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AN ESSAY ON DISCRETE FOUNDATIONS FOR PHYSICS*

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ABSTRACT

We base our theory of physics and cosmology on the five principles of finiteness, discreteness, finite computability, absolute non-uniqueness, and strict construction. Our modeling methodology starts from the current practice of physics, constructs a self-consistent representation based on the *ordering operator calculus* and provides rules of correspondence that allow us to test the theory by experiment. We use *program universe* to construct a growing collection of bit strings whose initial portions (*labels*) provide the quantum numbers that are conserved in the *events* defined by the construction. The labels are followed by *content strings* which are used to construct event-based finite and discrete coordinates. On general grounds such a theory has a limiting velocity, and positions and velocities do not commute. We therefore reconcile quantum mechanics with relativity at an appropriately fundamental stage in the construction. We show that events in different coordinate systems are connected by the appropriate finite and discrete version of the Lorentz transformation, that 3-momentum is conserved in events, and that this conservation law is the same as the requirement that different paths can "interfere" only when they differ by an integral number of deBroglie wavelengths.

The labels are organized into the four levels of the *combinatorial hierarchy* characterized by the cumulative cardinals $3, 10, 137, 2^{127} + 136 \simeq 1.7 \times 10^{38}$. We justify the identification of the last two cardinals as a first approximation to $\hbar c/e^2$ and $\hbar c/Gm_p^2 = (M_{Planck}/m_p)^2$ respectively. We show that the quantum numbers associated with the first three levels can be rigorously identified with the quantum numbers of the first generation of the standard model of quarks and leptons, with color confinement and a first approximation to weak-electromagnetic unification. Our cosmology provides an event horizon, a zero velocity frame for

the background radiation, a fireball time of about 3.5×10^6 years, about the right amount of visible matter, and 12.7 times as much "dark matter". A preliminary calculation of the fine structure spectrum of hydrogen gives the Sommerfeld formula and a correction to our first approximation for the fine structure constant which leads to $1/\alpha = 137.0359674....$ We can now justify the earlier results $m_p/m_e = 1836.151497...$ and $m_\pi/m_e \lesssim= 274$. Our estimate of the weak angle is $sin^2\theta_{Weak} = \frac{1}{4}$ and of the Fermi constant $\frac{G_F}{m_p^2} = \frac{1}{\sqrt{2}(256)^2}$. Our finite particle number relativistic scattering theory should allow us to systematically extend these results.

ETERIS PARIBUS, CAVEAT LECTOR

1. INTRODUCTION

Physics is an *experimental* science that relies on *counting*. For instance, Galileo counted the number of (equal "by construction", and presumably by experiential comparison) intervals a ball rolling down a smooth groove in an inclined plane passed while water flowed into a receptacle during the same interval. He then *counted* the number of (equal "by construction", and presumably by experiential comparison) weights which would balance the water content of the receptacle. We could now say that from these experiments he proved the invariance of the local acceleration due to gravity. We start our discussion by insisting that finite and discrete counting is the proper starting point for any fundamental theory of physics.

Physicists have long known that counting is not enough to achieve consensus. Sometimes the counts differ under the "same" circumstances; the scatter in the results is not always easy to understand "after the fact", let alone to allow for before. So a "theory of errors" has grown up, which is partly pragmatic, and more recently relies on "statistical theory". As a first rate experimental physicist has remarked "you can't measure errors". Current practice in high energy physics tries to estimate errors by simulating the experimental setup on a computer and making a finite number of pseudo-random runs to compare with the "real time" data. In this specific practice, the estimate of errors is also based on finite counting.

Until recently the legacy inherited by physicists from continuum mathematics, which some of their most illustrious predecessors had helped to create, dominated thinking about "measurement" and "errors". In particular, continuum models for "probability" — which can never be tested in a finite amount of time — dominated the theory of errors just as Euclidean geometry and its multidimensional extensions dominated the model space into which physical theories were thrown. Bridgman made a heroic effort to get out of this trap (but never went so far as to abandon the continuum). Eddington attacked the problem from a point of view historically connected to the approach adopted here, but was never able to carry any substantial body of physicists along with him. Much that is relevant to our work was going on in minority views about the foundations of mathematics at the same time. We leave the investigation of that background to others.

Computer scientists do not have the luxury of relying on "existence proofs" which they cannot demonstrate on a computer within budget and within a deadline. They have evolved a new science, which differs in significant ways from conventional continuum mathematics, in order to meet their specific needs. It is from this background that the most productive work in the theory presented here has arisen. We leave that aspect of the scientific revolution we hope to help initiate to other papers. This paper is addressed to *physicists.*

In the next chapter we review those aspects of the historical practice of physics which we find most relevant to our enterprise. Cosmology relies on particle physics for most of its quantitative "observational" data. So Chapter 3 sketches the aspects of "elementary particle physics" we feel need to be modeled accurately if our alternative theory is to be taken seriously by particle physicists and cosmologists. The basic methodology for our alternative approach is presented in Chapter 4. What in an older terminology might be called the "formal structure", and in ours is called the "representational framework" follows in Chapter 5. Here we find that many of the *ad hoc* attempts to fit relativity and quantum mechanics into the historically established framework — attempts which some distinguished physicists still find fall far short of their conceptual requirements —can be replaced by a finite and discrete alternative. Chapter 6 compares our results with experience. Chapter 7 steps back and looks at what we have and have not accomplished as part of a research program that has been going on for some of us for over three decades. It is here that we try to justify, or at least explain, the claims made in the abstract.

2. THE HISTORICAL PRACTICE OF PHYSICS

2.1. Scale Invariant Physics

Physics was a minor branch of philosophy until the seventeenth century. Galileo started "physics" in the contemporary sense. He emphasized both mathematical deduction and precise experiments. Some later commentators have criticized his *a priori* approach to physics without appreciating his superb grasp of the experimental method which he created, – including reports of his experiments that still allow replication of his accuracy using his methods. He firmly based physics on the *measurement* of *length* and *time*; from our current perspective he established the uniform acceleration of bodies falling freely near the surface of the earth.

A century later, Newton entitled what became the paradigm for "classical" physics *The Mathematical Principles of Natural Philosophy*, recognizing the roots that physics has in both disciplines. He also was a superb experimentalist[#1]. To a greater extent than Galileo, Newton had to create "new mathematics" in order to express his insight into the peculiar connection between experience, formalism, and methodology that still remains the core of physics. To length and time, he

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#1 Consider, for instance, his demonstration that gravitational and inertial mass are proportional using pendulum bobs of the same weight and exterior size and shape but composed of different materials. Eötvos had to rely on two centuries of technological development to construct a better technique; some physicists are still struggling to go beyond Eötvos (Cf., *Physics Today,* July 1988).

added the concept of *mass* in both its inertial and its gravitational aspect, and tied physics firmly to astronomy through universal gravitation. For philosophical reasons he introduced the concepts of absolute space and time, and thought of actual measurements as some practical approximation to these concepts.

It is often thought that Einstein's special relativity rejects the concept of absolute space-time, until it is smuggled back in through the need for boundary conditions in setting up a general relativistic cosmology. In fact, the concept of the homogeneity and isotropy of space used by Einstein to analyse the meaning of distant simultaneity in the presence of a limiting signal velocity is very close to Newton's absolute space and time. What Einstein shows is rather that it is possible to use local, consequential time to *replace* Newton's formulation of the concept. This was pointed out to HPN by David McGoveran[1] in the context of our fully finite and discrete approach to the foundations of physics, and our derivation of the Lorentz transformations using "information- transfer" velocities that are rational fractions of the limiting velocity. This same analysis shows that in a discrete physics, the universe has to be multiply connected. The space-like separated "supraluminal" correlations predicted by quantum mechanics — and recently demonstrated experimentally to the satisfaction of many physicists — can be anticipated for spins and for *any* set of countable degrees of freedom more impoverished that those needed to *specify* a "material object".

2.2. Breaking scale invariance

Nineteenth century physicists saw the triumph of the electromagnetic field theory. "Classical" physics was still firmly based on historical units of mass, length and time. Quantized atomic masses had been discovered by chemists early in the century, and quantized charges related to them by Faraday, but physicists only began to take them seriously after the discovery of the electron and "canal rays". Prior to the discovery of Planck's constant and the recognition that mass and charge were separately quantized, together with the understanding that the propagation velocity in free space required by Maxwell's equations was a *universal* limiting velocity, classical physics provided no way to question *scale invariance.*

Quantum theory and relativity were born at the beginning of this century. Quantum mechanics did not take on its current form until nearly three decades of work had passed. Although one route to quantum mechanics (that followed by deBroglie and Schrödinger) started from the continuum relativistic wave theory, the currently accepted form breaks the continuity by an interpretive postulate due to von Neumann sometimes called "the collapse of the wave function".

Criticism of this postulate as conceptually inconsistent with the time reversal invariant continuum dynamics of wave mechanics has continued ever since. This criticism was somewhat muted for a while by the near consensus of physicists that Bohr had "won" the Einstein-Bohr debate and the continuing dramatic technical successes of quantum mechanics. Scale invariance is gone because of the quantized units of mass, action and electric charge. These specify in absolute (i.e. countable) terms what is meant by "small". Explicitly $r_{Bohr} = \hbar^2/m_e e^2$ (with m_e the electron mass) specifies the atomic scale, $\lambda\!\!\!^{-}_{Compton} = (e^2/\hbar c) r_{Bohr} = \hbar/m_e c$ specifies the quantum electrodynamic scale, and the "classical electron radius " $e^2/m_e c^2 = (e^2/\hbar c)\lambda\!\!\!^{-}_{Compton} \simeq 2\hbar/m_\pi c \simeq 14\hbar/m_p c$ specifies the nuclear scale; here m_p is the proton mass, and $m_\pi \lesssim 2 \times 137 m_e$ is the pion mass. The elementary particle scale $\hbar/m_p c$ is related to the gravitational scale by $\lambda\!\!\!^{-}_G = (G\hbar^3/c)^{\frac{1}{2}} = \hbar/M_{Planck}c = (Gm_p^2/\hbar c)^{\frac{1}{2}}(\hbar/m_p c)$

The expanding universe and event horizon specify what is meant by "large". Here the critical numbers any *fundamental* theory must explain are: "Mass" of the universe as about $3 \times 10^{76} m_p$ — or at least ten times that number if one includes current estimates for "dark matter" ; "Size" of the universe or *event horizon* — naively the maximum radius which any signal can attain (or arrive from) transmitted at the limiting signal velocity c during the Age of the universe; "Age" of the universe as about 15 billion (15×10^9) years. Backward extrapolation using contemporary "laws of physics" to the energy and matter density when the radiation breaks away from the matter (size of the "fireball") is consistent with the observed $2.7^o K$ cosmic background radiation. The cosmological parameters are numerically related to the elementary particle scale by the fact that the visible mass in the currently observable universe is approximately given by $M_{vis.U} \simeq (\hbar c/Gm_p^2)^2 m_p$, and that linearly extrapolating backward from the fireball to the "start of the big bang" gives a time $T_{fireball} \simeq (\hbar c/Gm_p^2)(\hbar/m_p c^2)$ =3.5 million years. Any theory which can calculate all these numbers has a claim to being a fundamental theory.

For a while it appeared that reconciliation between quantum mechanics and special relativity would resist solution; the uncertainty principle and second quantization of classical fields gave an infinite energy to each point in space-time! During World War II, Tomonaga, and afterwards Schwinger and Feynman, developed formal methods to manipulate away these infinities and obtain finite predictions in fantastically precise agreement with experiment. Recently the non–Abelian gauge theories have made everything calculated in the "standard model" finite. Weinberg asserted at the Schrödinger Centennial in London that there is a practical consensus – but no proof – that second quantized field theory is the *only* way to reconcile quantum mechanics with special relativity. He also pointed out that the finite energy due to vacuum fluctuations is then 10^{120} too large compared to the cosmological requirements; the universe should wrap itself up and shut itself down almost as soon as it starts expanding.[2] Anyone who is willing to swallow this camel will still have to strain at the gnats of inflationary scenarios and the difficulties associated with including strong gravitational fields in any quantum theory. Continued attention to foundations seems fully justified.

2.3. EVENTS AND THE VOID: AN ALTERNATIVE?

The concept on which most of elementary particle physics rests has moved a long way from the mass points of post-Newtonian dynamics. For us, a paraphrase of the concept used by Eddington[3] is more useful:

a **PARTICLE** is

"A conceptual carrier of conserved 3-momentum and quantum numbers between events."

This definition applies in the practice of elementary particle physics (1) in the high energy particle physics laboratory, and in the theoretical formulations of either (2) second quantized field theory or (3) analytic S-matrix theory. In (1), the experimental application, "events" refer to the detection of any finite number of incoming and outgoing "particles" localized in macroscopic space-time volumes called "counters", or some conceptual equivalent. In (2), "events" start out as loci in the classical Minkowski 4-space continuum at which the "interaction Lagrangian" acting on a state vector creates and destroys particle states in Foch space. Since this prescription, naively interpreted, assigns an infinite energy and momentum to each space-time point, considerable formal manipulation and reinterpretation is needed before these "events" can be connected to laboratory practice. In (3), "events" refer to momentum-energy space "vertices" which conserve 4-momentum in "Feynman diagrams". These diagrams were originally introduced in context (2) as an aid to the systematic calculation of renormalized perturbation theory. S-matrix theory makes a strong case for viewing continuous "space-time" as a mathematical artifact produced by Fourier transformation. Like any scattering theory, or any application of second quantized field theory to discrete and finite particle scattering experiments, S-matrix theory includes rules for connecting amplitudes calculated from these diagrams directly to laboratory practice (1).

An alternative approach to the problem, which is beginning to be called *discrete* and/or *combinatorial* physics, is focused on constructed, discrete processes.[4] A quick characterization of the theory could be:

Chance, events and the void suffice.

Only discrete, finitely computable, combinatorial connectivities are allowed. But the multiple connectivity and the indistinguishables which our approach requires introduce subtle differences from conventional mathematics and physics at an early stage.

The connectivity can be provided by a growing universe of bit strings. The "events" generated by *Program Universe*[5] connecting bit strings use part of the string, called the *label*, to define conserved quantum numbers. The bits not used as the label can be called the *content* of the string. Looking back to our first pass at what we mean by **a PARTICLE**, one "carrier" connecting events is the evolving labeled string. Yet, once the universe is mature enough to allow a meaningful discrimination between label and content, there are many strings with the same label. The arbitrary evolution connects shorter to longer strings, or for strings of the same length connects two "3-events" to form a "4-event". Thanks to the "counter paradigm", this discrete model also accounts for the conservation of 3-momentum and quantum numbers consistent with laboratory practice (1), and serves the same purposes as the theoretical constructs in second quantized relativistic field theory (2) or analytic S-Matrix theory (3).

The next chapter reviews the language — supposedly adequate to describe the relevant phenomena — which elementary particle physicists employ, and expect others to employ when entering on their turf.

3. CONTEMPORARY PARTICLE PHYSICS

3.1. Yukawa Vertices

With the exception of *gluons*, the standard model of quarks and leptons starts from conventional interaction Lagrangians of the form $g\overline{\psi}\psi\phi$, into which various finite spin,isospin,... operators may be inserted. Here g is the "coupling constant" which measures the strength of the interaction relative to the mass terms in the "free particle" part of the Langrangian, ψ ($\overline{\psi}$) is a fermion (anti-fermion) second quantized field and ϕ a boson or "quantum" field. All three fields can be expanded in terms of creation and destruction operators acting on "particle" or "Foch space" states which in the momentum space representation contain separate 4-momentum vector variables for each fermion, anti-fermion or quantum.

Fortunately for us, in one of the first successful efforts to tame the infinities in this theory, Feynman introduced a diagrammatic representation for the terms generated by such interaction Lagrangians in a perturbation theory (powers of g) expansion of the terms which need to be calculated and summed in order to obtain a finite approximation for the predictions of the theory. These "Feynman Diagrams" have taken on a life of their own; they bring out the symmetries and conservation laws of the theory in a graphic way. This can be a trap, particularly if they are reified as representing actual happenings in space time. If used with care they can short circuit a lot of tedious calculation (or suggest viable additional approximations) and provide a powerful aid to the imagination.

In the usual theory, Minkowski continuum space-time is assumed and any interaction Lagrangian is constructed to be a Lorentz scalar. Consequently the quantum theory conserves 4-momentum at each 3-vertex. Here one must use care because of the uncertainty principle. If 4-momentum is precisely specified, the uncertainty principle prevents any specification of position; the vertex can be anywhere in space-time. This is the most obvious way in which the extreme non-locality of quantum mechanics shows up in quantum field theory. If we use a momentum space basis, we can still have precise conservation laws at the vertices for which the masses have an unambiguous interpretation. In practical applications of the theory momentum cannot be precisely known; quasi-localization is allowed as long as the restrictions imposed by the uncertainty principle are respected. In a careful treatment, this is called "constructing the wave packet"; actually specifying this construction requires some care as can be seen, for instance, by consulting Goldberger and Watson's *Collision Theory.* In practice, one usually works entirely in momentum space, knowing that the orthogonality and completeness of the basis states will allow the construction of appropriate wave packets in any currently encountered experimental situation. We have made a start on the corresponding construction in our theory [4].

Although 4-momentum conservation is insured in the conventional treatment, this is not the end of the problem. For a particle state with energy ϵ and 3-momentum $\vec{p}$ the formalism insures that $\epsilon^2 - \vec{p} \cdot \vec{p} = M^2$; here M is any invariant with the dimensions of mass and need not correspond to the rest mass of the particle m. In the usual perturbation theory this is simply accepted. The dynamical calculations are made "off mass shell", and the specialization to physical values appropriate to the actual laboratory situations envisaged is reserved to the end of the calculation. S-Matrix theory sticks closer to experiment in that all amplitudes refer to physical (realizable) processes with all particles "on mass shell". The dynamics is then supposed to be supplied by imposing the requirement of flux conservation ("unitarity") — a non-linear constraint — and by relating particle and anti-particle processes through "crossing". The analytic continuation of the amplitudes for distinct physical processes which gives dynamical content to the equations then makes S-matrix theory into a self-consistent or "bootstrap" formalism. There is no known way to guarantee a solution of this bootstrap problem short of including an infinite number of degrees of freedom – if then; of course, it is also well known that there is no known way to prove that quantum field theory possesses any rigorous solutions of physical interest. One must have recourse to finite approximations which may or may not prove adequate to particular situations.

The finite particle number scattering theory[6–9] keeps all particles on mass shell, and hence has 3-momentum conservation at 3-vertices. This theory insures unitarity for finite particle number systems by the form of the integral equations; these also provide the dynamics. The uncertainty principle is respected because of the "off-energy-shell" propagator, as it is in non-relativistic scattering theory; the approximation is the truncation in the number of particulate degrees of freedom.

If we put the "Feynman Diagrams" of the second quantized perturbation theory on mass shell, we can talk about 3-vertices and 4-events using a common language for all three theories. The rules are easy to state, particularly if we do so in the (cosmological) "zero momentum frame". We are justified in using any description derived from this cosmological frame within the mathematical models because we have restricted ourselves to free particle, mass shell kinematics. We can use a corresponding statement in the laboratory because this frame is empirically specified as the frame at rest with respect to the $2.7^o K$ background radiation. Then the Poincaré invariance of the theories allows us to go from this description to any other convenient Galilean frame.

As we show in Section 5.4, the 3-momenta at a 3-vertex add to zero. Diagrammatically we have three "vectors" which are "incoming" or "outgoing". By putting one of each together we obtain the generic 2-2 channel 4-event, as indicated in figure 1. Clearly for 4-events the total momentum of the two outgoing lines has to equal the total momentum of the two incoming lines, but the plane of the outgoing 3-event can be any plane obtained by rotating the outgoing vectors in the planar figure about the axis defined by the single line connecting them. By associating quantum numbers with each line, we can extend this description of 3-momentum conservation in Yukawa vertices and the 4-events constructed from them to the conservation of quantum numbers which "flow" along the lines.

The idea of associating physical particles with the lines as carriers of both momentum and quantum numbers which comes from this pictorial representation is almost irresistible. The reader is warned once again to resist this temptation. The diagram is in 3+1 momentum-energy space and *not* in space-time. In fact if we insist on interpreting it as a space-time diagram representing the motion of particles, the quantum theory will blow up! It will force us to assign an infinite energy and momentum to each point of that space time, and simplicity of interpretation becomes elusive.

Once we have this picture in hand, "crossing" is easy to define: if reversing a line and at the same time changing all its quantum numbers to their negatives does not alter the conservation laws, the new diagram also represents a possible physical process. The "particle" whose quantum numbers are the negative of another is called its "anti-particle". So "crossing" can also be stated as the requirement that the reversal of a reference direction and the simultaneous change from particle to antiparticle represents another possible physical process. The manner in which a single diagram in which momenta and quantum numbers add to zero at a general 3-vertex generates emission, absorption, decay and annihilation vertices by successive applications of this rule is illustrated in figure 2. The manner in which a single diagram in which momenta and quantum numbers add to zero in a general 4-event generates six physically observable processes by this rule is illustrated in figure 3.

Since one of the quantum numbers ("spin") is a pseudovector, "time reversal" — which changes the sign of velocity and hence the direction — is not the same as the "parity" operation which changes all coordinates to their negatives. In quantum electrodynamics or QED, the theory in which the diagrams originated, the quantum number which distinguishes particle from anti-particle is electric charge; these rules are a consequence of the "CPT invariance" of the theory. They generalize to other types of "charge", eg "color charge" in quantum chromodynamics (QCD). Spin is of great interest since it has a "space-time" significance as well as sharing the discrete, quantized character of other quantum numbers.

Before going on to the other quantum numbers, we note that the form of the Yukawa vertex couples the particle and anti-particle field in such a way that in the "time ordered" interpretation of the diagrams the number of fermions minus the number of anti-fermions is conserved; this is called the conservation of fermion number. Clearly the diagrams respect this conservation law; so far as we know f-number conservation, is followed in nature.

3.2. The Standard Model

The fermions encountered in nature fall into two classes: leptons and baryons. So far as we know to date, lepton number and baryon number are separately conserved. The lifetime for the decay of the proton into leptons and other particles has been shown to be greater than 10^{35} years; the experimental upper limit for the value depends on which decay mode was searched for. This fact has already ruled out many proposed schemes for "grand unification".

The existence of the enormous underground detectors constructed to test the hypothesis of proton decay had an unexpected payoff when two of them detected, "simultaneously", neutrino bursts from a supernova explosion 50,000 parsecs (1 parsec = 3.3 light-years) away. Individual neutrinos within the burst were cleanly resolved, but the time spread of the burst itself was so short that information about upper limits for the masses of the neutrinos could be obtained only by sophisticated statistical analysis. Although the time for the actual production of the neutrinos is supposed to be very short, the spread induced by the subsequent diffusion of the neutrinos out through the bulk of the star makes the calculation sensitive to the model used for calculating the explosion. Empirically, we can take the three types of neutrinos to be massless with an upper limit of 30 electron volts/c^2.

The quanta which couple via elementary Yukawa vertices in the standard model all have spin one. The earliest coupling explored in quantum field theory was the electromagnetic coupling between electrons (e^-), positrons (e^+) and the massless electromagnetic quanta; the theory, which can be extended to other charged fermions, is called quantum electrodynamics (QED). The masslessness of the electromagnetic quanta is imposed within the second quantized relativistic field theory by requiring the theory to be "gauge invariant". A lower limit to the mass of either fermions or quanta with specified quantum numbers defines a well understood experimental problem; if all such lower limits had to be finite, this would kill "gauge invariance". The requirement of gauge invariance is not compelling for us prior to some rough consensus as to what additional, independent tests (at an accuracy specified in advance) are relevant. We know of no proposed experimental program that could test gauge invariance within realistic error bounds; the concept of gauge invariance does not meet Popper's requirement. The upper limits on the mass of electromagnetic quanta are very good; empirically, we can assume photons to be massless.

The skepticism just implied makes our explanatory problem difficult. The current fashion in high energy elementary particle physics starts from "non-Abelian" gauge theories. Their broken "symmetries" generate "mass" from a "spontaneous breakdown of the vacuum". With care, this mechanism is claimed to be a guaranteed way to remove the infinities from a tightly constrained version of second quantized field theory. Without those constraints, which start from the necessity to get rid of the "classical" infinity of the e^2/r potential (infra-red divergence) and the infinity of energy-momentum at each space-time point forced on us classically by "point particles" and retained in the second quantized field theory in spite of the uncertainty principle (ultra-violet divergence), these theories are *prima facie* non-sensical. Self-consistency *within* the mathematical theory is disputed by some who take the "rigour" of continuum mathematics seriously.

Following a conventional route in a 4-dimensional formalism one runs into trouble because a massless photon with momentum has only two chiral states (γ_{LL} and γ_{RR}) while the formalism requires 4 components for a 4-vector. For a massive spin 1 "particle" (i.e something that can "carry" 3-momentum between two events in any coordinate system, and whose mass defines a rest system) there is no problem. The three states which quantum mechanics requires for spin 1 can be resolved along, against or perpendicular to the direction of motion, while the fourth component of the 4-vector is related to these three components "on shell" by the invariant mass. When the invariant mass is zero, we are left with only two chiral 3-momentum carrying states. For fermions this is no problem, once parity conservation is abandoned. But for spin 1 massless bosons, the "third" and "fourth" component of the "4-vector" have to combine to yield an undirected $1/r$ "coulomb potential" in a gauge invariant and manifestly covariant manner. In a classical theory with extended sources this was no problem because the transformation between the 4-vector notation and the "coulomb gauge" was always well defined, although coordinate system dependent. But in second quantized field theory achieving consistency between the classical substrate and the Feynman rules requires all kinds of technical artifices (indefinite metrics and the like). In a finite particle number theory, one can avoid some of these technical difficulties by always using transverse photons and the coulomb interaction in a well defined coordinate system, provided the (no longer manifest) "covariance" can be maintained. Of course this removes some of the (we believe superficial) formal simplicity of the "manifestly covariant" 4-vector formalism. Since the theory we have developed commits us to 3-momentum conservation as fundamental, this is a natural route for us to take.

Once this is understood, the particular crossing symmetric Yukawa vertices $e_L^-(Q = -e, s_h\hbar = -\frac{1}{2}\hbar)$, $e_R^-(Q = -e, s_h\hbar = +\frac{1}{2}\hbar)$ specifying massive leptonic QED for a single flavor (in this case e) coupled to $\gamma_{LL}, \gamma_{RR}, \gamma_c$ are given in figure 4. We note that for electromagnetic coupling charge and lepton number go together; the conservation law for one implies the conservation law for the other. We represent the combined conservation laws of $2s_h \in 0, \pm 1, \pm 2$ and $\ell = -Q/e \in 0, \pm 1$, by the vector states in a plane in figure 5. A Yukawa (QED) vertex requires three quantum number "vectors" consisting of a fermion, an antifermion and a quantum which add to zero, plus the temporally ordered processes derived from the fundamental diagram by crossing. The field theory notation for this QED coupling is[10] $-iQ\bar{e}\gamma_\lambda e A_\lambda$, with $Q^2/\hbar c = e^2/\hbar c \simeq 1/137$.

In contrast to the parity conserving electromagnetic vertices, the "weak" interactions violate parity conservation maximally. An easy way to represent this is to use a massless neutrino (ν_L), conventionally called "left handed". Consider an arrow in front of you with the head on the right. If you slip your right hand under the arrow to pick it up, your thumb will point in the same direction as the head; if you pick it up by slipping your left hand under the arrow, your thumb will point in the opposite direction to the head. The latter case is called "left-handed". By the Feynman rule the anti-neutrino $\bar{\nu}_L$ is then right-handed. The charged quantum which couples to the electron and neutrino is called W (the weak vector boson) and is also chiral, since in the zero momentum frame $e_L^- + \bar{\nu}_L \rightarrow W_{LL}^-$; in field theory notation the coupling is

$$-i(G_F M_W^2/\sqrt{2})^{\frac{1}{2}}\bar{\nu}\gamma_\lambda(1-\gamma_5)eW_\lambda$$

The Weinberg-Salam-Glashow "weak-electromagnetic unification" requires in addition to this electrically charged weak boson, which was a convenient way to parameterize the parity-nonconserving theory of β-decay, the neutral weak boson Z_0 responsible for "neutral weak currents". The reasons had to do initially with the removal of infinities from the theory, and go through a complicated sequence of arguments that predict, in addition, one or more scalar "Higgs bosons", for which there is at present no laboratory evidence. Since our theory is born finite and cannot produce the infinities of second quantized field theory, we have no need for these hypothetical particles in the first place. If they should be discovered (thanks to current efforts at many laboratories which are now consuming a large fraction of their experimental and computational resources), we will be faced with some difficult conceptual problems in our discrete theory. Fortunately, for the moment, we can ignore them, which makes our presentation of the conservation laws in the leptonic sector considerably simpler.

The coupling of the Z^0 to neutrinos is chiral and is given by

$$(-i/\sqrt{2})(G_F M_Z^2/\sqrt{2})^{\frac{1}{2}}\overline{\nu}\gamma_\lambda(1-\gamma_5)eZ_\lambda$$

The coupling to electrons is more complicated because it brings in the "weak angle" θ_W that distinguishes the coupling to left and right handed electrons in the following way:

$$(-i/\sqrt{2})(G_F M_Z^2/\sqrt{2})^{\frac{1}{2}}\overline{e}\gamma_\lambda[R_e(1+\gamma_5)+L_e(1-\gamma_5)]eZ_\lambda$$

Here $R_e = 2sin^2\theta_W$, $L_e = 2sin^2\theta_W - 1$. If $sin^2\theta_W = 1/4$, which is not too bad an approximation to the experimental value, Z couples to electrons like a heavy gamma ray, except that it is a pseudovector rather than a vector. The mixing angle is not independent of the masses of the weak bosons, because

$$M_W sin\theta_W = [\pi e^2/\hbar c G_F \sqrt{2}]^{\frac{1}{2}} = 37.3 Gev/c^2 = M_Z sin\theta_W cos\theta_W$$

Since there were estimates of the weak mixing angle available before the discovery of the weak bosons, their masses could be estimated to be around 84 and 94 Gev/c^2 respectively, which aided greatly in their experimental isolation. Since the W's are charged, they couple to photons and also directly to the Z. These couplings are, given in Ref. 10, p. 116. Eventually the more complicated four-vertices given in the same reference should provide a critical test of the standard model, and conceivably might also distinguish between our theory and the standard model even in the absence of experimental evidence for the Higgses. We ignore this complexity in what follows.

The conservation law situation is now considerably more complicated than it was for electromagnetic quanta. Charge, lepton number, and helicity are still conserved, but the pattern is not easy to follow if written in those terms. Following a strategy that was first introduced into nuclear physics to describe the approximate symmetry between neutron and proton as an "isospin doublet", we form a "weak isospin doublet" from the left-handed electron ($i_z = -\frac{1}{2}$) and left-handed neutrino ($i_z = +\frac{1}{2}$), and, assuming lepton number conservation, can talk about either charge conservation or "z component of isospin conservation" by introducing an appropriate version of the Gell Mann-Nishijima formula, namely $Q = \ell/2 + i_z$, for the left handed doublet. To include the right handed electron, which does not couple to neutrinos, we make it an isospin singlet. To couple it to γ-rays, we assign it a "weak hypercharge" $Y = -2$ and modify the Gell Mann-Nishijima formula to read $Q = Y/2 + i_z$. Our quantum numbers are now conveniently described in the 3-space picture given in figure 6. The numerical specifications are given in Table 1.

Although the type of spacial representation of the quantum numbers presented in figure 6 suggests that there might be rotational invariance in this space, actually only the values on the axes have precise meaning in terms of conservation laws. Total isospin is only approximately conserved; it is a "broken symmetry". Perhaps this should not be a surprise in a relativistic theory; if we take the four independent generators of the Poincaré group to be mass, parallel and perpendicular components of 3-momentum and helicity (i.e. the component of angular momentum along the parallel direction), the total angular momentum cannot be simultaneously diagonalized. People often forget that "total spin" is not a well defined concept in a relativistic theory.

Now that we have looked at the weak-electromagnetic unification of electrons, whose mass is 0.511 Mev/c^2, and their associated massless neutrinos, the full weak-electromagnetic unification scheme is easy to state. In addition to the electrons, we have two systems of leptons with much larger masses, the muon with mass 105.66 Mev/c^2 and the tau lepton with mass 1784 Mev/c^2. Associated with each are left handed $(\nu_\mu)_L$ and $(\nu_\tau)_L$ neutrinos whose interactions can be experimentally distinguished from those of the electron neutrinos $(\nu_e)_L$ and from each other. They may well be massless, but the upper limits on their masses were much higher than for the electron type neutrinos prior to the supernova measurement. As already noted, all three upper limits are now comparable. The coupling scheme is the same as that we have already discussed above within each "generation" $(e, \mu, \tau = 1^{st}, 2^{nd}, 3^{rd})$. The coupling between generations, specified by the Kobiyashi–Maskawa mixing angles, is weak.

To complete the scheme for the weak interactions we must bring in the quarks. There are two "flavors" (up and down) for the first (electron) generation, and two (charmed and strange) for the second (muon) generation; there are supposed to be two more in the third (tau) generation to complete the picture. The existence of the beautiful (or bottom) quark is well established, but searches for the true (or top) quark are still under way. It is the only particle missing from the scheme, other than the Higgses, if you stick to three generations. The quarks are fermions and have electric charge $Q_{u,c,t} = \pm\frac{2}{3}, Q_{d,s,b} = \mp\frac{1}{3}$ and baryon number $\frac{1}{3}$. Each forms a weak isodoublet and an isosinglet in the now familiar pattern. This completes the weak interaction picture at the level we will discuss it here.

The quarks differ markedly from the leptons in several respects. To begin with, they carry a conserved "color charge" with 3 colors, 3 anticolors and an eightfold symmetry we will describe in more detail in Section 5.5. They couple strongly at low energy to eight spin 1 colored "gluons". Color conservation is given a vector representation in figure 7.

Remarkably both quarks and gluons are "confined": they show up like internal particulate degrees of freedom in high energy experiments (parton model), but never have been liberated to be studied as free particles. Hence the definition of their masses is indirect; recent calculations would seem to indicate that the "mass" of an up or down quark is about 1/3 the mass of a proton at low energy, but falls off like $1/p^2$ as the momentum with which they interact increases.[11] One up quark combined with an up-down pair in a spin singlet state to form an overall color singlet state form a proton with charge 1, while a down quark combined with the pair in the same way forms a neutron with charge 0. Consequently the β-decay properties of the neutron can be related to the weak isodoublet description given above.

So far as quantum number conservation goes, we can talk about baryon number (B) spin and (strong) isospin with charge conservation given by $Q = B/2 + I_z$ in the same way we talked about weak hypercharge and weak isospin conservation above. Quark-antiquark pairs describe the mesons (pions, etc) which older theories used to explain nuclear forces, but the details of how the quark-nuclear physics interface actually works quantitatively is a very controversial field of research. The easiest way to picture all this is to write the "color" vertices separately as vectors in a plane and assume that they add to form a color singlet (which can be a neutral colored or anti-colored triplet, or any one of the color-anticolor pairs). Then we can return to the familiar picture of neutron, proton, their anti-particles and associated mesons in the (s_h, I_z, B) space pictured in figure 8. Note the symmetry of the diagram for these parity-conserving strong interactions in contrast to the asymmetric diagram which pictures the parity non-conserving weak-electromagnetic unification.

For any theory to get the quantitative details right is obviously a major research program. A useful reference that gives some idea of the magnitude of the task is the Proceedings of the 1986 SLAC Summer Institute.[12] Clearly we must stop at some point short of that effort here.

4. AN ALTERNATIVE STARTING POINT?

The last chapter has only skimmed the surface of the phenomena that elementary particle physicists expect to be discussed, quantitatively, in their own terms before they will take a rival approach seriously. Since cosmology, condensed matter physics, etc.,etc.,etc..... rest on the same foundations, and must confront much richer experiential detail, a serious alternative appears to be hard to construct. Nevertheless, a start has been made.

