

An equivalent-circuit model for MEMS electrostatic actuator using open-source software Qucs

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Abstract: We report an equivalent circuit model for MEMS (microelectromechanical systems) electrostatic actuator using open-source circuit simulator Qucs (quite universal circuit simulator). Electrostatic force, equation of motion, and Kirchhoff's laws are implemented by using the EDD (equation defined device) function of Qucs. Mathematic integral operation in the equation of motion is interpreted into electrical circuits by using an ideal electrical capacitor that read input signal as current and returns accumulation result in terms of voltage. Seamless multi-physics mixed signal simulation between micro mechanics and electronics has become possible on the single platform of the circuit simulator.

Keywords: MEMS, micro actuator, electrostatic, simulation, Qucs

Classification: Micro- or nano-electromechanical systems

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1 Introduction

Microelectromechanical system (MEMS) actuators on the early stages of history were stand-off transducers, and simple analytical co-solver models were used to predict their electromechanical characteristics [1]. In contrast, MEMS actuators today are mostly integrated in the microelectronics to compose a complex system such as accelerometer [2], digital micromirror array [3], and silicon microphone [4], and a system level simulator is indispensable to design the entire MEMS in a top-down manner. Apart from the commercially available MEMS simulation tools [5], which are usually a black box to users, we have newly adopted electrical circuit simulation software Qucs (Quite Universal Circuit Simulator) [6] for the first time to implement an equivalent-circuit model for electrostatic micro actuator. Qucs is open-source software subject to the GPL (general public license) agreement, and users have free access to the device models and the source codes.

In this paper, we present the detail diagram of a Qucs-compatible equivalent circuit model for electrostatic micro actuator that can deal with both small-amplitude oscillation as well as the large-amplitude hysteresis including the electrostatic pull-in and release. Thanks to the seamless interface between electrostatic mechanism and electrical circuit, multi-physics mixed signal simulation has become possible on the single platform of Qucs.

2 Equivalent-circuit model for electrostatic actuator

2.1 Governing equations of electrostatic actuator

Figure 1 illustrates a schematic of parallel-plate electrostatic actuator that is suspended with a spring of elastic constant k and electrically biased at voltage V through an electrical resistance R . Three sets of simultaneous equations are used to describe the plate's motion x :

$$\text{Electrostatics: } \begin{cases} F = \frac{1}{2}\epsilon_0 \frac{S}{(g-x)^2} V_A^2, & (1a) \\ Q = \epsilon_0 \frac{S}{g-x} V_A, & (1b) \end{cases}$$

$$\text{Equation of Motion: } \ddot{x} = (F - c \cdot \dot{x} - k \cdot x) \frac{1}{m}, \quad (2)$$

$$\text{Kirchhoff's Laws: } \begin{cases} V = R \cdot i + V_A, & (3a) \\ i = \frac{d}{dt}(C \cdot V_A), & (3b) \end{cases}$$

where nomenclature is listed in Table I. In a system-level description model such as Matlab/Simulink™ [7], the variables in the equations are processed in a block diagram using the Laplace transform of $1/s$ as an integrator. Nevertheless, co-solving electrostatic actuator with a peripheral electrical circuit is not a straightforward task but extra labor is needed to interpret electrical circuits into block diagrams using add-on software [8, 9].

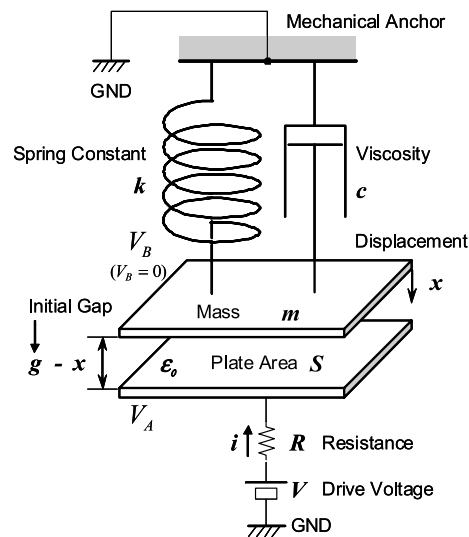


Fig. 1. Analytical model of electrostatic micro actuator.

2.2 Qucs equivalent circuit model

We newly adopt an electrical simulator QuCS to develop an equivalent circuit model for electrostatic actuator. Using a circuit simulator as a platform is beneficial in making a seamless interface between micromechanical component and electronics. Figure 2 shows the QuCS circuit diagram in which three

Equations (1) ~ (3) are implemented. Qucs has a built-in current-source called EDD (equation defined device), which has been originally prepared to emulate an I - V curve of arbitrary form. The output current of an EDD is described by an equation using the input voltages as variables, and a total maximum of eight EDD blocks can be stacked to make a cross-referenced EDD column to handle multiple voltage-input and current-output.

Table I. List of parameters.

Parameter	Symbol	Value used in Simulation
Dielectric Constant	ϵ_0	8.85×10^{-12} F/m
Young's Modulus (silicon)	E	160 GPa
Density (silicon)	ρ	2.33×10^3 kg/m ³
Plate Displacement (position)	x	---
Electrostatic Force	F	See Equation (1a)
Induced Electrical Charge	Q	See Equation (1b)
Plate Area	S	$(200 \mu\text{m})^2$
Suspension Width	w	10 μm
Suspension/Plate Thickness	h	2 μm
Suspension Length	l	100 μm
Spring Constant	$k = E (w h^3 / l^3) \times 4$	51.2 N/m
Plate Mass	$m = \rho S h$	9.32×10^{-10} kg
Viscosity	c	1×10^{-7} N s / m
Electrostatic Initial Gap	g	2 μm
Stopper Position	$LIM = g \times 0.8$	1.6 μm
Capacitance	$C = \epsilon_0 S / (g - x)$	---
Drive Voltage to Fixed Plate	V_A	Max 20 V
Drive Voltage to Movable Plate	V_B	GND 0 V
Input Resistance	R	50 Ω or 2 M Ω

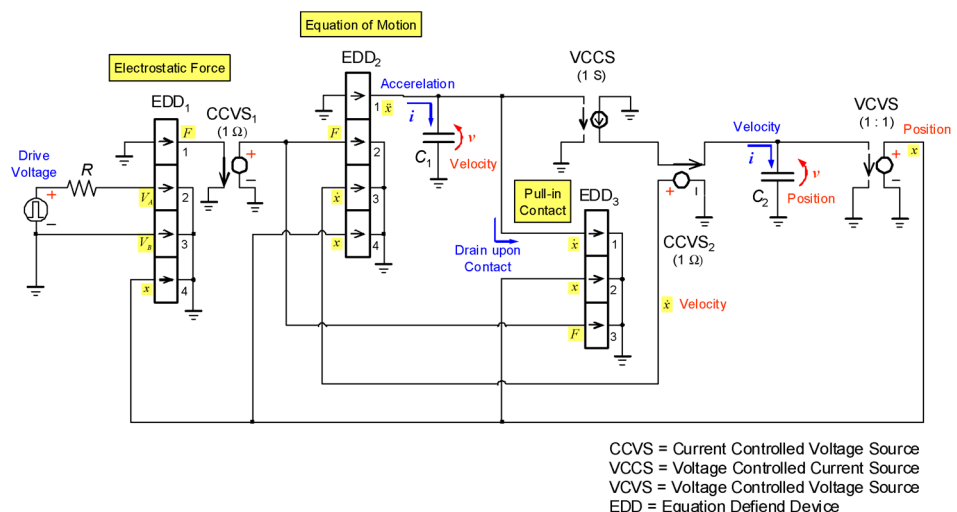


Fig. 2. Qucs equivalent circuit model for electrostatic micro actuator.

EDD₁ in Figure 2 is an implementation of electrostatic force F and induced electrical charge Q . Block 1 and 2 of EDD₁ receive drive voltages V_A and $V_B (= 0)$ applied to the actuator's plates. Also, block-3 takes the value

of the plate position x , which will be calculated in the main block of the diagram. These three inputs are read as voltages V_2 , V_3 , and V_4 , respectively. In the EDD configuration properties, the electrostatic force (2a) is rewritten as electrical current:

$$I_1 = (1/2) * \epsilon_0 * S * ((V_2 - V_3) \wedge 2) / ((g - V_4) \wedge 2). \quad (4)$$

The index numbers correspond to each block stacked in the EDD column. The constants and dimensional parameters are passed as arguments in the Qucs schematic file. Note that we only look at the unit-less values of the voltages and currents in a similar manner as we use electrical operational amplifier in analog computation. Because the EDD has been designed to accept only voltage as input, the output current of block 1 (that is force F) is converted into voltage by using a CCVS (current controlled voltage source) without changing the signal magnitude, i.e., the conversion resistance is set to be 1Ω .

