# AN EVALUATION OF POPULATION PROJECTIONS BY AGE* 

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#### Abstract

A number of studies have evaluated the accuracy of projections of the size of the total population, but few have considered the accuracy of projections by age group. For many purposes, however, the relevant variable is the population of a particular age group, rather than the population as a whole. We investigated the precision and bias of a variety of age-group projections at the national and state levels in the United States and for counties in Florida. We also compared the accuracy of state and county projections that were derived from full-blown applications of the cohort-component method with the accuracy of projections that were derived from a simpler, less data-intensive version of the method. We found that age-group error patterns are different for national projections than for subnational projections; that errors are substantially larger for some age groups than for others; that differences in errors among age groups decline as the projection horizon becomes longer; and that differences in methodological complexity have no consistent impact on the precision and bias of age-group projections.


Population projections are used for a wide variety of planning and budgeting purposes. In many instances, projections of demographic characteristics are at least as important as projections of the total population. Age is a particularly important characteristic and is commonly used when projecting births (Lapkoff 1993), school enrollment (Fishlow 1994), residential care for children (Dunton 1994), hospital services (Rives 1994), social security revenues and expenditures (Lee and Tuljapurkar 1997), and many other policy-relevant variables. How accurate are age-group projections? Do forecast errors differ from one age group, time period, or level of geography to another? Are some projection methods more accurate than others? These are the questions we address in this article. ${ }^{1}$

By their very nature, population projections are uncertain. We cannot know precisely what the population will be one year from now, much less in 10,20 , or 30 years. This uncertainty can be assessed in two complementary ways (Alho and Spencer 1997). One is to estimate ex ante, or beforehand, how accurate a projection is likely to be by developing stochastic forecasting models that attach explicit statements of probability to population projections (e.g., Alho 1990; Lee 1992; Lee and Tuljapurkar 1994; Lutz, Sanderson, and Scherbov 1998). The other is to investigate the accuracy of projections ex post, or after future values have become known (e.g., Keyfitz 1981; Smith and Sincich 1992; Stoto 1983). Both approaches reflect the uncertainty inherent in population projections and both can be used to develop confidence limits. Only the second,

[^0]however, provides information on the actual performance of previous projections. This is the approach we followed in our study. ${ }^{2}$

Despite the importance of age-group projections for many purposes, few analysts have conducted ex post evaluations of their accuracy. In a study of national population projections in the Netherlands, Keilman (1990) reported that forecast errors were much larger for the youngest and oldest age groups than for any other groups. Projections tended to be too high for the youngest groups and too low for the oldest, reflecting a tendency to underestimate the magnitude of future declines in fertility rates and future increases in survival rates. When Keilman (1998) evaluated United Nations' projections for seven regions of the world, he again found the largest errors for the youngest and oldest groups, although the direction of the errors in these age groups was not the same in every region. Summarizing the evidence from a number of countries and regions of the world, Bongaarts and Bulatao (2000) also reported that the largest errors typically occurred in the youngest and oldest age groups; in most instances, projections for the youngest were too high and projections for the oldest were too low.

There have been fewer studies of age-group projections at the subnational level than at the national or regional levels. Campbell (1991) used the Index of Dissimilarity (D) to evaluate a set of short-range projections for states. Although he did not show errors for specific age groups, he reported that projected age distributions for whites were not markedly dissimilar to the 1990 census counts. In contrast, D values were much higher for blacks than for whites and for persons of other races than for blacks. Within all racial groups, D values for males and females were similar.

Smith and Shahidullah (1995) evaluated 10-year projections for a group of census tracts in Florida. They reported mean absolute errors of $20 \%-29 \%$ for projections of age groups, compared to $18 \%$ for projections of the total population. The largest errors were found for ages 25-34 and 65 and older and the smallest for ages 45-54 and 55-64. Smith and Shahidullah noted a tendency for projections to be too low for the youngest group and too high for the oldest.

We believe that these differences in empirical results were caused primarily by differences in launch years, projection horizons, and levels of geography. In this article, we evaluate the forecast accuracy of several sets of age-group projections in the United States. We analyze national and state projections that were produced by the U.S. Census Bureau (USCB) and county projections in Florida that were produced by the Bureau of Economic and Business Research (BEBR) at the University of Florida. All these projections were constructed using the well-known cohort-component method, in which births, deaths, and migration are projected separately for each age-sex cohort in the population. Our first objective is to present an array of empirical evidence on errors in age-group forecasts and to provide some explanations for the error patterns that are observed. We believe that firm conclusions can be drawn only after errors for a number of launch years, projection horizons, and levels of geography are evaluated.

Most applications of the cohort-component method require a great deal of effort to collect, verify, and organize input data and to develop and implement a projection model. This is a costly and time-consuming process. Furthermore, the fertility, mortality, and migration data needed to apply a full-blown model are often unavailable for small areas. A simpler version of the cohort-component method was developed that requires data only from two consecutive censuses and a set of simple calculations (Hamilton and Perry 1962). Our second objective is to compare the forecasting performance of this simpler method with the performance of more complex cohort-component models. We

[^1]believe that the simpler method is adequate for many purposes-especially for small areas and short-to-medium projection horizons-and offers a useful alternative to a fullblown cohort-component model.

