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Article

An Equity-Based Optimization Model to Solve the Location Problem for Healthcare Centers Applied to Hospital Beds and COVID-19 Vaccination

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Abstract: Governments must consider different issues when deciding on the location of healthcare centers. In addition to the costs of opening such centers, three further elements should be addressed: accessibility, demand, and equity. Such locations must be chosen to meet the corresponding demand, so that they guarantee a socially equitable distribution, and to ensure that they are accessible to a sufficient degree. The location of the centers must be chosen from a set of possible facilities to guarantee certain minimum standards for the operational viability of the centers. Since the set of potential locations does not necessarily cover the demand of all geographical zones, the efficiency criterion must be maximized. However, the efficient distribution of resources does not necessarily meet the equity criterion. Thus, decision-makers must consider the trade-off between these two criteria: efficiency and equity. The described problem corresponds to the challenge that governments face in seeking to minimize the impact of the pandemic on citizens, where healthcare centers may be either public hospitals that care for COVID-19 patients or vaccination points. In this paper, we focus on the problem of a zone-divided region requiring the localization of healthcare centers. We propose a non-linear programming model to solve this problem based on a coverage formula using the Gini index to measure equity and accessibility. Then, we consider an approach using epsilon constraints that makes this problem solvable with mixed integer linear computations at each iteration. A simulation algorithm is also considered to generate problem instances, while computational experiments are carried out to show the potential use of the proposed mathematical programming model. The results show that the spatial distribution influences the coverage level of the healthcare system. Nevertheless, this distribution does not reduce inequity at accessible healthcare centers, as the distribution of the supply of health centers must be incorporated into the decision-making process.

Keywords: facility location problem; Gini index; linear programming; multi-objective optimization; SARS-CoV-2; simulations; spatial accessibility; two-step floating catchment area method

MSC: 90C05; 90C29

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1. Introduction and State-of-the-Art

In this section, we discuss introductory aspects of our investigation, detail the bibliographical review, give an illustrative example to facilitate understanding of the problem under analysis, and describe the contribution of this study, as well as detailing the contents of the sections of the paper.

1.1. Introductory Aspects

In public policy, social investment is one major concern for governments, especially when improving public healthcare systems. Access to these systems embraces a number of dimensions, including acceptability, accommodation, affordability, availability, and geographical location [1–3]. Some authors have addressed other barriers to be considered when measuring accessibility, such as dimensions related to age, behavior, ethnicity, financial status, information access, sex, and social class [4,5]. These dimensions allow the clustering of different accessibility measures according to whether spatial barriers are included in their quantification or not.

Certain decisions related to adequate health access focus on locating healthcare centers/facilities. This location is chosen, for example, to situate a set of additional centers or to provide a good supply of services, such as hospital beds, which improves the relationship between supply and demand. Unlike other facility location problems that focus on minimizing costs, decision-makers face complex challenges when deciding on locating the best sites for healthcare centers due to their possible inadequate spatial distribution or an insufficient number of them. These possibly counterproductive decisions can have a direct impact on the healthcare workers, increase the mortality and morbidity of the community [6], as well as limiting access to medical treatment [5,7–10].

Although the location of healthcare centers may meet an efficiency criterion, it must also guarantee an equitable distribution of resources [8], which can increase the inequity of access to healthcare centers [11,12]. If the focus is on a zone-divided region requiring the localization of healthcare centers, in each zone, it is necessary to select a geographical site (discrete center) where the demand is concentrated. Then, the decision-makers must determine where to situate the healthcare centers based on a set of possible locations that guarantee minimum and maximum thresholds for the operational viability of these centers. Since the set of potential locations does not necessarily cover the demand of all zones, decision-makers must maximize an efficiency criterion. However, efficient distribution of resources does not necessarily meet the equity criterion. Therefore, decision-makers must also consider the trade-off between these two criteria: efficiency and equity.

There are two types of accessibility: (i) revealed, which focuses on measuring the employment of the healthcare system; and (ii) potential, which measures the probable use of healthcare centers and not their utilization [5]. Some spatial measures have been studied on healthcare accessibility [13], many of which are based on the gravity model given by $A_i = \sum_j a_j / f(c_{ij})$, where i and j denote the healthcare demand i and supply j points, respectively; a_j is the healthcare supply of the point j; c_{ij} is the total cost between points i and j; and f is an impedance function. This function is decreasing and weights the supply from point j captured by the demand point i, that is, as the cost between points i and j increases, the supply of point j captured by point i decreases.

Some variants of the gravity model have been analyzed [1]. One of them is the two-step floating catchment area (2SFCA) method [5]. This method involves two stages: (i) the first calculates the ratio between the supply of healthcare centers of location j and the population that is distanced (or has a travel time) to a maximum threshold D of point j; and (ii) in the second, the accessibility of each demand point i is obtained as the sum of the supply-to-population ratio (calculated at the first stage) of all providers that are separated (or have a travel time) at most D from point i. Several studies adopted the 2SFCA method, or a variant of it, for analyzing accessibility to healthcare centers [14–16]. This accessibility is a non-medical factor that influences inequities in healthcare outcomes [11]; that is, there is a relation between inequity access to healthcare and inequitable distribution of illness [11,12,17,18]. For this reason, the World Healthcare Organization has issued recommendations for government actions. These actions are related to strengthening equity by improving investment in healthcare infrastructure, among other initiatives, to close the gap in healthcare outcomes [12].

1.2. Bibliographical Review

Several optimization models have been formulated for locating a set of healthcare facilities. Some of these models optimize an efficiency criterion, for example, to minimize the total travel distance or time [19–21], or to minimize the total operating and installation costs [22,23]. Other studies have focused on characterizing the location of healthcare facilities as a maximal covering location problem, which aims to situate a fixed number of healthcare facilities to maximize the demand coverage by these facilities [24]. The notion of covering a demand point refers to whether there is at least one facility at a maximum travel distance or threshold (or travel time) D from the demand point.

Various applications have adopted different coverage thresholds for demand points by healthcare facilities. For example, a travel distance threshold of fifteen miles was stated in [25] when studying spatial accessibility to healthcare centers in Illinois, United States (US), using a geographic information system. Similarly, a travel time threshold of three hours was assumed in [26] when analyzing spatial accessibility to major cancer care facilities in the US. In addition, a threshold defined as the third quartile of the data set was employed in a mobility survey in Florida, US [15]. A variant of the maximal covering location problem that included capacity and budget constraints was addressed in [27,28]. These authors assumed the number of facilities as a decision variable.

Other studies have adopted a multi-objective approach when locating a set of facilities. These objectives include costs, environment, equity concerns, and service level. For example, a two-criteria mathematical model was proposed in [29] for solving a multi-objective facility location problem. This model deals with efficiency (to minimize total costs) and equity concerns (to minimize the maximum distances from the facilities to the demand points). For an extensive literature review of the facility location problem under a multi-criteria approach, see [30], and for other p-center variants in a 2D Pareto front, see [31].

In the context of healthcare facility locations, under a multi-objective approach, a mathematical model was proposed in [23] for locating and sizing a set of medical departments based on finding a trade-off among costs, total travel distance, uncovered demand, and the number of changes to be implemented in the system. A similar approach was adopted in [21] for an application in a hierarchical healthcare system. Other investigations on this topic can be found in [32–34]. Similarly, a three-objective mathematical model was presented in [35] to locate preventive healthcare facilities, which included financial, coverage, and service level (waiting time) concerns.

Various metrics have been adopted to measure resource distribution and inequity. Among the measures of equity used in the context of facility location analysis, we have the standard deviation, variance, mean absolute deviation, and sum of absolute deviation [36,37]. These statistical metrics cannot be employed in any comparative study since they are not normalized metrics. Among normalized equity metrics, we have the Gini index, the Schutz index, and the coefficient of variation [38,39].

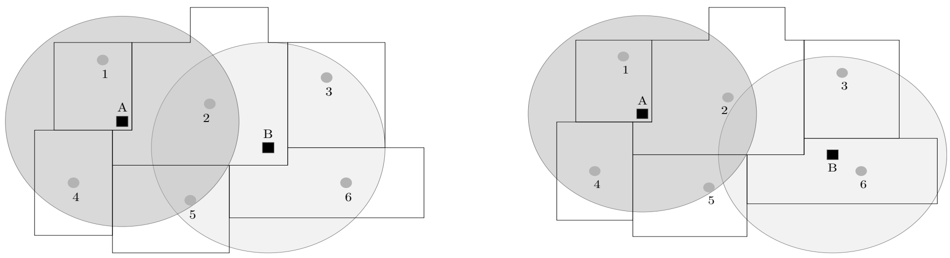
In the context of equity, the Gini index has been used in the selection of socioeconomic variables, in the establishment of relations of econometric models [40], and in the determination of conditioning factors for environmental equity [41], as a helpful method to establish equity in network programming problems [42].

1.3. Illustrative Example

To illustrate the problem that we address, let us suppose that a study region has been divided into six zones, each characterized by a discrete center in which a person is concentrated; see Figure 1a,b. Two healthcare centers, namely A and B, are located, with a healthcare supply of 1 and 2 units, respectively. The figures also show the coverage zones of each healthcare center (to which this center potentially offers its service).