4.1. Modeling methodology for physics

The practice of physics cannot get off the ground without essential agreement among the practitioners as to what they are about, how to go about it, and what constitutes progress in their common effort. Often this is clear enough to the "inside group", but in times of change the boundary shifts to include others. Then more formal — and more discursive — attention to these essential aspects of practice can be helpful. Keep in mind that the basic presenting problem we are tackling is to find a common origin for the structure of both quantum mechanics and relativity.

We adopt David McGoveran's modeling methodology (Ref. 1). This has three critical elements:

(1) an *epistemological framework* ("E-frame"), which is a set of loosely defined agreements made explicit by those injecting information into the model formulation; Gefwert[13] would call this a *practical understanding* of physics;

(2) a *representational framework* ("R-frame"), which is an abstract formalism consisting of a set of symbols and a set of rules for manipulation; to formulate such a frame is, for Gefwert, to practice *syntax*;

(3) a *procedural framework* ("P-frame"), which is an algorithm that serves to establish *rules of correspondence* between the observations agreed on in the E-frame and the symbols of the R-frame. Gefwert would describe this activity as the practice of *semantics.* Through recursion the P-frame serves to modify the rules of correspondence, the E-frame and the R-frame until a sufficient level of agreement concerning accuracy is achieved, – or the model fails. Kuhn[14] would call such a failure a "crisis", which in the fullness of time could lead to a "paradigm shift".

Note that we halt the infinite regress of the analysis of terminology in constructive modeling by recognizing the epistemology. We deny the validity and the value of any attempt to analyze "theory-laden" language. Such an analysis lies outside our task when we engage in generating a specific model. Attempting to make such an analysis would require us to generate a model which would contain the specific model as an instance. We *cannot* do so within our methodology. Analysis of that sort would involve non-constructive methods: the analyst *must* work from a specific model by generalization — having failed to construct the general model first.

In an earlier paper[15] we illustrated Gefwert's analysis of the role of the *participator* in a research program as is shown in figure 9. The comparison with McGoveran's modeling methodology (9b) is supposed to bring out the fact that the possible legal walks of the diagram are the same, but that the research program is contained *within* the methodology and that the methodology contains routes (arrows) that are *outside* the program. Thus the entry of the participator from a direction outside the box, and of the empirical confrontation (represented by Poseidon's trident Ψ) from a different direction remain the same; so does the fact that corroboration leaves the participator inside, while falsification takes him outside, in yet another direction. The methodology implies iteration in the EPR or ERP sequence or any inter-leaving of such sequences. The practitioner (and hopefully the reader of our papers) should keep on asking after each iteration how far our E-frame has gone toward expressing the aspects of contemporary physics which he can accept as a starting point.

The modeling methodology presupposes that the community adopting it commits itself, individually and collectively, to:

1. agreement of cooperative communications
    * commonly defined terms as fundamental
    * fundamental vs. derived terms
    * agreement of pertinence
2. agreement of intent
3. agreement on observations
4. agreement of explicit assumptions
5. The Razor
    * agreement of minimal generality
    * agreement of elegance
    * agreement of parsimony

Our agreed upon intent is to model the practice of physics. We take as fundamental the commonly defined terms of laboratory physics, treating terms denoting non-observables as derived or theoretical terms. We recognize that it is very unlikely that agreement on the distinction between observable and theoretical terms can be reached before several passes through the whole scheme have been made.

We take laboratory events as a sufficient set of observations to be modeled without requiring the standard theoretical interpretation. We take as understood that an experimental (laboratory) measurement may encompass many acts of observation. In other words, we are not committed to accept the how and why of the observations, only the observations themselves, operationally understood.

4.2. Five Principles

In the last section we have spelled out our modeling methodology with more attention to underlying ideas than physicists usually employ. We believe that this methodology is close to that customarily employed by the best physicists. Where we part company with standard practice in contemporary theoretical physics — and much of the mathematics physicists employ— is that we reject, from the start, the concept of the *continuum.* Physics has always rested on *counting* when it came to experiment; we are being "conservative" in taking discrete, numerical practice as our starting point.

The R-frame theory is constructed with the intent to meet the following five principles:

Principle I: The theory possesses the property of strict finiteness.

Principle II: The theory possesses the property of discreteness.

Principle III: The theory possesses the property of finite computability.

Principle IV: The theory possesses the property of absolute non-uniqueness.

Principle V: The formalism used in the theory is strictly constructive.

McGoveran (Ref. 1) has chosen these five principles, and the order in which they are presented, with particular care; their current form came into existence after an all day discussion of the theory with Kilmister, in which he remarked "The reader should be warned that this is a damned subtle theory". Since HPN has never before known Clive (Kilmister) to use such strong language, this warning should be taken to heart. We will not attempt here to give the precision to these principles which mathematicians, philosophers and computer scientists require; consult McGoveran's discussion if you desire that!

A few casual remarks for physicists are in order. Finiteness comes before discreteness. This requires us to specify *in advance* how far we intend to count; there is always some finite ordinal N_{Max}. If we exceed this initial bound, *all* arguments must be re-examined. Finite computability requires all algorithms to terminate within this N_{Max} and require no more memory for the storage of their coding and results than can be bounded by some cardinal $N log_2 N + N$. Absolute non-uniqueness requires us to assign equal prior probabilities to cases in the absence of further information; it also introduces *indistinguishables* whose cardinal number can exceed their ordinal number. Strict constructivism puts us firmly on one side of many debates about the foundations of mathematics. All of these requirements make sense to practicing computer scientists, and should also appeal to high energy experimental physicists who get frustrated by the vagueness of the "predictions" their theoretical colleagues often make.

McGoveran goes on to use these principles in the construction of an *ordering operator calculus* and a finite and discrete geometry based on "derivates" (i.e. finite differences) rather than the derivatives of continuum theories. Since one of us (DMcG) has broken new ground for the construction, and his methods are unfamiliar, we do not attempt here to match his precision of thought This paper is aimed at being "introductory"; unfortunately it cannot, under the circumstances, be "obvious".

5. EVENTS, CONSERVATION LAWS, and "(anti-)PARTICLES"

Our next task is to actually construct a self-consistent representational framework which embodies our principles. As Gefwert would put it, we will now practice *syntax*. Our intent is to reconcile quantum mechanics with relativity in a consistent way. We should exercise care *not* to introduce theory-laden language into the representational frame. The self-consistency must not rely on intuitive ideas drawn from physics. I (HPN) fear that I have not succeeded in avoiding this trap altogether; forty years of practicing theoretical physics in a conventional way has left me with some bad habits. The formalism presented here has been scrutinized Kilmister and Bastin, who are more sensitive than HPN to this trap; they support at least the essential aspects of the result.

5.1. THE COMBINATORIAL HIERARCHY

Historically, the line of research that has led to the results presented here began with Eddington, and Bastin's thinking about Eddington's fundamental theory. Bastin realized that when we go to the very large (distant galaxies, early times...) or the very small (quantum events, elementary particles...) the information available to us becomes extremely impoverished compared to the phenomena modeled by classical physics. He concluded that this fact should be reflected in the theory in such a way that this restriction is respected.

The route into the theory initially followed by Bastin and Kilmister concentrated on the problem of modeling discrete events.[16,17] Ordered strings of zeros and ones gave a powerful starting point for analysing this problem. Attention eventually centered on the question of whether bit strings were the same or different. Define a bit string by

$$(a)_n \equiv (..., b_i^a, ...)_n; \; b \in 0, 1; \; i \in 1, 2, ..., n$$

An economical way to compare an ordered sequence of two distinct symbols with other sequences of the same bit length is to use the operator XOR ("exclusive or", symmetric difference, addition (mod 2)= $+_2$, OREX, ...). Since we sum (or count) the 1's in the string to specify a measure we can treat the symbols "0", "1" as integers and only in some contexts can we think of them as bits; hence our "bit strings" can be more complicated conceptually than those encountered in standard computer practice. We therefore use the more general *discrimination* operation "$\oplus$", and a short hand notation for it. Define the symbol $(ab)_n$ and the discrimination operation $\oplus$ by

$$(ab)_n \equiv S^a \oplus S^b \equiv (..., (b_i^a - b_i^b)^2, ...)_n = (..., b_i^a +_2 b_i^b, ...)_n$$

The name comes from the fact that the same strings combined by discrimination yield the null string, but when they differ and $n \geq 2$ they yield a third distinct string which differs from either; thus the operation discriminates between two strings in the sense that it tells us whether they are the same or different.

We define the *null string* $(0)_n$ by $b_i^0 = 0, i \in 1,2,...,n$ and the *anti-null string* $(1)_n$ by $b_i^1 = 1, i \in 1,2,...,n$. Since the operation $\oplus$ is only defined for strings of the same length we can usually omit the subscript n without ambiguity. The definition of discrimination implies that

$$(aa) = (0);\ (ab) = (ba);\ ((ab)c) = (a(bc)) \equiv (abc)$$

and so on.

The importance of closure under this operation was recognized by John Amson. It rests on the obvious fact that $(a(ab)) = (b)$ and so on. We say that any finite and denumerable collection of strings, where all strings in the collection have a distinct tag $i, j, k...$, are *discriminately independent* iff

$$(i) \neq (0):\ (ij) \neq (0),\ (ijk) \neq (0), ...(ijk...) \neq (0)$$

We define a *discriminately closed subset* of non-null strings $\{(a),(b),...\}$ as the set with a single non-null string as member or by the requirement that any two different strings in the subset give another member of the subset on discrimination. Then two discriminately independent strings generate three discriminately closed subsets, namely

$$\{(a)\},\ \{(b)\},\ \{(a),(b),(ab)\}$$

Three discriminately independent strings give seven discriminately closed subsets, namely

$$\{(a)\},\ \{(b)\},\ \{(c)\}$$

$$\{(a),(b),(ab)\},\ \{(b),(c),(bc)\},\ \{(c),(a),(ca)\}$$

$$\{(a),(b),(c),(ab),(bc),(ca),(abc)\}$$

In fact x discriminately independent strings generate $2^x - 1$ discriminately closed subsets because this is simply the number of ways one can take x distinct things one, two, three,...,x at a time.

The discovery of the combinatorial hierarchy[18] was made by Parker-Rhodes in 1961. The history is fast receding[19]. Frederick (P-R) did indeed generate the sequence $3, 10, 137, 2^{127} + 136 \simeq 1.7 \times 10^{38}$ in suspiciously accurate agreement with the "scale constants" of physics. This was a genuine discovery; the termination is at least as significant! The sequence is simply $(2 \Rightarrow 2^2 - 1 = 3), (3 \Rightarrow 2^3 - 1 = 7)$ $[3 + 7 = 10], (7 \Rightarrow 2^7 - 1 = 127)$ $[10 + 127 = 137], (127 \Rightarrow 2^{127} - 1 \simeq 1.7 \times 10^{38})$. The real problem is to find some "stop rule" that terminates the construction.

The original stop rule was due to Parker-Rhodes. He saw that if the discriminately closed subsets at one level, treated as sets of vectors, could be mapped by non-singular (so as not to map onto zero) square matrices having uniquely those vectors as eigenvectors, and if these mapping matrices were themselves linearly independent, they could be rearranged as vectors and used as a basis for the next level. In this way the first sequence is mapped by the second sequence $(2 \Rightarrow 2^2 = 4)$, $(4 \Rightarrow 4^2 = 16)$, $(16 \Rightarrow 16^2 = 256)$, $(256 \Rightarrow 256^2)$. The process terminates because there are only $256^2 = 65,536 = 6.5536 \times 10^4$ d.i. matrices available to map the fourth level, which are many too few to map the $2^{127} - 1 = 1.7016... \times 10^{38}$ DCsS's of that level. By now there are many ways to achieve and look at this construction and its termination.[20–23] The (unique) combinatorial hierarchy is exhibited in Table 2.

5.2. THE LABEL-CONTENT SCHEMA

For some time the only operation used in the theory was discrimination. Kilmister eventually realized that one should also think about where the strings came from in the first place, He met this problem by introducing a second operation which he called "generation". As he and HPN realized, this operation eventually generates a universe which goes beyond the bounds of the combinatorial hierarchy. Once this happens, we can separate the strings into some finite initial segment that represents an element of the hierarchy, which we call the *label*, and the portion of the string beyond the label which we now call the *content*. It is clear that from then on the content ensemble for each label grows in both number and length as the generation operation continues. Since it takes 2+3+7+127=139 linearly independent basis strings to construct the four levels of the combinatorial hierarchy, the labels will be of at least this length; if we use the mapping matrix construction, they will be of length 256. Call this *fixed* length L, the length of any content string n, and the total length at any TICK (see next section) in the evolution of the universe $N_U = L + n$. Then the strings will have the structure $S^a = (L_a)_L \| (A^a_x)_n$ where a designates some string of the $2^{127} + 136$ which provide a representation of the hierarchy and x designates one of the 2^n possible strings of length n; the symbol "$\|$" denotes string concatenation.

5.3. PROGRAM UNIVERSE

In order to generate a universe of strings which grows, sequentially, in either number (SU) or length (N_U) Mike Manthey and HPN created *program universe*. Recently Manthey realized that the criterion we used to increase the string length (TICK) was unjustifiably selective. The previously published version of the program[5], called *program universe 1*, is compared with Manthey's new proposal, called *program universe 2*, in figure 10. A potentially significant effect of the change is to allow the bit string universe to contain, ephemerally in many cases, distinct strings which are indistinguishable under discrimination. The difference between PU1 and PU2 does not affect anything in this paper, but might eventually provide alternative cosmological models that make observationally different predictions.

The program is initiated by the arbitrary choice of two distinct bits, which become the first two strings in the universe. Whether insisting that one be "0" and the other "1", as in done in the flow chart, rather than allowing both to be arbitrary will eventually produce a significantly different cosmology (or choice among cosmologies) at our epoch is an open question.

Entering the main routine at $PICK$, we choose two strings (i) and (j) and discriminate them: $(ij) \equiv (i) \oplus (j)$. Whenever the two strings picked are identical, $(ij) = (0)_{N_U}$ and we go to TICK. TICK concatenates a single bit, arbitrarily chosen for each string, to the growing end, notes the increase in string length, and the program returns to $PICK$. The alternative route, which occurs when discrimination generates a non-null string, simply ADJOINs the newly created string to the universe, and the program returns to $PICK$.

In the older version we proved that TICK had to be "caused" (in the computer simulation) either by the occurrence of the "3-event" configuration $S^a \oplus S^b \oplus S^c = 0_{N_U}$ or by the configuration $S^a \oplus S^b \oplus S^c \oplus S^d = 0_{N_U}$, which we called a "4-event". But this implied a uniqueness which has no known demonstrable counterpart in events as modeled by contemporary physics; there can be many "simultaneous" events. At ANPA 9 HPN extended the definition of "event" to include all cases in which, at a given string length (or TICK), 3 or 4 strings combine under discrimination to produce the null string. This definition of "event" is retained here, but in Program Universe 2 is no longer the "cause" of TICK. Instead we TICK whenever two strings "interact" without producing any novelty. This is as close as we need to get to defining what would be called a "point" in a continuum theory. We will see in Section 6.2 that this construction of a "point" is consistent with our development of Einstein synchronization, and hence, to the extent possible in our discrete theory, consistent with the conventional use of the term "event" in relativity theory.

The method Manthey and HPN used to "construct" the hierarchy is much simpler than the original matrix construction given by Parker-Rhodes; in fact some might call it "simple-minded". The objection we now find cogent is that the method is non-constructive and hence violates our fundamental principles; new efforts to meet this objection are under way. Manthey and Noyes claimed that all we had to do was to demonstrate explicitly (i.e. by providing the coding) that any run of PROGRAM UNIVERSE contained (if we entered the program at appropriate points during the sequence) all we needed to extract some representation of the hierarchy and the label content scheme from the computer memory *without* affecting the running of the program. [Subsequently DMcG has pointed out that this way of meeting the problem is not strictly constructive and should be replaced by a generation scheme that develops the hierarchy constructively.] The obvious intervention point exists where a new string is generated, i.e. at ADJOIN. The subtlety here is that if we assign the tag i to the string $U[i]$ as a *pointer* to the spot in memory where that string is stored, this pointer can be left unaltered from then on. It is of course simply the integer value of $SU+1$ at the "time" in the simulation [sequential step in the execution of that run of the program] when that memory slot was first needed. Of course we must take care in setting up the memory that *all* memory slots are of length $N_{max} > N_U$, i.e. can accommodate the longest string we can encounter during the (necessarily finite) time our budget will allow us to run the program. Then, each time the program TICKs, , the bits which were present at that point in the sequential execution of the program when the slot $[i]$ was first assigned will remain unaltered; only the growing head of the string will change. Thus if the strings $i, j, k....$ tagged by these slots are discriminately independent at the time when the latest one is assigned, they will remain discriminately independent from then on.

Once this is understood the coding Manthey and HPN gave for our labeling routine should be easy to follow. We take the first two discriminately independent strings and call these the basis vectors for *level 1*. The next vector which is discriminately independent of these two starts the basis array for *level 2*, which closes when we have 3 basis vectors discriminately independent of each other and of the basis for level 1, and so on until we have found exactly $2+3+7+127$ discriminately independent strings. The string length when this happens is then the *label length* L; it remains fixed from then on. During this part of the construction we may have encountered strings which were *not* discriminately independent of the others, which up to now we could safely ignore. Now we make one *mammoth* search through the memory and assign each of these strings to one of the four levels of the hierarchy; it is easy to see that this assignment (if made sequentially passing through level 1 to level 4) has to be unique.

From now on when the program generates a new string, we look at the first L bits and see if they correspond to any label already in memory. If so we assign the content string to the *content ensemble* carrying that label. If the new string also has a new label, we simply find (by upward sequential search as before) what level of the hierarchy it belongs to and start a new labeled content ensemble. Because of discriminate closure, the program must eventually generate $2^{127} + 136$ distinct labels, which can be organized into the four levels of the hierarchy. Once this happens, the label set cannot change, and the parameters i for these labels will retain an *invariant* significance no matter how long the program continues to TICK. It is this invariance which will later provide us with the formal justification for assigning an invariant mass parameter to each string. We emphasize once more that *what* specific representation of the hierarchy we generate in this way is irrelevant; any "run" of PROGRAM UNIVERSE will be good enough for us.