## DATA SOURCES

The USCB published its first set of national projections in the 1940s and has published several sets each decade since then. In our study, we evaluated the first set of national age-group projections that was published after each decennial census from 1950 through 1990 (Day 1992; U.S. Census Bureau 1953, 1962a, 1962b, 1972, 1983). Each set included several alternative series. Those published in the 1950s, 1960s, and 1970s included series that were based on different assumptions about fertility (each series used the same assumptions about mortality and migration). For these projections, we evaluated an average that was based on the middle two series (or, for those published in 1962, the only two series). For the projections that were published in the 1980s and 1990s, we evaluated the projections that were specifically designated as the middle series. We refer to these projections as N1, N2, N3, N4, and N5, in chronological order.

Several changes in the coverage of national projections have occurred since 1950. The 1953 projections did not include Alaska and Hawaii, whereas all projections since 1960 have included these states. Projections before 1992 included armed forces abroad; projections since then have covered only the resident population. We accounted for these changes by comparing each set of projections with population data that are consistent with the assumptions of coverage on which the projections were based.

The USCB published its first set of state projections in the 1950s, but did not include details on ages until the 1970s. We evaluated the first set of age-group projections that were published after the 1970, 1980, and 1990 censuses. These projections refer solely to the resident population.

Although the procedures that were used to project births and deaths were similar in all three sets of state projections, those that were used to project interstate migration differed considerably from one set to another. The projections that were published in the 1970s (U.S. Census Bureau 1979) accounted for in- and out-migration separately by using state-specific out-migration rates and in-migration proportions. This set consisted of three alternative series: one that was based on 1965-1975 trends in interstate migration, one that was based on 1970-1975 trends in interstate migration, and one that was based on the hypothetical assumption that there would be no interstate migration. We evaluated an average that was based on the first two series. The projections that were published in the 1980s (U.S. Census Bureau 1983) had only a single series that was based on net migration rates between 1970 and 1980. Those that were published in the 1990s (Campbell 1994) contained several alternative series. We evaluated the series that was designated as "preferred" by the Census Bureau; this series used a matrix of state-to-state migration rates and a combination of time-series and other projection techniques. We refer to the state projections as $\mathrm{S} 1, \mathrm{~S} 2$, and S3, in chronological order.

The BEBR began making county population projections in the 1970s but did not publish details on ages until the mid-1980s. We evaluated one set of age-group projections from the 1980s (Smith and Ahmed 1986) and one from the 1990s (Smith, Shahidullah, and Bayya 1992). These projections were made in a two-step process. First, projections of the total population were made using several simple extrapolation techniques and several alternative historical base periods. The final projection for each county was based on a trimmed mean of the individual projections, controlled to an independent projection of the state population. Second, projections by age and sex were made using the cohortcomponent method and were controlled to the county projections of the total population described in the first step. We evaluated the middle series in each set of projections and refer to them as C 1 and C 2 , in chronological order.

The BEBR makes population projections in five-year age groups through age 85 and older, but USCB publications have followed several different age-reporting formats. Some have used single-year age groups, some have used five-year age groups, and some have used 10 -year age groups. Some had 85 or older as the oldest age group, whereas others had 65 or older (hereafter $65+$ ). We evaluated errors using categories that covered the full age spectrum and that allowed for consistent comparisons of all sets of projections: younger than age 5 (hereafter < 5), 5-14, 15-24, 25-34, 35-44, 45-54, 55-64, and 65+. ${ }^{3}$

## ERROR MEASURES

We evaluated age-group projections by comparing them with census counts for each decennial census year for which the projections were made. We refer to the differences as "forecast errors," although they may have been caused partly by errors in the census counts themselves. ${ }^{4}$ For the total population and each age group, percentage errors (PE) were calculated by subtracting the census count (A) from the corresponding projection (F) for each target year ( $t$ ), dividing the difference by the census count, and multiplying by 100 :

$$
\mathrm{PE}_{t}=\left[\left(\mathrm{F}_{t}-\mathrm{A}_{t}\right) / \mathrm{A}_{t}\right] \times 100
$$

We used two measures of the accuracy of forecasts. The mean absolute percentage error (MAPE) is the average when the direction of error is ignored. This is a measure of precision, or how close the projections were to census counts, regardless of whether they were too high or too low. The mean algebraic percentage error (MALPE) is the average when the direction of error is accounted for. This is a measure of bias, or the tendency of projections to be too high or too low. Both measures have been widely used to evaluate the accuracy of population projections (e.g., Keilman 1990; Long 1995; Smith and Sincich 1992; Tayman and Swanson 1996). ${ }^{5}$

## EXPECTATIONS

The growth or decline in an area's population is determined by the interplay of births, deaths, and migration. In most countries, international migration accounts for only a small proportion of the growth in the national population (Bongaarts and Bulatao 2000:9). Furthermore, it generally influences the size of all age groups (albeit not equally), rather than a single age group. Errors in projecting international migration rates

[^2]would therefore be expected to have relatively little impact on the accuracy of age-group projections in most countries.