Analyzing the scenario represented in Figure 1a, note that healthcare center A potentially offers one center to the four patients in its coverage region (zones 1, 2, 4, and 5). Hence, each zone receives 1/4 health units of center A, while healthcare center B potentially offers two centers to the four patients in its coverage region (zones 2, 3, 5 and 6).

Thus, each patient receives from center B $2/4 = 1/2$ health units. Given that zones 2 and 5 can potentially access health centers offered by both centers A and B, the potential spatial accessibility to such centers of these zones is the sum of what they potentially receive from the centers A and B. Then, the potential spatial accessibility from zones 1, 2, 3, 4, 5 and 6 to healthcare centers is 1/4, 3/4, 1/4, 1/4, 3/4, and 1/4, respectively, yielding a Gini index of 0.267, as computed in [38,39]. Moreover, if the locations of the health centers are those shown in Figure 1b, center A offers to each patient in zones 1, 2, 4 and 5 a value of 1/4 units of healthcare centers, while center B offers to each patient in zones 3 and 6 a value of $2/2 = 1$ unit of healthcare center. Therefore, the potential spatial accessibility of zones 1, 2, 3, 4, 5 and 6 to healthcare centers is 1/4, 1/4, 1, 1/4, 1/4 and 1, respectively, which produces a Gini index of 0.333. Although all the zones of the hypothetical study region are covered by the health system in both scenarios, it is evident that an inadequate location of the centers could increase the inequity in the distribution of resources, evidenced by the difference in the value of the Gini index.



(**a**) (**b**)

Figure 1. Hypothetical spatial distribution of health centers and their coverage zone in: (**a**) scenario 1; and (**b**) scenario 2; where "A" and "B" in both plots are the healthcare centers, whereas "1–6" are the geographical zones, with ● and ■ sketching a zone discrete center and a center location, respectively.

1.4. Contributions and Description of Sections

Since an efficient resource distribution does not necessarily guarantee equity and vice-versa [8], we propose a multi-objective approach. Our main contribution here is to solve the problem of location for healthcare centers considering, in addition to the costs of opening such centers, the trade-off among three elements: accessibility, demand, and equity, based on a mathematical programming model. To the best of our knowledge, these three elements have not been considered together until now in a multi-objective nonlinear programming model to solve this problem based on a coverage formulation using the Gini index to measure equity and accessibility. Our proposal seeks to set an efficiency criterion when covering the locations of healthcare centers subject to the criteria of equity and accessibility. The problem described fits perfectly with the challenge that governments face to minimize the impact of the pandemic on citizens, where healthcare centers can be either public hospitals that care for COVID-19 patients or vaccination points [43–46]. In this study, we focus on a type of accessibility named potential, due to confidentiality, and on the problem of locating healthcare centers in a zone-divided region.

From an algorithmic point of view, the problem to be solved here is relevant, since this is a discrete p-center problem, which has been proven to be NP-hard [47,48]. Although some variants of this problem can be solved in polynomial time [49,50], unfortunately, in the variant that we want to solve, this does not happen. Then, approximation algorithms are necessary to solve instances in a reasonable time.

Our study is structured in the following way. First, in Section 2, we formulate the proposed mathematical model. Then, in Section 3, an algorithm is presented for solving the optimization model. Next, in Section 4, numerical experiments are carried out based on simulations and examples. Finally, we end the article in Section 5 with discussion of some advantages and limitations of our study, as well as providing recommendations for further research.

2. Mathematical Model

In this section, we describe some notations and propose a three-objective mathematical programming model for the problem considered in this study that embeds efficiency and equity in the decision-making process of locating a set of healthcare centers.

2.1. Notations and Symbols

Before formulating the model, Table 1 presents a summary of some notations to be used throughout the document.

Table 1. Acronyms, notations and symbols.

Acronym Notation/Symbol	Definition
Sets	
$N \equiv \{1, \ldots, n\}$	Set of demand zones.
$M \equiv \{1, \ldots, m\}$	Set of potential locations for a healthcare center.
Parameters	
k	Number of healthcare facilities to be installed.
D	Threshold of travel distance (or travel time).
w_i	Demand of zone i.
d_{ij}	Distance between zone i and zone j.
$s^{\min}(s^{\max})$	Minimum (maximum) supply of a healthcare center measured in terms of hospital bed units.
$\text{HBI}^{\min}(\text{HBI}^{\max})$	Minimum (maximum) hospital bed index measured in terms of hospital bed units per 1000 inhabitants.
B	A big number.
Decision variables	
y_j	One (1) if a healthcare center is located at j, and zero (0) otherwise.
x_i	One (1) if the demand zone i is covered by the healthcare center, that is, there is at least one operational healthcare center located at most D of zone i), and zero (0) otherwise.
$s_j \geq 0$	Supply of healthcare center located at zone j measured in terms of hospital beds.
$a_i \geq 0$	Potential spatial accessibility of zone i measured in terms of hospital beds.
$r_j \geq 0$	Supply-to-population ratio within the coverage zone of the healthcare center j measured in terms of hospital beds per inhabitants.
$A_i \geq 0, h_{il} \geq 0, z \geq 0$	Auxiliary variables.

2.2. Objective Functions

We define a three-objective mathematical model according to:

(i) Coverage. Maximize the coverage of the demand points, that is, maximize an efficiency criterion given by

$$\max\left\{ f_1 = \sum_{i=1}^{n} w_i x_i \right\}, \tag{1}$$

with n, w_i and x_i being defined in Table 1.

(ii) Equity. Minimize inequity in access to healthcare centers. For this purpose, we use the Gini index as an inequity measure [51], which is defined as one-half of the relative mean absolute difference between all pairs of potential spatial accessibility (a measure of the potential use of the healthcare center) a_i, with $i \in N$, that is, one-half of the mean absolute difference between all pairs of potential spatial accessibility a_i divided by the average of the values of a. Specifically, to minimize inequity in access to healthcare centers, we formulate the objective function given by

$$\min\left\{ f_2 = \sum_{i=1}^{n}\sum_{l=1}^{n} |a_i - a_l| / \left(2n\sum_{i=1}^{n} a_i\right)\right\}, \tag{2}$$

with n and a_i being defined in Table 1.

(iii) Accessibility. The third objective incorporated into our model is focused on maximizing the minimum potential spatial accessibility difference to zero and is stated as

$$\max\left\{ f_3 = \min_{i\in N: a_i \neq 0} \{a_i\}\right\}, \tag{3}$$

with N and a_i being defined in Table 1.

2.3. Constraints

To meet the three objectives described previously in (i)–(iii), a set of constraints must be satisfied, which defines its domain set detailed below.

The following inequalities allow us to identify, through variable x_i, whether a demand zone i is covered by the healthcare center; that is, if there is at least one operational healthcare center at most distance D of zone i. In fact, if there is not a healthcare center located at most D of zone i, then $\sum_{j=1:d_{ij}\leq D}^{m} y_j = 0$, which leads to $x_i = 0$. Reciprocally, if $x_i = 0$, then it leads to $y_j = 0$. Therefore, we have the constraints established as

$$\sum_{j=1:d_{ij}\leq D}^{m} y_j \geq x_i, \quad \forall i \in N, \tag{4}$$

$$x_i \geq y_j, \quad \forall i \in N, \quad \forall j \in M, \quad d_{ij} \leq D,$$

with d_{ij}, D, m, M, and y_j being defined in Table 1. The constraints also ensure the opening of a fixed number of facilities specified in advance, which are given by $\sum_{j=1}^{m} y_j = k$. To compute the potential spatial accessibility of a zone i according to the 2SFCA method, the constraints are stated as

$$r_j = \frac{s_j}{\sum_{i=1:d_{ij}\leq D}^{n} w_i}, \quad \forall j \in M, \tag{5}$$

$$a_i = \sum_{j=1:d_{ij}\leq D}^{m} r_j, \quad \forall i \in N, \tag{6}$$

with r_j being defined in Table 1. Specifically, the constraint given in (5) calculates the supply-to-population ratio (within the coverage zone of a healthcare center j) of the healthcare center j, while the constraint defined in (6) computes the potential spatial accessibility of zone i to the healthcare centers.

To guarantee the minimum and maximum supply of a health center, based on the constraint given in (7), and the minimum and maximum global index of supply at the analyzed region, based on the constraint expressed in (8), the corresponding constraints are specified as

$$s^{\min} y_j \leq s_j \leq s^{\max} y_j, \quad \forall j \in M, \tag{7}$$

$$\text{HBI}^{\min} \leq \sum_{j=1}^{m} s_j \leq \text{HBI}^{\max}, \tag{8}$$

with s_j being defined in Table 1 and HBI denoting the hospital bed index also stated in this table. The basic domain constraints of our proposed model are presented as

$$\begin{aligned} y_j, x_i &\in \{0,1\}, \quad \forall i \in N, \quad \forall j \in M, \\ s_j &\in \mathbb{Z}^+ \cup \{0\}, \quad \forall j \in M, \\ a_i, r_j &\geq 0, \quad \forall i \in N, \quad \forall j \in M, \end{aligned} \tag{9}$$

3. Solution Algorithm for the Three-Objective Optimization Model

In this section, we describe the ε-constraint approach for obtaining the Pareto optimal set of the multi-objective mathematical model and the corresponding algorithm to optimize the model.