Electrical charge induced on the actuator's plates can also be modeled in the identical EDD block. In the configuration properties of EDD₁, we set the following equations:

$$\begin{cases} Q_2 = \epsilon_0 * S * (V_2 - V_3) / (g - V_4), & (5a) \\ Q_3 = \epsilon_0 * S * (V_3 - V_2) / (g + V_4). & (5b) \end{cases}$$

The electromechanical coupling expressed by the Kirchhoff's laws on voltage (3a) and current (3b) are thus interpreted by using the charge induction (5a) and (5b) and by simply wiring a resistance R placed at the input port of the actuator.

2.3 Equation of motion

EDD₂ in Figure 2 represents the equation of motion to calculate the acceleration \ddot{x} . Block 2, 3, and 4 receive the electrostatic force F , plate velocity \dot{x} , and position x , respectively. By using these variables represented by voltages, we rewrite the equation of motion (2) in the Qucs EDD form of electrical current:

$$I_1 = (V_2 - c * V_3 - k * V_4) / m. \quad (6)$$

Mathematic integration operation from acceleration \ddot{x} to velocity \dot{x} is electrically interpreted by using a capacitor C_1 that accumulates the electrical current \ddot{x} and returns velocity \dot{x} in terms of voltage. Another integration from velocity \dot{x} to position x is made in the same manner by using capacitor C_2 . Both capacitor values are set to be 1 Farad. They may look excessive for a realistic value of electronic component but Qucs computation can simply process it as a numeral. The VCCS (voltage controlled current source) inserted between C_1 and C_2 is to decouple the two integration components. Another CCVS before C_2 is to tap the current signal to synthesize the velocity \dot{x} in terms of voltage. Integration result of C_2 is also read as voltage by using

a VCVS (voltage controlled voltage source) of a conversion factor of unity. These values of \dot{x} and x are passed to EDD₁ and EDD₂ through the feedback loop for the referenced calculation of electrostatic force and acceleration.

2.4 Electrostatic Pull-in process

Parallel-plate electrostatic micro actuators are known to have the pull-in effect, where electrostatic attractive force exceeds the mechanical restoring force of the suspending spring, and that the movable plate is spontaneously pulled into physical contact with the opposite electrode. Mechanical stoppers are usually inserted in the electrostatic gap to avoid electrical short circuit. The pull-in phenomenon provides electrostatic actuator with hysteresis in the displacement-voltage characteristic, which is used in low-voltage latching and digital motion limiter [10]. Despite the importance of the pull-in phenomenon, it is usually disregarded in most numerical simulation model due to the difficulty in translating the large non-linear behavior of non-conservation of energy into analysis [11, 12].

In our Qucs equivalent model, we simply judge the mechanical contact by using an “if”-clause in the description of EDD₃, and realize the $\dot{x} = 0$ condition by promptly discharging C_1 through a bypassing route. The virtual drain current I_1 from C_1 is programmed to be

$$\text{abs}(V_2) \geq LIM \ \&\& \ \text{abs}(V_3) > k * V_2 * V_1 * DF : 0, \quad (7)$$

where $\text{abs}(V_2)$ returns the absolute value of V_2 , for instance. The if-clause before the question mark is to test if the position $x(= V_4)$ is greater than the stopper position LIM (which is passed as an argument) and that the electrostatic force $F(= V_3)$ is greater than the mechanical restoring force $k * V_2$ at the present position $x(= V_2)$. If these two conditions are simultaneously fulfilled, the capacitor C_1 is discharged by the drain current of $I_1 = V_1 * DF$. If not fulfilled, on the other hand, the charge is maintained by virtually choking the drain current to be zero. Here, DF is an artificially introduced factor called a “drain factor” in our program to adjust the prompt discharge of C_1 . We empirically found an appropriate value of DF to be 10^8 for most micro actuators whose resonant frequency is in a kHz range. An appropriate value of DF brings a stable contact position during the pull-in. If any drift of contact position is seen in the simulation results, one is advised that DF should be chosen in a range from 10^4 to 10^{10} by a trial-and-error manner. The drain factor DF remains to be an only experimentally determined parameter at this moment, and it will be replaced with theoretical one in our future improvement.

3 Verification

We compare the Qucs equivalent model with a known analytical model for verification. The solid curve in Figure 3 (a) shows the result of pseudo-static Qucs simulation (displacement-voltage) for an electrostatic parallel plate actuator; the parameters used in the simulation are listed in Table I. The circles

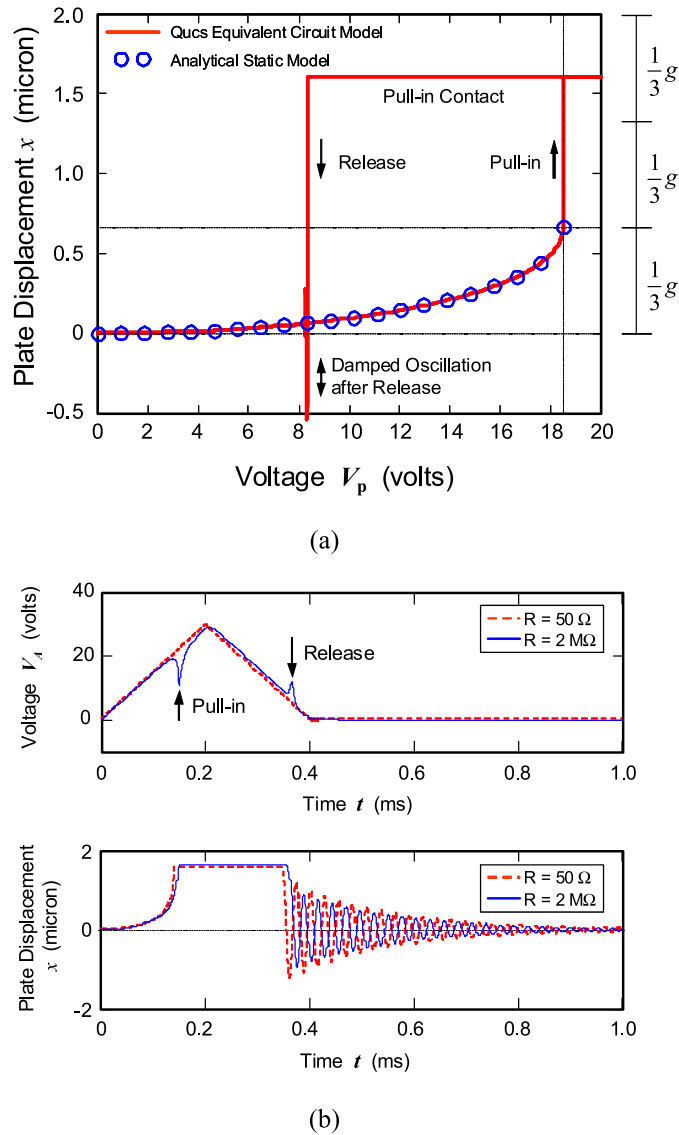


Fig. 3. Simulation results: (a) Qucs simulation result (solid curve) and static analytical model based on Runge-Kutta method (circles). (b) Dynamic analysis of Qucs simulation model with different input resistance values: $50\ \Omega$ (dashed red curves) and $2\ \text{M}\Omega$ (solid blue curves).

in the same plot are the results of numerical simulation of the static model by using the 4th-order Runge-Kutta method [13]. As predicted by the analytical model, electrostatic pull-in is seen at the $1/3$ of the initial gap g , and the value of pull-in voltage agreed well with the theoretical value of $\sqrt{8kg^3/(27\epsilon_0 S)}$. While the static analytical model can handle only the region before the pull-in, our equivalent circuit model is good through all regions including the electrostatic pull-in, contact, and release. The displacement-voltage curve has been re-plot from a temporal sequence, and thus the damped oscillation after the contact-release is shown superposed at $V = 8.2$.

Figure 3 (b) shows the results of dynamic behavior calculation for a given triangular drive voltage. We compare two different input resistance R to see

the effect of electromechanical coupling implemented by the charge induction through Equation (5a) and (5b). For a small input resistance of $R = 50 \Omega$, the voltage measured on the plate V_A (dashed red curve) follows the drive voltage V , and the pull-in occurs exactly at the 1/3-gap condition. For a large input resistance of $R = 2 \text{ M}\Omega$, on the other hand, the plate voltage V_A (solid blue curve) is observed to fall momentarily at the pull-in timing due to the finite time required to recharge the actuator through the resistance against the sudden increase of capacitance. A sharp increase of voltage is also seen at the release timing for the same reason. The resistance R and the input capacitance of the actuator make an LPF (low pass filter) for the actuation voltage, and thus the response of the actuator is delayed compared with the $R = 50 \Omega$ model, as shown in the displacement-time plot. From these observations, we have concluded that the Qucs equivalent model can be used to describe the static and dynamic model of electrostatic micro actuators.

4 Conclusion

We used open-source circuit simulation software Qucs to implement an equivalent circuit model for electrostatic micro actuator and demonstrated multiphysics mixed signal simulation of electrostatics and electrical circuits. The equivalent circuit model is under improvement to make up sub-circuit models of mass, spring, and electrostatic actuator such that each could be dealt with like an electrical component on the circuit simulator. Sub-circuit models, variations of electrostatic actuator models, and their application models are reported elsewhere.