepsilon_k \left[\frac{\sigma}{\sigma_k(T)} \right]^{k\ (t)} \quad [9]$$

in which σ_k plays the role of a temperature dependent deformation modulus, the exponent $k > 1$ is usually little affected by the temperature, while ε_k is an arbitrary small standard strain unit introduced only for convenience in calculation and plotting of data.

For a test at a given constant temperature T, the numerical values of σ_k and k are obtained from a log-log plot of the pseudo-instantaneous stress-strain curve, after subtracting the elastic portion, because Eq. [9] linearizes as

[10] $\log [\varepsilon^{(ip)}/\varepsilon_k] = k (\log \sigma - \log \sigma_k)$.

Finally, in case of loading, the total pseudo-instantaneous strain takes the form

[11] $\varepsilon^{(i)} = \varepsilon^{(ie)} + \varepsilon^{(ip)} = \sigma/E + \varepsilon_k (\sigma/\sigma_k)^k \equiv F(\sigma, T)$.

Because $\varepsilon^{(ip)}$ is an irreversible strain quantity, the second term should be deleted in case of unloading.

Alternatively, it may be convenient to represent $F(\sigma, T)$ by a single mathematical expression, such as a hyperbola passing through the origin (Kondner 1963; Kondner and Krizek 1965).

Similarly, the creep law, $G(\sigma, T)$, may often be written as a simple power expression (Hult 1966; Marin *et al.* 1951; Norton 1929; Odquist 1966), which seems to be supported by experimental evidence particularly in the lower and intermediate stress range (Laks *et al.* 1957).

[12] $$\dot{\varepsilon} = d\varepsilon/dt = \dot{\varepsilon}_c \left[\frac{\sigma}{\sigma_c (T)}\right]^{n (T)}$$

where $\sigma_c(T)$ and $n(T)$ are creep parameters, both depending on the temperature, the latter, however, much less than the former. The quantity $\dot{\varepsilon}_c$ is a small arbitrary standard strain rate, introduced into Eq. [12] to put it into a normalized form.

The stress quantity $\sigma_c (T)$ in Eq. [12] is the uniaxial stress that causes a constant creep rate equal to $\dot{\varepsilon}_c$, and is often called the creep proof stress (Hult 1966). The magnitude of $\sigma_c (T)$ depends on the value chosen for $\dot{\varepsilon}_c$. Conventionally, for metals, $\dot{\varepsilon}_c$ is chosen to be 10^{-9} s^{-1} or 3.16 %/yr. For frozen soils, it may be convenient to take, e.g., $\dot{\varepsilon}_c = 10^{-5}$ min^{-1} which corresponds to 1.44 %/day. For a constant temperature, the numerical values of σ_c and n are obtained from a log-log plot of experimental stress-strain rate curves (Fig. 5).

Once experimentally determined for a given material, the functions $F(\sigma, T)$ and $G(\sigma, T)$ should be substituted in Eq. [5] to give the constitutive equation of the material.

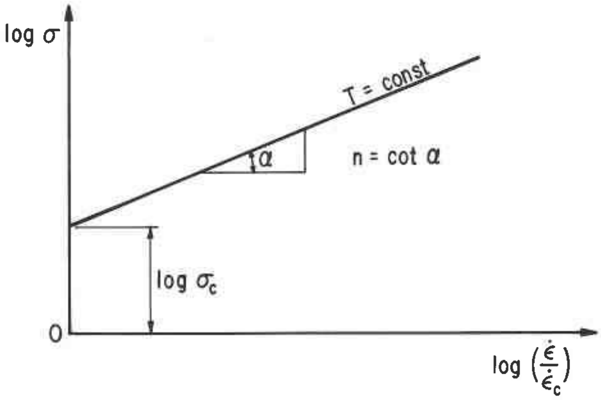

FIG. 5. Log-log plot of the creep law, Eq. [12].

In its integrated form, which can be used in connection with step loading problems, Eq. [5] becomes

[13] $\varepsilon = \sigma/E + \varepsilon_k (\sigma/\sigma_k)^k + \dot{\varepsilon}_c (\sigma/\sigma_c)^n t$.

Equation [13] represents a family of non-linear isothermal and isochronous stress-strain curves (isocurves), each valid for a constant time t. Experimental evidence in frozen soils (Vialov 1959) shows that for time intervals greater than about 24 h the two instantaneous strain terms together become less than 10% of the creep strain. For time intervals longer than about one day it may be sufficient, for practical purposes, to retain only the third term in Eq. [13] for describing the isocurves:

[14] $$\varepsilon \approx \varepsilon_c\, t \left[\frac{\sigma}{\sigma_c (T)}\right]^n.$$

Creep Strength

Creep strength is defined as the stress level at which, after a finite time interval, either rupture or instability leading towards rupture occurs in the material. In tensile creep testing, the creep strength is mostly taken as the stress at which actual rupture occurs. In compression creep testing, however, especially of ductile materials such as high temperature metals and frozen soils, in which only a plastic type of failure occurs that is much less clearly defined, the creep strength is most often identified with the moment of the test at which the first sign of instability occurs. In constant stress creep testing, this moment coincides with the passage from steady state creep to accelerating creep of the material (point C, Fig. 1). In a constant strain rate test, in turn, this sign of instability would coincide with the first drop of strength after the peak of the stress-strain curve.

In general terms, the problem of creep strength prediction consists of finding a relationship between the creep strength σ_f and the magnitudes such as: time to failure, t_f; steady state creep rate, $\dot{\varepsilon}^{(c)}$; strain at failure, ε_f; and temperature, T.

From the geometry of a constant stress creep test, (Fig. 1a and 2), it follows that

[15] $$\varepsilon_f = \varepsilon^{(i)} + t_f\,\dot{\varepsilon}^{(c)}$$

from which

[16] $$t_f = \frac{\varepsilon_f - \varepsilon^{(i)}}{\dot{\varepsilon}^{(c)}}.$$

One of the easiest ways of relating the time to failure to the secondary creep rate is to consider the numerator in Eq. [16] to be a constant,

[17] $$t_f = C/\dot{\varepsilon}^{(c)}.$$

This simple relationship, saying that the time to failure in creep is inversely related to the steady state creep rate, which in turn can be related to the applied stress and temperature, has proven to be very successful in predicting the creep rupture behavior of high temperature metals and has been adopted as the basis for a number of creep rupture criteria (Garofalo 1965). In fact, however, for larger creep rate intervals it is found that the numerator in Eq. [16] is not a constant but is itself some function of creep rate, strain, and temperature. A more general form of Eq. [17] is, therefore:

[18] $$t_f = F(\varepsilon, \dot{\varepsilon}, T)/\dot{\varepsilon}$$

According to Monkman and Grant (1956), for a large number of alloys at high temperature, Eq. [18] can be approximated very closely by the empirical expression

[19] $$t_f = C\,\dot{\varepsilon}^{-m}$$

where C and m are parameters, constant for a given temperature. Comparing Eqs. [18] and [19], it follows that, according to the two authors

[20] $$F(\varepsilon, \dot{\varepsilon}, T) = C\,\dot{\varepsilon}^{1-m}.$$

They find, however, that the exponent m is close to unity ($0.77 < m < 0.93$) for a large variety of materials which justifies the replacing of the function F in Eq. [18] by a constant, at least within a limited interval of strain rate variation.

In compression creep of frozen soils it is often found that the amount of permanent strain at the onset of tertiary creep is approximately constant for a given temperature and type of test, at least in the quasi-static loading range (Sayles and Epanchin 1966, Vialov 1962). Physically, the phenomenon may be interpreted by saying that instability in creep occurs when total damage done by straining attains a certain value. There seems to be some experimental justification, therefore, for using a constant permanent strain as a basis for creep failure criterion in frozen soils. The criterion, in addition to being convenient in application to problem solving, has the advantage of limiting the total strain to acceptable values in the design.