3.1. ε-Constraint Approach

To meet the different objectives described in the expressions given in (1)–(3), we adopt an ε-constraint method [52] for obtaining the Pareto optimal set and establishing the equilibrium point where we find the highest achievable well-being for society. For this purpose, we define as our primary objective to maximize the coverage of the demand points stated in (1). In contrast, the objectives of inequity defined in (2) and accessibility established in (3) are set as additional constraints. Thus, we define ε_{acc} and ε_{GI} as lower and upper bounds of inequity in minimum potential spatial accessibility and access to healthcare centers, respectively. Based on the above, the constraints derived when defining upper and lower bounds to the objectives defined in (2) and (3) are stated as

$$f_2 \leq \varepsilon_{\text{GI}}, \quad f_3 \geq \varepsilon_{\text{acc}}, \tag{10}$$

respectively. Note that the constraints defined in (10) are non-linear, so we proceed to linearize them by introducing auxiliary variables and constraints.

Given that there is a minimum supply of healthcare centers, $\sum_{i=1}^{n} a_i > 0$ namely, the expression given in (10) can be formulated as

$$\sum_{i=1}^{n} \sum_{l=1}^{n} |a_i - a_l| \leq \varepsilon_{\text{GI}} \left(2n \sum_{i=1}^{n} a_i \right). \tag{11}$$

Let h_{il} be a positive variable for each $i, l \in N$. Then, the constraints stated in (11) can be linearized by the expressions defined as

$$\sum_{i=1}^{n} \sum_{l=1}^{n} h_{il} \quad = \quad \varepsilon_{\text{GI}} \left(2n \sum_{i=1}^{n} a_i \right), \tag{12}$$

where $-h_{il} \leq a_i - a_l \leq h_{il}, \forall i, l$. Note that we must choose the minimum potential spatial accessibility among all zones. Hence, we define, for each zone i, a positive variable A_i that takes the value a_i providing that zone i is covered by a healthcare center, which is characterized by $x_i = 1$ or $x_i = 0$. Let B be a large number and z be a positive variable such that $z = \min_{i \in N: a_i \neq 0} \{a_i\}$. Hence, the inequality stated in (10) can be replaced by $z \geq \varepsilon_{\text{acc}}$ as long as for each zone i, $A_i = a_i + B(1 - x_i)$, and $z \leq A_i$, that is,

$$\varepsilon_{\text{acc}} \leq z \leq a_i + B(1 - x_i), \quad \forall i \in N. \tag{13}$$

Based on the above, the Pareto optimal set can be obtained by solving it iteratively. Then, the single-objective model is given by

$$
\begin{aligned}
& \max\{f_1\}, \qquad (14)\\
\text{subject to} \quad & \sum_{j=1:d_{ij}\le D}^{m} y_j \ge x_i, \quad \forall i \in N,\\
& x_i \ge y_j, \quad \forall i \in N, \quad \forall j \in M, \quad d_{ij} \le D,\\
& \sum_{j=1}^{m} y_j = k,\\
& r_j = s_j / \sum_{i=1:d_{ij}\le D}^{n} w_i, \quad \forall j \in M,\\
& a_i = \sum_{j=1:d_{ij}\le D}^{m} r_j, \quad \forall i \in N,\\
& s^{\min} y_j \le s_j \le s^{\max} y_j, \quad \forall j \in M,\\
& \mathrm{HBI}^{\min} \le \sum_{j=1}^{m} s_j \le \mathrm{HBI}^{\max},\\
& \sum_{i=1}^{n}\sum_{l=1}^{n} h_{il} = \varepsilon_{\mathrm{GI}}\Big(2n\sum_{i=1}^{n} a_i\Big),\\
& \varepsilon_{\mathrm{acc}} \le z \le a_i + B(1 - x_i), \quad \forall i \in N,\\
& y_j, x_i \in \{0,1\}, \quad \forall i \in N, \quad \forall j \in M,\\
& s_j \in \mathbb{Z}^+ \cup \{0\}, \quad \forall j \in M,\\
& a_i, r_j, h_{il}, z \ge 0, \quad \forall i, l \in N, \quad \forall j \in M,
\end{aligned}
$$

for different values of $\varepsilon_{\mathrm{GI}}$ and $\varepsilon_{\mathrm{acc}}$, where f_1 is defined in (1).

One of the disadvantages of the ε-constraint approach for solving a multi-objective mathematical model is the computational time spent when exploring $\varepsilon_{\mathrm{GI}}$ and $\varepsilon_{\mathrm{acc}}$ values that generate infeasible solutions. Hence, we must define a range for both values. By definition, the Gini index has a maximum value of one, whereas the minimum index Gini can be obtained by solving the mathematical model stated as

$$
\begin{aligned}
\varepsilon_{\mathrm{GI}}^{\mathrm{lower}} \;=\; & \max\{f_2\}, \qquad (15)\\
\text{subject to} \quad & \sum_{j=1:d_{ij}\le D}^{m} y_j \ge x_i, \quad \forall i \in N,\\
& x_i \ge y_j, \quad \forall i \in N, \quad \forall j \in M, \quad d_{ij} \le D,\\
& \sum_{j=1}^{m} y_j = k,\\
& r_j = s_j / \sum_{i=1:d_{ij}\le D}^{n} w_i, \quad \forall j \in M,\\
& a_i = \sum_{j=1:d_{ij}\le D}^{m} r_j, \quad \forall i \in N,\\
& s^{\min} y_j \le s_j \le s^{\max} y_j, \quad \forall j \in M,\\
& \mathrm{HBI}^{\min} \le \sum_{j=1}^{m} s_j \le \mathrm{HBI}^{\max},\\
& y_j, x_i \in \{0,1\}, \quad \forall i \in N, \quad \forall j \in M,\\
& s_j \in \mathbb{Z}^+ \cup \{0\}, \quad \forall j \in M,\\
& a_i, r_j \ge 0, \quad \forall i \in N, \quad \forall j \in M,
\end{aligned}
$$

where f_2 is defined in (2).

Note that the lower minimum potential spatial accessibility is zero, whereas its upper minimum value can be obtained by solving the mathematical model expressed as

$$\begin{aligned}
\varepsilon_{\text{acc}}^{\text{upper}} &= \max\{f_3\}, \qquad (16)\\
\text{subject to} \quad & \sum_{j=1:d_{ij}\leq D}^{m} y_j \geq x_i, \quad \forall i \in N,\\
& x_i \geq y_j, \quad \forall i \in N, \quad \forall j \in M, \quad d_{ij} \leq D,\\
& \sum_{j=1}^{m} y_j = k,\\
& r_j = s_j / \sum_{i=1:d_{ij}\leq D}^{n} w_i, \quad \forall j \in M,\\
& a_i = \sum_{j=1:d_{ij}\leq D}^{m} r_j, \quad \forall i \in N,\\
& s^{\min} y_j \leq s_j \leq s^{\max} y_j, \quad \forall j \in M,\\
& \text{HBI}^{\min} \leq \sum_{j=1}^{m} s_j \leq \text{HBI}^{\max},\\
& y_j, x_i \in \{0,1\}, \quad \forall i \in N, \quad \forall j \in M,\\
& s_j \in \mathbb{Z}^+ \cup \{0\}, \quad \forall j \in M,\\
& a_i, r_j \geq 0, \quad \forall i \in N, \quad \forall j \in M,
\end{aligned}$$

so that $\varepsilon_{\text{GI}} \in [\varepsilon_{\text{GI}}^{\text{lower}}, 1]$ and $\varepsilon_{\text{acc}} \in [0, \varepsilon_{\text{acc}}^{\text{upper}}]$, where f_3 is defined in (3).

3.2. Algorithm

Since ε_{GI} and ε_{acc} belong to a continuous set, we discretize their respective domains in intervals δ_1 and δ_2 defined in advance and we choose ε_{GI} and ε_{acc} as the limits for each interval. Thus, the procedure for obtaining the Pareto optimal set of the problem, Ω namely, is defined as $\{\max\{f_1\}, \min\{f_2\}, \max\{f_3\}\}$, subject to the constraints stated in (4) to (9), and formulated in Algorithm 1.

Algorithm 1 ε-constraint approach for obtaining the Pareto optimal set of the multi-objective mathematical model

begin
  input : δ_1, δ_2 numbers of discrete intervals of $[\varepsilon_{\text{GI}}^{\text{lower}}, 1]$ and $[0, \varepsilon_{\text{acc}}^{\text{upper}}]$, respectively
  output: Pareto optimal set Ω
  Initialize $\Omega = \emptyset$
  Compute $\varepsilon_{\text{GI}}^{\text{lower}}$ by solving (15)
  Calculate $\varepsilon_{\text{acc}}^{\text{upper}}$ by solving (16)
  $\varepsilon_{\text{GI}} \leftarrow 1$
  while $\varepsilon_{\text{GI}} > \varepsilon_{\text{GI}}^{\text{lower}}$ **do**
    $\varepsilon_{\text{acc}} \leftarrow 0$
    $\varepsilon_{\text{GI}} \leftarrow \varepsilon_{\text{GI}} - (1 - \varepsilon_{\text{GI}}^{\text{lower}})/\delta_1$
    while $\varepsilon_{\text{acc}} < \varepsilon_{\text{acc}}^{\text{upper}}$ **do**
      $\varepsilon_{\text{acc}} \leftarrow \varepsilon_{\text{acc}} + \varepsilon_{\text{acc}}^{\text{upper}}/\delta_2$
       Find the optimal value of (14) denoted by f_1^*
       Determine f_2 by means of the argument of (2)
       Obtain f_3 by means of the argument of (3)
       $\Omega \leftarrow \Omega \cup \{f_1^*, f_2, f_3\}$
    end
  end
end

4. Numerical Results

In this section, we propose an algorithm to generate instances of the location problem of healthcare centers and another algorithm to solve them. In addition, application examples of the proposed mathematical model are included and discussed. Furthermore, the computational burden to evaluate the efficiency of the proposed procedure is reported.

4.1. Simulation Algorithm

We propose Algorithm 2 to generate an instance of the problem defined in Section 2. To build the algorithm, we need to establish:

- h: Upper limit on the horizontal axis and the vertical axis.
- p: Number of inhabitants who may require healthcare.
- l_1: Real number between 0 and 1 that allows us to determine the radius of the interval to generate the demand of the zones.
- l_2: Real number between 0 and 1 that permits us to state the radius of the interval for the total zone demand.
- K: Set of possible values for k.
- $s_u^{\min}$: Upper limit for $s^{\min}$.
- $s_l^{\max}$: Lower limit for $s^{\max}$.
- U: A random variable with a continuous uniform distribution in (α, β), denoted as $\mathrm{U}(\alpha, \beta)$.

Regarding other input and output parameters of Algorithm 2, see Table 1. It is important to mention that, on the one hand, Step 3 of Algorithm 2 allows us to assign a uniformly distributed demand to each zone in an interval centered on the ratio p/n with a radius of $l_1\, p/n$. On the other hand, its Step 4 permits us to ensure that the value of the total demand of the zones is in an interval centered on p and with a radius of $l_2\, p$. Similarly, $s^{\min}$ must satisfy the condition $s^{\min} \leq s_u^{\min}$, while $s^{\max}$ must satisfy the condition $s^{\max} \geq s_l^{\max}$, with $s^{\min} < s^{\max}$.

Algorithm 2 Simulation algorithm for generating instances of the location problem for healthcare centers

begin

input : $n, h, p, \mathrm{HBI}^{\min}, \mathrm{HBI}^{\max}, l_1, l_2, K$.
output: $s_u^{\min}, s_l^{\max}, \forall i \in \{1, \ldots, n\} : (u_i, v_i), w_i$.
Step 1: Generate a value u_i from $\mathrm{U}(0, h)$ for $i = 1$ to n
Step 2: State a value v_i from $\mathrm{U}(0, h)$ for $i = 1$ to n
Step 3: Determine a value w_i from $\mathrm{U}(p(1 - l_1)/n, p(1 + l_1)/n)$ for $i = 1$ to n
Step 4: Set $P = \sum_{i=1}^{n} w_i$
Step 5: Go back to Step 3 if $P < p(1 - l_2)$ or $P > p(1 + l_2)$
Step 6: Fix $s_u^{\min} = \min_{k \in K} \{\mathrm{HBI}^{\max} P/1000k\}$
Step 7: Assume $s_l^{\max} = \max_{k \in K} \left\{\mathrm{HBI}^{\min} P/1000k\right\}$

end

To solve instances of the location problem for healthcare centers, we design Algorithm 3 that uses a sensor in a database. This algorithm achieves its objective by relying on a simulation algorithm, a database, a sensor located in it, and a solver responsible for obtaining the solution of the corresponding mathematical model.

Algorithm 3 Approach for solving instances of the location problem for healthcare centers using a database-sensor.

1: Generate an instance of the problem based on Algorithm 2 and store it in a database.

2: Detect the generation of the instance by means of a sensor in the database and build the corresponding mathematical model with the data from the instance at run-time.

3: Send the built mathematical model to a solver by using a sensor.

4: Receive the mathematical model from the solver and compile it.

5: Store the results in the database and use them for later analysis and discussion.

4.2. Application Example

To analyze the impact of this novel approach when locating healthcare centers, an instance was obtained when applying Algorithm 2 with the values $n = 10, h = 1.7, p = 1$ million of inhabitants, $l_1 = 0.4, l_2 = 0.1$, and $k \in \{3, 4, 5, 6\}$. The values of these parameters have been set for experimental purposes. However, they depend on the criteria of decision-makers when looking for a balance between the service level and available budget.

To test with an adequate minimum and maximum global index of healthcare center supply (HBI^{min} and HBI^{max}), we analyzed data from the World Bank Group [53] related to hospital beds in several countries worldwide. For example, in 2018, Colombia reported that the HBI was 1.7 hospital beds per 1000 inhabitants, while it was 1.4 in Ecuador in 2014. Based on the above, we adopted 1 and 2 hospital beds per 1000 inhabitants as the minimum and maximum global index of healthcare centers, respectively, a more realistic condition in South America. The execution of Algorithm 2 does not only provide us with the spatial location of the study zones and their respective demands, but it also offers upper and lower bounds for s^{min} and s^{max}, respectively. Based on the bounds calculated by the algorithm, the values $s^{\text{min}} = 250$ and $s^{\text{max}} = 400$ are obtained.

Figure 2 shows the spatial distribution of ten zones, denoted from 1 to 10, each one represented by a discrete center whose diameter is proportional to the demand for healthcare centers in each zone. In other words, zone 8 is the area with the least demand, while zone 5 has the highest demand. Each line in Figure 2 indicates a connection between a pair of zones whose length is given by the Euclidean metric. Otherwise, the distance between each pair of nodes is provided by the shortest path between the nodes.

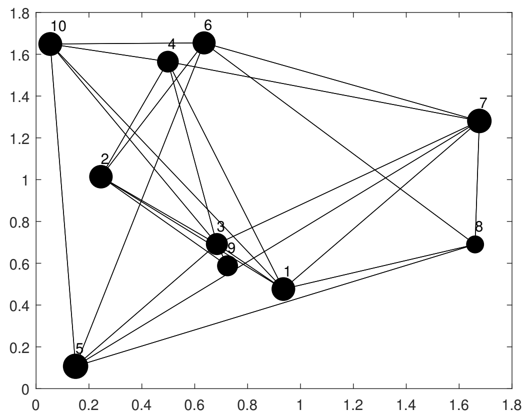


Figure 2. Spatial location of zones in a hypothetical instance 1, where 1 to 10 are the geographical zones, with ● sketching a discrete center whose diameter is proportional to the demand for healthcare centers in the indicated zone.

As described in Algorithm 1, the Pareto optimal set of the instance is obtained from the application of the ε-constraint method. We vary both ε_{acc} and ε_{GI} in a fine grid. In this application example, we assume $\delta_1 = \delta_2 = 500$ as the number of intervals in which the domain of ε_{GI} and ε_{acc} is divided. We implement the models in GAMS 24.0.2 and use

ILOG CPLEX 12.5 on a computer equipped with an Intel Core i7-5500U 2.66 Ghz processor and 8 GB of RAM.

The application results of the proposed instance for $k \in \{3,4,5,6\}$ are shown in Table 2. For each value of k, the Pareto-frontier set is shown, in which each point in the set is identified as a scenario. In addition, Table 3 presents the values of objective functions for each scenario described in Table 2.

Table 2. Pareto frontier set for the listed k, scenario, zone, and other indicators in instance 1.

k	Scenario	ε_{GI}	ε_{acc} (Hospital Beds/ 1000 Inhabitants)	Coverage (Inhabitants)	Healthcare Supply (Hospital Beds by Zone)										Total (Hospital Beds)
					1	2	3	4	5	6	7	8	9	10	
3	3–1	0.707400	2.78100	315,742	0	0	0	0	400	0	395	250	0	0	1045
	3–2	0.401325	1.94052	597,625	0	0	400	0	0	0	362	0	399	0	1161
	3–3	0.401325	1.79220	633,727	0	0	347	0	0	400	0	0	400	0	1147
	3–4	0.330100	1.32870	708,572	400	0	0	0	0	350	0	297	0	0	1047
	3–5	0.212675	1.25454	803,882	400	0	0	0	0	0	250	0	0	400	1050
	3–6	0.214600	1.24218	807,832	400	0	0	400	0	0	250	0	0	0	1050
	3–7	0.166475	0.97026	915,367	0	0	400	0	0	0	251	0	0	394	1045
4	4–1	0.301225	1.94052	727,037	0	0	400	0	252	0	362	0	399	0	1413
	4–2	0.201125	1.79220	820,057	0	0	344	0	0	400	337	0	400	0	1481
	4–3	0.237700	1.32870	837,984	400	0	0	0	250	298	250	0	0	0	1198
	4–4	0.110650	1.25454	915,367	0	0	250	0	0	0	250	0	275	400	1175
	4–5	0.151075	1.25454	933,294	400	0	0	0	250	0	252	0	0	400	1302
	4–6	0.153000	1.24218	937,244	400	0	0	400	250	0	251	0	0	0	1301
	4–7	0.114500	0.97026	1,044,779	0	0	0	0	250	0	252	0	400	400	1302
5	5–1	0.337800	1.80456	729,037	0	250	343	0	0	263	0	0	400	400	1656
	5–2	0.339725	1.92198	729,037	0	262	394	0	0	297	0	0	399	400	1752
	5–3	0.341650	1.94052	729,037	0	269	400	0	0	285	0	0	399	400	1753
	5–4	0.343575	2.14446	729,037	352	0	0	377	0	260	0	0	400	310	1699
	5–5	0.345500	2.21244	729,037	372	0	0	399	0	276	0	0	400	309	1756
	5–6	0.106800	1.79220	949,469	0	0	343	0	250	400	336	0	399	0	1728
	5–7	0.052900	1.25454	1,044,779	0	0	302	0	250	0	250	0	250	400	1452
6	6–1	0.339725	2.49054	729,037	371	0	267	397	0	274	0	0	250	400	1959
	6–2	0.245400	2.17536	858,449	362	0	400	388	282	268	0	0	0	309	2009
	6–3	0.247325	2.21244	858,449	372	0	400	400	287	277	0	0	0	308	2044
	6–4	0.145300	2.10738	915,367	341	0	0	366	0	253	0	393	400	309	2062
	6–5	0.147225	2.13210	915,367	348	0	0	374	0	258	0	400	400	309	2089
	6–6	0.158775	1.34724	949,469	250	0	306	0	278	400	0	400	250	0	1884
	6–7	0.160700	1.39050	949,469	250	0	250	0	278	400	400	0	324	0	1902
	6–8	0.162625	1.43994	949,469	250	0	250	0	277	400	400	0	343	0	1920
	6–9	0.164550	1.48938	949,469	250	0	363	0	278	400	400	0	250	0	1941
	6–10	0.166475	1.79220	949,469	0	0	400	0	252	400	250	250	400	0	1952
	6–11	0.047125	1.25454	1,044,779	310	0	0	250	260	0	374	0	400	399	1993
	6–12	0.087550	1.32870	1,044,779	400	274	0	400	252	400	363	0	0	0	2089

Where GI: Gini index.

Table 3. Objective values in the listed scenario, zone, and other indicators for instance 1.

Scenario	Coverage	GI $^{(**)}$	Potential Spatial Accessibility (Hospital Beds by Zone/1000 Inhabitants)										MPSA $^{(*)}$
			1	2	3	4	5	6	7	8	9	10	
3–1	315,742	0.70740	0	0	0	0	3.0909	0	3.4616	3.4616	0	0	3.0909
3–2	597,625	0.40001	1.9426	1.9426	1.9426	0	0	0	1.9428	1.9428	1.9426	0	1.9426
3–3	633,727	0.40132	1.8162	1.8162	1.8162	0	0	1.7983	0	0	1.8162	1.7983	1.7983
3–4	708,572	0.32976	1.3342	0	1.3342	0	0	1.5735	1.5939	1.5939	1.3342	1.5735	1.3342
3–5	803,882	0.21166	1.3342	0	1.3342	1.2589	0	1.2589	1.3417	1.3417	1.3342	1.2589	1.2589
3–6	807,832	0.21394	1.3342	1.2434	1.3342	1.2434	0	0	1.3417	1.3417	1.3342	1.2434	1.2434
3–7	915,367	0.16646	0.9725	0.9725	0.9725	1.2400	0	1.2400	1.3471	1.3471	0.9725	1.2400	0.9725
4–1	727,037	0.30021	1.9426	1.9426	1.9426	0	1.9473	0	1.9428	1.9428	1.9426	0	1.9426
4–2	820,057	0.20089	1.8089	1.8089	1.8089	0	0	1.7983	1.8086	1.8086	1.8089	1.7983	1.7983
4–3	837,984	0.23756	1.3342	0	1.3342	0	1.9318	1.3397	1.3417	1.3417	1.3342	1.3397	1.3342
4–4	915,367	0.11063	1.2765	1.2765	1.2765	1.2589	0	1.2589	1.3417	1.3417	1.2765	1.2589	1.2589
4–5	933,294	0.15089	1.3342	0	1.3342	1.2589	1.9318	1.2589	1.3524	1.3524	1.3342	1.2589	1.2589
4–6	937,244	0.15295	1.3342	1.2434	1.3342	1.2434	1.9318	0	1.3471	1.3471	1.3342	1.2434	1.2434
4–7	1,044,779	0.11421	0.9725	0.9725	0.9725	1.2589	1.9318	1.2589	1.3524	1.3524	0.9725	1.2589	0.9725

Table 3. *Cont.*

Scenario	Coverage	GI (**)	Potential Spatial Accessibility (Hospital Beds by Zone/1000 Inhabitants)										MPSA (*)
			1	2	3	4	5	6	7	8	9	10	
5–1	729,037	0.33775	1.8065	2.4422	2.4422	1.8946	0	2.4413	0	0	2.4422	2.4413	1.8065
5–2	729,037	0.33971	1.9281	2.5943	2.5943	1.9251	0	2.5941	0	0	2.5943	2.5941	1.9251
5–3	729,037	0.34162	1.9426	2.6266	2.6266	1.9429	0	2.5402	0	0	2.6266	2.5402	1.9426
5–4	729,037	0.34350	2.1466	2.1445	2.1466	2.1476	0	2.1445	0	0	2.1466	3.3165	2.1445
5–5	729,037	0.34451	2.2133	2.2129	2.2133	2.2128	0	2.2133	0	0	2.2133	3.4536	2.2128
5–6	949,469	0.10677	1.8041	1.8041	1.8041	0	1.9318	1.7983	1.8033	1.8033	1.8041	1.7983	1.7983
5–7	1,044,779	0.05128	1.3421	1.3421	1.3421	1.2589	1.9318	1.2589	1.3417	1.3417	1.3421	1.2589	1.2589
6–1	729,037	0.33972	2.4945	2.4911	2.4945	2.4930	0	2.4907	0	0	2.4945	3.7248	2.4907
6–2	858,449	0.24536	2.1800	2.1787	2.1800	2.1786	2.1791	2.1773	0	0	2.1800	3.3835	2.1773
6–3	858,449	0.24614	2.2133	2.2160	2.2133	2.2128	2.2177	2.2147	0	0	2.2133	3.4581	2.2128
6–4	91,536	0.14529	2.1099	2.1103	2.1099	2.1102	0	2.1099	2.1092	2.1092	2.1099	3.2476	2.1092
6–5	91,536	0.14629	2.1333	2.1351	2.1333	2.1351	0	2.1324	2.1467	2.1467	2.1333	3.2950	2.1324
6–6	949,469	0.15877	2.1857	1.3518	2.1857	0	2.1482	1.7983	2.1467	2.1467	2.1857	1.7983	1.3518
6–7	949,469	0.16062	2.2295	1.3956	2.2295	0	2.1482	1.7983	2.1467	2.1467	2.2295	1.7983	1.3956
6–8	949,469	0.16261	2.2756	1.4418	2.2756	0	2.1405	1.7983	2.1467	2.1467	2.2756	1.7983	1.4418
6–9	949,469	0.16450	2.3243	1.4904	2.3243	0	2.1482	1.7983	2.1467	2.1467	2.3243	1.7983	1.4904
6–10	949,469	0.16634	1.9451	1.9451	1.9451	0	1.9473	1.7983	2.6834	2.6834	1.9451	1.7983	1.7983
6–11	1,044,779	0.04704	2.0065	1.7497	2.0065	2.0329	2.0091	1.2557	2.0072	2.0072	2.0065	2.0329	1.2557
6–12	1,044,779	0.08755	1.3342	1.9401	2.0309	1.9401	1.9473	1.7983	1.9482	1.9482	2.0309	3.0417	1.3342

Where (*) MPSA: Minimum potential spatial accessibility; GI (**): Gini index.

The results of the proposed instance for $k = 6$ show that the Pareto optimal set corresponds to the mesh sets (Figure 3a), where the Pareto-frontier set is made up of twelve non-dominated solutions (Figure 3b). Figure 4 presents the location of healthcare centers when $k = 6$ for different coverage values, where a black square represents the healthcare center.