What was *not* realized when this program was created was that this simple algorithm provides us with the minimal elements needed to construct a finite particle number scattering theory. The increase in the number of strings in the universe by the creation of novel strings from discrimination is our replacement for the "particle creation" of quantum field theory. It is not the same, because it is both finite and irreversible; it also changes the "state space". Note that the string length N_U is simply the number of TICKs that have occurred since the start up of the universe; this order parameter is irreversible and monotonically increasing like the cosmological "time" of conventional theories. Our events are unique, indivisible and global, in the computer sense; consequently events cannot be localized, and will be "supraluminally" correlated.

5.4. "VECTOR" CONSERVATION LAWS

So far we have a gross structure based on bit strings, and two operations which generate them via a specific program: (1) ADJOIN, which adjoins a non-null string produced by discrimination to the extant bit string universe and (2) TICK which increases the string length by concatenating a single bit, arbitrarily chosen for each string, at the growing end of each string. We have two kinds of connectivity which result from this construction. One is the label-content schema. Once the label basis has closed under discrimination to form 2+3+7+127 linearly independent strings, program universe will necessarily generate some representation of the combinatorial hierarchy at that label length; this will close with $3 + 7 + 127 + 2^{127} - 1$ labels of that length. Once the label basis (and label string length) is fixed, program universe assigns each novel content string to a specific label when it is created by discrimination, and augments each content string by an arbitrary bit at each TICK. The second is the connectivity between strings of the same length (i.e. "between ticks") which we have characterized as 3-vertices $(abc)_{L+n} = (0)_{L+n}$ and 4-events $(abcd)_{L+n} = (0)_{L+n}$.

To come closer to what we need for physics in the sense of relating the (R-frame) model to measurement ("counting") in the laboratory, we need to introduce a quantitative measure and a norm for such measures. Once we have done this, we can introduce a third operation connecting bit strings ("inner product") that allows us to derive conservation laws. Define a measure $\|x\|$ on (x) by

$$\|x\| \equiv \Sigma_{i=1}^{n} b_i^x, x \in a, b, c...$$

This is the usual Hamming measure. $\|x\|/n$ is McGoveran's normalized attribute distance relative the reference string (0) ($b_i^0 = 0$ for all i; $\|0\| = 0$), and $(n - \|x\|)/n$ is the distance relative to the anti-null string (1) ($b_i^1 = 1$ for all i; $\|1\| = n$).

Consider a *3-vertex* defined by $(abc) = (0)$, or equivalently by $\|abc\| = 0$.

Theorem 1. The measure $\|x\|$ is a norm, i.e.

$$(abc) = (0) \Rightarrow \mid \|a\| - \|b\| \mid \leq \|c\| \leq \mid \|a\| + \|b\| \mid, \ \textit{cyclic on } a, b, c$$

Argument: From the definition of discrimination, if we consider the three bits at any ordered position i in the three strings of a three vertex, we can only have either one 0 and two 1's in the three strings, or three zeros. If the single zero is $b_i^a = 0$, call the number of times this occurs n_{bc} (cyclic on a, b, c), and the number of times we have three 0's n_0. Clearly $n_{bc} + n_{ca} + n_{ab} + n_0 = n$ and $\|a\| = n_{bc} + n_{ca}$, cyclic on a, b, c, from which the desired inequalities follow.

Note that this theorem depends on a computer memory. It is *static* in that it depends only on a particular type of configuration that is "wired in" by the program. It is *dynamic*, in the sense that the three strings are brought together as a consequence of past sequences that are *arbitrary* from the point of view of the local vertex. It is *global* in that any single three-vertex (or four-event) *could* lead to a TICK which affects the whole bit string universe.

If we now define the inner product $< (x) \cdot (y) >$ between two strings $(a), (b)$ connected by a three vertex$(abc) = (0)$ with the equality

$$2 < (a) \cdot (b) > \equiv \|a\|^2 + \|b\|^2 - \|c\|^2$$

it follows immediately that

Corollary 1.1.

$$\|ab\|^2 = < (a) \cdot (ab) > + < (b) \cdot (ab) > = < (ab) \cdot (ab) >$$

$$\|a\|^2 = < (ab) \cdot (a) > + < (b) \cdot (a) > = < (a) \cdot (a) >$$

$$\|b\|^2 = < (ab) \cdot (b) > + < (a) \cdot (b) > = < (b) \cdot (b) >$$

If we define a *4-vertex* by $(abcd) = (0)$, or equivalently by $\|abcd\| = 0$, with an obvious extension of the notation it also follows that

Theorem 2.

$$(abcd) = (0) \Rightarrow \|a\| = \|bcd\|, \; \textit{cyclic on abcd}$$

$$\|ab\| = \|cd\|; \; \|ac\| = \|db\|; \; \|ad\| = \|bc\|$$

Argument: $(abcd) = (0) \Rightarrow (abc) = (d)$, etc. and $\Rightarrow (ab) = (cd)$ etc., from which the result follows.

Corollary 2.1. For any pair taken from the ensemble *abcd* the appropriate version of Corrolary 1.1 follows.

Corollary 2.2.

$$< (a) \cdot (cd) > + < (b) \cdot (cd) > = \|ab\|^2 = \|cd\|^2 = < (c) \cdot (ab) > + < (d) \cdot (ab) >$$

and so on for any of the three pairs. It follows that we can put two three events together to make a four event in the six different ways required by 2-2 crossing, as discussed in our presentation of the practice of particle physics.

As Kilmister has pointed out to us, this is not sufficient for us to go from these results and our earlier definition of the inner product to the conclusion that a 4-vertex defines the vector conservation law

$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$$

in all cases. Fortunately, all we need for the physics we develop below is the 2-2 crossing in observable events, which *does* follow from what we have developed above when clothed with the appropriate rules of correspondence; that is, we can justify what, in a vector theory would be written as the three interpretations

$$\vec{a} + \vec{b} = \vec{c} + \vec{d};\ \vec{a} + \vec{c} = \vec{b} + \vec{d};\ \vec{a} + \vec{d} = \vec{b} + \vec{c}$$

Since a 4-vertex $(abcd) = (0)$ can be decomposed in seven different ways, namely

$$(ab) = (cd); (ac) = (bd); (ad) = (bc)$$

$$(a) = (bcd); (b) = (cda); (c) = (dab); (d) = (abc)$$

we can — under appropriate circumstances—still make seven different temporally ordered interpretations of the single 4-vertex given above: three (2,2) channels, four (3,1) channels and the unobservable (4,0) channel. Note that all eight relationships are generated by one 4-vertex.

5.5. THE STANDARD MODEL FOR QUARKS AND LEPTONS USING COMBINATORIAL HIERARCHY LABELS

Our next step is to recall that we can always separate a string into two strings $(a)_{L+n} = (L_a)_L \| (A_a)_n$ where "$\|$" denotes string concatenation. We call the first piece the *label* and the second the *content.* There is a simple correlation between the two pieces. If we take some content string A_a and call its velocity $\beta_a = 2\|A_a\|/n-1$, the string $(a1)$ has the opposite velocity. Further, if we use the string (a) as the reference string for a conservation law defined by the inner product relations given above, the reversal of the velocity achieved by discrimination with the anti-null string can be correlated with the definition of label quantum numbers and conservation laws in such a way that physically observable crossing symmetry is respected. Then the theory is invariant under the *arbitrary* choice of reference direction.

It can be seen that the string for which both label and address are the anti-null string plays a special role in the theory, since it specifies the relationship between particle and anti-particle, and interacts with everything whether it is massive or massless. Since it is unique among the $2^1 27 + 136$ labels, it is readily identified as the Newtonian gravitational interaction. It is the only level 4 label we will refer to explicitly, for reasons discussed below.

Physical interpretation of the labels naturally starts with the simplest structures, which are the weak and electromagnetic interactions. We can get quite a long way just by looking at the leading terms in a perturbation theory in powers of $e^2/\hbar c \simeq 1/137$ for quantum electrodynamics and of $G_F \simeq 10^{-5}/m_p^2$ for the low energy weak interactions such as beta decay. As Lee and Yang saw, if the neutrino is massless and chiral, the Fermi β-decay theory will violate parity conservation maximally; this is still the simplest accurate description of low energy weak interactions.

Since level 1 has only two basic entities, we identify these with the neutrino ν and the anti-neutrino $\overline{\nu}$. One might think that their closure would be the zero helicity component of the spin 1 neutral weak boson Z^0, but if we take the neutrinos to be massless, and hence their content strings to be null or antinull, they cannot form a 3-vertex with a massive particle. Actually the Z_0 and W must couple to all of the first three levels and hence must be assigned to level 4, which we are not attempting to model in detail in this article. Further, although massive, they are also unstable — as are all massive level 4 entities — and hence require us to go beyond the simple modeling of Yukawa vertices for stable, elementary particles developed in this article. If we follow the usual convention of defining the chirality of the neutrino as "left-handed", once we have added content strings and defined directions, we still need a convention as to whether the label is to be concatenated with the string $(1)_n$ with velocity $+c$, or the string $(0)_n$ with velocity $-c$. We can take the bit string state $(\nu_L)_{L+n} = (\nu_\lambda)_L \| (1)_n$ and the right-handed (i.e. anti-) neutrino $(\nu_R)_{L+n} = (\overline{\nu}_\rho)_L \| (0)_n$. Then if we use a representation in which $(\nu_\rho)_L = (1\nu_\lambda)_L$, the Feynman rules will be obeyed. The vertex can be interpreted as the gravitational interaction of a neutrino or an anti-neutrino. Note that for massless particles ($\beta = \pm c$), we cannot specify a direction until we connect them to slower particles whose directions can be assigned. Thus we are forced to adopt a Wheeler-Feynman type of theory in which all massless "radiation" emitted by charged particles must be absorbed.

Interpretation of level 2 as modeling the vertices of quantum electrodynamics for electrons, positrons and photons follows the following scheme. We take as the linearly independent basis strings $(e_\lambda^+), (e_\lambda^-), (\Gamma_{\lambda\lambda})$ and define the non-null string which guarantees their independence as $(\Gamma_c) = (e_\lambda^+ e_\lambda^- \Gamma_{\lambda\lambda})$. The remaining 3 label strings which close level 2 are then defined by

$$(e_\rho^+) = (\Gamma_c e_\lambda^-);\ (e_\rho^-) = (\Gamma_c e_\lambda^+);\ (\Gamma_{\rho\rho}) = (\Gamma_c \Gamma_{\lambda\lambda})$$

We take the same convention for positive direction and chirality as we did for level 1, using the negative, left handed electron as our reference string and the velocity $\beta_{e_L^-} = 2k_{e_L^-}/n - 1$ as positive when this number is positive. The physical states, where we omit the subscripts on β, are then given by

$$(\gamma_c)_{L+n} = (\Gamma_c)_L \| (1)_n;\ (e_L^-) = (e_\lambda^-) \| (-\beta)_n;\ (e_L^+) = (e_\lambda^+) \| (-\beta)_n$$

$$(e_R^+) = (e_\rho^+) \| (\beta)_n = (\gamma_c e_L^-);\ (e_R^-) = (e_\rho^-) \| (\beta)_n = (\gamma_c e_L^+)$$

$$(\gamma_{RR}) = (\Gamma_{\rho\rho}) \| (1)_n;\ (\gamma_{LL}) = (\Gamma_{\lambda\lambda}) \| (0)_n = (\gamma_c \gamma_{RR})$$

and the Feynman rules are obeyed for all 3-vertices.

The 4-vertex $(e\bar{e}\gamma\gamma_c) = (0)$ cannot be readily discussed until we have the configuration space theory nailed down. It is related to our finite treatment of Bremstrahlung in a "coulomb field". The vertex $(\gamma_{LL}\gamma_{RR}\gamma_c) = (0)$ would seem to imply an interaction between photons and the "coulomb field", — a vertex that vanishes in the conventional theory because of the masslessness of the photon and gauge invariance.

A related problem arises with the vertices implied by our connection between particles and anti-particles, namely

$$(\nu\overline{\nu}1) = (0);\ (e\overline{e}1) = (0):\ (\gamma\overline{\gamma}1) = 0$$

A little thought shows that such vertices will occur for *any* particle-antiparticle pair. Hence the anti-null label string "interacts" with everything and must be assigned to level 4. This unique label string, which occurs with probability $1/(2^{127} + 136)$, is identified with Newtonian gravitation. It leads to the bending of light in a "gravitational field". Of course, to get the experimentally observed result, we will have to identify the "spin 2" gravitons as well, and show that they double this deflection.

We conclude this chapter by identifying the level 3 structure with the quarks and gluons of quantum chromodynamics. This discussion follows along the lines already laid down in discussing the first two levels. We take as our basis label strings a quark part $(u^+), (u^-), (d^+)$ or (d^-) concatenated with a color part $(r), (y), (b)$ which gives us the seven independent strings needed to form level 3. The color strings are linearly independent, so we can define (analogous to what we did at level 2)

$$(ryb) = (w);\ (\overline{r}) = (rw);\ (\overline{y}) = (yw);\ (\overline{b}) = (bw)$$

from which it follows that

$$(ry\overline{b}) = (0);\ (r\overline{y}b) = (0);\ (\overline{r}yb) = (0);\ (\overline{r}\overline{y}\overline{b}) = (0)$$

Similarly, the linear independence of the quark parts allows us to define

$$(u^+u^-d^+d^-) = (Q);\ (\overline{q}) = (qQ), q \in u^+, u^-, d^+, d^-$$

Then a colored quark label $(q_c^{\pm}) = (q^{\pm})\|(c)$ and a colored gluon label $(g_c) = (Q)\|(c)$, $c \in r, y, b$, allow us to recognize the label part of the Yukawa vertex for QCD as $(q_{c_1}\overline{q}_{c_2}g_{c_3}) = (0)$. The essential point here is that, as proved above, $(c_1\overline{c}_2c_3) = (0)$ for any three distinct colors. We can then attach content labels and helicity in the same way as we did in QED, and once again the Feynman rules apply. Any one familiar with lowest order QCD can now immediately derive from our formalism the "valence quark" structure of the proton and neutron in terms of three quarks, and the structure of the π, ρ and ω in terms of quark-antiquark pairs. In contrast to the level 2 situation, the three gluon vertex does not vanish, and implies a 4-gluon vertex. So we find that we have constructed *all* the lowest order vertices of QCD with the correct conservation laws.

The problem of "color confinement" is solved, in principle, by *McGoveran's Theorem*,[24,25] i.e. the conclusion that in any finite and discrete theory there can be no more than three "homogeneous and isotropic dimensions" that remain indistinguishable as the (finite and discrete) cardinals and ordinals keep on increasing. (We discuss this theorem with more care in Section 6.1.) Because our labels are tied to contents, and hence via the counter paradigm to macroscopic directions, we can only have three quantum number "dimensions" asymptotically. These are saturated by the three absolutely (so far as we know currently) conserved quantum numbers: lepton number, baryon number and charge (or "z-component" of isospin), leaving no room for free quarks or gluons conserving asymptotic "color charge". They can occur at short distance as degrees of freedom in the scattering theory — as we showed above — but eventually they have to "compactify" and become distinguishable from free particle quantum numbers. We can conclude this immediately without any detailed dynamical argument.

6. COMPARISON WITH EXPERIMENT

We use the traditional phrase for the title of this chapter. In McGoveran's terminology, we provide here the rules of correspondence, or a procedural framework by means of which we can connect our formal representational framework (Gefwert's *syntax*) developed in the last chapter to the informal epistemological framework — the practice of experimental and theoretical physics in the laboratory — which it is our intent to model. In so doing we provide meaning, or, as Gefwert would put it, practice *semantics.*

6.1. THE COUNTER PARADIGM

Bastin has insisted for decades that the primal contact between a (computable) formalism and the empirical "world" can only be made once. This was a basic reason why he and Kilmister [16,17] fastened on steps of a scattering process as a likely point at which to investigate the connection between finite mathematics and physical theory. HPN started thinking of the elementary scattering process as fundamental thanks to his early involvement in Chew's S-Matrix theory; for him this gave specific content to Bridgman's operationalism and Heisenberg's very early ideas. At ANPA 2 and 3 some of us saw that Stein's "random walk" derivation of the Lorentz transformation and the Uncertainty Principle[26] must somehow connect to scattering processes; others recognized the seminal nature of his work because of his ontological viewpoint.

The specific genesis of the "counter paradigm" occurred after HPN's presentation[27] at the conference honoring deBroglie's 90^{th} birthday. Fortunately, HPN had an opportunity to start working on the final version of that paper[28] in consultation with Ted Bastin before it was published. HPN realized that if he thought of Stein's "random walk" as a model for two sequential events in two spatially separated laboratory counters with the discrete step length being the deBroglie relativistic phase wavelength that, by representing Stein's random walks as bit strings with the bit 1 taken as a step toward the final counter and the bit zero a step away from it, he had the right point of contact between the bit strings used in the *combinatorial hierarchy* and the start of a scattering theory.

So far we have only discussed 3- and 4-vertices for a fixed value of n. But each time program universe TICKs, each content string in each labeled ensemble acquires an arbitrary bit at the growing end. In the absence of further information, each content string therefore represents a sequence of Bernoulli trials with 0 and 1 representing the two possibilities. This has an extremely important consequence, which we call *McGoveran's Theorem*[24,25]. As has been noted by Feller,[29] if we have D *independent* sequences of Bernoulli trials, the probability that after n trials we will have accumulated the same number (k) of 1's is $p_D(n) = (\frac{1}{2^{nD}})\Sigma_{k=0}^{n}\binom{n}{k}^D$. He then shows that the probability that this situation will repeat N times is strictly bounded by

$$P_D(N) = \Sigma_{n=1}^{N} p_D(n) < [\frac{2}{\pi D}]^{-\frac{1}{2}}\Sigma_{n=1}^{N} n^{\frac{1}{2}(D-1)}$$

Consequently for $D = 2, 3$, where $p_D(n) < n^{-\frac{1}{2}}, n^{-1}$, such repetitions can keep on occurring with finite probability, but for four or more independent sequences, this probability is strictly bounded by zero in the sense of the law of large numbers.

McGoveran uses finite attributes, which can always be mapped onto ordered strings of zeros and ones, as the starting point for his ordering operator calculus. As is discussed in more detail in Ref.1, these can be used to construct a finite and discrete metric space. In order to introduce the concept of *dimensionality* into this space, he notes that we need some metric criterion that does not in any way distinguish one dimension from another. (In a continuum theory, we would call this the property of "homogeneity and isotropy"; we need it in our theory for the same reason Einstein did in his development of special relativity.) McGoveran discovered that by interpreting the coincidences $n = 1, 2, ..., N$ in Feller's construction as "metric marks" the metric space so constructed has precisely the discrete property corresponding to "homogeneity and isotropy" as just defined. Consequently Feller's result shows that in *any* finite and discrete theory, the number of independent "homogeneous and isotropic" dimensions is bounded by three! If we start from a larger number of independent dimensions using *any* discrete and finite generating process for the attribute ensembles, we find that the metric will, for large numbers, continue to apply to only three of them, and that what may have looked like another dimension is not; the probability of generating the next "metric" mark in any of the others (let alone all of them) is strictly bounded by $1/N_{MAX}$!

Of course the argument depends on the theory containing a *universal ordering operator* which is isomorphic to the ordinal integers. Further, since we know empirically that "elementary particles" are *chiral*, we will need three rather than two "spacial" dimensions. Thus *any* discrete and finite theory such as ours when applied to physics must be globally described by three dimensions and a monotonically increasing order parameter. Consequently we are justified in constructing a "rule of correspondence" for our theory which connects the large number properties of our R-frame to *laboratory* (E-frame) 3 + 1 space-time.

We begin with the paradigmatic case of a single particle entering a space-time volume (detector) $\Delta V \Delta T$, causing a count, and a time T later entering a second detector with similar resolution a macroscopic distance L from the first and causing a second count. We then say that the (average) velocity of the particle between the two detectors is $V = L/T$; empirically this number is always less than or indistinguishable from the limiting velocity c.

This language is well understood by the particle physics experimentalist, but raises a number of problems for others. To begin with he uses "cause" in a philosophically vague but methodologically precise sense, which includes a host of practical experience about "background", "spurious counts", "real counts", "goofs", "GOK's" (i.e. "God only knows"),....

The actual practice of experimental particle physics implies the concept of *indistinguishability* in a critical way; the experimentalist uses, often without conscious analysis, finite collections whose cardinal number may exceed their ordinal number; this fact is diagnostic for *sorts* that are not reducible to *sets*[23]. To put it more formally in terms of "background" and "counts", in the absence of a constructive definition of the two subsets — which is often unavailable in practice, and in our theory we would claim can be unavailable in principle — the two collections are *sorts* rather than *sets*.

The rule of correspondence in the counter paradigm case (two sequential counts spatially separated) applies to a labeled string with label L_a which at the TICK with the content string length n_0 was part of a 3- or 4-vertex and again part of a vertex at content string length $n_0 + n_a$, AND WHICH IS APPROPRIATELY ASSIGNED TO THEORETICALLY RELEVANT DATA RATHER THAN TO BACKGROUND. We ask how many 1's were added to the content string; we call these k_a. We identify the (average) laboratory velocity of the particle ($V = L/T$) with the R-frame quantity by the equation $V = (\frac{2k_a}{n_a} - 1)c$. The sign of this velocity defines the positive or negative sense of the direction between the counters in the laboratory (or visa versa: a choice must be made *once*). Since the evolution of the bit string universe will provide many candidates for the strings which meet these criteria within the time and space resolution of the counters, we will have to provide more and more precise definitions of these criteria as the analysis develops.

6.2. EVENT-BASED COORDINATES AND THE LORENTZ TRANSFORMATIONS

As is discussed with much more care in Reference 1, any theory satisfying our principles can be mapped onto ensembles of bit strings simply because, with respect to *any* attribute, we can say whether a collection has that attribute or does not. To introduce a metric, we need a distance function *relative* to some reference ensemble. Because of our finite and discrete principles, any allowed program can only take a finite number of steps to bring any ensemble into local isomorphism with the reference ensemble *in respect to that attribute.* Note that there can be many attributes, many distance functions and that the space can be multiply connected. Note that this definition also provides a (dichotomous, eg $\pm$) *sense* to the computation steps: they must increase the attribute distance or decrease it. Calling the number of increments I and the number of decrements D, using a well defined computational procedure, the attribute distance is, clearly, $d_A = I - D$, and the total number of steps $N = I + D$. Then we can also define the *attribute velocity* with which the two ensembles are "separating" or "coming together" $v_A = (I - D)/(I + D)$. Thus there always is a "limiting velocity" for each attribute, which is attained when all steps are taken in the same direction.

If we wish to model the events of which contemporary physics takes cognizance, we know that all physical attributes are directly or indirectly coupled to electromagnetism. Therefore the limiting velocity of physics, c, will be the *smallest* of these limiting attribute velocities simply because it refers to the attribute with the maximum cardinality. Any ensemble of attributes specified by a more limited description involves a "supraluminal" velocity without allowing supraluminal communication of information. Hence we can expect to find correlation between and synchronization of events in space-like separated regions; from our discrete point of view the existence of the effects demonstrated in Aspect's and other EPR-Bohm experiments is anticipated and in no way paradoxical. We guarantee Einstein locality for *causal* events, that is for those initiated by the transfer of *physical* information.[30]

In order to go from this general proof of the limiting velocity to the laboratory practice of relativistic particle quantum mechanics, we need a more specific formalism than the general derivation given in Ref. 1. We start from the 3- and 4- vertices already mentioned and consider how they can be used to model the "laboratory" situation given in figure 11. The initial 4-vertex $(abcd)_{L+n_0} = 0$ is followed sequentially by 5 vertices involving "soft" photons, as is explained below. In the laboratory neither vertices, nor elementary events nor soft photons can be observed; limiting cases in which the disturbance caused by the firing of counters connected with these 5 events is negligibly small are easy to envisage. We use a specific example of labels that can, if we wish, be given a specific interpretation in which particles a, b, c have spin $\frac{1}{2}$ and the photons have left or right spin 1 helicity.

We assume that it takes n_i TICKs of program universe beyond $L + n_0$ to generate the strings involved in the i^{th} event. Since all strings will have the portion through content string length n_0 unaltered, we need use only these *relative* values: $n_i = N_U(i) - L - n_0$ and the corresponding terminal pieces of the strings for our contents. For Event 1, we take the three strings to be

$$(a) = (1000)\|(A_1^a)_{n_1};\ (a') = (0100)\|(A_1^a)_{n_1};\ (\overline{\gamma}) = (1100)\|(0)_{n_1}$$

Hence $(aa'\overline{\gamma}) = (0)$ defines a 3-vertex in which the velocity of a does not change; we could call it a "soft photon" vertex. By crossing (cf. Sections 3.1 and 5.5 above) this also can be interpreted as a vertex in which a flips its spin and emits a photon with the appropriate helicity, i.e. $(\gamma) = (0011)\|(1)_{n_1}$. The laboratory direction between events 1 and 2 then defines the reference direction for all subsequent discussion. The remaining vertices can be consistently represented by using

$$(b) = (1000)\|(A_2^b)_{n_2};\ (\gamma) = (0011)\|(1)_{n_2}:\ (b') = (0111)\|(A_2^b)_{n_2}$$

$$(\gamma') = (1100)\|(1)_{n_2}$$

$$(c) = (1000)\|(A_3^c)_{n_3};\ (\gamma') = (1100)\|(1)_{n_3};\ (c') = (0111)\|(A_3^c)_{n_3}$$

$$(\overline{\gamma}') = (0011)\|(0)_{n_3}$$

$$(b') = (0111)\|(A_4^{b'})_{n_4};\ (\overline{\gamma}') = (0011)\|(0)_{n_4};\ (b'') = (1000)\|(A_4^{b'})_{n_4}$$

$$(\overline{\gamma}) = (1100)\|(0)_{n_4}$$

$$(a') = (0100)\|(A_5^{a'})_{n_5};\ (a'') = (1000)\|(A_5^{a'})_{n_5};\ (\overline{\gamma}) = (1100)\|(0)_{n_5}$$

We now trust that our rule of correspondence between 3- and 4- vertices and a standard "laboratory" situation used in the derivation of the Lorentz transformations is clear.

For simplicity, we consider here that particle a is, *on the average*, "at rest" between events 0, 1 and between events 1, 5:

$$k_0^a = n_0/2;\ k_1^a = n_1/2;\ k_5^a = n_5/2$$

We also assume, again *on the average*, that b and c have constant velocity over the appropriate intervals:

$$\beta_b = 2k_0^b/n_0 - 1 = 2k_2^b/n_2 - 1 = 2k_4^b/n_4 - 1$$

$$\beta_c = \beta = 2k_0^c/n_0 - 1 = 2k_3^c/n_3 - 1$$

Our next simplification is to assume that all the events lie on a single "line", reducing this to a 1+1 dimensional problem. None of these simplifications are needed, as can be seen from the general discussion in Ref.1.

In conventional terms, we are asking the question of how the coordinates of an event at $x = \beta ct$ in one coordinate system (the one in which particle a is at rest) transform to the coordinate system in which particle b is at rest. We are forced by our principles to assume, as in conventional treatments, that the velocity of light is the same in all coordinate systems and that the time at which event 3 occurs is the average between when the light signal that defines event 3 was emitted by a and returns to it. Introducing a parameter with the dimensions of length, whose value we will discuss later, these statements follow immediately from the definitions of attribute distance and velocity, since

$$x/\lambda = 2k - n; ct/\lambda = n; \beta = (2k/n) - 1$$

for any particle, and $k = 0$ or n specifies a connection with the limiting velocity for any set of strings. This is even clearer when we introduce "light cone" coordinates:

$$d_+ = n + (2k - n) = 2k; d_- = n - (2k - n) = 2(n - k)$$

The relationship between the two descriptions is illustrated in figure 12.

One way to derive the Lorentz transformations is to require that the interval s between events 0 and 3 be invariant, where

$$s^2/\lambda^2 = (c^2t^2 - x^2)/\lambda^2 = n^2 - (2k - n)^2 = 4k(n - k)$$

In light cone coordinates this relationship becomes

$$d_+d_- = 4k(n - k) = s^2/\lambda^2$$

which makes one way of insuring the invariance requirement particularly simple, namely

$$k' = \rho k, n' - k' = \rho^{-1}(n - k) \Rightarrow 4k'(n' - k') = 4k(n - k)$$

Note that if we are to compare the integer *bit string* coordinates, this restricts k' to be a rational multiple of k. One of the great successes of our theory is precisely this restriction that keeps events an integral number of deBroglie wavelengths apart. A fundamental explanation of why our theory can contain "interference" phenomena starts here.

If we now note that

$$d_\pm = (1 \pm \beta)n$$

the invariance requirement gives us that

$$(k'/k)[(n-k)/(n'-k')] = \rho^2 = [(1+\beta')/(1+\beta)][(1-\beta)/(1+\beta']$$

Hence

$$\beta_\rho = (\beta'-\beta)/(1-\beta\beta') \Leftrightarrow \rho^2 = [1+\beta_\rho]/[1-\beta_\rho]$$

From the fact that when transforming from a system at rest ($d_+/d_- = 1$),we see that the relative velocity between the two systems is simply β_ρ. We have derived the velocity composition law for rational fraction velocities in any system. Tom Etter arrived at this composition law for attribute velocities on general grounds, as is discussed in Ref.1. With

$$\gamma = (1/2)[\rho + \rho^{-1}]$$

we have that

$$x' = \gamma(x + \beta_\rho ct) : t' = \gamma(ct + \beta_\rho x) \qquad QED$$

6.3. QUANTUM MECHANICS

Program universe provides an *invariant* significance for the label strings, once they close (in some length with at least 139 bits) to form some basis for some realization of the combinatorial hierarchy. For each of the $2^{127} + 136$ labels L_ℓ we can assign a dimensional parameter λ_0^ℓ which is the step length when the particle is "at rest", i.e. when, on the average $2k_\ell = n_\ell$. Since program universe increases the string length one arbitrary bit at a time, this requirement can at best be satisfied only at every other step. We have seen that when all steps are in the same direction (i.e. when the content string is either the null string or the anti-null string), this corresponds to a "light signal". In any string evolution all steps are executed at the limiting velocity c - a finite and discrete "zitterbewegung". The invariance of λ_0^ℓ allows us to associate with each label an invariant parameter with the dimensions of mass m_0^ℓ, and relate the two by $\lambda_0^\ell = h/m_0^\ell c$, where h is a universal constant with the dimensions of action. We will now show that h can, indeed, be identified with Planck's constant.

The extension of our Lorentz transformations to momentum space is now immediate. We simply define $E = \gamma m_0 c^2, p = \gamma\beta m_0 c$. For $p_\pm = E/c \pm p$ we have that $p_+ p_- = m_0^2 c^2, p_+/p_- = k/(n-k)$ and $\frac{1}{2}(p_+ x_- + p_- x_+) = Et - px$. The justification of calling this "momentum" is more than definitional; we showed above that 3- and 4- vertices support "vector" conservation laws and "crossing symmetry". We have 3-momentum conservation in any allowed event-based reference frame. Clearly $m_0 c \lambda_0 = h = E\lambda/c$ in any allowed coordinate system, and we have recovered the initial identification of the step length in the "random walk" as $\lambda = hc/E$, the deBroglie phase wavelength with which our initial statement of the "counter paradigm" began. We can now *derive* the quantum mechanical commutation relations from our model.

We note that if we consider a system that evolves with constant velocity $\beta_0 \equiv 2k_0/n_0 - 1$, strings which grow subject to this constraint, i.e. $n = n_T n_0, k = n_T k_0, 1 \leq n_T \leq n/n_0$ will have a periodicity $T \equiv n_T \Delta t = n_T \lambda/c$ specifying the events in which this condition can be met. Hence, in more complicated situations where there can be more than one "path" connecting strings with the same velocity to a single event, this event can occur only when the paths differ by an integral number of "d-wavelengths" λ. Thus our construction already contains the seeds of "interference" and a conceptual explanation of the "double slit experiment".

We have already seen that any system with "constant velocity" – at those "ticks" when events can occur – evolves by discrete steps $\pm\lambda_a$ in $x = q_a$ *between* ticks. McGoveran's ordering operator calculus[1] which specifies the connectivity between events allows these discrete happenings to occur in a *void* where space and time are meaningless. Since $\lambda/\Delta t = c$, each step occurs forward or backward with the limiting velocity. Thus we deduce a discrete *Zitterbewegung* from our theory. If we think of this as a "trajectory" in the pq phase space, each time step induces a step $\pm\lambda$ in q correlated with a step $\pm mc$ in p. Even in the case of a particle "at rest", this must be followed by two steps of the opposite sign to return the system to "rest". Thus there is, minimally, a four-fold symmetry to the "trajectory" in phase space corresponding to the generation periodicity we discovered above.

If we now recall from classical mechanics[31] that for any momentum which is a constant of the motion we can transform to angle and action variables with $\oint p_J dq_J = J$ where J has the dimensions of action, $p_J = J/2\pi$ and q_J is cyclic, we have an immediate interpretation. In the classical case the "period" goes to infinity for a free particle; for us we have already seen that we have a *finite* period $T = \lambda/c$. Therefore we can immediately identify $m_a c\lambda_a = J = n_T h$; we have constructed Bohr-Sommerfeld quantization within our theory.

To go on to the commutation relations, we can replace the geometrical description of periodic trajectories in phase space by using complex coordinates $z = (q, ip)$ [or by $(q_J, in_T h/2\pi)$], where q_J is restricted to $2n + 1$ values with $-n_T \leq n \leq +n_T$. Then the steps around the cycle in the order $qpqp$ are proportional to $\pm 2\pi(1, i, -1, -i)$ where $\pm$ depends on whether the first step is in the positive or negative direction or equivalently whether the circulation is counter-clockwise or clockwise. We have now shown why $qp - pq = \pm i\hbar$ for free particles in our theory; this result holds for any theory satisfying our principles which uses a discrete free particle basis.

In order to go to a detailed three dimensional description, we must supply three discriminately independent reference strings, define inner products with respect to them (cf. Section 5.4) and go to a "coordinate" description. There will then be three independent periodicities (velocities and momenta) which will commute with each other but not with their conjugate position variable. The commutation relations for angular momentum follow immediately. Since this has already been shown in quite general terms in Ref. 1, we will leave the details to future publications. An alternative is to develop the "radial coordinate" (n, l, m) description using "bound states" as the basis.

Now that we have two ($\hbar$ and c) of the three dimensional constants needed to connect a fundamental theory to experiment in the 3-space in which physics operates, and which we have proved must be the asymptotic space of our theory, all that remains is to determine the unit of mass. But this has already been done for us by the combinatorial hierarchy result $2^{127} + 136 \simeq 1.7 \times 10^{38} \simeq \hbar c/Gm_p^2 = (M_{Planck}/m_p)^2$ which tells us that we can either identify the unit of mass in the theory as the proton mass, in which case we can calculate (to about 1 % in this first approximation) Newton's gravitational constant, or if we take the Planck mass as fundamental, calculate the proton mass. From now on we have to compute everything else. If we fail to agree with experiment to the appropriate accuracy (one of the rules of correspondence!), we must either revise or abandon the theory.

6.4. A DISCRETE MODEL FOR THE BOHR ATOM

We have seen that any bit string has the deBroglie periodicity h/mc^2 for each digital "time step" $\Delta n = 1$, and that when it evolves with "constant velocity" also has the longer digital period n_0 connected to the velocity by $\beta = 2k_0/n_0 - 1$ at each finite "position" $N_{ph}n_0\beta = N_{ph}(2k_0 - n_0)$ where an event can (but need not) occur after the initial vertex at $N_{ph} = 0$. We define $\Delta k_0 = k_0 - n_0/2$ and hence $\beta = \Delta k_0/n_0$. Only one integer can be added to the string at each step. This must happen Δk_0 times before the periodic pattern can be completed. Therefore the number of step lengths in the periodic pattern — the *coherence length* — is $n_0 = 1/\beta$. Since, as we saw above, the step length is $\lambda = hc/E$, we find that the coherence length required for periodic phenomena at constant velocity is $\lambda_g = hc/\beta E = h/p$.

By adding a constraint representing a second periodicity we can now model the periodicity representing a "closed orbit around some fixed center". Clearly this periodicity must use the coherence length derived above if we are to have a stable, repeating, pattern that starts from some "origin" and closes after N_B coherence lengths. This model, which only describes the average "motion", will persist from the time when we start the model off to the time when some vertex — for example the absorption of a "hard" photon — ends the finite sequence of periods. Of course this can only occur at one of the positions allowed for events. In the average sense we can image this "trajectory" as a regular polygon with N_B sides of length λ_g. With the usual "geometrical" image in mind, we call the distance traversed in this period "$2\pi R$"$= N_B\lambda_g$ and hence $mvR = N_B\hbar$. Afficionados of the early history of quantum mechanics will recognize that we have constructed a digital version of deBroglie's analysis of the geometry of the Bohr atom, and produced a reason for angular momentum quantization. For the meaning of "π" in a discrete and finite theory, refer to the discussion in Ref. 1.

Although this part of the derivation of the Bohr atom should be reasonably familiar, our introduction of the "electromagnetic interaction" will be radically different from the conventional approach. We have seen above that the coulomb interaction is represented by only 1 out of 137 labels in the combinatorial hierarchy construction, and that strings evolve by the arbitrary selection of strings from memory to calculate the vertices; thanks to the counter paradigm, these vertices have now become "events". In the case at hand, 136 of these choices can only provide a "background" which will cause fluctuations of the position of our particle; on the average these must cancel out. Only once in 137 times will the step correspond to the vertex that serves to keep the particle in its orbit. We can think of this as happening at the vertices of the polygon, i.e. N_B times in one full period. So, compared to the basic evolution time, we find that for this electromagnetic orbit, $\beta = 1/137N_B$. Making the hierarchy identification $137 = \hbar c/e^2$, our quantization condition derived above then gives us the standard result $R = N_B^2\hbar^2/me^2$, and an explanation of the old puzzle of why the Bohr radius is 137 times the Compton wavelength!

To calculate the binding energy, consider the energy change between this average motion and the particle at rest caused, for example by the emission or absorption of a photon. We must use the average velocity because, in the absence of other information, we cannot know "where" in the orbit the interaction occurs. Our theory can readily accommodate emission and absorption of photons, conserving both momentum and energy, as we have seen in our derivation of the Lorentz transformations, and can include the usual recoil correction if we so desire. Thus, we argue that the binding energy ϵ_{N_B} is related to the velocity $\beta_{N_B} = 1/137N_B$ by $(\epsilon_{N_B} + m_0c^2)^2 = m_0^2c^4/(1 - \beta_{N_B}^2)$ from which all the usual results for the Bohr atom follow to order β^2.

6.5. SCATTERING THEORY

To construct a scattering theory, we need to provide the connectivity between events. To obtain a statistical connection between events, we start from our counter paradigm, and note that because of the macroscopic size of laboratory counters, there will always be some uncertainty $\Delta\beta$ in measured velocities, reflected in our integers k_a by $\Delta k = \frac{1}{2}N\Delta\beta > 0$. A measurement which gives a value of β outside this interval will have to be interpreted as a result of some scattering that occurred among the TICK's that separate the event (firing of the exit counter in the counter telescope that measures the initial value of $\beta = \beta_0$ to accuracy $\Delta\beta$) which defines the problem and the event which terminates the "free particle propagation"; we must exclude such *observable* scatterings from consideration.

What we are interested in is the probability distribution of finding two values k, k' within this allowed interval, and how this correlated probability changes as we tick away. If $k = k'$ it is clear that, when we start, both lie in the interval of integral length $2\Delta k$ about the central value $k_0 = \frac{N}{2}(1+\beta_0)$. When $k \neq k'$ the interval in which both can lie will be smaller, and will be given by

$$[(k+\Delta k) - (k' - \Delta k)] = 2\Delta k - (k' - k)$$

when $k' > k$ or by $2\Delta k + (k' - k)$ in the other case. Consequently the correlated probability of encountering both k and k' in the "window" defined by the velocity resolution, normalized to unity when they are the same, is $f(k,k') = \frac{2\Delta k \mp (k'-k)}{2\Delta k \pm (k'-k)}$, where the positive sign corresponds to $k' > k$. The correlated probability of finding two values k_T, k'_T after T ticks in an event with the same labels and same normalization is $\frac{f(k_T, k'_T)}{f(k,k')}$. This is 1 if $k' = k$ *and* $k'_T = k_T$. However, when $k' \neq k$, a little algebra allows us to write this ratio as

$$\frac{1 \pm \frac{2(\Delta k - \Delta k_T)}{(k'-k)} + \frac{4\Delta k \Delta k_T}{(k'-k)^2}}{1 \mp \frac{2(\Delta k - \Delta k_T)}{(k'-k)} + \frac{4\Delta k \Delta k_T}{(k'-k)^2}}$$

If the second measurement has the same velocity resolution $\Delta\beta$ as the first, since $T > 0$ we have that $\Delta k_T < \Delta k$. Thus, if we start with some specified spread of events corresponding to laboratory boundary conditions, and tick away, the fraction of connected events we need consider diminishes. If we now ask for the correlated probability of finding the value β' starting from the value β for the sharp resolution approximation (i.e. ignoring terms smaller than $1/T$ or proportional to $1/T$ and smaller) this is 1 if $\beta = \beta'$ and bounded by $\pm 1/T$ otherwise. That is we have shown that in our theory a free particle propagates with constant velocity with overwhelming probability – our version of Newton's first law, and Descartes' principle of inertia.

Were it not for the $\pm$, the propagator in a continuum theory would simply be a δ-function. In our theory we have already established relativistic "point particle" scattering kinematics for discrete and finite vertices connecting finite strings. We also showed that the order in which we specify position and velocity introduces a sign that depends on which velocity is greater, which in turn depends on the choice of positive direction in our laboratory coordinate system, and hence in terms of the general description on whether the state is incoming or outgoing. In order to preserve this critical distinction in our propagator, and keep away from the undefined (and undefinable for us) expression *const.*/0, we write the propagator as

$$P(\beta, \beta') = \left[\frac{-i\eta\lambda}{\beta' - \beta \mp i\eta/T}\right]$$

where η is a positive constant less than T. The normalization of the propagator depends on the normalization of states, and is best explored in a more technical context, such as the relativistic Faddeev equations for a finite particle number scattering theory in the momentum space continuum approximation being developed elsewhere[6–9].

7. CONCLUSIONS AND A LOOK FORWARD

The research program discussed here started, so far as some current participators go, in the 1950's — and earlier if you look back to Eddington. By now there is a solid body of results, both conceptual and numerical. One aspect that conventional physicists find puzzling is that we can reach some fundamental results very easily — results that for them require enormously complicated calculations, and a generous (though often unrecognized) input of empirical data. For instance, to "prove" the 3+1 asymptotic structure of space time starting from conventional "string theory" requires the "compactification" of an initially 26-dimensional structure whose uniqueness can, mildly speaking, be questioned. For us, this 3+1 structure for events follows directly from McGoveran's Theorem, once our basic principles and rules of correspondence are understood. For those familiar with Kuhn's model for scientific revolutions, this should come as no surprise. Any new fundamental theory finds some problems easier to solve, and for other problems loses (sometimes for a long while) some of the explanatory power of the theory it is attempting to replace.

At a somewhat less fundamental level than the global "irreversibility of time" and the "3-dimensionality of space", all conventional theories take the existence of a limiting velocity and the quantization of action as a "just so story". We show why *any* theory satisfying our principles has to have both a limiting velocity and non-commutativity. We show that our positions and velocities for our events must be connected by a discrete form of the Lorentz transformations. We derive 3-momentum conservation, quantum number conservation and "on-shell" 4-momentum conservation at our elementary vertices. We also show that when one compares position and velocity in the connected circumstances implicit in the physics of "conjugate variables", the resultant non-commutative structure can be mapped onto that employed in quantum mechanics.

Moving on up to more concrete aspects of conventional theories, given c and $\hbar$ — and the scale-invariant *laboratory* methods of relating them to arbitrary standards of mass, length, and time — conventional physicists need some mass or coupling constant that has to be taken from experiment. Once again the existence of this unique constant — let alone a means of computing it within the theory — is not an obvious structural requirement of conventional practice. In contrast, we obtain a first order estimate $\hbar c/e^2 \simeq 137$ and $\hbar c/Gm_p^2 \simeq (M_{Planck}/m_p)^2 \simeq 2^{127} + 136$. As has been emphasized above, *any* fundamental theory of MLT physics must compute everything else as physically dimensionless ratios once these constants are fixed.

It is sometimes suggested that ours is a "Pythagorean" or *a priori* theory. This criticism implies a lack of understanding of our modeling methodology. We *start* from the current practice of physics, both theoretical and experimental, and try to construct (a) a self-consistent formal structure guided by that prior knowledge and (b) rules of correspondence that bring us back to laboratory practice, including empirical tests. In this sense, we are trodding a well worn path followed by many physicists engaged in constructing fundamental theories.

Another, related, criticism assumes that the high degree of structural information we must ascribe to counting finite integers is a very loose mesh. Changes of interpretation *seemed* possible before the program produced a coherent lump of concepts and structure and numerical correlations. Bastin was often able to be sure that some of HPN's early attempts at interpretation had to be wrong; unfortunately these objections had to be made at a level of generality that prevented the specific technical line of argument from being developed. We now have a 35 year "track record" of meeting honest criticism and modifying our ideas to meet the challenges posed. Some challenges come from the explosion of precise information provided by contemporary high energy particle physics and observational cosmology. Others come from questions of self-consistency and coherence that can only be met by a "paradigm shift". Perhaps the best way to meet these challenges is to summarize the positive predictions that stem from our program, — predictions whose failure would require us to modify or abandon the theory. We summarize these predictions in Table 3.

The conventional physicist accepts all the structural results we have listed; in his practice he uses numbers which satisfy (to an accuracy discussed below) the numerical consequences of the algebraic relations given. At this point we would like to ask this "conventional reader" why *he* accepts the structural results we have "predicted" from our principles. The unconventional reader may accept some, but not all of our structural results; we ask him *how* he makes that selection. We ask either type of reader what would cause them to *reject* any of these results which they now accept. We also ask them to *explain* why they accept or reject any of our results.

Many people are uncomfortable with a theory that rests on what appears to be so little empirical foundation. Of course, there are tried and true routes out of the problems our theory poses: naked empiricism, "just so stories", laws of thought, uncontrolled skepticism, solopsism, logic, quantum logic, infinity,.... We believe we are close to the current practice of physics when we reject such escape hatches as likely to dump us in a still more unfortunate situation. We part company with most contemporary practice only by insisting that it is important to ask these fundamental questions. We are comfortable with the ways, sketched in this paper, we arrive at our conclusions. We are prepared to scrub the theory if there is clear evidence that any piece of this structure fails, and will look to such failures for clues to where to look in starting an new approach.

Physicists tend to be impatient with "philosophical" challenges. We turn next to the cosmological predictions. Ours have both a structural and a quantitative aspect. Conventional cosmology breaks into two parts: the evolution of the universe after the radiation breaks away from the matter, which we call "fireball time", and the model-sensitive earlier history. Since the combinatorial hierarchy result set the gravitational and electromagnetic scales back in 1961, and we have subsequently given detailed proof that we can calculate atomic and nuclear problems in close enough agreement with experiment for most cosmological purposes, conventional extrapolation of the $2.7^o K$ background radiation back to that time works as well for us as for anyone else — given the 50 % empirical uncertainties in the critical parameters. There is an event horizon beyond which even radio galaxies disappear, and behind that the fireball; this backward extrapolation is reasonably consistent with contemporary physics as it works here and now.

All of this works for us because our estimate of the visible matter within the event horizon is an order of magnitude smaller than the amount of matter needed to "close the universe" in conventional (general) relativistic cosmologies. Since we have established the conservation laws of the standard model, and our labels are created either by discrimination or TICK in order to form the labels in the first place, we can estimate the number of vertices in which two different labels participate for the first time as $2^{127} + 136$. Once the labels are formed, the construction retains each of them independently as labels for content ensembles. Hence there are something like $(2^{127} + 136)^2$ quantum number conserving labels generated before the "space-time content" has much meaning. In the absence of further information, the average mass must be our unit mass m_p, from which the estimate follows. This prediction is in agreement with observation, since the observed visible mass within the event horizon is about what we estimate. A more precise estimate will require a more detailed statistical calculation of the probability of formation of lepton and baryon labels. With such an "open" universe, Newtonian gravitation is good enough for post-fireball time cosmological calculations.

This same estimate gives us our next prediction. Mike Manthey noted that the fact that it takes $2^{127} + 136$ TICKs to form the labels defines a time, and HPN identified it as "fireball time". The problem here is an old one. As we go back earlier, we have to rely more and more on what we mean by "the laws of physics" or whatever phrase describes the methodology used for extrapolation. Once one tries to extrapolate backward from fireball time using a linear time scale, one rapidly approaches extreme conditions that currently occur only in the interiors of stars, in the cosmic radiation when it interacts with matter, in the neighborhood of massive "black holes" or in high energy physics laboratories. When one tries to get back inside of "the first three minutes" the empirical evidence vanishes and only disciplined conjecture provides a guide. We simply assert that "time" loses any useful "model independent" meaning somewhere between fireball time and the first three minutes. In our model, if we use the appropriate unit of time ($\hbar/m_pc^2$), our backward extrapolation gives us roughly 3 and a half million years back to the first discrimination. Other models give roughly similar results back to the first three minutes. Before that I see no way for a physicist to make testable statements as to whether the universe "always existed" or "came into being at a finite time". As we have already commented on above, the conventional wisdom is in much worse shape here than we are. Most of their model universes are buried under a pile of (BLEEP) that weighs 10^{125} times too much for them to dig their way out from under it - except by the observation that we nevertheless exist, and that human ingenuity should be able to find an explanation.

The prejudice of most cosmologists is that the universe should be closed, or "just closed", for reasons that escape me. [I find an open universe much more satisfactory, particularly after reading Dyson's scientific eschatological analysis.][32] The "deficit" from the conventional perspective is now to be made up by "dark matter". Here they have a good observational case in that ten times as much of the mass of galaxies, as measured by Newtonian gravitation and the Doppler shift, is "dark" rather than electromagnetically visible. How much more there is depends, once again, on details of the cosmological model rather than on observation.

Here our theory makes a new prediction. Visible matter can only be understood by us in terms of the 137 labels for the first three levels of the hierarchy. But there are 3+7=10 labels that cannot be interpreted prior to the formation of the "background" of the 127 labels which make up level 3. Whatever they are, they must be electrically neutral and will occur, statistically, 12.7 times more frequently than the level 3 labels. They could form electromagnetically inert structures at any scale compatible with our finite scheme (quantum geons?). So our estimate of the amount of "dark matter" left over from the "big bang" to the visible matter is 12.7; a better estimate will depend on what version of the early stages of *program universe* we use. Quantitatively, the prediction for the gravitational constant (using m_p, c and $\hbar$ to connect our units of mass length and time to experiment) fails by a little less than 1 %. We anticipate a correction of order α, and hope to be able to compute it once we have sorted out the experimental effects usually ascribed to general relativity.

Most of this cosmology is predicated on the assumption that we have got the atomic and nuclear physics right. If one believes the six results given in Table 3, which we compare with experiment below, as "elementary particle predictions" and accepts our finite particle number scattering theory as both unitary and crossing symmetric, we can do as least as well as most practitioners in reproducing one or another currently accepted phenomenology for atomic, nuclear, and high energy particle physics. This will be "obvious" to readers with an S-Matrix background; we will never be able to convince some physicists who are not used to that type of practice. So we concentrate here on where these six numbers come from, what estimate of theoretical uncertainty we ascribe to them, and how they compare with experimental values.

The calculation of the fine structure constant is due to David McGoveran.[33] It is preliminary, and was discussed at ANPA 10. The calculation came out of an examination of the Sommerfeld formula for the fine structure spectrum.[34] HPN argued that since we now have a fully relativistic theory, including angular momentum conservation and non-commutativity, a non-relativistic combinatorial model for the Bohr Atom (Section 6.4), and Bohr-Sommerfeld quantization (Section 6.3), we should be able to get this relativistic correction by including two different periods in the calculation. This is indeed the case, but then HPN realized that our approximation of 137 for $1/\alpha$ is no longer good enough. He feared we would have to do all of QED to order α^2 in order to sort this out, but McGoveran realized that the existence of two frequencies in the problem gave us a combinatorial argument that leads to the result quoted above. Numerically this formula predicts $1/\alpha = 137.0359674..$ as compared to the two values quoted in the particle properties data booklet:[35] (old) 137.03604(11) and (new) 137.035963(15). So far as we can see, any correction to our prediction should be of order G_F/M_W^2 or that number times —$sin^2\theta_{Weak} - \frac{1}{4}$—; if this estimate of the uncertainty is correct, we do not find the close agreement with experiment surprising.

The m_p/m_e formula is due to Parker-Rhodes[23]. Since our theory differs from his, in the past we could only provide heuristic justification for the calculation. Now that we have a fully developed relativistic quantum mechanics, with 3-momentum conservation, these past arguments become rigorous when we view the calculation as a calculation of the mass in the electron propagator — for us, a finite "self-energy". One puzzle was the extreme accuracy of the result, using 137 rather than the empirical value for $1/\alpha$. However, now that we have found that the "empirical value" comes about in systems which lack spherical symmetry, or in combinatorial terms have two independent frequencies, and recognize that in the m_p/m_e calculation there is no way to define a second frequency, we have a rigorous justification for the formula as it stands. Numerically, we predict $m_p/m_e = 1836.151497...$ as compared with[35]: (old) 1836.15152(70) and (new) 1836.152701(100). We see that the proposed revision in the fundamental constants has moved the empirical value outside of our prediction by a presumably significant amount. For the m_p/m_e calculation the correction due to non-electromagnetic interactions could be large enough to affect our results.

The calculation of the neutral pion mass was made long ago.[36] The model is due to our interpretation of Dyson's argument[37] that the maximum number of charged particle pairs which can be *counted* within their own Compton wavelength is 137. Taking these to be electron-positron pairs, we get the result. The argument in the past rested on the use of the Coulomb "potential". Now that we have a combinatorial calculation of the Bohr atom, we no longer need this extraneous element. If one looks at the *content* strings minimally needed to describe the possible states of the bound system, the saturation at 137 pairs emerges. As we can see from the Bohr atom calculation (eg by considering one electron or positron interacting with the average charge of the rest of the system), the first approximation for the binding energy is non-relativistic. Consequently the estimate for the system mass, interpreted as the neutral pion mass, is just the sum of the masses, or 274 m_e, in agreement with experiment to better than ten electron masses. It will be interesting to calculate the α relativistic corrections (including the virtual electron-positron annihilation) and the neutral pion lifetime. Adding an electron-antineutrino pair to get the π^-, or a positron-neutrino pair to get the π^+, will be a good problem for sorting out our understanding of weak-electromagnetic unification.

The weak-electromagnetic unification needs more work, as has already been indicated. The first order prediction of $sin^2\theta_{Weak} = 0.25$ as compared to the experimental result[38] of 0.229 ± 0.004 is firm, and reasonably satisfactory at the current stage of development. The identification of the weak coupling constant (without the factor of $\sqrt{2}$) was a suggested by Bastin long ago[18]; our formula predicts a result which is about 7 % too large. Since this is roughly the amount by which we fail to get the weak angle, the two discrepancies might find a common explanation.

The quantum number structure of the quarks has been discussed in Section (5.5), and does lead to the usual 3 "valence" quark structure of the baryons, which gives us the usual "non-relativistic quark model" as a starting point. As already noted, McGoveran's theorem does not allow more than three "asymptotic" degrees of freedom, so we do predict color confinement. This means that we cannot use our standard "free particle" states to describe quarks or gluons and define their mass. We suspect that we can eventually obtain "running masses" analagous to those Namyslowski[11] gets out of the conventional theories, but have only just started thinking about the problem. Another challenge will be to relate the pion model discussed above to a quark-antiquark pair.

By now we hope the reader will grant that we have made a case for discrete physics as a fundamental theory. We have been led to many conceptual and numerical results that can only be obtained with difficulty, or not at all, by more conventional approaches. We believe the program will prove to be useful even if it ultimately fails. So far we have run into no insuperable barriers — frankly somewhat to HPN's surprise. We have nailed down the quantum numbers in agreement with the standard model, and have computed reasonable values for the basic masses and coupling constants. Thanks to the high degree of overdetermination of elementary particle physics due to crossing and unitarity — Chew's bootstrap — we can expect to do about as well as conventional strong interaction theories. This means that when a difficulty *does* arise, it will suggest an area of phenomena that will deserve detailed experimental and theoretical examination. Again, we share this strategy with more conventional approaches.

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30. H.P.Noyes, remark in *Proc. ISQM 2*, Namiki, et al, eds., Phys.Soc.Japan, 1986, p.78; this argument was made by Noyes, McGoveran, Etter, Manthey and Gefwert in an abstract submitted to (but not published by) the 2^{nd} *INTERNATIONAL SYMPOSIUM ON FOUNDATIONS OF QUANTUM MECHANICS -In the Light of New Technology*, Tokyo, September 1-4, 1986; the text is given in footnote 4, p. 50 of *Proc. ANPA 7*. A more detailed explanation is given in Ref.1 and also in D.O.McGoveran and H.P.Noyes, "On the Computer Simulation of the EPR-Bohm Experiment", SLAC-PUB-4729, submitted to Bell's Theorem Workshop, George Mason Univ., Oct. 1988.

31. H.Goldstein, *Classical Mechanics*, 2nd Ed., Addison-Wesley, 1980, pp. 445-471.

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Table 1

Quantum numbers for weak-electromagnetic unification

Particle		Q	Y	$2i_z$	ℓ	$2h$	m in Gev/c^2
fermion	ν_L	0	−1	+1	−1	−1	0
	$\overline{\nu}_L$	0	+1	−1	+1	+1	0
	e_L^-	−1	−1	−1	−1	−1	$.511 \times 10^{-3}$
	$\overline{e}_L^-$	+1	+1	+1	+1	+1	"
	e_R^-	−1	−2	0	−1	−1	"
	$\overline{e}_R^-$	+1	+2	0	+1	+1	"
quantum	W_{LL}^-	−1	0	−2	0	−2	$37.3/\sin\theta_W$
	$\overline{W}_{LL}^-$	+1	0	+2	0	+2	"
	Z_{LL}^0, γ_{LL}	0	0	0	0	−2	$37.3/\sin\theta_W \cos\theta_W$,0
	$\overline{Z}_{LL}^0, \overline{\gamma}_{LL}$	0	0	0	0	+2	"
	Z_0^0, γ_c	0	0	0	0	0	"

Table 2

The combinatorial hierarchy

hierarchy level	ℓ	$B(\ell+1) = H(\ell)$	$H(\ell) = 2^{B(\ell)} - 1$	$M(\ell+1) = [M(\ell)]^2$	$C(\ell) = \Sigma_{j=1}^{\ell} H(j)$
	(0)	-	2	(2)	-
	1	2	3	4	3
	2	3	7	16	10
	3	7	127	256	137
	4	127	$2^{127} - 1$	$(256)^2$	$2^{127} - 1 + 137$

-Level 5 cannot be constructed because $M(4) < H(4)$

Table 3

Predictions made by discrete and combinatorial physics

Structural Predictions

- 3+1 asymptotic space
- limiting velocity
- discrete events
- supraluminal synchronization and correlation *without* supraluminal signaling
- discrete Lorentz transformations (for event-based coordinates)
- non-commutativity between position and velocity (for event-based coordinates)
- transport (exponentiation) operator
- recognizable conservation laws for 3- and 4- events
- quantum numbers of the standard model for quarks and leptons
- event horizon
- zero-velocity frame for the cosmic background radiation
- color confinement — quark and gluon masses not directly observable

Algebraic Cosmological Predictions

$$\left(\frac{M_{Planck}}{m_p}\right)^2 \simeq 2^{127} + 136 \simeq \frac{\hbar c}{G m_p^2}$$

$$M_{Vis.U} \simeq (2^{127} + 136)^2 \, m_p$$

$$\text{Fireball time} \simeq (2^{127} + 136) \frac{\hbar}{m_p c^2}$$

$$M_{Dark} \simeq 12.7 \, M_{Vis.U}$$

Algebraic Elementary Particle Predictions

$$\frac{1}{\alpha} = \frac{137}{\left[1 - \frac{1}{127 \times 30}\right]}$$

$$\frac{m_p}{m_e} = \frac{137\pi}{\frac{3}{14}\left(1 + \frac{2}{7} + \frac{4}{49}\right)\frac{4}{5}}$$

$$m_\pi \lesssim 2 \times 137 \, m_e$$

$$\sin^2 \theta_{\text{Weak}} = \frac{1}{4}$$

$$\frac{G_F}{m_p^2} = \frac{1}{\sqrt{2}\,(256)^2}$$

$$m_{u,d}(p_0) = \frac{1}{3} m_p$$

FIGURE CAPTIONS

1) The connection between 3-vertices and 4-events.
2) The generic Yukawa vertex and crossing.
3) Four-leg crossing.
4) Quantum electrodynamics.
5) Quantum electrodynamic conservation laws as planar vectors.
6) Weak-electromagnetic unification in terms of weak hypercharge, weak isospin and helicity.
7) Colors and anticolors as discrete vectors.
8) Spin, isospin, and baryon number conservation for color singlet neutrons and protons $p = u(ud), n = d(ud)$.
9) 9a) The participator model for a research program in physics; 9b) comparison with McGoveran's modeling methodology.
10) Program Universe 1 and 2 compared.
11) A 4-event followed by 5 events involving limiting velocity signals which can be used to establish the Lorentz transformations for event no. 3.
12) The connection between space-time and light-cone coordinates in terms of bit string distances and velocities for the physical situation envisaged in figure 11.

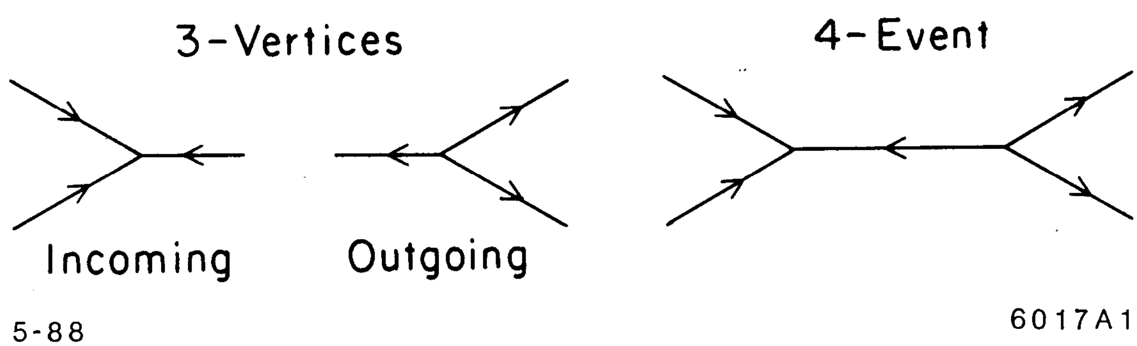


Fig. 1

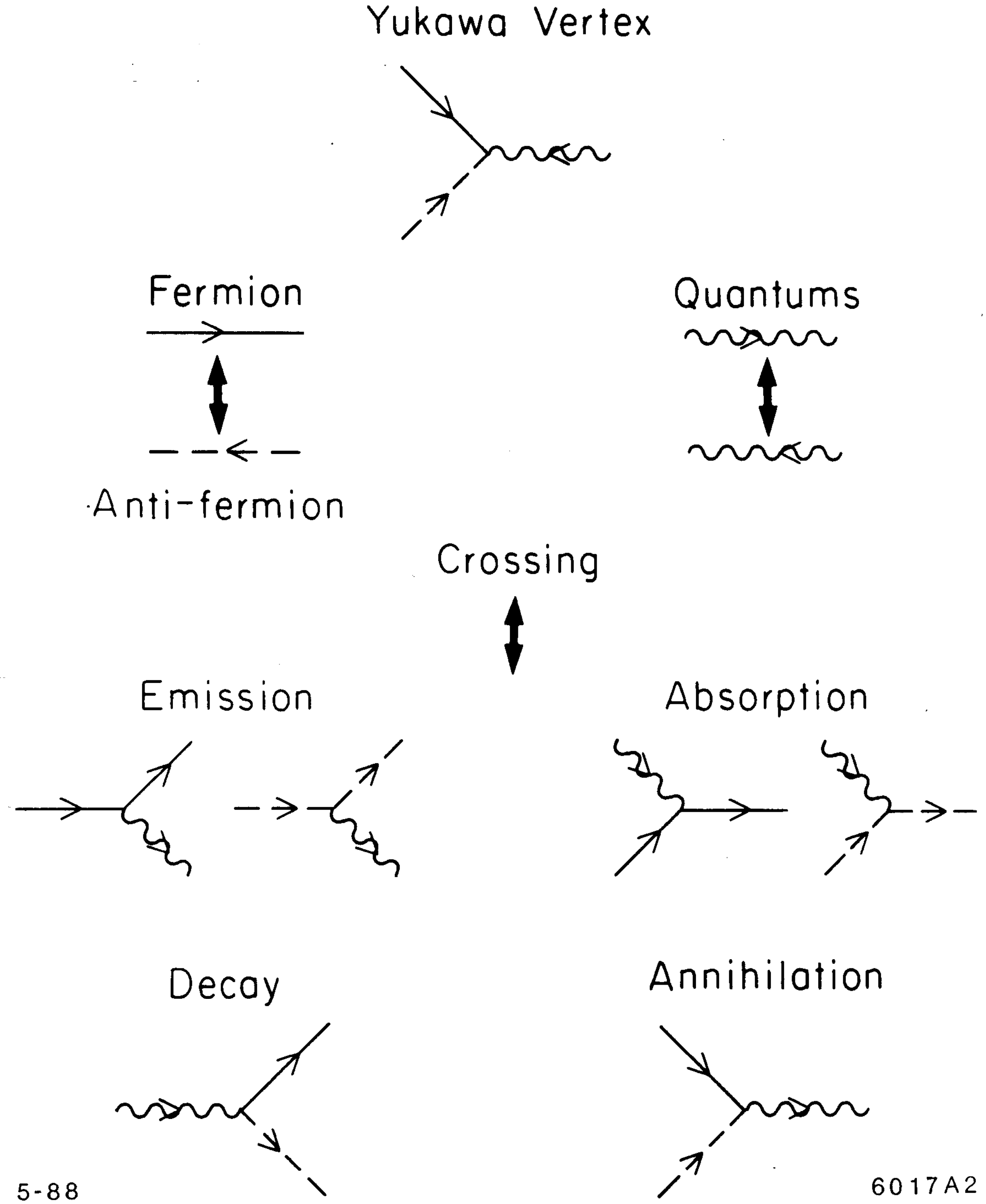

Yukawa Vertex
Fermion
Quantums
Anti-fermion
Crossing
Emission
Absorption
Decay
Annihilation
5-88
6017A2

Fig. 2

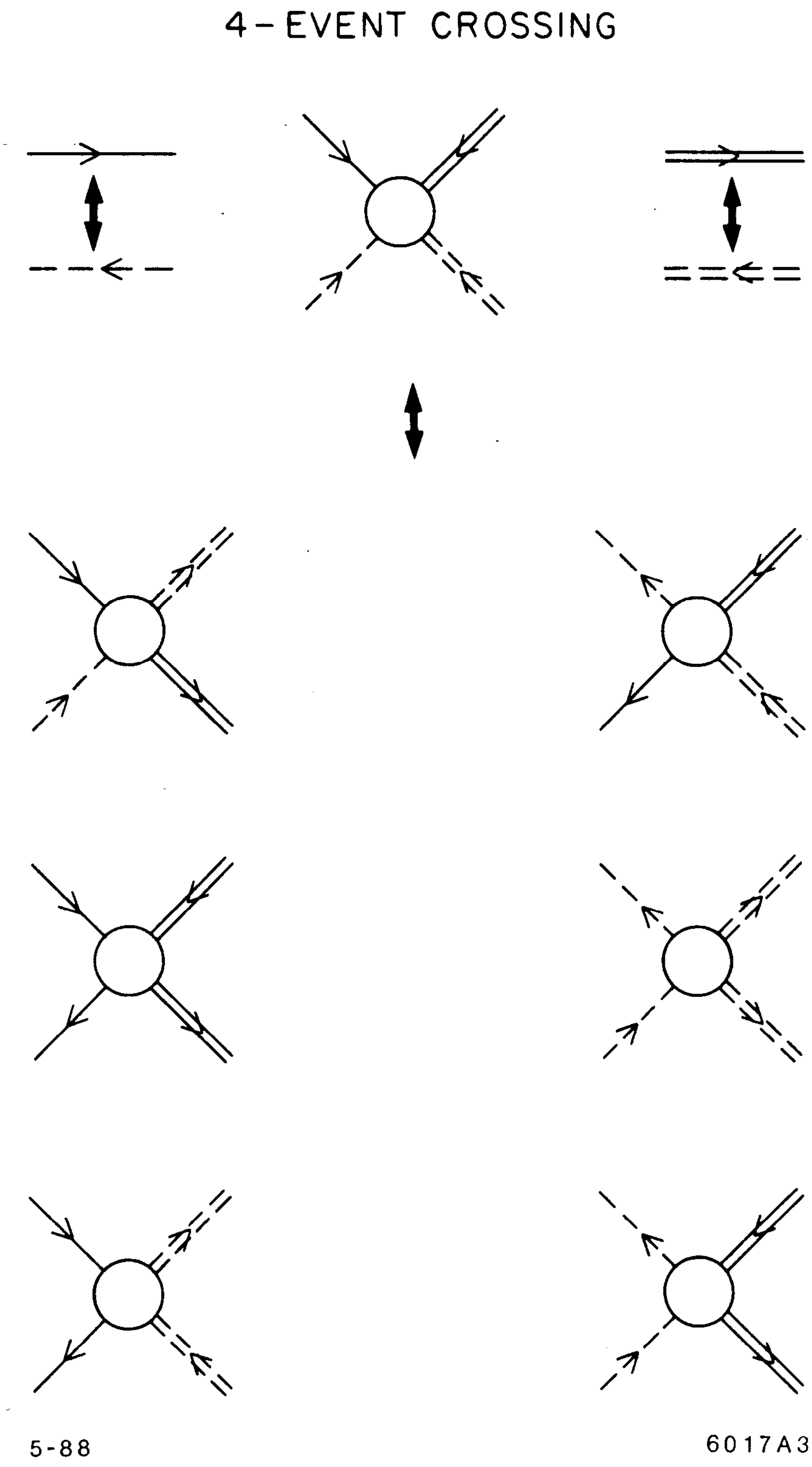


Fig. 3

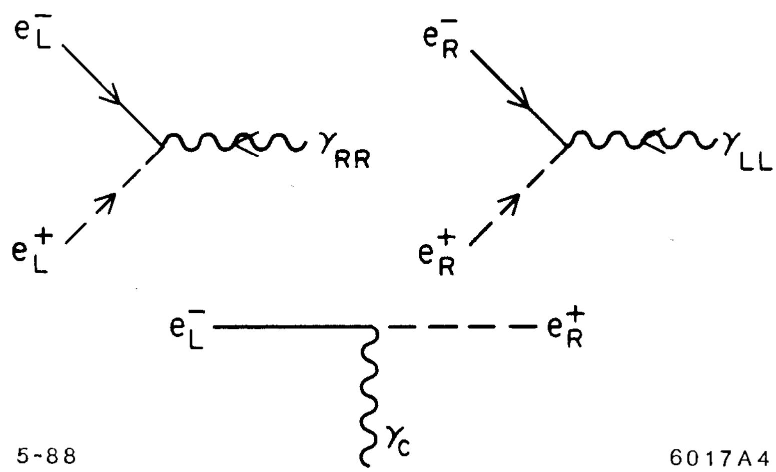


Fig. 4

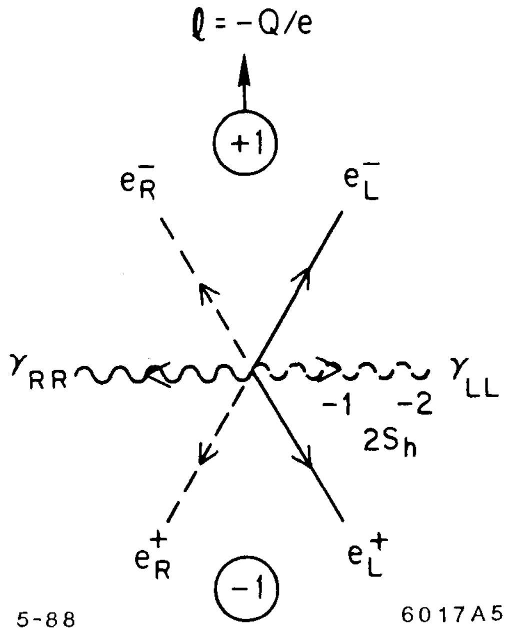
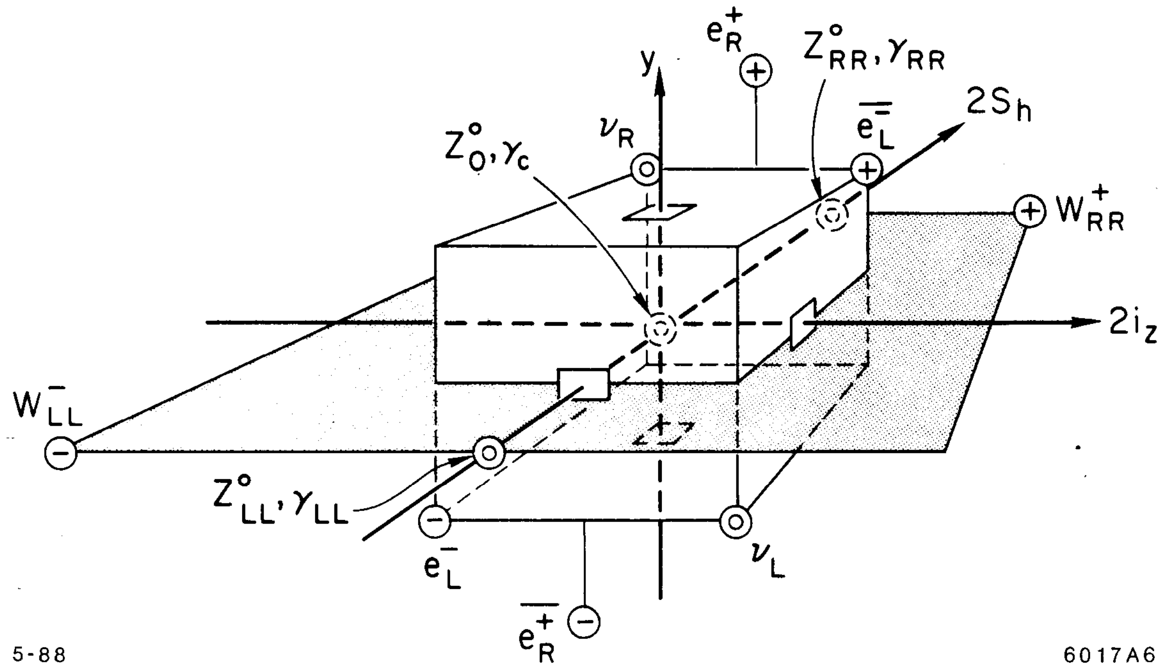


Fig. 5



e_R^+
$Z^\circ_{RR}, \gamma_{RR}$
y
$2S_h$
ν_R
$\overline{e_L^-}$
Z°_0, γ_c
W^+_{RR}
$2i_z$
W^-_{LL}
$Z^\circ_{LL}, \gamma_{LL}$
e_L^-
ν_L
$\overline{e_R^+}$
5-88
6017A6

Fig. 6

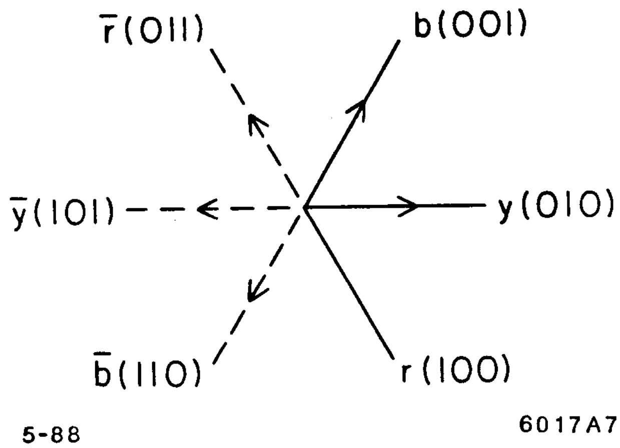


Fig. 7

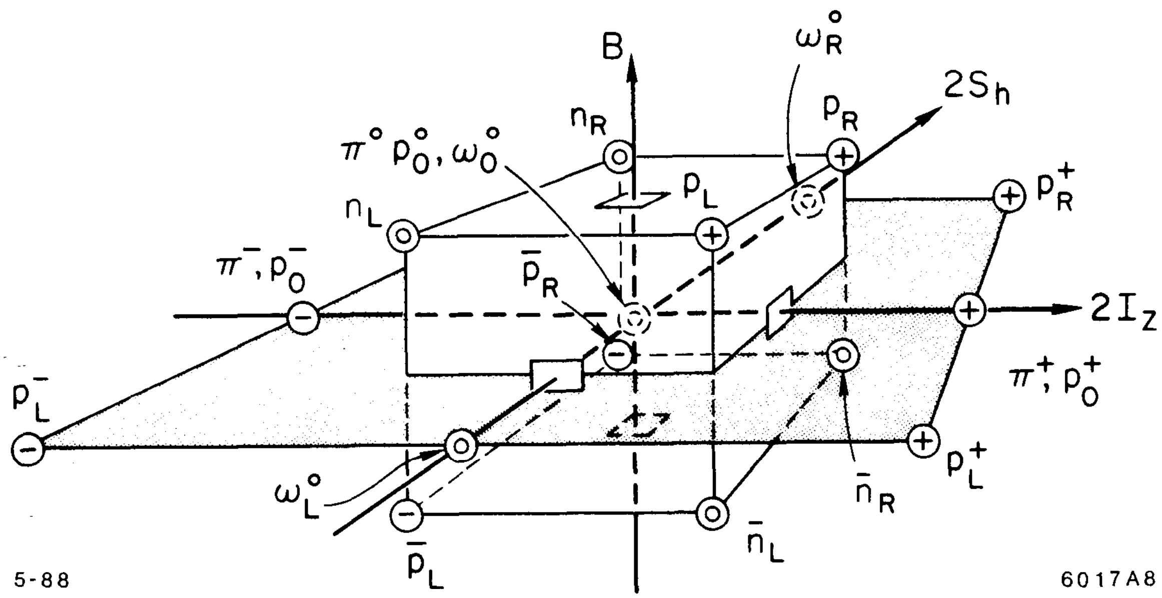

B
ω_R°
$2S_h$
n_R
p_R
$\pi^\circ\ p_0^\circ, \omega_0^\circ$
p_R^+
p_L
n_L
π^-, p_0^-
$\bar{p}_R$
$2I_z$
π^+, p_0^+
p_L^-
p_L^+
ω_L°
$\bar{n}_R$
$\bar{n}_L$
$\bar{p}_L$
5-88
6017A8

Fig. 8

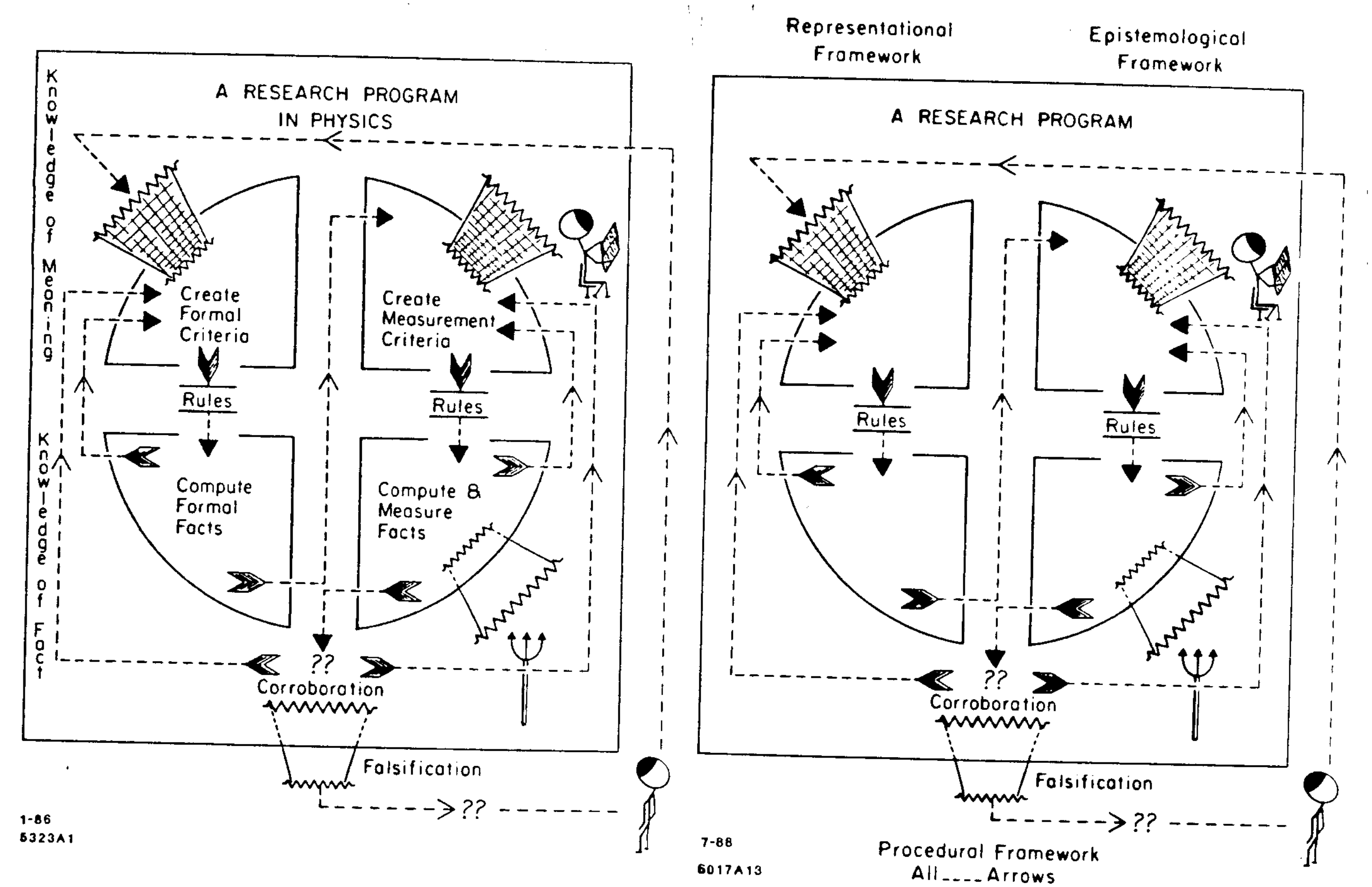

A RESEARCH PROGRAM IN PHYSICS
Knowledge of Meaning
Knowledge of Fact
Create Formal Criteria
Create Measurement Criteria
Rules
Rules
Compute Formal Facts
Compute & Measure Facts
??
Corroboration
Falsification
??
1-86
5323A1
Representational Framework
Epistemological Framework
A RESEARCH PROGRAM
Rules
Rules
??
Corroboration
Falsification
??
7-88
6017A13
Procedural Framework
All ---- Arrows

Fig. 9

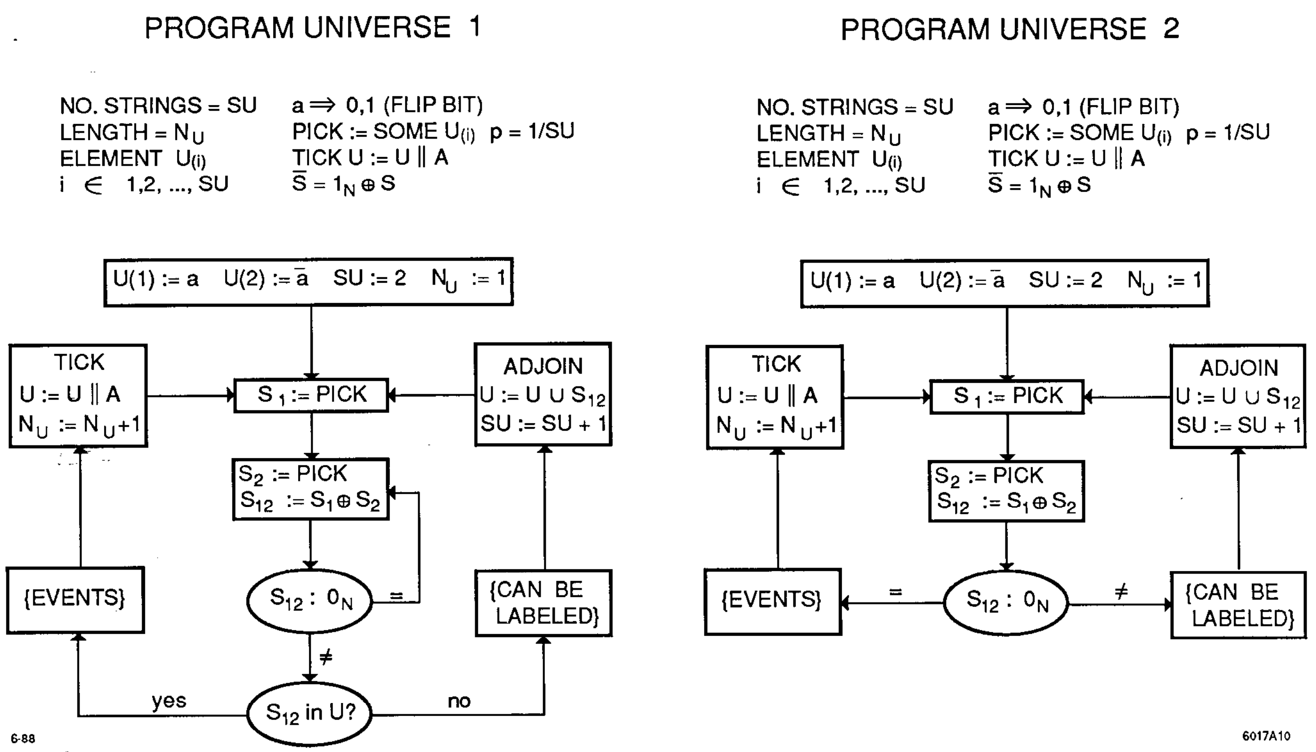

PROGRAM UNIVERSE 1
NO. STRINGS = SU
LENGTH = N_U
ELEMENT U(i)
i ∈ 1,2, ..., SU
a ⟹ 0,1 (FLIP BIT)
PICK := SOME U(i) p = 1/SU
TICK U := U ∥ A
S̄ = 1_N ⊕ S
U(1) := a U(2) := ā SU := 2 N_U := 1
TICK
U := U ∥ A
N_U := N_U+1
S_1 := PICK
ADJOIN
U := U ∪ S_12
SU := SU + 1
S_2 := PICK
S_12 := S_1 ⊕ S_2
{EVENTS}
S_12 : 0_N
=
{CAN BE LABELED}
≠
yes
S_12 in U?
no
6-88
PROGRAM UNIVERSE 2
NO. STRINGS = SU
LENGTH = N_U
ELEMENT U(i)
i ∈ 1,2, ..., SU
a ⟹ 0,1 (FLIP BIT)
PICK := SOME U(i) p = 1/SU
TICK U := U ∥ A
S̄ = 1_N ⊕ S
U(1) := a U(2) := ā SU := 2 N_U := 1
TICK
U := U ∥ A
N_U := N_U+1
S_1 := PICK
ADJOIN
U := U ∪ S_12
SU := SU + 1
S_2 := PICK
S_12 := S_1 ⊕ S_2
{EVENTS}
=
S_12 : 0_N
≠
{CAN BE LABELED}
6017A10

Fig. 10

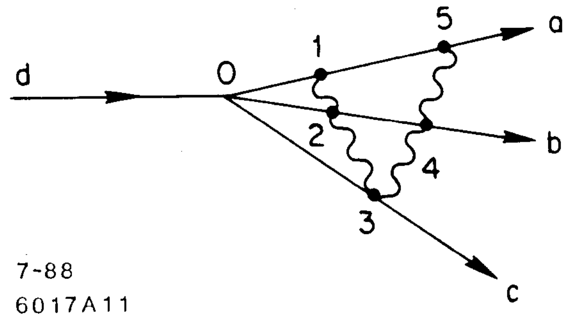


Fig. 11

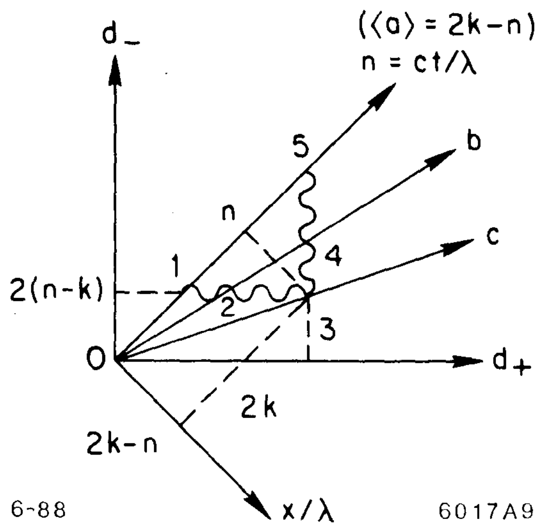


Fig. 12