We believe that the same is true for national projections in the United States. Although net international migration has increased considerably during recent decades, natural increase was the major determinant of growth in the U.S. population during the period covered by this study. Consequently, errors in projecting fertility and mortality rates are likely to have had a greater impact on the accuracy of national age-group projections than have errors in projecting international migration. Furthermore, since errors in projecting fertility rates immediately affect only the youngest group and errors in projecting mortality rates affect primarily the oldest groups, we expect that forecast errors for short-range U.S. projections will be the largest at the youngest and oldest ages, as previous studies of national and regional projections (e.g., Bongaarts and Bulatao 2000; Keilman 1990, 1998) have found. As the projection horizon becomes longer, however, the relatively large errors for the youngest age group will spread to successively older groups.

We do not believe that the direction of forecast errors for the youngest and oldest groups can be generalized. Rather, it will vary according to trends in fertility and mortality rates. If fertility rates are projected to decline but decline more rapidly than expected, projections of the youngest groups will be too high; if they decline less rapidly (or increase), they will be too low. Similarly, the tendency for projections of the older groups to be too high or too low will depend on whether mortality rates decline more or less rapidly than expected. We do not believe it is possible to know in advance which tendency will prevail for any given set of projections.

For subnational areas, migration is often the major determinant of population growth (Smith and Ahmed 1990). It is also the most difficult component of growth to forecast accurately because state and local migration rates are subject to much greater volatility than are either fertility or mortality rates (e.g., Kulkarni and Pol 1994; Long and McMillen 1987; Nakosteen 1989). Since migration rates vary markedly by age, forecast errors are likely to be larger for some age groups than for others. Typically, migration rates are high for young children, decline as those children reach adolescence, rise to a peak for young adults, and decline steadily thereafter (sometimes with a small increase around retirement age). For subnational areas, then, we expect forecast errors to be relatively large not only for young children but also for young adults. If an area is characterized by unusual migration patterns (e.g., a large inflow of retirees), errors may be relatively large for other age groups as well.

## EMPIRICAL RESULTS

## National

Table 1 summarizes forecast errors for projections of the U.S. population for each projection series and target year, by age group. Errors varied considerably from one age group to another, from one projection series to another, and from one target year to another. For most projection series, the average error for all age groups was greater than the error for the total population (disregarding the direction of error). In many instances, however, errors for some age groups were smaller than the error for the projection of the total population.

These results largely confirm our expectations. For projections covering approximately 10-year horizons (N1-1960, N2-1970, N3-1980, N4-1990, and N5-2000), errors were almost always the largest for the youngest age group (sometimes by a wide margin). The direction of error was not uniform, however. N1 substantially underprojected the $<5$ age group in 1960, whereas N2 and N3 substantially overprojected that group in 1970 and 1980, respectively. This finding reflects the difficulty of accurately projecting fertility rates during times of rapid change. It should be noted that the $<5$ age group had

Table 1. Percentage Errors for U.S. Population Projections, by Age Group

| Series and <br> Target Year | $<$ | 5 | $5-14$ | $15-24$ | $25-34$ | $35-44$ | $45-54$ | $55-64$ | $65+$ | Total |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | Average $^{2}$.

${ }^{2}$ Average of all age groups (disregarding the signs).
a small error for N 5 ; this finding can be attributed to the relatively high degree of stability in U.S. fertility rates over the past 20 years.

The population aged $65+$ was underprojected in four of the five sets of 10 -year projections; this finding is consistent with the findings of the national and regional studies cited previously. In contrast to those studies, however, errors for this age group were particularly large only for N 1 and N 3 . In three of the five sets of projections, errors for age 65+ were similar to (or smaller than) the average age-group error. These small errors reflect the relative stability in U.S. mortality trends over the past few decades.

Projections for the middle age groups were quite accurate. With only two exceptions, errors for the 25-34, 35-44, 45-54, and 55-64 age groups were smaller than average in every set of projections. In many instances, errors for these groups were $1 \%$ or less.

Forecast errors tend to build on themselves over time. This pattern can be seen when age groups are viewed from either a cohort or a period perspective. In Series N3, for example, there was a large error for the < 5 age group in 1980, the 5-14 age group in 1990, and the 15-24 age group in 2000 (i.e., the cohort perspective). For each age group older than 5-14, errors became larger as the horizon became longer (i.e., the period perspective). Similar results were found in the other series as well.

For every series, the average age-group error (disregarding the signs) became progressively larger as the projection horizon lengthened. This finding is not surprising, of course: the longer the horizon, the greater the chances that unforeseen developments will lead to unexpected changes in fertility, mortality, and migration patterns (Bongaarts and Bulatao 2000:41).