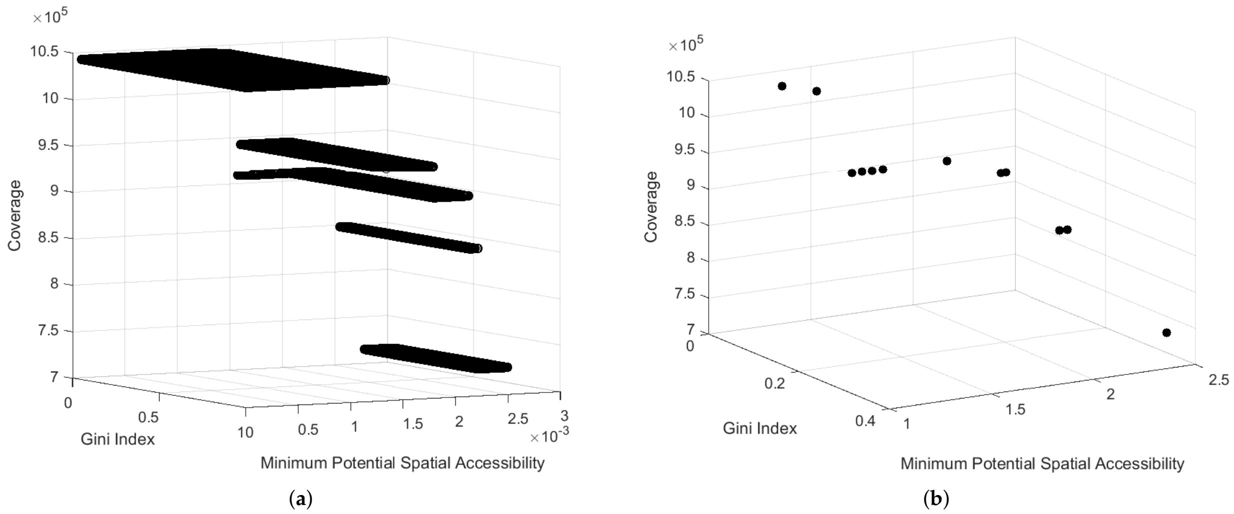
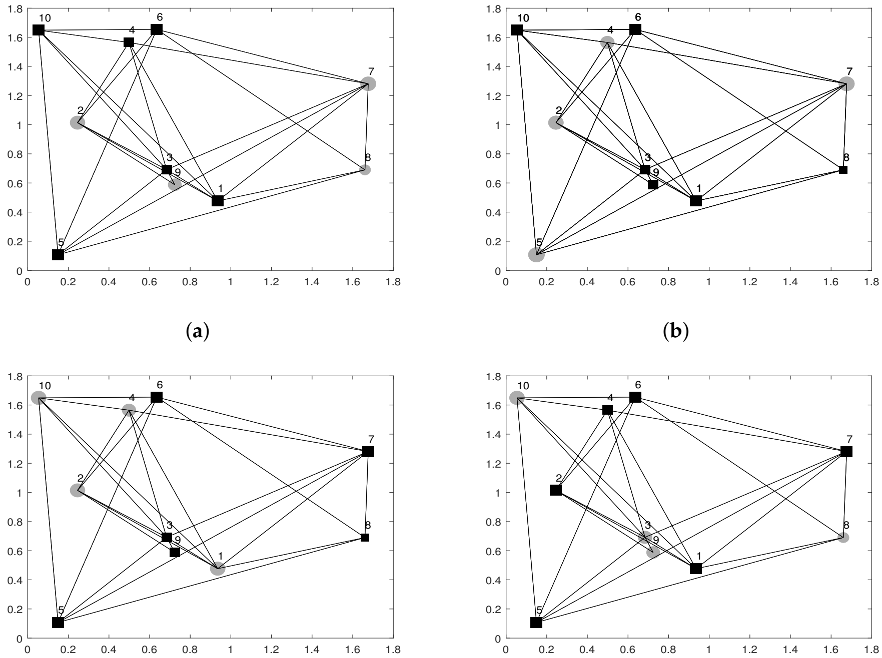


Figure 3. Solution for $k = 6$ with: (**a**) Pareto optimal set; and (**b**) Pareto-frontier set; where black planes to the left indicate the mesh sets and black dots to the right are the non-dominated solutions.

An aspect worth noting is that the increase in the number of healthcare centers to be installed does not necessarily increases the level of system coverage and additionally it could increase inequity. Indeed, taking the results of Table 2, note that the maximum coverage level obtained when $k = 3$ (scenario 3–7) is less compared to that obtained when $k = 4$ (scenario 4–7). From there, when $k = 5$ and $k = 6$, the maximum coverage level remains constant because it is the maximum value that can be achieved in instance 1. Nevertheless, when increasing from $k = 5$ to $k = 6$ healthcare facilities, the Gini index might improve, although the minimum potential spatial accessibility decreases (see Table 3 scenarios 5–7 and 6–11). However, the opposite effect happens if we compare scenarios 5–7 and 6–12 in Table 3.

The location of the facilities and healthcare center supply play an essential role in reducing inequity. From Figure 5c,d, note that, in both cases, the same level of coverage exists. Nonetheless, the inequity level in scenario 6–12 is greater compared to scenario 6–11 (Table 3). Thus, we can see that, although a zone has the largest population, a health center should not necessarily be installed (Figure 5b), even if its sanitary supply is the largest allowed when it is installed (see scenario 6–12 on Table 2 and Figure 5d).

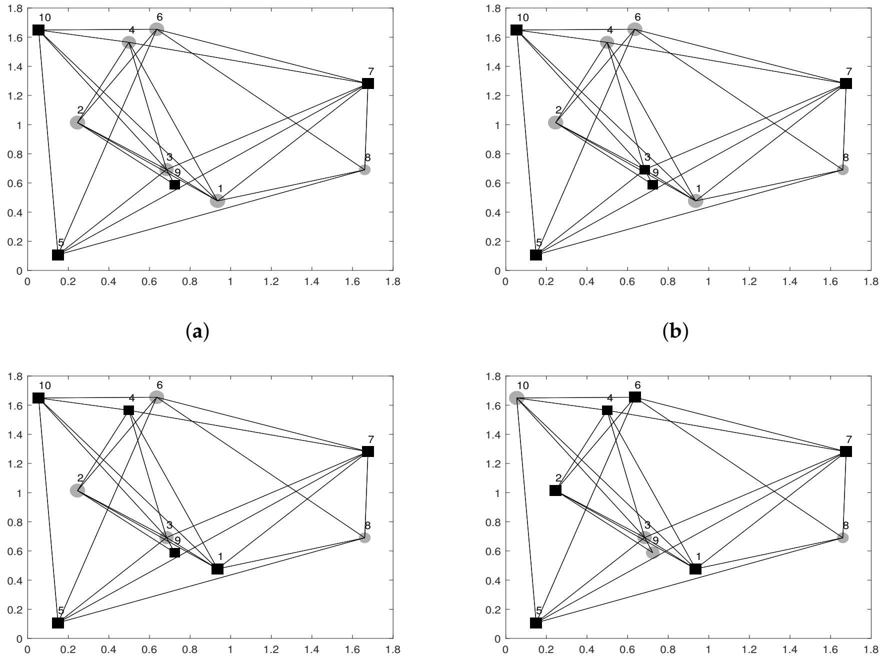


(**a**) (**b**) (**c**) (**d**)

Figure 4. Spatial location of healthcare centers for different coverage levels with $k = 6$ and: (**a**) 858,449 inhabitants; (**b**) 915,367 inhabitants; (**c**) 949,469 inhabitants; and (**d**) 1,044,779 inhabitants; where 1 to 10 are the geographical zones, with ● and ■ sketching a zone discrete center and a healthcare center location, respectively.

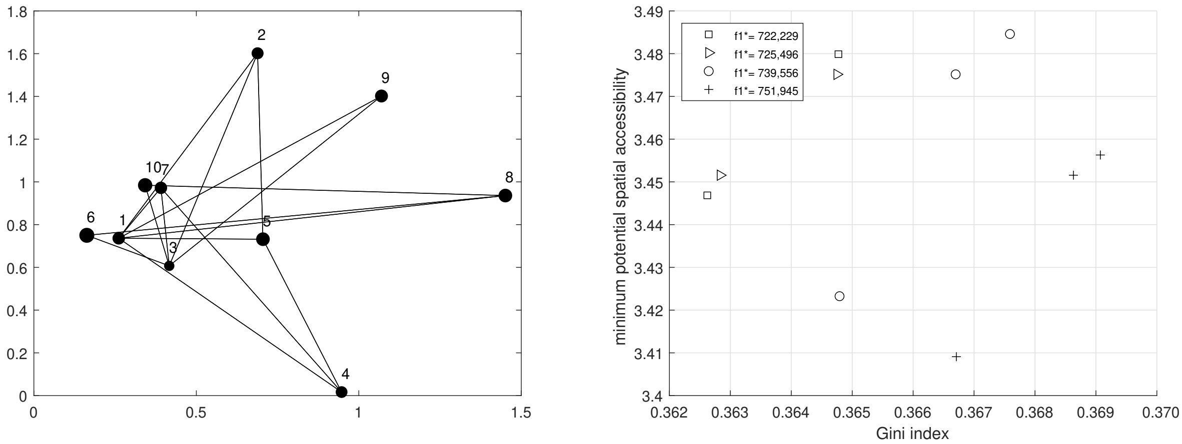
A similar pattern can be seen in another random instance (Figure 6a), which was generated according to Algorithm 2 with its parameters such as instance 1. To solve that, we applied Algorithm 1. By analyzing a non-dominated Pareto solution set (Figure 6b), observe that, when $k = 7$, the coverage level increases from 722,229 (square marks) to 751,945 (plus marks), producing a decrease in both inequity and accessibility of healthcare systems (increasing in the Gini index) and also a decrease in the minimum potential accessibility in some cases (see Tables 4 and 5).

An exhaustive analysis of the Pareto-frontier sets shows that, in some scenarios, an increase in the total healthcare center supply of the system does not necessarily cause a decrease in inequity. For example, the total healthcare center supply in scenario 5–1 is 1723 hospital beds (Table 4) and its Gini index is 0.439 (Table 5). However, in scenario 5–7, its Gini index decreases to 0.289 although its total healthcare center supply goes down to 1549 hospital beds. A similar situation can be seen in scenarios 6–1 and 6–8 reported in Tables 4 and 5. In addition, we can observe that, although the total healthcare center supply of the system is the same, its efficiency (measured by coverage) and inequality (measured by the Gini index) are different, which clearly highlights the need to adequately manage the available resources (see all scenarios where $k = 7$ in Tables 4 and 5).



(**a**) (**b**) (**c**) (**d**)

Figure 5. Spatial location of healthcare centers with a maximum coverage level (1,044,779) for different values of k in instance 1 with: (**a**) $k = 3$; (**b**) $k = 4$; (**c**) $k = 5$; and (**d**) $k = 6$; where 1 to 10 are the geographical zones, with ● and ■ sketching a zone discrete center and a healthcare center location, respectively.



(**a**) (**b**)

Figure 6. A randomly generated instance 2 for: (**a**) spatial zone distribution; and (**b**) non-dominated Pareto solutions when $k = 7$; where 1 to 10 are the geographical zones, with ● sketching a discrete center whose diameter is proportional to the demand for healthcare centers in the indicated zone; and the indicated optimum coverage level f_1^*.

Table 4. Pareto frontier set for the listed k, scenario, zone, and other indicators in instance 2.

k	Scenario	ε_{GI}	ε_{acc} (Hospital Beds/ 1000 Inhabitants)	Coverage (Inhabitants)	Healthcare Supply (Hospital Beds by Zone)										Total (Hospital Beds)
					1	2	3	4	5	6	7	8	9	10	
3	3–1	0.701650	3.09000	305,751	0	315	0	0	0	0	0	400	365	0	1080
	3–2	0.641980	1.88490	429,267	0	0	0	0	400	0	0	332	296	0	1028
	3–3	0.555362	1.76130	522,146	0	0	400	0	0	0	395	0	0	329	1124
	3–4	0.557287	1.77366	522,146	0	0	400	0	0	0	400	0	0	347	1147
	3–5	0.430247	1.52646	637,082	0	0	308	0	324	400	0	0	0	0	1032
	3–6	0.432172	1.59444	637,082	0	0	344	0	339	400	0	0	0	0	1083
	3–7	0.434097	1.67478	637,082	0	0	384	0	355	400	0	0	0	0	1139
	3–8	0.436022	1.70568	637,082	0	0	400	0	365	400	0	0	0	0	1165
	3–9	0.476444	1.21128	657,008	0	0	0	0	378	0	0	250	0	400	1028
	3–10	0.416773	1.06914	751,945	400	0	0	0	0	0	0	250	0	378	1028
4	4–1	0.601558	3.09000	390,898	0	274	0	264	0	0	0	355	317	0	1210
	4–2	0.441796	1.73658	637,082	297	0	0	0	250	400	0	0	0	342	1289
	4–3	0.443721	1.86636	637,082	345	0	0	0	250	400	0	0	0	384	1379
	4–4	0.445646	1.98996	637,082	391	0	0	0	250	400	0	0	0	400	1441
	4–5	0.349404	1.70568	751,945	0	0	400	0	362	400	0	250	0	0	1412
	4–6	0.405224	1.21128	759,482	0	0	0	0	400	0	0	250	250	400	1300
	4–7	0.351329	1.06914	854,419	399	0	0	0	0	0	0	250	250	400	1299
5	5–1	0.437947	2.50290	637,082	297	0	400	0	362	400	0	0	0	264	1723
	5–2	0.439872	2.65122	637,082	352	0	400	0	362	400	0	0	0	312	1826
	5–3	0.441796	2.77482	637,082	400	0	398	0	361	400	0	0	0	354	1913
	5–4	0.347479	1.97142	751,945	386	0	0	0	250	400	0	254	0	400	1690
	5–5	0.349404	2.00850	751,945	400	0	0	0	256	398	0	262	0	400	1716
	5–6	0.268561	1.70568	854,419	0	0	400	0	370	400	0	250	250	0	1670
	5–7	0.289734	1.06914	942,833	399	250	0	0	0	0	0	250	250	400	1549
6	6–1	0.457195	2.71302	637,082	308	0	293	0	400	400	250	0	0	400	2051
	6–2	0.345554	2.6265	725,496	388	250	337	0	336	400	0	0	0	344	2055
	6–3	0.339780	2.28042	751,945	282	0	305	0	323	400	0	262	0	250	1822
	6–4	0.341705	2.53998	751,945	312	0	398	0	361	400	0	292	0	277	2040
	6–5	0.343630	2.5647	751,945	379	0	317	0	328	400	0	295	0	336	2055
	6–6	0.255087	2.0085	854,419	400	0	0	0	257	398	0	263	250	400	1968
	6–7	0.195417	1.70568	942,833	0	250	400	0	370	400	0	250	250	0	1920
	6–8	0.218515	1.06914	1,027,980	400	250	0	250	0	0	0	255	250	400	1805

Table 4. *Cont.*

k	Scenario	ε_{GI}	ε_{acc} (Hospital Beds/ 1000 Inhabitants)	Coverage (Inhabitants)	Healthcare Supply (Hospital Beds by Zone)										Total (Hospital Beds)
					1	2	3	4	5	6	7	8	9	10	
7	7–1	0.362878	2.33604	722,229	250	0	250	250	353	397	250	0	0	305	2055
	7–2	0.364803	2.36694	722,229	250	0	250	250	360	320	250	0	0	375	2055
	7–3	0.362878	2.32986	725,496	250	250	250	0	354	400	250	0	0	301	2055
	7–4	0.364803	2.36076	725,496	250	250	250	0	359	333	250	0	0	363	2055
	7–5	0.364803	2.31132	739,556	250	0	250	0	348	361	250	0	271	325	2055
	7–6	0.366728	2.36076	739,556	250	0	250	0	359	306	250	0	256	384	2055
	7–7	0.368652	2.37312	739,556	250	0	250	0	361	294	250	0	250	400	2055
	7–8	0.366728	2.29278	751,945	250	0	250	0	345	316	250	290	0	354	2055
	7–9	0.368652	2.33604	751,945	250	0	250	0	354	277	250	274	0	400	2055
	7–10	0.370577	2.34222	751,945	250	0	250	0	355	280	250	270	0	400	2055
	7–11	0.245463	2.26188	854,419	318	0	250	0	299	396	0	260	250	282	2055
	7–12	0.178093	1.86636	942,833	349	250	0	0	250	400	0	250	250	306	2055
	7–13	0.139597	1.55736	1,027,980	0	250	322	250	333	400	0	250	250	0	2055

Where GI: Gini index.

Table 5. Objective values in the listed scenario, zone, and other indicators for instance 2.

Scenario	Coverage	GI (**)	Potential Spatial Accessibility (Hospital Beds by Zone/1000 Inhabitants)										MPSA (*)
			1	2	3	4	5	6	7	8	9	10	
3–1	305,751	0.70152	0	3.5628	0	0	0	0	0	3.4824	3.5619	0	3.4824
3–2	429,267	0.64197	1.8874	0	0	0	1.8874	0	0	2.8904	2.8885	0	1.8874
3–3	522,146	0.55535	1.7637	0	2.7600	0	0	2.7600	1.7637	0	0	1.7624	1.7624
3–4	522,146	0.55719	1.7763	0	2.8271	0	0	2.8271	1.7763	0	0	1.8169	1.7763
3–5	637,082	0.43019	2.1187	0	1.5307	0	1.5288	1.5307	1.5307	0	0	1.5307	1.5288
3–6	637,082	0.43212	2.2584	0	1.5997	0	1.5996	1.5997	1.5997	0	0	1.5997	1.5996
3–7	637,082	0.43408	2.4105	0	1.6763	0	1.6751	1.6763	1.6763	0	0	1.6763	1.6751
3–8	637,082	0.43581	2.4883	0	1.7069	0	1.7223	1.7069	1.7069	0	0	1.7069	1.7069
3–9	657,008	0.47587	1.7836	0	1.2113	0	1.7836	1.2113	0	2.17651	0	1.2113	1.2113
3–10	751,945	0.41531	1.0736	0	2.2183	0	1.0736	1.1447	1.0736	2.17651	0	1.1447	1.0736
4–1	390,898	0.60029	0	3.0991	0	3.1005	0	0	0	3.09064	3.09347	0	3.0906
4–2	637,082	0.44171	1.9768	0	2.7737	0	1.9768	1.9765	1.7380	0	0	1.9765	1.7380
4–3	637,082	0.44372	2.1056	0	3.0297	0	2.1056	2.1037	1.8668	0	0	2.1037	1.8668
4–4	637,082	0.44561	2.2291	0	3.2016	0	2.2291	2.1522	1.9903	0	0	2.1522	1.9903
4–5	751,945	0.34917	2.4742	0	1.7069	0	1.7081	1.7069	1.7069	2.1765	0	1.7069	1.7069
4–6	759,482	0.40464	1.8874	0	1.2113	0	1.8874	1.2113	0	2.1765	2.4396	1.2113	1.2113
4–7	854,419	0.35122	1.0709	0	2.2823	0	1.0709	1.2113	1.0709	2.1765	2.4396	1.2113	1.0709
5–1	637,082	0.43793	3.2714	0	3.3036	0	2.5053	2.5064	2.5041	0	0	2.5064	2.5041
5–2	637,082	0.43987	3.4190	0	3.5965	0	2.6529	2.6517	2.6517	0	0	2.6517	2.6517
5–3	637,082	0.44143	3.5393	0	3.8487	0	2.7770	2.7751	2.7767	0	0	2.7751	2.7751
5–4	751,945	0.34747	2.2157	0	3.1882	0	2.2157	2.1522	1.9769	2.2113	0	2.1522	1.9769
5–5	751,945	0.34931	2.2816	0	3.2211	0	2.2816	2.1475	2.0098	2.2810	0	2.1475	2.0098
5–6	854,419	0.26844	2.5119	0	1.7069	0	1.7459	1.7069	1.7069	2.1765	2.4396	1.7069	1.7069
5–7	942,833	0.28905	1.0709	2.8276	2.2823	0	1.0709	1.2113	1.0709	2.1765	2.4396	1.2113	1.0709

Table 5. *Cont.*

Scenario	Coverage	GI (**)	Potential Spatial Accessibility (Hospital Beds by Zone/1000 Inhabitants)										MPSA (*)
			1	2	3	4	5	6	7	8	9	10	
6–1	637,082	0.45680	3.9066	0	4.1714	0	2.7141	3.3447	2.9601	0	0	2.7133	2.7133
6–2	725,496	0.34553	3.2723	2.8276	3.6694	0	2.6268	2.6280	2.6277	0	0	2.6280	2.6268
6–3	751,945	0.33977	2.8651	0	3.0390	0	2.2810	2.2821	2.2819	2.2810	0	2.2821	2.2810
6–4	751,945	0.34168	3.3031	0	3.3794	0	2.5408	2.5419	2.5405	2.5422	0	2.5419	2.5405
6–5	751,945	0.34361	3.1721	0	3.5827	0	2.5649	2.5655	2.5652	2.5683	0	2.5655	2.5649
6–6	854,419	0.25506	2.2863	0	3.2211	0	2.2863	2.1475	2.0098	2.2897	2.4396	2.1475	2.0098
6–7	942,833	0.19534	2.5119	2.8276	1.7069	0	1.7459	1.7069	1.7069	2.1765	2.4396	1.7069	1.7069
6–8	1,027,980	0.21850	1.0736	2.8276	2.2850	2.9361	1.0736	1.2113	1.0736	2.2200	2.4396	1.2113	1.0736
7–1	722,229	0.36262	3.4468	0	3.6386	2.9361	2.3367	2.9676	2.7150	0	0	2.3362	2.3362
7–2	722,229	0.36477	3.4799	0	3.6695	2.9361	2.3697	2.9985	2.5339	0	0	2.3671	2.3671
7–3	725,496	0.36283	3.4516	2.8276	3.6336	0	2.3414	2.9626	2.7220	0	0	2.3312	2.3312
7–4	725,496	0.36475	3.4752	2.8276	3.6637	0	2.3650	2.9927	2.5645	0	0	2.3613	2.3613
7–5	739,556	0.36479	3.4233	0	3.6145	0	2.3131	2.9435	2.6303	0	2.6446	2.3121	2.3121
7–6	739,556	0.36670	3.4752	0	3.6638	0	2.3650	2.9928	2.5009	0	2.4982	2.3614	2.3614
7–7	739,556	0.36759	3.4846	0	3.6841	0	2.3744	3.0130	2.4727	0	2.4396	2.3816	2.3744
7–8	751,945	0.36671	3.4091	0	3.5965	0	2.2989	2.9255	2.5245	2.5247	0	2.2941	2.2941
7–9	751,945	0.36863	3.4516	0	3.6441	0	2.3414	2.9730	2.4327	2.3855	0	2.3417	2.3414
7–10	751,945	0.36907	3.4563	0	3.6511	0	2.3461	2.9801	2.4398	2.3506	0	2.3487	2.3461
7–11	854,419	0.24537	2.7432	0	3.1177	0	2.2644	2.2642	2.2638	2.2636	2.4396	2.2642	2.2636
7–12	942,833	0.17798	2.1164	2.8276	2.8043	0	2.1164	1.8675	1.8776	2.1765	2.4396	1.8675	1.8675
7–13	1,027,980	0.13847	2.1880	2.8276	1.5575	2.9361	1.5713	1.5575	1.5575	2.1765	2.4396	1.5575	1.5575

Where (*) MPSA: Minimum potential spatial accessibility; GI (**): Gini index.

4.3. Computational Burden

To evaluate the computational performance of the proposed procedure, we apply Algorithm 2 to generate instances of different sizes. According to the inputs given in this algorithm, in all the generated instances, we considered $\text{HBI}^{\min} = 1$, $\text{HBI}^{\max} = 2$, $h = 2$, $p = 0.2$, $l_1 = 0.2$, $l_2 = 0.2$, and $k \in \{5, 6, 7, 8, 9\}$.

Table 6 reports the results of the application of our model in instances with 10, 15, 20 and 50 zones. Considering in each case different values of k, healthcare facilities are installed. Each of these instances was run within a maximum runtime of 2 days (48 h).

Table 6. Computational study results for the indicated n and k.

n	***k***	**Runtime (in Hours)**
10	5	9.33
	6	7.25
	7	8.30
	8	8.01
	9	9.20
15	5	11.15
	6	13.14
	7	12.42
	8	11.23
	9	12.31
20	5	19.76
	6	18.25
	7	21.24
	8	22.28
	9	23.77
30	5	32.21
	6	33.02
	7	36.12
	8	36.53
	9	37.10
50	5	>48
	6	>48

As can be seen in Table 6, the implementation of the model is adequate in small instances (less than 20 zones). However, it is evident that, in instances that exceed 50 zones, more than two days will be required to obtain the Pareto frontier, so the use of meta-heuristic-based approaches could be considered.

5. Conclusions, Limitations, and Future Research

In this section, we provide the conclusions of our study, as well as some of its limitations, and ideas for further investigation.

5.1. Concluding Remarks

In this paper, we addressed the location problem for healthcare centers under a multi-criteria approach that embeds efficiency and equity concerns. For this purpose, we have adopted the coverage of healthcare systems as an efficiency criterion, and the Gini index as a proxy variable that measures the inequity level on accessibility to healthcare centers.

The results obtained in this study show that the application of an efficiency criterion in the location of facilities does not guarantee equity in resource distribution and vice versa. Therefore, the adopted approach in this study was appropriate. Note that the spatial distribution influenced the coverage level of the healthcare system. Nevertheless, this is not decisive for reducing inequity in accessible healthcare centers since the distribution of healthcare center supply must be incorporated into the decision-making process. In fact, there could be situations with a low total healthcare center supply (see Table 2, scenario 6–11) that are more equitable than a scenario with a high healthcare center supply (see Table 2, scenario 6–12) for the same coverage level. However, the minimum potential spatial accessibility would be sacrificed. In contrast, a decrease in the total healthcare center supply could increase inequality and decrease minimum potential spatial accessibility, although it might increase the coverage of the healthcare system.

For example, results from the numerical application showed 10 non-dominated Pareto solutions with a coverage level of 915,367 inhabitants (circle marks in Figure 7), which performed poorly both in equity and minimum potential spatial accessibility compared to non-dominated Pareto solutions with a coverage level of 949,469 inhabitants (plus marks). Therefore, the performance of the suggested actions depends to a great extent on the number of facilities to be installed and on the total healthcare center supply as well. This gives us an opportunity to expand the study by incorporating budget constraints. In this context, from the results reported in Tables 2 and 3, we have noticed that an increase in total healthcare center supply could increase the coverage of the system. Nonetheless, this does not necessarily reduce inequity. For example, scenario 4–4 had a total healthcare center supply of 1175 hospital beds distributed in zones 3, 7, 9 and 10, which covered 915,367 inhabitants and produced a Gini index of 0.111. In addition, increasing the total healthcare center supply up to 1302 hospital beds (scenario 4–5) distributed into zones 1, 5, 7 and 10, the coverage increases up to 933,294 inhabitants, but the Gini index was 0.151. However, this fact cannot be considered a rule since there are cases in which an increase in the total healthcare center supply produced an increase in the coverage and a decrease in the Gini index, that is, the inequality declined. For example, this happened when comparing scenarios 3–1 and 3–2.

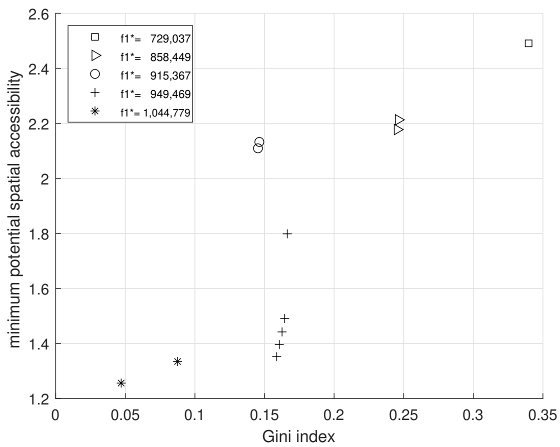


Figure 7. Spatial location of health centers for the indicated optimum coverage level f_1^* with $k = 6$.

5.2. Limitations and Future Research

A limitation of the present investigation is that the simulation model could not be used outside the limits of the parameters studied in which it was constructed. This may cause a false appreciation of the multi-objective linear programming problem when considering other scenarios. Undoubtedly, future research should analyze other efficiency criteria that contribute to reducing the inequality gap. In addition, some variants to this problem can be addressed in future research, such as the inclusion of decay functions in the expression measuring potential spatial accessibility. This could reflect a gradual supply capture according to the distance that separates a demand zone from each healthcare center. It is also important to consider restrictions on the budgetary availability of resources to install healthcare centers, which may vary according to geographical zones.

Since the method we use to solve our problem, that is, the ε-constraint method, deals with mixed integer linear problems at each iteration, it is important to mention that a study of techniques that employ, for example, reformulations or relaxations [2,54,55] might be applied to improve the performance of the method and its execution times.

Finally, due to what was stated in Section 4.3 about runtimes, the utilization of heuristic techniques is something that should be explored, even when there is no guarantee of obtaining an optimal solution when this approach is applied.

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