According to this latter criterion, which is analogous to Garofalo's (1965) criterion in the metal creep literature, the numerator in Eq. [16] is not a constant but is a function of the pseudo-instantaneous plastic strain $\varepsilon^{(ip)}$, that, in turn, can be related to stress. Eq. [16] becomes

[21] $$t_f = (\varepsilon_f - \varepsilon^{(ip)})/\dot{\varepsilon}^{(c)}.$$

Substituting in Eq. [21] for $\varepsilon^{(ip)}$ and $\dot{\varepsilon}^{(c)}$ from Eqs. [9] and [12], one gets

[22] $$t_f = \frac{\varepsilon_f - \varepsilon_k\,(\sigma/\sigma_k)^k}{\dot{\varepsilon}_c\,(\sigma/\sigma_c)^n}.$$

Sometimes, in frictional materials, it is found that the secondary creep rate becomes practically zero when the applied stress is lower than a finite value σ_{lt}, called "long term strength" (Vialov 1962). If σ_{lt} exists, Eq. [22] may be written as

[23] $$t_f = \frac{\varepsilon_f - \varepsilon_k\,(\sigma/\sigma_k)^k}{\dot{\varepsilon}_c\left(\dfrac{\sigma - \sigma_{lt}}{\sigma_c}\right)^n}$$

When t tends to infinity, the creep strength tends to zero according to Eq. [22], and to a finite value, σ_{lt}, according to Eq. [23] (Fig. 6).

For a very long time interval, the pseudo-instantaneous portion of strain can be neglected relative to the time-dependent portion, and Eq. [22] becomes

$$[24] \qquad t_f = \varepsilon_f/\dot{\varepsilon}_c\,(\sigma/\sigma_c)^n$$

which now becomes analogous to Eq. [17]. As Eq. [24] has the same analytical form as Eq. [12], the time to failure, t_f, and the creep strength, σ_f, can be obtained directly from the plotted creep law in Fig. 5, as is shown in Fig. 7. For this purpose, it is only necessary to read the values at the abscissa in terms of log $(\dot{\varepsilon}_f/\dot{\varepsilon}_c)$, where

$$[25] \qquad \dot{\varepsilon}_f = \varepsilon_f/t_f.$$

From Eqs. [24] and [25], the creep strength after a long time interval and at a constant temperature T, is given by

$$[26] \qquad \sigma_f \approx \sigma_c\,(T)\,(\dot{\varepsilon}_f/\dot{\varepsilon}_c)^{1/n}.$$

In Fig. 8 experimental steady-state creep rate data obtained for a silty-sandy loam (Vialov 1962) in uniaxial compression have

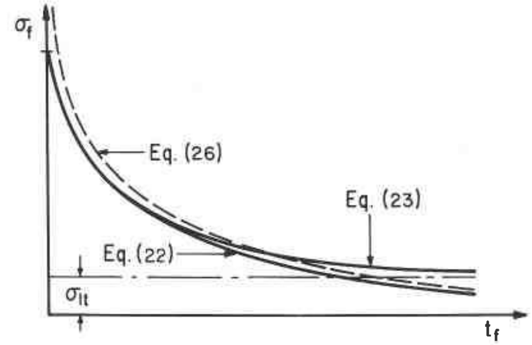

FIG. 6. Creep strength versus time curves.

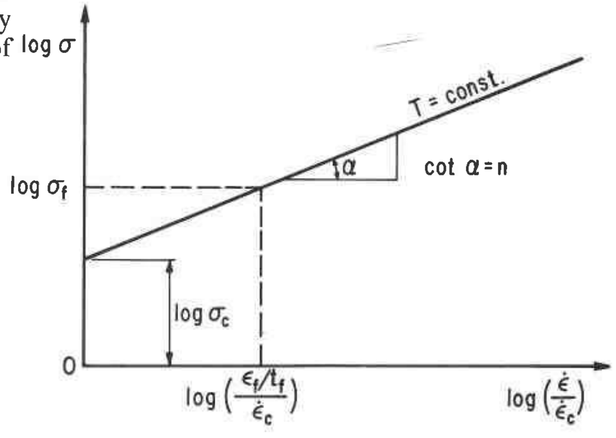

FIG. 7. Creep strength determination according to Eq. [26].

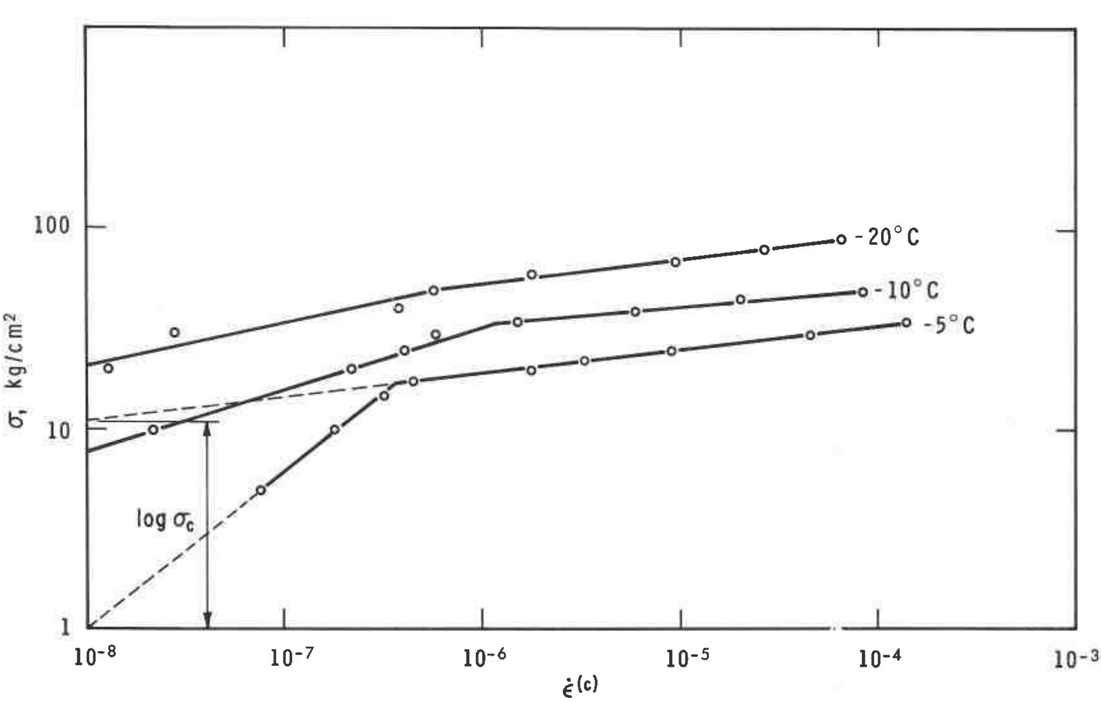

FIG. 8. Log-log plot of creep rate data for Callovian silty-sandy loam in uniaxial compression at three different temperatures, (from Vialov (1962), Table 7, p. 106).

been plotted against the corresponding stresses in a log-log plot. It is seen that for each particular temperature, the creep rate data can be approximated by two straight lines, defining two separate power creep laws. For example, at $T = -5$ °C, and with $\dot{\varepsilon}_c = 10^{-8}$ s^{-1}, one gets

[27] For $0 \leq \sigma < 17$ kg/cm^2, $\dot{\varepsilon}^{(c)} = 10^{-8}\,(\sigma/1.0)^{1.256}$ and

[28] For $17 \leq \sigma < 35$ kg/cm^2, $\dot{\varepsilon}^{(c)} = 10^{-8}\,(\sigma/11.07)^{8.28}$.

For the creep strength evaluation according to the proposed method, additional information about the average creep failure strain, ε_f, is needed. From Fig. 40 in (Vialov 1962), it is found that for the soil considered, at -5 °C, $\varepsilon_f \approx 0.12$ is a reasonable average value of the failure strain for the range of $\sigma > 20$ kg/cm^2. Using Eq. [28] as a basis, Eq. [26] yields

[29] $$\sigma_f = 11.07\,(10^8\,\dot{\varepsilon}_f)^{0.121}$$

in which

[30] $$\dot{\varepsilon}_f = 0.12/t_f$$

and t_f is in seconds.

The values of σ_f for times to failure between 10 min and 24 h, predicted according to Eq. [29] are shown graphically in Fig. 9, curve A.

It is obvious that Eq. [24] overestimates the time to failure, because it neglects the pseudo-instantaneous plastic strains in calculating the average creep rate. For the same reason, Eq. [26] overestimates the creep strength, especially for short time intervals. A better prediction should, therefore, be obtained by using Eq. [22] instead of Eq. [26].

For example, from the data in (Vialov 1962), for the same soil at -5 °C, one finds (Vialov 1962, Table 10, p. 120) for the instantaneous stress strain curve ($t = 1$ min)

[31] $$\sigma_k = A\,\varepsilon_k^m = 66.5\,(10^{-3})^{0.28} = 9.61 \text{ kg/cm}^2$$

[32] $$k = 1/m = 1/0.28 = 3.57$$

where A and m are Vialov's parameters.

With the above values of σ_k and k, Eq. [24] becomes

[33] $$t_f = 10^5\,\frac{120 - (\sigma/9.61)^{3.57}}{(\sigma/11.07)^{8.28}}$$

Curve B in Fig. 9 shows the creep strength

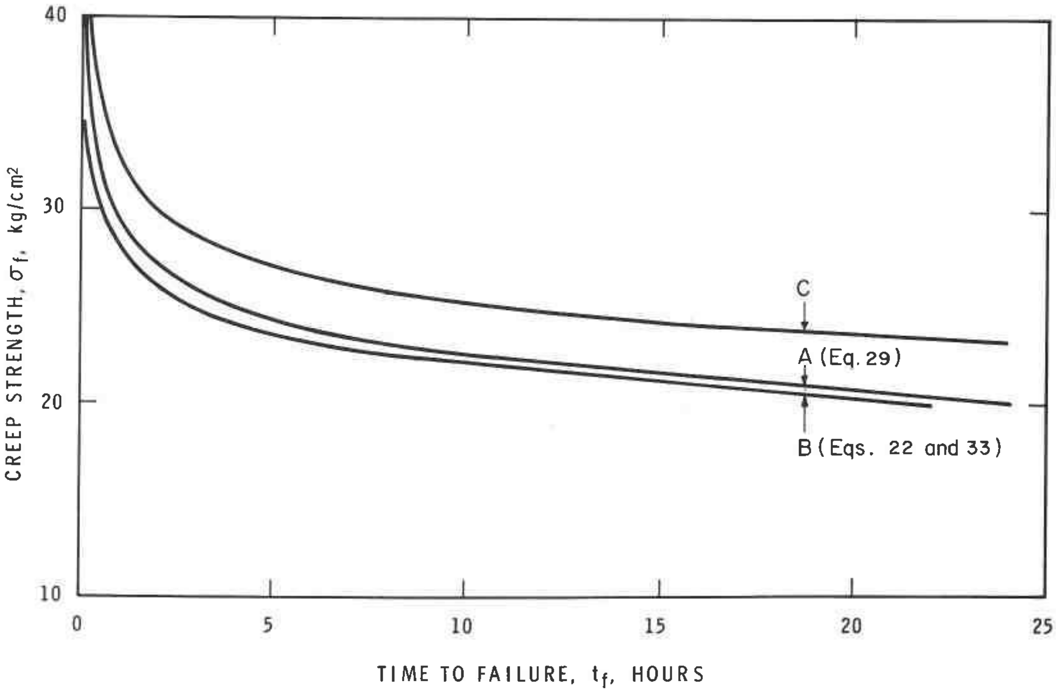

FIG. 9. Calculated creep strength curves for the Callovian silty-sandy loam at -5 °C (Curves A and B). Curve C: Measured creep strengths for Callovian sandy loam (Vialov 1962).

σ_f calculated according to Eq. [33]. Curve C, shown for comparison, represents actually measured creep strengths at rupture of a sandy loam at −5 °C (Vialov 1962, Table 11). It will be seen that the values of σ_f predicted by this method are lower, and correspond more to the onset of tertiary creep stage actually observed for this soil (Vialov 1962, Fig. 40 (A), (C)).

Finally, Fig. 10 presents schematically a unified plot of all creep information according to this concept.

Effect of Temperature on Creep Rate and Strength

Experimental evidence showing that creep in metals involves thermally activated processes has been available for some time (Kauzmann 1941). These processes show a rate dependence on temperature through the factor exp $(-U/RT)$, where U is the apparent activation energy for the process or processes which are controlling, R is the universal gas constant, and T is the absolute temperature.

For unfrozen soils, the validity of this type of relationship has been demonstrated by Mitchell *et al.* (1968). For frozen soils, the same relationship has been found valid by Andersland and Akili (1967), Akili (1970), and Andersland and AlNouri (1970).

For everything else constant, the temperature dependence of the creep rate according to the above theory is given by Andersland and AlNouri (1970):

[34] $$\dot{\varepsilon}^{(c)} = A \exp(-L/T)$$

where

[35] $$L = U/R$$

in units of temperature.

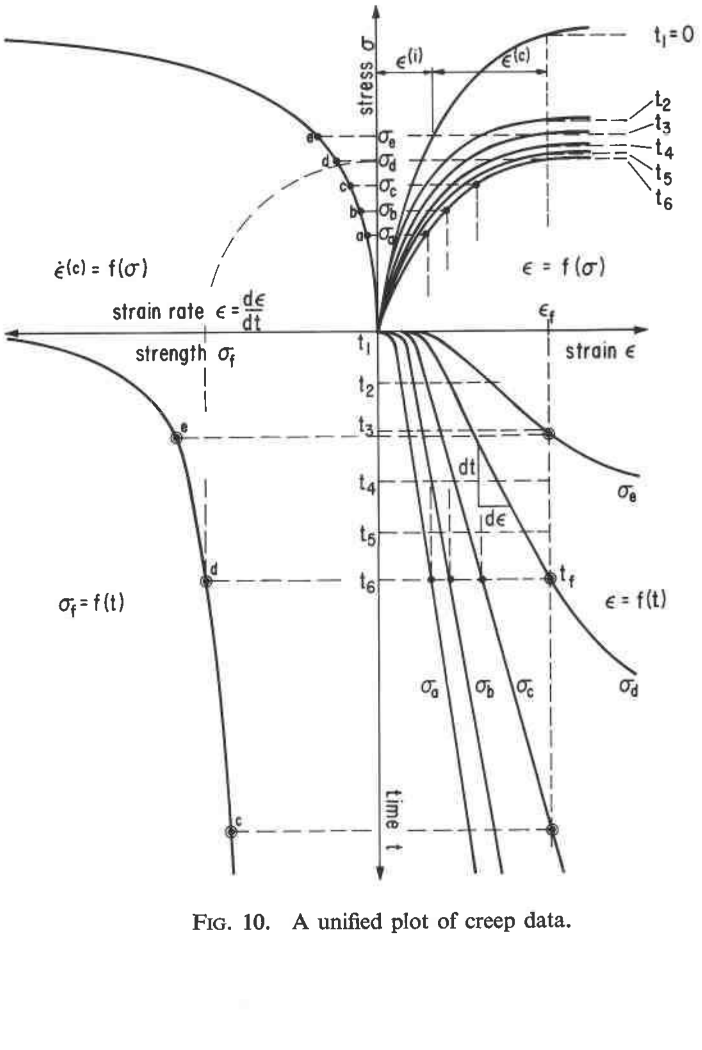

FIG. 10. A unified plot of creep data.

For varying temperature, the values of A and L in Eq. [34] can be found by plotting natural logarithms of the observed creep rates against the reciprocal of the absolute temperature.

If Eq. [34] is used together with the power creep law, such as Eq. [12], as has been done for ice (Glen 1955, Gold 1970) and for metals at high temperature and lower stress range (Laks *et al.* 1957), one can write

[36] $\dot{\varepsilon}^{(c)} = \dot{\varepsilon}_c (\sigma/\sigma_{cT})^n = A \exp(-L/T)$.

In order to eliminate A, let $\sigma_{cT} = \sigma_{co}$ when $T = 273$ °K. Then

[37] $\dot{\varepsilon}^{(c)} = \dot{\varepsilon}_c (\sigma/\sigma_{co})^n = A \exp(-L/273)$.

Dividing Eq. [36] by [37] one finds

[38] $$\left(\frac{\sigma_{cT}}{\sigma_{co}}\right)^{-n} = \frac{\exp(-L/T)}{\exp(-L/273)}$$

from which

[39] $$\sigma_{cT} = \sigma_{co} \exp\left[\frac{L(273-T)}{273\,nT}\right] \equiv \sigma_{co} f_1(T).$$

Eqs. [36] and [39] give

[40] $$\dot{\varepsilon}^{(c)} = \dot{\varepsilon}_c \left(\frac{\sigma}{\sigma_{co}}\right)^n \exp\left[-\frac{L(273-T)}{273\,T}\right].$$

The meaning of the term $\dot{\varepsilon}_c (\sigma/\sigma_{co})^n$ is seen to be the creep rate at a temperature close to the melting point. ($T \approx 273$ °K).

If, after Vialov, θ is defined as the absolute value of negative temperature in °C,

[41] $$\theta\,(°\text{C}) = 273 - T\,(°\text{K}),$$

then Eq. [39] can be written as

[42] $$\sigma_{cT} = \sigma_{co} \exp\left[\frac{L\,\theta}{273\,n(273-\theta)}\right] \equiv \sigma_{co} f_1(\theta).$$

Because, in practical frozen soil problems, θ is much smaller than 273 degrees, Eq. [42] can be written approximately as

[43] $$\sigma_{cT} \approx \sigma_{co} \exp\left[\frac{L\,\theta}{273^2\,n}\right] \equiv \sigma_{co} f_2(\theta)$$

where the term $273^2\,n/L$ is a constant temperature in °C. It is seen that both parameters, σ_{co} and L, can be determined by plotting logarithm of σ_{cT} against θ, as in Fig. 11 and 12.

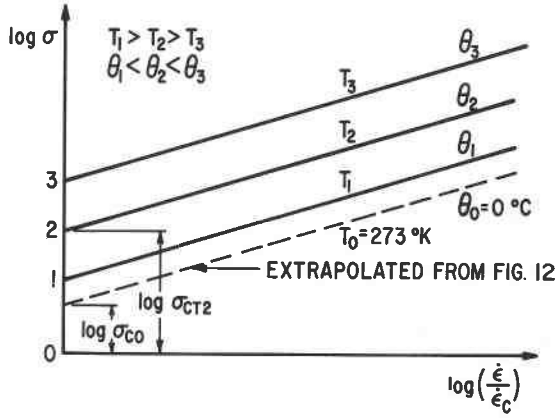

FIG. 11. Creep rate versus stress plot for different temperatures.

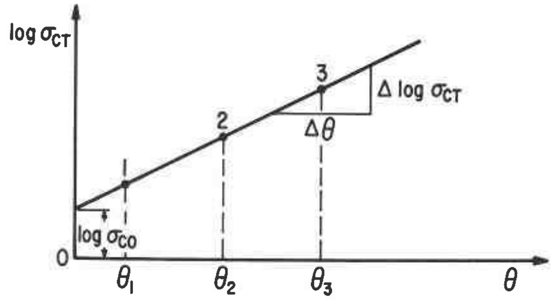

FIG. 12. Temperature dependence of σ_{cT} according to Eq. [43].

From Eq. [43],

[44] $$\frac{L}{273^2 n} = \frac{\Delta \ln \sigma_{cT}}{\Delta\,\theta}$$

and, using the slope of the line in Fig. 12,

[45] $$L \approx 2.303 \times 273^2\, n \frac{\Delta \log \sigma_{cT}}{\Delta\,\theta}$$

in °C. The apparent activation energy is then

[46] $$U = RL$$

where $R = 1.987$ cal/mol °C. Some reported values of $L = U/R$ are: $L = 4274$ °C for frozen saturated Ottawa sand (Andersland and AlNouri 1970); L = 56 000 °C for frozen Sault Ste. Marie clay (Akili 1970); L = 10 000 °C for polycrystalline ice (Gold 1970).

It should be noted, however, according to Hoekstra (1969), that the activation energy calculated by this method does not necessarily correspond to its physical definition, since in frozen soil not only the thermal energy of the moving molecules changes with temperature but in addition a gradual phase change also occurs. As a result, when the theory of rate processes is applied to creep of frozen soils, activation energies are obtained as high as 93 kcal/mol, which is much higher than 20 kcal/mol obtained for the creep of polycrystalline ice (Gold 1970).

Moreover, the same theory, Eq. [40], predicts an exponential increase of creep strength for a linear decrease in temperature, which is only partially supported by experimental results. For example, from the experimental evidence shown by Sayles (1966, 1968), (Fig. 13), it seems that this type of strength variation is limited mainly to clays. For coarser-grained frozen materials such as silts and sands, the observed strength increase with temperature decrease is found to be more nearly linear or even parabolic, levelling off after about −100 °C. The latter type of strength variation with temperature can be seen also in Vialov's work (1962). There may, therefore, be some justification in adopting alternatively for the effect of temperature, Vialov's power relationship which can be written in the following normalized form (Assur 1963)

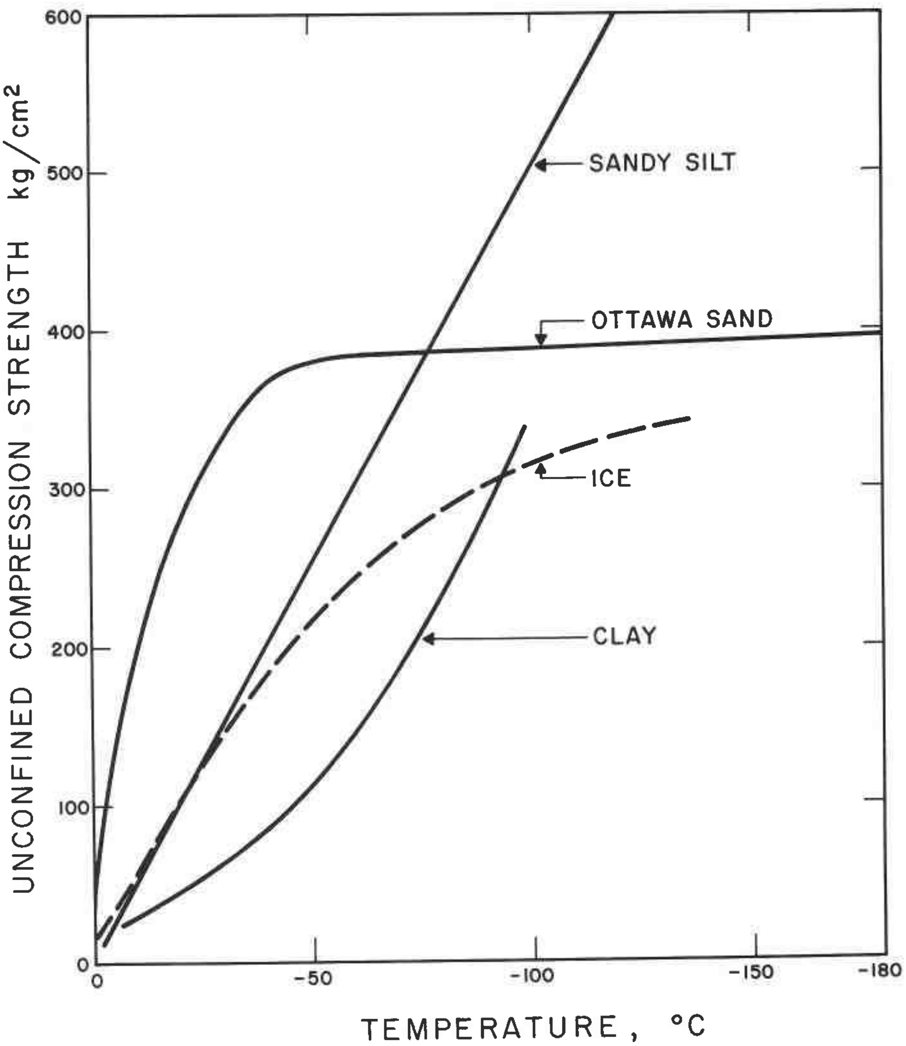

FIG. 13. Temperature dependence of unconfined (uniaxial) short term compression strength for various frozen materials (after Sayles 1966).

[47] $\sigma_{cT} = \sigma_{co}(1 + \theta/\theta_c)^{\omega} \equiv \sigma_{co} f_3(\theta)$

where θ_c is an arbitrary temperature, say 1 °C, and θ as defined by Eq. [41]. The value of the exponent ω in Eq. [47] can be obtained by plotting σ_{cT} vs $(1 + \theta/\theta_c)$ in a log-log plot as in Fig. 14.

[48] $$\omega = \frac{\Delta \log \sigma_{cT}}{\Delta \log(1 + \theta/\theta_c)} = \tan \alpha.$$

For limited temperature intervals, $\omega \approx 1$, and the power law Eq. [47] reduces to

[49] $\sigma_{cT} \approx \sigma_{co}(1 + \theta/\theta_o) = \sigma_{co} f_4(\theta)$.

The corresponding linear plot is shown in Fig. 15.

Finally, the temperature dependent uniaxial creep strength is obtained by substituting for $\sigma_c(T)$ in Eq. [26] σ_{co} multiplied by any of the temperature functions $f_1(\theta)$ to $f_4(\theta)$, giving

[50] $\sigma_f = \sigma_{co}(\dot{\varepsilon}_f/\dot{\varepsilon}_c)^{1/n} f(\theta)$.

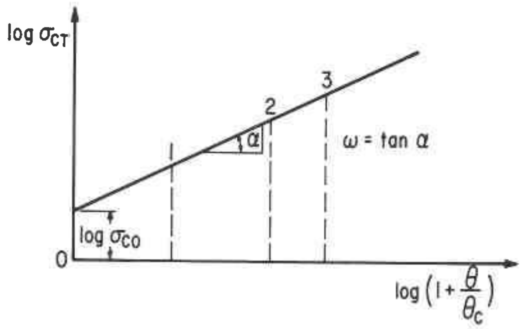

FIG. 14. Temperature dependence of σ_{cT} according to Eq. [47].

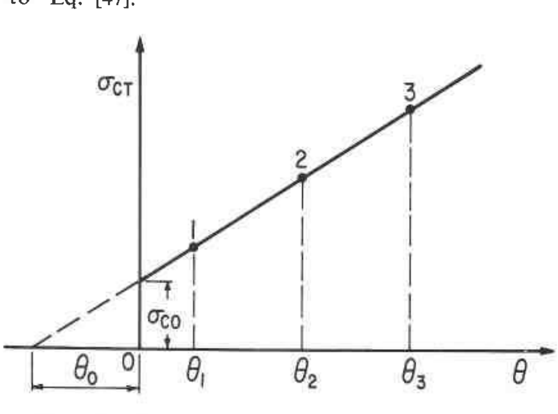

FIG. 15. Temperature dependence of σ_{cT} according to Eq. [49].

An experimental evidence of an approximately linear temperature dependence of frozen soil strength down to −20 °C can be found in Vialov (1962). From his plot (Vialov 1962, Fig. 51),

$\sigma_{co} = 4.56$ kg/cm² and $\theta_o = 3.5$ °C,

giving

[51] $\sigma_f = 4.56\,(10^8\,\dot{\varepsilon}_f)^{0.121}\,(1 + \theta/3.5)$.

For example, for $\theta = 5$ °C and $t_f = 1$ h, Eq. [51] gives $\sigma_f = 29.5$ kg/cm². The corresponding value of σ_f shown in Vialov (1962 Fig. 51) is 33.5 kg/cm². The small difference between the two values is due to a slightly different creep failure criterion adopted by Vialov.

Multiaxial States of Stress

No Effect of Hydrostatic Pressure

Assuming the validity of the von Mises plasticity rule and the volume constancy for all plastic deformations including the creep deformations, the power laws adopted in the uniaxial case (Eqs. [9] and [12]) can be generalized for the multiaxial state by expressing the power laws in terms of equivalent stresses, strains and strain rates, as shown by Odquist and Hult (1962),

[52] $\varepsilon_e^{(ip)} = \varepsilon_k(\sigma_e/\sigma_{ku})^k$

and

[53] $\dot{\varepsilon}_e^{(c)} = \dot{\varepsilon}_c(\sigma_e/\sigma_{cu})^n$

where σ_e, ε_e and $\dot{\varepsilon}_e$ are the equivalent stress, strain and strain rate respectively, defined in terms of stress and strain invariants or explicitly in terms of principal stresses and strains,

[54] $$\sigma_e^2 = \frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right],$$

[55] $$\varepsilon_e^2 = \frac{2}{9}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right],$$

[56] $$\dot{\varepsilon}_e^2 = \frac{2}{9}\left[(\dot{\varepsilon}_1 - \dot{\varepsilon}_2)^2 + (\dot{\varepsilon}_2 - \dot{\varepsilon}_3)^2 + (\dot{\varepsilon}_3 - \dot{\varepsilon}_1)^2\right].$$

In Eqs. [52] and [53], subscript u has been added to the stress parameters σ_k and σ_c to denote that they refer to the uniaxial state of stress.

The equivalent stress and strain parameters are defined so that, for the uniaxial state of stress, *i.e.*, for $\sigma_1 = \sigma$, $\sigma_2 = 0$, $\sigma_3 = 0$ $\varepsilon_1 = \varepsilon$, $\varepsilon_2 = -\frac{1}{2}\varepsilon$, $\varepsilon_3 = -\frac{1}{2}\varepsilon$, Eqs. [52] and [53] transform into previous uniaxial form of Eqs. [9] and [12] respectively.

For an axially symmetric state of stress, Eqs. [52] and [53] become

$$[57] \qquad \varepsilon_1^{(\mathrm{iD})} = \varepsilon_k \left(\frac{\sigma_1 - \sigma_3}{\sigma_{ku}} \right)^k \quad \text{and}$$

$$[58] \qquad \dot{\varepsilon}_1^{(\mathrm{c})} = \dot{\varepsilon}_c \left(\frac{\sigma_1 - \sigma_3}{\sigma_{cu}} \right)^n .$$

For a plane strain condition, *i.e.*, for σ_1 , $\sigma_2 = \frac{1}{2}(\sigma_1 + \sigma_3)$, σ_3 , and ε_1 , $\varepsilon_2 = 0$, $\varepsilon_3 = -\varepsilon_1$, one gets

$$[59] \quad \varepsilon_1^{(\mathrm{iD})} = \left(\frac{\sqrt{3}}{2} \right)^{(k+1)} \varepsilon_k \left(\frac{\sigma_1 - \sigma_3}{\sigma_{ku}} \right)^k \quad \text{and}$$

$$[60] \quad \dot{\varepsilon}_1^{(\mathrm{c})} = \left(\frac{\sqrt{3}}{2} \right)^{(n+1)} \dot{\varepsilon}_c \left(\frac{\sigma_1 - \sigma_3}{\sigma_{cu}} \right)^n .$$

Effect of Hydrostatic Pressure

Expected influence of hydrostatic pressure — Unconsolidated frictional earth materials yield usually at very low shear stresses, so that permanent strains occur practically throughout the whole straining up to failure. Consequently, the laws of plasticity rather than elasticity should be valid for describing their stress-strain behavior even in prefailure state. In frictional earth materials, it is expected therefore, that hydrostatic pressure would affect not only the peak strength but also the whole stress-strain-strain rate behavior in the pre-failure state. The effect can be expressed by using either a two or a three parameter failure theory, as shown in the following.

Two-principal-stress Failure Theory

Failure state (*peak strength*) — According to Vialov (1962), if the same testing procedure as for uniaxial compression is repeated for uniaxial tension and triaxial compression, one can derive a set of failure envelopes of Mohr circles at failure (or at the onset of the third stage of creep), where each envelope corresponds to a given time to failure (Fig. 16). Since the envelopes usually seem to have a parabolic shape, they can be expressed conveniently by an equation of the form proposed by Fairhurst (1964) for rocks

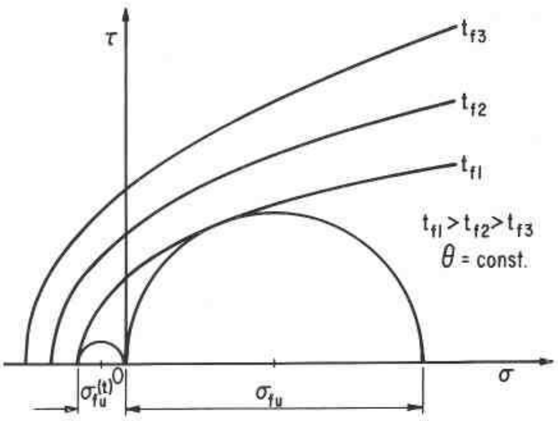

FIG. 16. Time dependence of failure envelopes.

$$[61] \qquad \tau = [(s - 1)/r]\, \sigma_{fu}\, [1 + r\, (\sigma/\sigma_{fu})]^{\frac{1}{2}}$$

where

$$[62] \qquad r = \sigma_{fu}/\sigma_{fu}^{(\mathrm{t})}$$

is the ratio between the uniaxial compressive and tensile creep strengths, both of which are time and temperature dependent, and

$$[63] \qquad s = (r + 1)^{\frac{1}{2}}.$$

According to the quoted test results (Vialov 1962, Figs. 63 and 66), while the position of the envelopes changes with time and temperature, the envelopes remain practically homothetic. This means that in Eq. [61] σ_{ru} varies with time and temperature while r and s remain practically constant. For example in Vialov (1962, Fig. 63 and 66) for the Callovian sandy loam one finds that, at a constant temperature of -10 °C, $3.5 < r < 3.8$ when $24^h > t > 1^h$, and r = 3.5 at $t = 24^h$ when the temperature varies from -5 °C to -20 °C.

This experimental fact enables Eq. [61] to be written as

$$[64] \quad \tau = \frac{s - 1}{r}\, \sigma_{fu}\,(t, \theta) \left[1 + r\, \frac{\sigma}{\sigma_{fu}\,(t, \theta)} \right]^{\frac{1}{2}}$$

in which according to Eq. [50], for a long time interval

$$[65] \qquad \sigma_{fu}\,(t, \theta) = \sigma_{cu0}\, (\dot{\varepsilon}_f/\dot{\varepsilon}_c)^{1/n}\, f\,(\theta).$$

The stress σ_{cu0} denotes as before the proof stress in uniaxial compression extrapolated to 0 °C. Some values of the ratio r reported in literature for various types of frozen soils are: Ottawa sand (Sayles 1969), $5 < r < 8$, Callovian sandy loam (Vialov 1962), $3 < r < 4$, Bat Baioss clay (Vialov 1962), $1.5 < r < 2$.

Sometimes, for practical purposes, it is convenient to approximate the parabolic failure envelopes by a set of straight lines, at least within a limited interval of normal pressure. The Coulomb-Mohr envelopes obtained in that way are defined by

[66] $$\tau = c(t, \theta) + \sigma \tan \Phi$$

or

[67] $$\tau = [H(t, \theta) + \sigma] \tan \Phi$$

where

[68] $$H(t, \theta) = c(t, \theta) \cot \Phi$$

and

[69] $$c(t, \theta) = \sigma_{fu}(t, \theta)/2\sqrt{f}$$

f being the flow value, defined by

[70] $$f = \frac{1 + \sin \Phi}{1 - \sin \Phi}.$$

Alternatively, in terms of principal stresses, the stress difference at failure

[71] $$(\sigma_1 - \sigma_3)_f = \sigma_{fu}(t, \theta) + \sigma_3(f - 1).$$

Again, similarly as the ratio r, the angle Φ is found to depend only little on time and temperature within their practically interesting interval of variation, and the effect remains mostly concentrated in the value of σ_{fu}. For example, for a long time interval, according to Sanger (1968), one may expect to find the following values of Φ: Sands: $\Phi = 30°$; Silts: $\Phi = 20°$; and Clays: $\Phi = 0$ to $10°$. A set of Mohr-Coulomb envelopes corresponding to Eq. [66], assuming a constant temperature and different times to failure, are shown in Fig. 17. Alternatively, a set of creep strength curves according to Eq. [71] for constant temperature and different confining pressures are shown schematically in Fig. 18. It is interesting to note that this analytical form gives, for an infinite time to failure or an infinitely slow creep rate, a finite value of a purely frictional threshold strength which agrees well with experimental findings in many earth materials.

Pre-failure state — If the Coulomb-Mohr theory of failure is assumed to be valid also

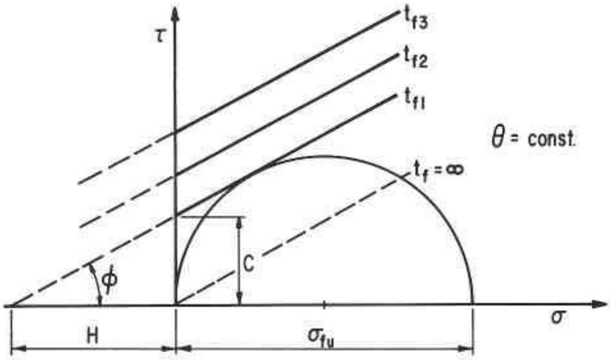

FIG. 17. Straight-line approximation of time dependent failure envelopes.

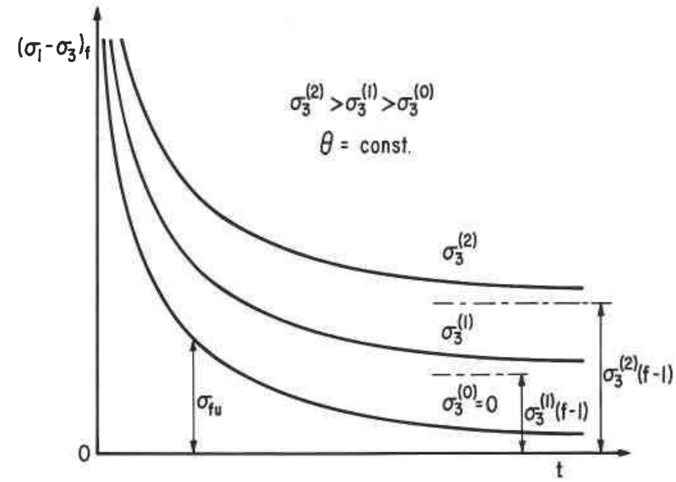

FIG. 18. Dependence of creep strength on confining pressure, Eq. [71].

in the pre-failure state, the dependence of strain and strain rate on normal, or confining, pressure can be deduced from Eqs. [65] and [71] as follows:

[72] $$\sigma_1 - \sigma_3 = \sigma_{cu0}(\dot{\varepsilon}^{(c)}/\dot{\varepsilon}_c)^{1/n} f(\theta) + \sigma_3(f - 1)$$

from which the steady state creep rate

[73] $$\dot{\varepsilon}^{(c)} = \dot{\varepsilon}_c \left[\frac{\sigma_1 - f\sigma_3}{\sigma_{cu0}\ \mathrm{f}\ (\theta)}\right]^n$$

and the creep strain after the time t

[74] $$\varepsilon^{(c)} = \dot{\varepsilon}_c\, t \left[\frac{\sigma_1 - f\sigma_3}{\sigma_{cu0}\, f\ (\theta)}\right]^n.$$

For $f = 1$, Eq. [73] reduces to the originally adopted form of Eq. [12] and coincides with Eq. [58]. Figure 19 shows schematically a set of isochronous stress-strain curves implied by Eq. [74] for the pre-failure and Eq. [71] for the failure state, respectively.

It is noted that Eq. [73] can also be written in terms of the mean normal pressure $\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$. For example, if $\sigma_2 = \sigma_3$, one gets

[75] $$\dot{\varepsilon}^{(c)} = \dot{\varepsilon}_c \left[\frac{(f+2)(\sigma_1 - \sigma_3) - 3(f-1)\sigma_m}{3\ \sigma_{cu0}\ f\ (\theta)}\right]^n.$$

Another equation, based on exponential forms but serving the same general purpose as Eq. [75], has been proposed by Andersland and AlNouri (1970). In our notation, the latter can be written as,

[76] $$\dot{\varepsilon}^{(c)} = \frac{A\ \exp N\ (\sigma_1 - \sigma_3)}{F(T)\ \exp\ (m\ \sigma_m)}$$

where A, N and m are experimental parameters, and

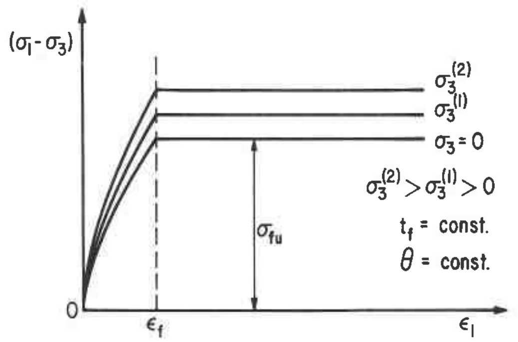

FIG. 19. Stress-strain curves according to Eqs. [71] and [74].

[77] $$F(T) = \exp(L/T).$$

It can be shown, for example, that the experimental data on strain-rate-dependent strength behavior of frozen saturated Ottawa sand at −12 °C (Andersland and AlNouri 1970, Fig. (20)), presented in the form of a $q = (\sigma_1 - \sigma_3)/2$ versus $p = (\sigma_1 + \sigma_3)/2$ plot, with $\dot{\varepsilon}$ as parameter, can be approximated fairly well not only by the exponential law proposed by the authors but also by the power law as shown in Eqs. [65] to [75].

Using the authors' exponential form, Eq. [76], and the values of creep parameters given in their paper, it is found that the mentioned experimental data can be approximated by

[78] $$q = 61.7\ \ln(1.1\times10^5\ \dot{\varepsilon}) + 0.595\ p$$

in which q and p are in p.s.i., and $\dot{\varepsilon}$ is in $\min^{-1}$. On the other hand, the same data can be approximated very closely by Eq. [75] by taking: $n = 2.485$, $\sigma_{cu0}\ f(\theta) = 228$ p.s.i., $\dot{\varepsilon}_c = 10^{-5}\ \min^{-1}$ and $f = 4$ ($\Phi = 36.6°$), which gives the expression

[79] $$q = 54.75\ (10^5\ \dot{\varepsilon})^{0.402} + 0.595\, p.$$

Unfortunately, neither Eq. [75] nor Eq. [76], as formulated above, are able to cover the whole region of pre-failure strain rates, as they both give non-zero strain rates at zero stress difference, if f and m are constants, as assumed. Their application should, therefore, be limited either to strains close to failure or to those contained within a narrow range of mobilization of internal friction, the latter with the corresponding reduced values of f and m.

The limitation may be overcome by writing, instead of Eq. [72,]

[80] $$\sigma_1 - \sigma_3 = (\varepsilon_{(c)}/\dot{\varepsilon}_c)^{1/n}\,[\sigma_{cu0} f(\theta) + \sigma_3(f - 1)]$$

from which

[81] $$\dot{\varepsilon}^{(c)} = \dot{\varepsilon}_c \left[\frac{\sigma_1 - \sigma_3}{\sigma_{cu0} f(\theta) + \sigma_3\ (f - 1)}\right]^n.$$

Equation [81] yields zero strain rate at $\sigma_1 = \sigma_3$ as required. However, at failure, instead of Eq. [71] one gets from Eq. [81]:

[82] $$(\sigma_1 - \sigma_3)_f = \sigma_{fu}\ (t,\ \theta) + \sigma_3\ (f - 1)\ (\dot{\varepsilon}_f/\dot{\varepsilon}_c)^{1/n}$$

which implies a time-dependent angle of friction if ε_f is kept constant, or an ε_f increasing linearly with time, if the angle of friction is made independent of time. For the former assumption, f in Eq. [81] denotes the flow value at the strain rate $\dot{\varepsilon}_f = \dot{\varepsilon}_c$.

A still simpler form may be obtained if only the strength and not the whole stress-strain behavior is made dependent on normal pressure. This is in fact a usual assumption made for dense solid materials, such as rocks and concrete and may be found acceptable also for many practical problems in frozen soil mechanics. One evidence supporting the latter assumption can be seen in Fig. 9 in the paper by Andersland and AlNouri (1970), showing a relatively small effect of confining pressure on pre-failure behavior of a sand-ice material.

If the latter assumption is retained, the isochronous curves will have the shape similar to those in Fig. 20, and will be defined by Eq. [58] in the pre-failure state and by Eq. [71] in the failure state.

It is obvious that the adoption of the Mohr's two-parameter failure criterion implies that the projection on the π-plane in the principal stress space of any failure surface is an irregular hexagon. Alternatively, as shown in the following, one may adopt the extended von Mises criterion of failure, which is represented by a circular cone in the principal stress space. There is, however, a large amount of experimental evidence in unfrozen soil mechanics literature showing that, in frictional earth materials, the former criterion may be closer to reality. It is clear that, even in the opposite case, the former may still be of interest because it is simpler and contains an additional factor of safety against failure.

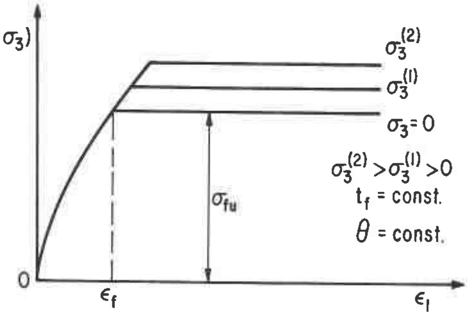

FIG. 20. Stress-strain curves according to Eqs. [58] and [71].

Three-principal-stress Failure Theory

Failure state — For materials with low friction such as frozen clays or sands with high ice content, the use of a three-principal-stress failure theory such as the von Mises failure criterion may be more appropriate than the Mohr criterion. The extended von Mises criterion is represented in the principal stress space, by a circular cone which can be defined by

$$[83] \quad \sigma_{fe} = \frac{2}{r+1}\sigma_{fu}(t, \theta) + \frac{3(r-1)}{r+1}\sigma_m$$

in which σ_{fe} is the failure value of equivalent stress σ_e given by Eq. [54], σ_{fu} is given by Eq. [65], r is the strength ratio given by Eq. [62], and σ_m is the mean (or octahedral) normal stress.

For a constant value of r, and a constant temperature, Eq. [83] represents a set of concentric, homothetic circular conical surfaces in the principal stress space, each valid for a given time to failure.

Pre-failure state — For the pre-failure state as long as r remains constant, Eq. [83] can be written as

$$[84] \quad \sigma_e = \frac{2}{r+1}\sigma_{cu0}\, \mathrm{f}(\theta) (\dot{\varepsilon}_e^{(c)}/\dot{\varepsilon}_c)^{1/n} + \frac{3(r-1)}{r+1}\sigma_m$$

from which the creep rate equation is

$$[85] \quad \dot{\varepsilon}_e^{(c)} = \dot{\varepsilon}_c\left[\frac{(r+1)\sigma_e - 3(r-1)\sigma_m}{2\,\sigma_{cu0} f(\theta)}\right]^n$$

Equation [85] is a proper extension of Eq. [53] to hydrostatic pressure dependent materials, and it reduces to the latter for $r = 1$.

For the special case of axial symmetry, $(\sigma_2 = \sigma_3)$, Eq. [85] becomes

$$[86] \quad \dot{\varepsilon}_1^{(c)} = \dot{\varepsilon}_c\left[\frac{(r+1)(\sigma_1 - \sigma_3) - 3(r-1)\sigma_m}{2\,\sigma_{cu0}\,\mathrm{f}(\theta)}\right]^n$$

or in terms of confining pressure σ_3

$$[87] \quad \dot{\varepsilon}_1^{(c)} = \dot{\varepsilon}_c\left[\frac{\sigma_1 - \sigma_3(3r-1)/2}{\sigma_{cu0} f(\theta)}\right]^n$$

which is similar in form to Eq. [73].

Again, similarly as before, if in Eqs. [85] to [87] the ratio r is assumed to be constant, the validity of these equations is limited to a narrow range of strain close to failure.

Another form of creep equation avoiding this difficulty is

$$[88] \quad \dot{\varepsilon}_e^{(c)} = \dot{\varepsilon}_c \left[\frac{(r+1)\sigma_e}{2\sigma_{cu0} f(\theta) + 3(r-1)\sigma_m} \right]^n$$

This would, however, lead to a creep strength equation

$$[89] \quad \sigma_{fe} = \frac{2}{r+1}\sigma_{fu}(t, \theta) + 3\sigma_m \frac{r-1}{r+1}(\dot{\varepsilon}_f/\dot{\varepsilon}_c)^{1/n}$$

with the same implications as in case of Eq. [82]. If ε_f is kept constant, the parameter r in Eq. [89] denotes the ratio of uniaxial compressive to uniaxial tensile strength at the strain rate $\dot{\varepsilon}_f = \dot{\varepsilon}_c$.

Conclusions

A unified engineering theory of time, temperature and normal pressure dependent deformation and strength of frozen soils has been developed and compared with the existing theories and the available experimental information.

In developing the theory, basic concepts and methods used in creep theories for metals have been followed. This enabled the creep and creep failure information to be expressed in a relatively simple analytical form using a minimum number of experimental parameters. Due to its simple and systematically normalized form, it is hoped that the theory will find useful application as a framework for generalizing experimental information as well as a basis for solving various frozen soil mechanics problems.

Finally, it is found that, in spite of the apparent abundance of published experimental data on frozen soil behavior, a clear answer to a number of questions is still lacking. In particular, it is felt that the effect of mean normal pressure on creep and creep failure of frozen soils requires a systematic and thorough investigation.

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Appendix: Symbols

c = cohesion intercept
E = Young's modulus
f = flow value, Eq. [70]
$F(\ )$ = function
$G(\ )$ = function
H = $c \cot \Phi$
k = exponent in stress-strain equation
L = U/R
m = exponent
n = exponent in creep equation
r = ratio of uniaxial compressive to uniaxial tensile strength
R = universal gas constant
s = $(r + 1)^{\frac{1}{2}}$
t = time
t_f = time to failure
T = absolute temperature
U = activation energy
$\varepsilon^{(c)}$ = creep strain
$\varepsilon^{(i)}$ = pseudo-instantaneous (intercept) strain
$\varepsilon^{(ie)}$ = elastic portion of $\varepsilon^{(i)}$
$\varepsilon^{(ip)}$ = plastic portion of $\varepsilon^{(i)}$
ε_k = arbitrary strain in stress-strain equation
$\dot{\varepsilon}_c$ = arbitrary strain rate in creep equation
ε_e = equivalent strain, Eq. [55]
ε_f = creep failure strain (at the onset of tertiary creep)
$\varepsilon_1\varepsilon_2\varepsilon_3$ = principal normal strains
θ = absolute value of negative temperature, Eq. [41]
θ_c = arbitrary temperature (positive)
θ_o = temperature intercept in Fig. [14] (positive)
σ = uniaxial normal stress
σ_c = proof stress in creep equation
σ_{co} = σ_c for freezing temperature close to 0 °C
σ_{cT} = σ_c for temperature T
σ_{cu} = σ_c for creep in uniaxial compression
σ_{cuO} = σ_{cu} for freezing temperature close to 0 °C
σ_{cuT} = σ_{cu} for temperature T
σ_e = equivalent stress, Eq. [54]
σ_k = proof stress in pseudo-instantaneous stress strain equation
σ_{lt} = long term strength
σ_f = creep failure stress
σ_{fu} = σ_f in uniaxial compression
$\sigma_{fu}^{(t)}$ = σ_f in uniaxial tension
$\sigma_1\sigma_2\sigma_3$ = principal normal stresses
τ = shear stress
Φ = angle of internal friction
ω = exponent in temperature equation

Note: Dot over strain symbols denotes time rate.