## State

Table 2 summarizes the forecast errors for state projections for each series and target year. The projections covered horizons of approximately 10 years (S1-1980, S2-1990, and S3-2000), 20 years (S1-1990 and S2-2000), and 30 years (S3-2000). ${ }^{6}$ Several patterns

[^3]Table 2. Mean Absolute Percentage Errors (MAPEs) and Mean Algebraic Percentage Errors (MALPEs) for State Population Projections, by Age Group

| Series and <br> Target Year | $<5$ | $5-14$ | $15-24$ | $25-34$ | $35-44$ | $45-54$ | $55-64$ | $65+$ | Total | Average $^{2}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| MAPE |  |  |  |  |  |  |  |  |  |  |
| S1-1980 | 8.7 | 4.9 | 5.5 | 6.5 | 4.4 | 2.9 | 4.3 | 3.0 | 4.1 | 5.0 |
| S1-1990 | 11.8 | 9.4 | 6.9 | 8.5 | 8.5 | 6.4 | 5.6 | 5.4 | 6.0 | 7.8 |
| S1-2000 | 12.7 | 11.3 | 10.9 | 12.6 | 10.9 | 9.8 | 8.9 | 10.5 | 9.4 | 11.0 |
| S2-1990 | 14.9 | 7.9 | 7.5 | 10.1 | 7.9 | 5.3 | 3.2 | 4.8 | 6.9 | 7.7 |
| S2-2000 | 19.3 | 16.2 | 13.1 | 15.0 | 12.9 | 9.4 | 7.1 | 8.1 | 11.2 | 12.6 |
| S3-2000 | 7.4 | 4.2 | 4.9 | 7.3 | 3.3 | 5.1 | 4.7 | 2.7 | 3.4 | 4.9 |
| MALPE |  |  |  |  |  |  |  |  |  |  |
| S1-1980 | -3.3 | -3.0 | -3.1 | -4.3 | -0.8 | -0.2 | -1.7 | -2.8 | -2.8 | -2.4 |
| S1-1990 | 6.9 | 2.5 | -3.6 | -2.1 | -1.8 | 1.6 | -0.7 | -4.8 | -1.1 | -0.3 |
| S1-2000 | -1.3 | -0.5 | -4.8 | -8.9 | -6.3 | -4.6 | -4.1 | -10.1 | -5.5 | -5.1 |
| S2-1990 | 9.6 | 3.8 | -0.2 | 6.7 | 4.7 | 3.1 | 0.1 | 1.1 | 3.7 | 3.6 |
| S2-2000 | 2.8 | 1.5 | -3.8 | -0.6 | 3.1 | 1.2 | -2.1 | -2.7 | 0.0 | -0.1 |
| S3-2000 | 2.0 | -1.0 | -2.6 | -0.5 | 0.0 | -5.0 | -4.5 | -0.8 | -1.6 | -1.6 |

${ }^{\text {a }}$ Average of all age groups.
stand out. MAPEs by age group were generally larger than those reported in the corresponding sets of national projections. This result is consistent with the common empirical finding that larger populations can generally be projected more accurately than smaller populations (e.g., Bongaarts and Bulatao 2000; Smith and Sincich 1992; White 1954). The average MAPE for all eight age groups was larger than the MAPE for the total population for all projection series and all target years; on the average, it was more difficult to develop precise forecasts for specific age groups than for the population as a whole.

Errors differed considerably from one age group to another. For all three sets of 10year projections, MAPEs were the largest for the $<5$ age group and the second largest for the $25-34$ age group. This finding is consistent with our expectation that errors will be particularly large for young adults because they have high migration rates and for young children because the uncertainty in projecting fertility rates is combined with the uncertainty in projecting the number of women of childbearing age. MAPEs for age groups older than 35 were uniformly lower than MAPEs for age groups younger than 35 .

For projections with two or more target years (i.e., Series S1 and S2), MAPEs increased with the length of the projection horizon. This finding can be seen both when age groups are viewed from a cohort perspective (e.g., 5-14 in 1980 and 15-24 in 1990) and when they are viewed from a period perspective (e.g., 5-14 in both 1980 and 1990). It is consistent with many studies that have found that errors for projections of the total population increase with the length of the projection horizon (e.g., Keilman 1990; Smith and Sincich 1992; White 1954).

For longer horizons, errors for older age groups incorporate the impact of previous errors for younger age groups, averaging together the effects of larger and smaller errors over time. Table 3 shows the largest and smallest age-group MAPEs for state projections. For S1, the largest MAPE was three times larger than the smallest for 1980, a little more than twice as large for 1990 , and $40 \%$ larger for 2000 . For S2, the largest MAPE was

Table 3. Comparison of the Largest and Smallest Age-Group Mean Absolute Percentage Errors (MAPEs) for State Projections

| Series and Target Year | Largest Error |  | Smallest Error |  | Ratio ${ }^{\text {a }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Age Group | MAPE | Age Group | MAPE |  |
| S1-1980 | < 5 | 8.7 | 45-54 | 2.9 | 3.0 |
| S1-1990 | < 5 | 11.8 | 65+ | 5.4 | 2.2 |
| S1-2000 | < 5 | 12.7 | 55-64 | 8.9 | 1.4 |
| S2-1990 | < 5 | 14.9 | 55-64 | 3.2 | 4.7 |
| S2-2000 | < 5 | 19.3 | 55-64 | 7.1 | 2.7 |
| S3-2000 | < 5 | 7.4 | $65+$ | 2.7 | 2.7 |

${ }^{\text {a }}$ The largest MAPE/the smallest MAPE.
almost five times larger than the smallest for 1990 but less than three times larger for 2000. We believe that differences in MAPEs among age groups tend to decline as the projection horizon becomes longer. For very long horizons, errors are likely to be much the same for all age groups.

MAPEs were generally larger for S2 than for the other two projection series. There are several possible explanations for this finding. First, S2 was based on a net-migration model, whereas the others were based on gross-migration models. Several studies have shown that the choice of the migration model can have a substantial impact on the resulting projections (e.g., Isserman 1993; Plane 1993; Smith 1986). Second, the projections in S2 covered a full 10-year horizon, whereas those in S1 and S3 were based on postcensal estimates and covered horizons of fewer than 10 years (especially for S 1 ). As we noted previously, forecast errors tend to increase with the length of the projection horizon. Finally, some periods are simply characterized by more demographic instability than others. A number of studies have found that forecast errors vary-sometimes substantially-by launch year (e.g., Keilman 1990; Long 1995; Smith and Sincich 1992). The relatively large errors for S2 may have been caused by any (or all) of these factors.

Two sets of projections had a downward bias and one had an upward bias. S1 and S3 had negative MALPEs for the total population and most age groups, whereas S2 had a positive MALPE for the total population and most age groups. We do not believe that cohort-component projections have any inherent upward or downward biases, either for the total population or for any specific age group. Rather, the tendency for projections to be too high or too low is determined by the correspondence between the underlying assumptions and the ensuing demographic trends. For some sets, the majority of projections turn out to be too high; for others, the majority turn out to be too low. There is no way to know in advance which tendency will prevail for any given set of projections (Smith and Sincich 1988).

## County

Table 4 summarizes the forecast errors for BEBR's Florida county projections. Some of the results were similar to those reported for the state projections, but others were considerably different. The $25-34$ age group had the largest MAPE in two of the three sets of projections and the second largest in the third; this finding is consistent with our expectations and with the state results. However, in two of the three sets of projections, the 55-64 and 65+ age groups also had larger-than-average MAPEs and the $<5$ age group had a smaller-than-average MAPE. These results are considerably different from those reported for states.

Table 4. Mean Absolute Percentage Errors (MAPEs) and Mean Algebraic Percentage Errors (MALPEs) for County Population Projections in Florida, by Age Group

| Series and <br> Target Year | $<5$ | $5-14$ | $15-24$ | $25-34$ | $35-44$ | $45-54$ | $55-64$ | $65+$ | Total | Average $^{2}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| MAPE |  |  |  |  |  |  |  |  |  |  |
| C1-1990 | 9.6 | 9.1 | 11.2 | 13.3 | 10.4 | 9.5 | 9.3 | 11.0 | 6.7 | 10.4 |
| C1-2000 | 16.4 | 13.7 | 13.1 | 18.0 | 14.7 | 12.3 | 15.5 | 17.8 | 10.6 | 15.2 |
| C2-2000 | 10.3 | 8.0 | 10.2 | 12.7 | 11.6 | 11.2 | 13.3 | 10.1 | 7.5 | 10.9 |
| MALPE |  |  |  |  |  |  |  |  |  |  |
| C1-1990 | -4.0 | 2.5 | 2.3 | -2.4 | -0.7 | 0.2 | 0.2 | 5.4 | 0.9 | 0.4 |
| C1-2000 | -9.0 | -6.8 | -6.0 | -12.3 | -9.1 | -1.7 | -4.8 | 4.3 | -5.4 | -5.7 |
| C2-2000 | 4.7 | -2.8 | -4.5 | -7.0 | -8.6 | -4.2 | -8.7 | 0.7 | -4.6 | -3.8 |

${ }^{2}$ Average of all age groups.

We believe that these differences were caused primarily by the large volume and unusual nature of migration into and out of Florida. Florida has been one of the fastestgrowing states in the nation for many years. In recent decades, almost $90 \%$ of the growth in Florida's population has been due to net migration. Changes in fertility and mortality rates thus have less of an impact on the size and composition of the population in Florida than in most states.

In addition, the age structure of Florida's migration stream differs from that found in most other states. Although young adults account for a large proportion of in-migrants in Florida, older persons also account for a large proportion. Over the past 50 years, the migration of elderly people has given Florida the highest proportion of persons aged $65+$ of any state in the nation. (In 2000, more than $20 \%$ of the population was aged $65+$ in 19 of the state's 67 counties.) Given these migration characteristics, it is not surprising that MAPEs were relatively large for the 55-64 and 65+ age groups, as well as for the 25-34 age group.

MALPEs varied considerably from one age group to another and from one projection series to another. These results are consistent with those reported for national and state projections, in the sense that they show no tendency for cohort-component projections to be consistently too high or too low, either for the total population or any particular age group.

## ALTERNATIVE APPROACH: HAMILTON-PERRY

## Methodology

The cohort-component method is widely used because it accounts separately for the components of growth, can accommodate a number of different theoretical models and projection techniques, and provides projections of demographic characteristics and the size of the total population. However, it is a data-intensive method that requires the collection or calculation of many age- and sex-specific fertility, mortality, and migration rates. Thus, it not only is time consuming but also creates problems for projections of small areas where fertility, mortality, and migration data are difficult or impossible to obtain.

A simpler version of the cohort-component method has been developed that requires only population counts by age ( $o r$ by age and sex) from two consecutive censuses (Hamilton and Perry 1962). Under this method (HP), cohort-survival rates are calculated
by dividing the population age $i$ in year $t$ by the population age $i-10$ in year $t-10$. These rates are applied to each age group in year $t$ to provide projections by age in year $t+10$. Projections of children younger than age 10 can be made using child-to-woman ratios or age-specific fertility rates. A step-by-step illustration of the HP method can be found in Smith, Tayman, and Swanson (2001:153-58).

The HP method has often been used for small-area projections, but its accuracy has seldom been tested or compared with the accuracy of a full-blown cohort-component model. To conduct such a test, we constructed HP population projections for all states and for counties in Florida and compared them with census counts for each target year. We made two sets of HP projections. One used 1970 and 1980 census data to make projections for 1990 and 2000, and the other used 1980 and 1990 census data to make projections for 2000. Both used cohort-survival rates that were calculated separately for males and females. Children younger than age 10 were projected using child-to-woman ratios from the launch year (i.e., males or females younger than age 5/females aged 15-44; males or females aged 5-9/females aged 20-49). ${ }^{7}$

The census-survival rates used in the HP method combine the effects of mortality and net migration. In essence, they are decade change rates for each age-sex cohort. Because the application of constant growth rates often leads to projections that are too high (especially for rapidly growing areas), we believe it is advisable to control HP agegroup projections to an independent set of total population projections (Smith et al. 2001:159). We used such controls for both sets of HP projections. For states, the control totals were an average of projections of the total population that was derived from five simple extrapolation techniques: linear, exponential, constant share, shift share, and share of growth. For counties, the control totals were those produced by the BEBR, using similar extrapolation techniques. In both instances, controlling was done using simple proportionate adjustments. ${ }^{8}$

## Empirical Results

Figure 1 shows MAPEs for each set of HP state projections, by age group, along with the corresponding MAPEs for the USCB projections described previously. For 10-year projections to 1990 and 20-year projections to 2000, the HP projections had smaller errors in six of the eight age groups. For 10 -year projections to 2000 , the USCB projections had smaller errors in six of the eight age groups. In many instances, differences in errors between the two methods were quite small; in a few instances, they were fairly large. In all three sets of projections, the patterns of errors by age group were about the same for both methods.

Figure 2 shows MAPEs by age group for the county projections. For 10 -year projections to 2000 , the HP projections had smaller errors than the BEBR projections in six of the eight age groups. For the other two sets of projections, HP and BEBR each had smaller errors in about half of the age groups. In many instances, errors within age groups were about the same for each of the two methods.

We also evaluated MALPEs by age group (not shown here). For 10-year state projections to 1990, both the HP and USCB methods produced positive errors for most age groups. For 10 -year state projections to 2000, both methods produced negative errors for
7. This represents the most basic application of the HP method. A number of refinements could be made, such as using age-specific fertility rates, rather than child-to-woman ratios, and adjusting for events that disproportionately affect a particular segment of the population (e.g., the opening of a large prison). In some circumstances, these refinements may substantially improve the performance of the HP method, especially for small areas.
8. We also evaluated HP projections that were not controlled to independent projections of the total population. The uncontrolled projections generally had larger errors than the controlled projections (especially for counties), but patterns of errors by age group were similar for both sets of projections.

Figure 1. Mean Absolute Percentage Errors (MAPEs) for HP and USCB Projections for States, by Age Group


Base Year $=1990$, Target Year $=2000$


Base Year $=1980$, Target Year $=2000$


[^4]Figure 2. Mean Absolute Percentage Errors (MAPEs) for HP and BEBR Projections for Florida Counties, by Age Group


Base Year $=1990$, Target Year $=2000$


Base Year $=1980$, Target Year $=2000$


[^5]Table 5. Comparison of HP and Cohort-Component Projections for States and Florida Counties: Average Errors for All Age Groups

| Series and Target Year | MAPE | MALPE |
| :--- | :---: | :---: |
| States |  |  |
| S2-1990 | 7.7 | 3.6 |
| HP-1990 | 7.0 | 2.3 |
| S2-2000 | 12.6 | -0.1 |
| HP-2000 | 10.7 | -1.3 |
| S3-2000 | 4.9 | -1.6 |
| HP-2000 | 5.6 | -2.3 |
| Florida Counties |  |  |
| C1-1990 | 10.4 | 0.4 |
| HP-1990 | 10.6 | 0.7 |
| C1-2000 | 15.2 | -5.7 |
| HP-2000 | 15.4 | -5.0 |
| C2-2000 | 10.9 | -3.8 |
| HP-2000 | 9.5 | -3.8 |

Note: MAPE = mean absolute percentage error, MALPE = mean algebraic percentage error, and $\mathrm{HP}=$ the Hamilton-Perry method.
most age groups. For 20 -year state projections to 2000 , the two methods were about evenly split between positive and negative errors. In some instances, the HP projections displayed more upward or downward bias than the USCB projections; in other instances, the opposite was true.

For 10 -year county projections to 1990 , age-group errors were about evenly split between positive and negative for both the HP and BEBR projections. For the $10-$ and 20 -year county projections to 2000 , both methods produced negative errors for most age groups. In all three sets of projections, errors for both methods generally had the same signs within each age group. In some instances, the HP projections displayed more upward or downward bias than the BEBR projections; in other instances, the opposite was true.

These results show that the forecasting performance of the HP method is similar to that of a full-blown cohort-component model. Averaging over all eight age groups, we found MAPEs of $7.0 \%$ and $5.6 \%$ for the two sets of 10 -year HP state projections, compared to $7.7 \%$ and $4.9 \%$ for the corresponding USCB projections (see Table 5). For 20 -year state projections, MAPEs were $10.7 \%$ and $12.6 \%$ for HP and USCB, respectively. For counties, MAPEs were $10.6 \%$ and $9.5 \%$ for the two sets of 10 -year HP projections, compared to $10.4 \%$ and $10.9 \%$ for the BEBR projections. For 20 -year county projections, MAPEs were $15.4 \%$ for HP and $15.2 \%$ for BEBR. In all instances, MALPEs for the HP projections were similar to MALPEs for the corresponding USCB and BEBR projections. This evidence strongly suggests that simple applications of the cohort-component method can produce age-group projections that are every bit as accurate as those produced using more complex techniques. ${ }^{9}$

[^6]
## CONCLUSIONS

## Patterns of Errors

At the national level, births and deaths are typically the major determinants of population change. Because changes in fertility and mortality rates have their greatest impact at the youngest and oldest ages, the largest errors in national projections typically occur in those age groups (especially for countries that are undergoing the transition from high to low fertility and mortality rates). The results reported here for national projections in the United States are consistent with this pattern, particularly for the youngest age group. If the increases in immigration and the relative stability of fertility rates that have been observed over the past several decades were to persist, however, that pattern would most likely become less distinct in future U.S. projections.

At the subnational level, migration is the most volatile component of population change and is often the major component as well. Since young adults typically have the highest migration rates, young-adult age groups often have relatively large errors in subnational population projections. Projections of young children also tend to have large errors because they are affected by uncertainty about both future fertility rates and the future number of women of childbearing age. For the state-level projections evaluated in this study, the largest MAPEs consistently occurred in the $<5$ age group, and the second largest frequently occurred in the 25-34 age group. For the county-level projections, the 25-34 age group had the largest MAPE in two of the three projection series and the second largest in the third. In general, the impact of migration on population change becomes larger as the geographic unit becomes smaller.

For any given area, the largest forecast errors are likely to be found in the age groups that are the most strongly affected by the major determinants of population change in that area. For national projections, these age groups are typically the youngest and-to a lesser extent-the oldest. For subnational projections, they are typically children and young adults. For some areas, however, location-specific characteristics may have a substantial impact as well (e.g., relatively large errors for older age groups in areas with high levels of migration by retirees).

For short projection horizons (e.g., 5-10 years), errors for some age groups are typically two, three, or four times larger than errors for other age groups. As the projection horizon becomes longer, cohorts pass through succeeding age groups, experiencing the error characteristics of each group as they do. Relatively large and small errors are thus averaged together, causing differences in errors among age groups to become smaller. For very long horizons, we believe that forecast errors are likely to be much the same for all age groups.

## Simple Versus Complex

A longstanding controversy in demography is whether complex projection methods produce more accurate forecasts than do simpler methods (e.g., Keyfitz 1981; Long 1995; Rogers 1995; Smith and Sincich 1992). For projections of the total population, we believe the empirical evidence is clear: complex methods do not consistently produce more accurate forecasts than do simple extrapolation methods (for a review of the evidence, see Smith et al. 2001:307-16). For projections of demographic characteristics, however, few empirical tests have been performed. In this article, we reported the results of a comparison of simpler and more complex applications of the cohort-component method for states in the United States and counties in Florida. At both geographic levels, we found that simpler and more complex applications produced similar projections of the age-sex structure of the population; neither approach consistently produced more accurate projections than the other.

We believe this finding is important not only for what it adds to the discussion of simplicity versus complexity but also for what it means to the production of state and local population estimates and projections. Full-blown applications of the cohort-component method require major efforts to collect, organize, and adjust the data and to write the necessary computer programs. The data required for the HP method, on the other hand, are available directly from the decennial census, and the method can be applied relatively quickly and easily. The results presented in this article suggest that using the HP method, rather than a full-blown cohort-component model, is not likely to reduce the accuracy of age-sex estimates and projections covering short-to-medium horizons. This is good news for practitioners who are operating with inadequate data or binding time and budgetary constraints.

Furthermore, the age- and sex-specific fertility, mortality, and migration data that are required for a full-blown cohort-component model are seldom available for subcounty areas, such as cities, census tracts, school districts, or market areas. Attempting to collect these data is expensive and time consuming; in some instances (e.g., out-migration data), it is virtually impossible. Constructing proxy measures for missing data is also expensive and time consuming and introduces additional sources of error. Given these deficiencies in data, the HP method is particularly useful for subcounty projections.

We are not suggesting that the HP method should replace more complex applications of the cohort-component method, of course. The HP method does not provide separate projections of births, deaths, in-migrants, and out-migrants. It cannot determine the effects of changes in assumptions about fertility, mortality, or migration on projected growth rates or demographic composition. It cannot be directly related to theories of population growth or readily used in structural or stochastic projection models. Its performance for long-range projections has not yet been evaluated. Clearly, there are purposes for which the HP method will not be useful. When projections are used solely as forecasts of future population change, however, we believe that the HP method is a viable alternative to more complex models, particularly for small areas and for short-tomedium projection horizons.

## REFERENCES

Alho, J.M. 1990. "Stochastic Methods in Population Forecasting." International Journal of Forecasting 6:521-30.
Alho, J.M. and B.D. Spencer. 1997. "The Practical Specification of the Expected Error of Population Forecasts." Journal of Official Statistics 13:203-25.
Bongaarts, J. and R.A. Bulatao, Eds. 2000. Beyond Six Billion. Washington, DC: National Academy Press.
Campbell, P. 1991. "Evaluating the Age Distribution in State Population Projections." Federal Forecasters Conference: Papers and Proceedings. Washington, DC: U.S. Government Printing Office.
——. 1994. "Population Projections for States, by Age, Sex, Race, and Hispanic Origin: 1993 to 2020." U.S. Census Bureau, Current Population Reports, Series P-25, No. 1111. Washington, DC: U.S. Government Printing Office.
Day, J.C. 1992. "Population Projections of the United States, by Age, Sex, Race, and Hispanic Origin: 1992 to 2050." U.S. Census Bureau, Current Population Reports, Series P-25, No. 1092. Washington, DC: U.S. Government Printing Office.
Dunton, N. 1994. "Planning for Children's Residential Care." Pp. 327-42 in Demographics: A Casebook for Business and Government, edited by H.J. Kintner, T.W. Merrick, P.A. Morrison, and P.R. Voss. Boulder: Westview Press.
Fishlow, H. 1994. "Enrollment Projection in a Multicampus University System." Pp. 298-306 in Demographics: A Casebook for Business and Government, edited by H.J. Kintner, T.W. Merrick, P.A. Morrison, and P.R. Voss. Boulder: Westview Press.

Hamilton, C.H. and J. Perry. 1962. "A Short Method for Projecting Population by Age From One Decennial Census to Another." Social Forces 41:163-70.
Isserman, A. 1993. "The Right People, the Right Rates: Making Population Estimates and Forecasts With an Interregional Cohort-Component Model." Journal of the American Planning Association 59:45-64.
Keilman, N. 1990. Uncertainty in National Population Forecasting. Amsterdam: Swets and Zeitlinger.
-. 1998. "How Accurate Are the United Nations World Population Projections?" Population and Development Review 24(Suppl.):15-41.
Keyfitz, N. 1981. "The Limits of Population Forecasting." Population and Development Review 7:579-93.
Kulkarni, M. and L.G. Pol. 1994. "Migration Expectancy Revisited: Results for the 1970s, 1980s and 1990s." Population Research and Policy Review 13:195-202.
Lapkoff, S. 1993. "Projecting Births in a California School District." Paper presented at the annual meeting of the Population Association of America, Cincinnati, April 1-3.
Lee, R.D. 1992. "Stochastic Demographic Forecasting." International Journal of Forecasting 8:315-27.
Lee, R.D. and S. Tuljapurkar. 1994. "Stochastic Population Forecasts for the United States: Beyond High, Medium, and Low." Journal of the American Statistical Association 89:1175-89.
—_. 1997. "Death and Taxes: Longer Life, Consumption, and Social Security." