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An EOQ model with time dependent deterioration under discounted cash flow approach when supplier credits are linked to order quantity

by

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Abstract: This article deals with an inventory model under a situation in which the supplier offers the purchaser some credit period if the purchaser orders a large quantity. Shortages are not allowed. The effects of the inflation rate on purchase price, ordering price and inventory holding price, time dependent deterioration of units and permissible delay in payment are discussed. A mathematical model is developed when units in inventory are subject to time dependent deterioration under inflation when the supplier offers a permissible delay to the purchaser if the order quantity is greater than or equal to a pre-specified quantity. Optimal solution is obtained and algorithm is given to find the optimal order quantity and replenishment time, which minimizes the total cost of an inventory system in different scenarios. The paper concludes with a numerical example to illustrate the theoretical results and interdependence of parameters is studied for the optimal solutions.

Keywords: time dependent deterioration, discounted cash-flows (DCF) approach, supplier credit linked to order quantity.

1. Introduction

The classical inventory model deals with a constant demand rate. However, in real-life situations, there is inventory loss due to deterioration of units. Ghare and Schrader (1963) were the first to develop a model for an exponentially decaying inventory. Covert and Philip (1973) extended the above model to a two-parameter Weibull distribution. Shah and Jaiswal (1977) and Aggarwal (1978) developed an order level inventory model with a constant rate of deterioration. Dave and Patel (1981) considered an inventory model for deteriorating items with time proportional demand. Sachan (1984) extended the model of Dave and Patel (1981) by allowing shortages. Later, Hariga (1996) generalized the demand pattern to any log–concave function. Teng et al. (1999) and Yang et al. (2001) considered the demand function to include any non-negative continuous function that fluctuates with time. Raafat (1991), Shah and Shah (2000) and Goyal and Giri (2001) gave comprehensive surveys on the recent trends in modeling of deteriorating inventory.

The second stringent assumption of the classical EOQ model was that the purchaser must pay for items as soon as the items are received. However, in practice, the supplier may provide a credit period to their customers if the outstanding amount is paid within the allowable fixed credit period and the order quantity is large. Thus, indirectly, the delay in payment to the supplier is one kind of price discount to the buyer. Because paying later reduces the purchase cost, it can motivate customers to increase their order quantity. Goyal (1985) derived an EOQ model under the conditions of permissible delay in payments. Shah (1993), and Aggarwal and Jaggi (1995) generalized Goyal's model for constant rate of deterioration of units. Jamal et al. (1997) further generalized the model to allow for shortages. Liao et al. (2000) derived an inventory model for stock dependent consumption rate when delay in payments is permissible. Arcelus et al. (2001) compared price discount versus trade credit. Other related articles are those of Davis and Gaither (1985), Arcelus and Srinivasan (1993, 1995, 2001), Shah (1997), Khouja and Mehrez (1996), Hwang and Shinn (1997), Chu et al. (1998), Chung (1998), Teng (2002), and Gor and Shah (2003).

From a financial point of view, an inventory symbolizes a capital investment and must compete with other assets for an organization's limited capital funds. Thus, the effect of inflation on the inventory system plays an important role. Buzacott (1975), Bierman and Thomas (1977), Misra (1979a) investigated the inventory decisions under inflationary conditions for the EOQ model. Misra (1979b) derived an inflation model for the EOQ, in which the time value of money and different inflation rates were considered. Gor et al. (2002) extended the above model for deteriorating items when demand is decreasing with time by allowing shortages. Bhrambhatt (1982) derived an EOQ model under a variable inflation rate and marked-up prices. Chandra and Bahner (1985) studied the effects of inflation and time value of money on optimal order policies. Datta and Pal (1991) gave a model with linear time dependent rates and shortages to study the effects of inflation and time-value of money on a finite horizon policy. Shah and Shah (2003) gave pros and cons of classical EOQ model versus EOQ model under discounted cash flow approach for time dependent deterioration of units in an inventory system. Gor and Shah (2003) formulated a model with Weibull distribution deterioration when a delay in payments is permissible. Liao et al. (2000) proposed a model with deteriorating items under inflation when a delay in payment is permissible. Other relevant articles in this context are those by Chang and Tang (2004), Teng et al. (2005), Ouyang et al. (2006) and Teng (2006).

In practice, a supplier offers the purchaser either a quantity discount or a credit period if the purchaser orders a large quantity, which is greater than or equal to a pre-determined quantity (say Q_d). The articles on quantity discounts in the literature are reviewed by Dixit and Shah (2003). In this article, the focus is on how a purchaser obtains an optimal solution when a supplier offers a credit period for a large order. An EOQ model with time dependent deterioration of units under inflation, when a supplier gives a permissible delay of payments for a large order that is greater than or equal to the predetermined quantity Q_d is formulated. It is assumed that the purchaser will have to pay immediately on the receipt of the items in the inventory if the procurement order quantity is less than Q_d.

The paper is organized as follows: In Section 2, notations and assumptions used throughout this study are given. In Section 3, the mathematical models are derived under four different scenarios in order to minimize the total cost on the finite planning horizon. In Section 4, an algorithm is given to search for an optimal solution. Section 5 deals with a numerical example to demonstrate the applicability of the proposed model and study the interdependence of parameters. The effect of inflation rate, deterioration rate, credit period on the optimal replenishment cycle, order quantity and total cost are studied. Paper ends with conclusions and possible future extensions.

2. Notations and assumptions

The following notations and assumptions are used throughout this paper:

Notations:

H = the length of finite planning horizon.

R = the demand per unit time.

i = the inventory carrying charge fraction per unit per annum excluding interest charges.

r = constant rate of inflation per unit time, where $0 \leqslant r < 1$.

$P(t) = Pe^{rt}$ = the selling price per unit at time t, where P is the unit selling price at time zero.

$C(t) = Ce^{rt}$ = the unit purchase cost at time t, where C is the unit purchase price at time zero and $C < P$.

$A(t) = Ae^{rt}$ = the ordering cost per order at time t, where A is the ordering cost at time zero.

I_C = the interest charged per \$ in stock per year by the supplier.

I_e = the interest earned per unit per \$ ($I_C > I_e$).

M = the permissible trade credit period in settling account in a year.

Q = the order quantity (a decision variable).

Q_d = the prespecified minimum order quantity at which the delay in payments is permitted.

T_d = the time interval in which Q_d units are depleted to zero due to both demand and deterioration.

$I(t)$ = the level of inventory at time t, $0 \leqslant t \leqslant T$.

T = the cycle time (a decision variable).

$PV(T)$ = the present value of all cash out flows that occur during the time interval $[0, H]$. It consists of (a) cost of placing orders, OC; (b) cost of purchasing, PC; (c) cost of inventory holding excluding interest charges, IHC; (d) cost of interest charges for unsold items at the initial time or after the credit period M, IC; and minus (e) interest earned from sales revenue during the permissible delay period, IE.

Assumptions:

1. The system deals with single item only.
2. The demand for the item is known and constant during the period under consideration.
3. The inflation rate is constant.
4. Shortages are not allowed. Lead time is zero.
5. Replenishment is instantaneous.
6. If the order quantity is greater than or equal to pre-specified minimum quantity Q_d, then the delay period of M time units is allowed. During the trade credit period if the account is not settled, the generated sales revenue is deposited in an interest bearing account. At the end of the permissible delay, the purchaser pays off for all units ordered and thereafter pays interest charges on the items in stock.
   If the order quantity is less than Q_d, then the payment for the items received in system must be made immediately.
7. The deterioration rate is given by the Weibull distribution
   $$\theta(t) = \alpha\beta t^{\beta-1} \quad 0 \leqslant t \leqslant T \tag{1}$$
   where α denotes scale parameter, $0 \leqslant \alpha < 1$; β denotes shape parameter, $\beta \geqslant 1$; t denotes time to deterioration, $t > 0$.
8. There is no repair or replacement of deteriorated units during a given cycle.

3. Mathematical model

We assume that the length of planning horizon $H = nT$, where n is (an integer) number of replenishments to be made during H and T is an interval of time between two consecutive replenishments. Let $I(t)$ be the on-hand inventory at any instant of time t $(0 \leqslant t \leqslant T)$. The depletion of inventory occurs due to deterioration and due to demand simultaneously. The differential equation governing the instantaneous state of $I(t)$ at time t, $0 \leqslant t \leqslant T$ is given by

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -R, \quad 0 \leqslant t \leqslant T \tag{2}$$

with the boundary conditions $I(0) = Q$, $I(T) = 0$.

Consequently, the solution of (2) is

$$I(t) = R\left[T - t + \frac{\alpha T}{\beta+1}\left(T^{\beta} - (1+\beta)t^{\beta}\right) + \frac{\alpha\beta t^{\beta+1}}{\beta+1}\right] \tag{3}$$

and the order quantity is

$$Q = R\left[T + \frac{\alpha T^{\beta+1}}{\beta+1}\right]. \tag{4}$$

Using (3), we can obtain the time interval T_d during which Q_d units are depleted to zero due to both demand and deterioration. Trade credit is only permitted if $Q > Q_d$, equivalently $T > T_d$.

Since the lengths of time intervals are all the same, we have

$$I(kT+t) = R\left[T - t + \frac{\alpha T}{\beta+1}\left(T^{\beta} - (1+\beta)t^{\beta}\right) + \frac{\alpha\beta t^{\beta+1}}{\beta+1}\right]$$
$$0 \leqslant k \leqslant n-1, \quad 0 \leqslant t \leqslant T. \tag{5}$$

The different costs associated with the total cost in $[0, H]$ are as specified below:

- Cost of placing orders

$$OC = A(0) + A(T) + \ldots + A(n-1)T = A\left(\frac{e^{rH}-1}{e^{rT}-1}\right). \tag{6}$$

- Cost of purchasing

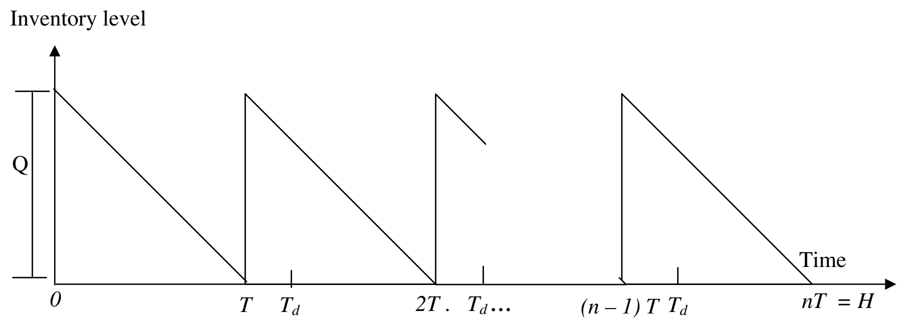
$$PC = Q[C(0)+C(T)+\ldots+C(n-1)T] = CR\left[T+\frac{\alpha T^{\beta+1}}{\beta+1}\right]\left(\frac{e^{rH}-1}{e^{rT}-1}\right). \tag{7}$$

- Cost of inventory holding

$$IHC = i\sum_{k=0}^{n-1} C(kT)\int_0^T I(kT+t)\,dt = CiR\left[\frac{T^2}{2} + \frac{\alpha\beta T^{\beta+2}}{\beta+1}\right]\left(\frac{e^{rH}-1}{e^{rT}-1}\right). \tag{8}$$

Regarding interests charged and earned, we have the following four scenarios based on the values of T, M and T_d:

Scenario 1: $0 < T < T_d$)



Here, the replenishment time interval T is less than T_d (i.e. the order quantity Q is less than Q_d), the delay in payments is not permitted. Hence, the purchaser will have to pay for items as soon as items are received. This is one of the assumptions of the classical EOQ model.

• Interest charges for all unsold items

$$IC_1 = I_C \sum_{k=0}^{n-1} C(kT) \int_0^T I(kT+t)\, dt = CI_C R\left[\frac{T^2}{2} + \frac{\alpha\beta T^{\beta+2}}{\beta+1}\right]\left(\frac{e^{rH}-1}{e^{rT}-1}\right). \quad (9)$$

• Interest earned

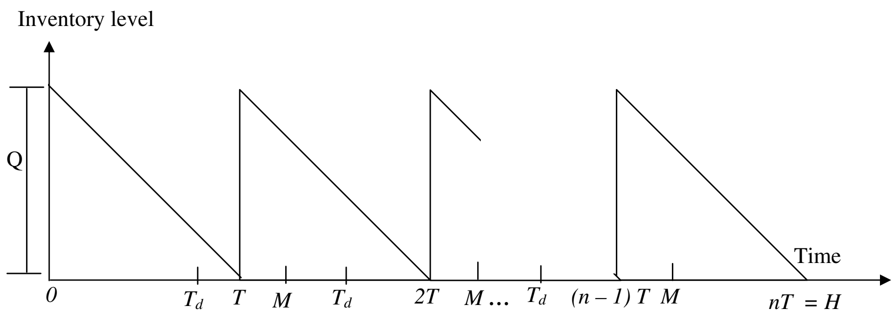
$$IE_1 = 0.$$

Using equations (6) to (9), the present value of all cash-out flows over $[0, H]$ is given by

$$PV_1(T) = OC + PC + IHC + IC_1$$

$$PV_1(T) = \left\{A + CR\left[T + \frac{\alpha T^{\beta+1}}{\beta+1}\right] + C(i+I_C)R\left[\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+1}\right]\right\} \times \times \left(\frac{e^{rH}-1}{e^{rT}-1}\right). \quad (10)$$

Scenario 2: $T_d \leqslant T < M$



Here, permissible delay period M is longer than the replenishment interval T. Hence,

• Interest charges paid during $[0, H]$ are

$$IC_2 = 0.$$

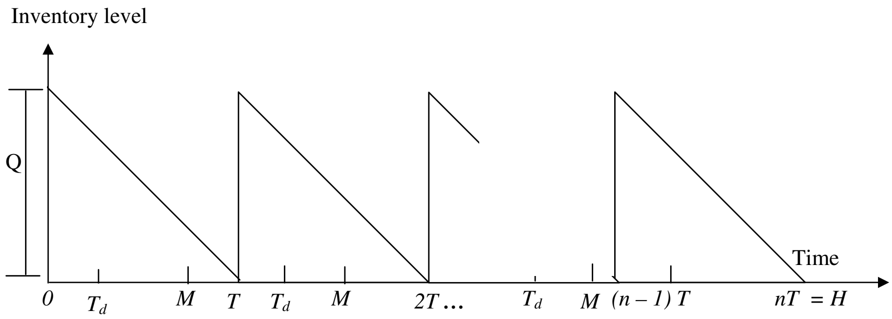
• Interest earned during $[0, H]$ is

$$\begin{aligned} IE_2 &= I_e \sum_{k=0}^{n-1} P\left(kT\right) \left[\int_0^T Rtdt + RT\left(M - T\right) \right] \\ &= PI_e RT \left(M - \frac{T}{2}\right) \left(\frac{e^{rH} - 1}{e^{rT} - 1}\right). \end{aligned} \tag{11}$$

Therefore, the present value of all cash-out flows over $[0, H]$ is

$$\begin{aligned} PV_2(T) &= OC + PC + IHC - IE_2 \\ PV_2(T) &= \left\{ A + CR\left[T + \frac{\alpha T^{\beta+1}}{\beta+1}\right] + CiR\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+1}\right) \right. \\ &\quad \left. - PI_e RT\left(M - \frac{T}{2}\right) \right\} \left(\frac{e^{rH} - 1}{e^{rT} - 1}\right). \end{aligned} \tag{12}$$

Scenario 3: $T_d \leqslant M \leqslant T$



Here, the replenishment cycle time T is greater than or equal to both T_d and M. Hence,

- Interest charges payable in $[0, H]$ are

$$IC_3 = I_C \sum_{k=0}^{n-1} C(kT) \int_M^T I(kT+t)\,dt = \frac{CI_C R}{2}\left[T^2 + M^2 - 2\alpha M T^{\beta+1} + \frac{2\alpha\beta T^{\beta+2}}{\beta+1} + \frac{2\left(\alpha M^{\beta+1} - \beta M - \beta\right)T}{\beta+1}\right]\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{13}$$

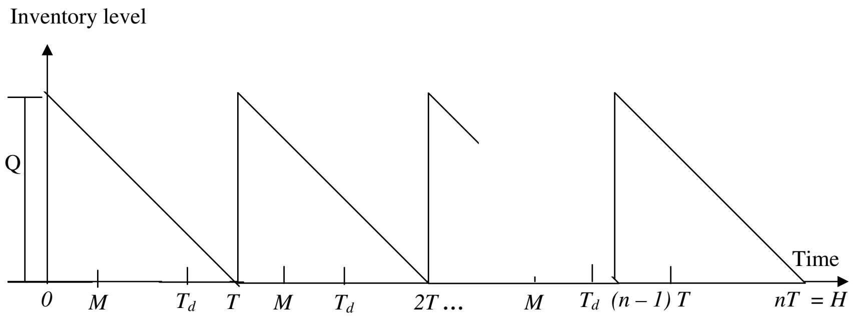
- The interest earned in $[0, H]$ is

$$IE_3 = I_e \sum_{k=0}^{n-1} P(kT) \int_0^M Rt\,dt = \frac{PI_e RM^2}{2}\left(\frac{e^{rH}-1}{e^{rT}-1}\right). \tag{14}$$

Hence, the present value of all cash-out flows over $[0, H]$ is

$$\begin{aligned} PV_3(T) &= OC + PC + IHC + IC_3 - IE_3 \\ &= \left\{A + CR\left[T + \frac{\alpha T^{\beta+1}}{\beta+1}\right] + CiR\left(\frac{T^2}{2} + \frac{\alpha\beta T^{\beta+2}}{\beta+1}\right)\right. \\ &+ \frac{CI_c R}{2}\left[T^2 + M^2 - 2\alpha M T^{\beta+1} + \frac{2\alpha\beta T^{\beta+2}}{\beta+1} + \frac{2\left(\alpha M^{\beta+1} - \beta M - \beta\right)T}{\beta+1}\right] \\ &\left. - \frac{PI_e RM^2}{2}\right\}\left(\frac{e^{rH}-1}{e^{rT}-1}\right). \end{aligned} \tag{15}$$

Scenario 4: $T_d \leqslant M \leqslant T$



The replenishment time interval T is greater than or equal to both T_d and M. Thus, Scenario 4 is similar to Scenario 3.

Therefore, the present value of all cash out-flows over $[0, H]$ is

$$\begin{aligned}
&PV_4(T) = OC + PC + IHC + IC_3 - IE_3 \\
&= \left\{A + CR\left[T + \frac{\alpha T^{\beta+1}}{\beta+1}\right] + CiR\left(\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+1}\right)\right. \\
&+\frac{CI_cR}{2}\left[T^2 + M^2 - 2\alpha M T^{\beta+1} + \frac{2\alpha\beta T^{\beta+2}}{\beta+1} + \frac{2\left(\alpha M^{\beta+1} - \beta M - \beta\right)T}{\beta+1}\right] \\
&\left.-\frac{PI_eRM^2}{2}\right\}\left(\frac{e^{rH}-1}{e^{rT}-1}\right).
\end{aligned} \tag{16}$$

The first order condition for $PV_1(T)$ in Eq. (13) is $\frac{dPV_1(T)}{dT} = 0$, where

$$\begin{aligned}
&\frac{dPV_1(T)}{dT} = \left(e^{rH}-1\right)\left[(3CRI_Cr + 3CRir)\frac{T^2}{8} + (-CRi + CRr - CRI_C)\frac{T}{2}\right. \\
&+\frac{Ar}{4} - \frac{CR}{2} + \frac{CRI_C\alpha\beta T^\beta}{r} + \frac{CR\alpha\beta T^{\beta-1}}{r(\beta+1)} - \frac{CR\alpha T^\beta}{2} \\
&+\frac{CR\alpha(\beta+2)rT^{\beta+1}}{4(\beta+1)} + \frac{CRi}{2r} + \frac{CRi\alpha\beta(\beta+3)rT^{\beta+2}}{4(\beta+1)} + \frac{CRI_C}{2r} \\
&+\frac{CRi\alpha\beta T^\beta}{r} - \frac{CRi\alpha\beta(\beta+2)T^{\beta+1}}{2(\beta+1)} - \frac{CRI_C\alpha\beta(\beta+2)T^{\beta+1}}{2(\beta+1)} \\
&\left.+\frac{CRI_C\alpha\beta(\beta+3)rT^{\beta+2}}{4(\beta+1)} - \frac{A}{rT^2}\right].
\end{aligned} \tag{17}$$

The second order condition is

$$\begin{aligned}\frac{d^2PV_1(T)}{dT^2} = \Bigg[& -\frac{1}{r}CRI_C\alpha\beta^2T^{\beta-1} + \frac{1}{2}CR\alpha\beta T^{\beta-1} + \frac{1}{2}e^{rH}CR\alpha rT^{\beta} \\
& -\frac{1}{2}CRr - \frac{1}{r}CRi\alpha\beta^2T^{\beta-1} - e^{rH}CRi\alpha\beta T^{\beta} + \frac{1}{2}e^{rH}CR\alpha rT^{\beta} - \frac{1}{2}CRr \\
& -\frac{CRi\alpha\beta^2T^{\beta-1}}{r} - e^{rH}CRi\alpha\beta T^{\beta} - \frac{1}{2}CR\alpha rT^{\beta} - \frac{1}{4}CR\alpha\beta rT^{\beta} \\
& -\frac{3CRI_C\alpha\beta rT^{\beta+1}}{2(\beta+1)} - \frac{1}{2}e^{rH}CRI_C\alpha\beta^2T^{\beta} - e^{rH}CRI_C\alpha\beta T^{\beta} \\
& +\frac{1}{4}e^{rH}CR\alpha\beta rT^{\beta} - \frac{1}{2}e^{rH}CRi\alpha\beta^2T^{\beta} - \frac{1}{2}e^{rH}CRi + \frac{1}{2}e^{rH}CRr \\
& -\frac{1}{2}e^{rH}CRI_C + CRi\alpha\beta T^{\beta} + \frac{1}{2}CRi\alpha\beta^2T^{\beta} + CRI_C\alpha\beta T^{\beta} \\
& +\frac{1}{2}CRI_C\alpha\beta^2T^{\beta} - \frac{CR\alpha\beta^2T^{\beta-2}}{r(\beta+1)} + \frac{CR\alpha\beta T^{\beta-2}}{r(\beta+1)} \\
& +\frac{e^{rH}CRI_C\alpha\beta^3rT^{\beta+1}}{4(\beta+1)} + \frac{CRi}{2} + \frac{CRI_C}{2} - \frac{e^{rH}CR\alpha\beta T^{\beta-2}}{r(\beta+1)} \\
& +\frac{e^{rH}CR\alpha\beta^2T^{\beta-2}}{r(\beta+1)} + \frac{3e^{rH}CRI_C\alpha\beta rT^{\beta+1}}{2(\beta+1)} + \frac{5e^{rH}CRI_C\alpha\beta^2rT^{\beta+1}}{4(\beta+1)} \\
& +\frac{e^{rH}CRi\alpha\beta^3rT^{\beta+1}}{4(\beta+1)} + \left(e^{rH}CRir + e^{rH}CRI_Cr - CRir - CRI_Cr\right)\frac{3T}{4} \\
& +\frac{2A\left(e^{rH}-1\right)}{rT^3} - \frac{1}{2}e^{rH}CR\alpha\beta T^{\beta-1} + \frac{1}{r}e^{rH}CRi\alpha\beta^2T^{\beta-1} \\
& +\frac{3e^{rH}CRi\alpha\beta rT^{\beta+1}}{2(\beta+1)} + \frac{5e^{rH}CRi\alpha\beta^2rT^{\beta+1}}{4(\beta+1)} \\
& +\frac{1}{r}e^{rH}CRI_C\alpha\beta^2T^{\beta-1} - \frac{5CRI_C\alpha\beta^2T^{\beta+1}}{4(\beta+1)} - \frac{CRI_C\alpha\beta^3rT^{\beta+1}}{4(\beta+1)} \\
& -\frac{5CRi\alpha\beta^2rT^{\beta+1}}{4(\beta+1)} - \frac{CRi\alpha\beta^3rT^{\beta+1}}{4(\beta+1)} - \frac{3CRi\alpha\beta rT^{\beta+1}}{2(\beta+1)}\Bigg]. \qquad (18)\end{aligned}$$

Which is > 0 at $T = T_1$. Therefore, T_1 is the optimal value of T for scenario 1 (having ensured that $T_1 < T_d$). Hence, the optimum procurement quantity is

$$Q^*(T_1) = R\left[T_1 + \frac{\alpha T_1^{\beta+1}}{\beta+1}\right]. \qquad (19)$$

Likewise, the first order condition for scenario 2 is $\frac{dPV_2(T)}{dT} = 0$, where

$$\frac{dPV_2(T)}{dT} = \left(e^{rH} - 1\right)\Bigg[(3CRir + 3PI_eRr)\,\frac{T^2}{8} + (-CRi + PI_eRMr+ \\ CRr - PI_eR)\,\frac{T}{2} + \frac{Ar}{4} - \frac{CR}{2} + \frac{CRi}{2r} + \frac{CR\alpha\beta T^{\beta-1}}{r\,(\beta+1)} - \frac{CR\alpha T^{\beta}}{2} \\ + \frac{CR\alpha\,(\beta+2)\,rT^{\beta+1}}{4\,(\beta+1)} - \frac{CRi\alpha\,(\beta+2)\,T^{\beta+1}}{2\,(\beta+1)} \\ + \frac{CRi\alpha\beta\,(\beta+3)\,rT^{\beta+2}}{4\,(\beta+1)} + \frac{PI_eRM}{2} + \frac{CRi\alpha\beta T^{\beta}}{r} + \frac{PI_eR}{2r} - \frac{A}{rT^2}\Bigg]. \tag{20}$$

Call this solution $T = T_2$ for which the second order condition is

$$\frac{d^2PV_2(T)}{dT^2} = \Bigg[\frac{1}{2}\,CR\alpha\beta T^{\beta-1} + \frac{1}{2}e^{rH}CR\alpha rT^{\beta} - \frac{1}{2}CRr \\ + \left(e^{rH}CRir - CRir + e^{rH}PI_eRr - PI_eRr\right)\frac{3T}{4} \\ -\frac{1}{r}CRir\alpha\beta^2T^{\beta-1} - e^{rH}CRi\alpha\beta T^{\beta} - \frac{1}{2}CR\alpha rT^{\beta} - \frac{1}{4}CR\alpha\beta rT^{\beta} \\ +\frac{1}{4}e^{rH}CR\alpha\beta rT^{\beta} - \frac{1}{2}e^{rH}CRi\alpha\beta^2T^{\beta} - \frac{1}{2}e^{rH}CRi + \frac{1}{2}e^{rH}CRr \\ +CRi\alpha\beta T^{\beta} + \frac{1}{2}CRi\alpha\beta^2T^{\beta} - \frac{CR\alpha\beta^2T^{\beta-2}}{r\,(\beta+1)} + \frac{CR\alpha\beta T^{\beta-2}}{r\,(\beta+1)} + \frac{1}{2}CRi \\ -\frac{e^{rH}CR\alpha\beta T^{\beta-2}}{r\,(\beta+1)} + \frac{e^{rH}CR\alpha\beta^2T^{\beta-2}}{r\,(\beta+1)} + \frac{e^{rH}CRi\alpha\beta^3rT^{\beta+1}}{4\,(\beta+1)} \\ +\frac{2A\left(e^{rH}-1\right)}{rT^3} - \frac{1}{2}e^{rH}CR\alpha\beta T^{\beta-1} + \frac{1}{2}PI_eRMr + \frac{1}{2}PI_eR \\ +\frac{1}{r}e^{rH}CRi\alpha\beta^2T^{\beta-1} + \frac{3e^{rH}CRi\alpha\beta rT^{\beta+1}}{2\,(\beta+1)} \\ +\frac{5e^{rH}CRi\alpha\beta^2rT^{\beta+1}}{4\,(\beta+1)} - \frac{5CRi\alpha\beta^2rT^{\beta+1}}{4\,(\beta+1)} - \frac{CRi\alpha\beta^3rT^{\beta+1}}{4\,(\beta+1)} \\ -\frac{3CRi\alpha\beta rT^{\beta+1}}{2\,(\beta+1)} - \frac{1}{2}e^{rH}PI_eRMr - \frac{1}{2}e^{rH}PI_eR\Bigg], \tag{21}$$

which is > 0 at $T = T_2$. Therefore, T_2 is the optimal value of T for scenario 2 (having ensured that $T_d \leqslant T_2 < M$). We can obtain optimum procurement quantity $Q^*(T_2)$ using Eq. (4).

The first order condition for scenario 3 is $\frac{dPV_3(T)}{dT}=0$, where

$$\frac{dPV_3(T)}{dT}=(e^{rH}-1)\Bigg[(3CRI_Cr+3CRir)\frac{T^2}{8}$$
$$+\left(CRr+\frac{CRI_C\alpha rM^{\beta+1}}{\beta+1}-CRI_C-CRi-\frac{CRI_CM\beta r}{(\beta+1)}-\frac{CRI_C\beta r}{(\beta+1)}\right)\frac{T}{2}$$
$$+\frac{CR\alpha\beta T^{\beta-1}}{r(\beta+1)}+\frac{CRI_C}{2r}+\frac{CRI_C\alpha\beta T^\beta}{r}+\frac{Ar}{4}-\frac{CRI_C\alpha\beta(\beta+2)T^{\beta+1}}{2(\beta+1)}$$
$$+\frac{CRi\alpha\beta(\beta+3)rT^{\beta+2}}{4(\beta+1)}-\frac{CRi\alpha\beta(\beta+2)T^{\beta+1}}{2(\beta+1)}+\frac{CR\alpha(\beta+2)rT^{\beta+1}}{4(\beta+1)}$$
$$-\frac{CRI_C\alpha(\beta+2)MrT^{\beta+1}}{4}+\frac{CRI_C\alpha(\beta+1)MT^\beta}{2}-\frac{CR}{2}-\frac{PI_eRM^2r}{8}$$
$$+\frac{CRi\alpha\beta T^\beta}{r}+\frac{CRI_CM^2r}{8}+\frac{CRI_C\beta}{2(\beta+1)}+\frac{CRI_C\alpha\beta(\beta+3)rT^{\beta+2}}{4(\beta+1)}$$
$$-\frac{CRI_C\alpha\beta MT^{\beta-1}}{r}-\frac{CRI_C\alpha M^{\beta+1}}{2(\beta+1}\Big)+\frac{CRi}{2r}-\frac{CR\alpha T^\beta}{2}+\frac{CRI_CM\beta}{2(\beta+1)}$$
$$+\left(\frac{PI_eRM^2}{2r}-\frac{A}{r}-\frac{CRI_CM^2}{2r}\right)\frac{1}{T^2}\Bigg], \tag{22}$$

which can be solved for $T=T_3$ by the Newton-Raphson method.

The second order condition is

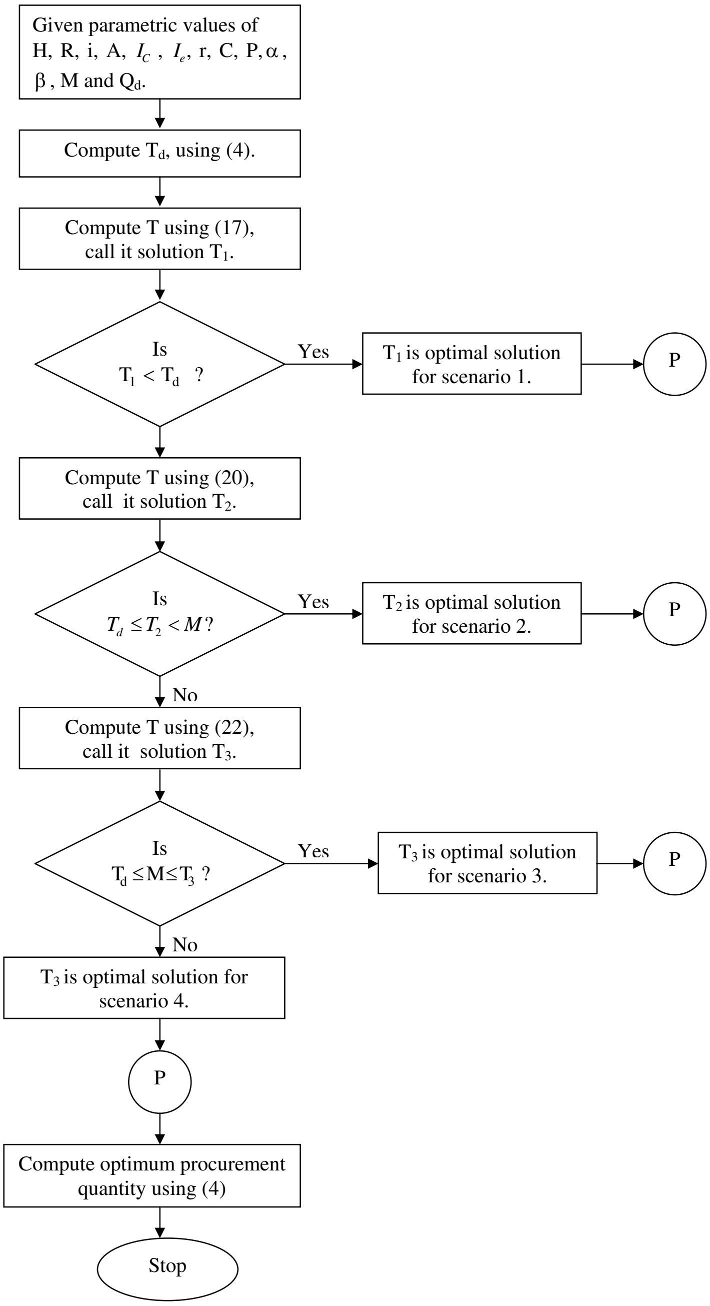
$$\frac{d^2PV_3(T)}{dT^2}=\Bigg[-\frac{1}{r}CRI_C\alpha\beta^2T^{\beta-1}+\Bigg(-\frac{1}{r}e^{rH}PI_eRM^2+\frac{1}{r}e^{rH}CRI_CM^2$$
$$-\frac{2A}{r}+\frac{1}{r}PI_eRM^2-\frac{1}{r}CRI_CM^2+\frac{2e^{rH}A}{r}\Bigg)\frac{1}{T^3}+\frac{1}{2}CR\alpha\beta T^{\beta-1}$$
$$+\frac{1}{2}e^{rH}CR\alpha rT^\beta-\frac{1}{2}CRr+\frac{1}{2}e^{rH}CRI_C\alpha\beta^2MT^{\beta-1}-\frac{1}{r}CRi\alpha\beta^2T^{\beta-1}$$
$$-e^{rH}CRi\alpha\beta T^\beta-\frac{1}{2}CR\alpha rT^\beta-\frac{1}{4}CR\alpha\beta rT^\beta+\frac{1}{r}CRI_C\alpha\beta^2MT^{\beta-2},$$
$$-\frac{1}{r}CRI_C\alpha\beta MT^{\beta-2}-\frac{1}{2}CRI_C\alpha\beta MT^{\beta-1}-\frac{3CRI_C\alpha\beta rT^{\beta+1}}{2(\beta+1)}$$
$$-\frac{1}{2}e^{rH}CRI_C\alpha\beta^2T^\beta-e^{rH}CRI_C\alpha\beta T^\beta+\frac{1}{4}e^{rH}CR\alpha\beta rT^\beta$$
$$-\frac{1}{2}e^{rH}CRi\alpha\beta^2T^\beta-\frac{1}{2}e^{rH}CRi+\frac{1}{2}e^{rH}CRr-\frac{1}{2}e^{rH}CRI_C+CRi\alpha\beta T^\beta$$
$$+\frac{1}{2}CRi\alpha\beta^2T^\beta+CRI_C\alpha\beta T^\beta+\frac{1}{2}CRI_C\alpha\beta^2T^\beta-\frac{CR\alpha\beta^2T^{\beta-2}}{r(\beta+1)}$$

$$
\begin{aligned}
&+\frac{CR\alpha\beta T^{\beta-2}}{r(\beta+1)}-\frac{1}{2}CRI_C\alpha\beta^2MT^{\beta-1}+\frac{e^{rH}CRI_C\alpha\beta^3rT^{\beta+1}}{4(\beta+1)}+\frac{1}{2}CRi\\
&+\frac{1}{2}CRI_C+\frac{1}{2}e^{rH}CRI_C\alpha\beta MT^{\beta-1}+\frac{CRI_CM\beta r}{2(\beta+1)}-\frac{e^{rH}CR\alpha\beta T^{\beta-2}}{r(\beta+1)}\\
&+\frac{e^{rH}CRI_C\alpha rM^{\beta+1}}{2(\beta+1)}+\frac{e^{rH}CR\alpha\beta^2T^{\beta-2}}{r(\beta+1)}+\frac{3e^{rH}CRI_C\alpha\beta rT^{\beta+1}}{2(\beta+1)}\\
&+\frac{5e^{rH}CRI_C\alpha\beta^2rT^{\beta+1}}{4(\beta+1)}-\frac{CRI_C\alpha rM^{\beta+1}}{2(\beta+1)}-\frac{1}{r}e^{rH}CRI_C\alpha\beta^2MT^{\beta-2}\\
&+\frac{e^{rH}CRi\alpha\beta^3rT^{\beta+1}}{4(\beta+1)}+\frac{1}{r}e^{rH}CRI_C\alpha\beta MT^{\beta-2}\\
&+\left(e^{rH}CRir+e^{rH}CRI_Cr-CRir-CRI_Cr\right)\frac{3T}{4}\\
&-\frac{1}{2}e^{rH}CR\alpha\beta T^{\beta-1}+\frac{1}{r}e^{rH}CRi\alpha\beta^2T^{\beta-1}+\frac{3e^{rH}CRi\alpha\beta rT^{\beta+1}}{2(\beta+1)}\\
&+\frac{5e^{rH}CRi\alpha\beta^2rT^{\beta+1}}{4(\beta+1)}+\frac{1}{2}CRI_C\alpha MrT^{\beta}+\frac{1}{4}CRI_C\alpha\beta^2MrT^{\beta}\\
&+\frac{3}{4}CRI_C\alpha\beta MrT^{\beta}+\frac{1}{r}e^{rH}CRI_C\alpha\beta^2T^{\beta-1}-\frac{5CRI_C\alpha\beta^2rT^{\beta+1}}{4(\beta+1)}\\
&-\frac{1}{2}e^{rH}CRI_C\alpha MrT^{\beta}-\frac{1}{4}e^{rH}CRI_C\alpha\beta^2MrT^{\beta}-\frac{3}{4}e^{rH}CRI_C\alpha\beta MrT^{\beta}\\
&-\frac{e^{rH}CRI_C\beta r}{2(\beta+1)}-\frac{CRI_C\alpha\beta^3rT^{\beta+1}}{4(\beta+1)}-\frac{e^{rH}CRI_CM\beta r}{2(\beta+1)}-\frac{5CRi\alpha\beta^2rT^{\beta+1}}{4(\beta+1)}\\
&-\frac{CRi\alpha\beta^3rT^{\beta+1}}{4(\beta+1)}-\frac{3CRi\alpha\beta rT^{\beta+1}}{2(\beta+1)}+\left.\frac{CRI_C\beta r}{2(\beta+1)}\right],
\end{aligned}
\tag{23}
$$

which is > 0 at $T = T_3$. Therefore, T_3 is the optimal value of T for scenario 3 (but ensure that $T_d \leqslant M \leqslant T_3$). We can obtain optimum procurement quantity $Q^*(T_3)$ using Eq. (4).

$PV_4(T)$ in scenario 4 is the same as that of scenario 3, therefore the optimal value of $T = T_4$ for scenario 4 is the solution of $\frac{dPV_3(T)}{dT} = 0$.

4. Computational algorithm



5. Numerical example

Consider the following parametric values in appropriate units:

$$[H, R, i, I_C, I_e, r, C, P, \alpha, \beta, M, A, Q_d]$$
$$= [1, 1000, 10\%, 9\%, 6\%, 5\%, 20, 35, 0.03, 1.2, 30/365, 100, 70].$$

We obtain $T_d = 0.06996$ years which is $< M$ $(= 0.082192$ years).

Using algorithm compute optimum T. The computational results and sensitivity analysis for different parameters are given below.

Table 1. Sensitivity analysis of ordering cost A

A	T^*	$Q(T^*)$	$PV(T^*)$	$\frac{d^2PV_4(T)}{dT^2}$ at $T = T^*$
100	0.3530	354.38	389586	4785.91
200	0.4940	496.89	394260	3495.88
300	0.5999	604.33	397870	2958.58

Table 2. Sensitivity analysis of minimum order quantity Q_d

Q_d	T^*	$Q(T^*)$	$PV(T^*)$	$\frac{d^2PV_1(T)}{dT^2}$ at $T = T^*$
60	0.05997	60	412528	951508.28
70	0.06996	70	407958	612226.04
80	0.07995	80	404565	599486.28

Table 3. Sensitivity analysis of credit period M

M	T^*	$Q(T^*)$	$PV(T^*)$	$\frac{d^2PV_4(T)}{dT^2}$ at $T = T^*$
15/365	0.354	355.39	390440	4703.74
30/365	0.353	354.38	389586	4888.48
45/365	0.350	352.36	388705	4843.66

Table 4. Sensitivity analysis of inflation rate r

r	T^*	$Q(T^*)$	$PV(T^*)$	$\frac{d^2PV_4(T)}{dT^2}$ at $T = T^*$
0.03	0.318	319.10	651470	6469.23
0.04	0.333	335.22	487813	5665.59
0.05	0.353	354.38	389586	4785.70

6. Conclusions

An EOQ model under inflation for time dependent deterioration of units is formulated to determine the optimal ordering policy when the supplier offers a credit period linked to order quantity to settle the accounts. Since expressions obtained are highly non-linear, Taylor series approximation is used. An easy to use algorithm is given to obtain the optimal replenishment cycle time. The following managerial issues are observed:

1. Increase in ordering cost increases optimal values of order quantity, replenishment cycle time and present value of future cost.
2. If minimum order quantity for availing the facility of credit period increases, optimum order quantity and replenishment cycle time increase but present value of future cost decreases.
3. Increase in credit period lowers the order quantity to be procured and replenishment cycle, it also results in a decrease in present value of future cost.
4. As inflation rate increases, optimum order quantity and replenishment cycle time increases but present value of future cost decreases.

The proposed model can be extended by taking demand as a function of time, selling price, product quality and stock. It can also be generalized to allow for shortages, partial lost-sales and quantity discounts.

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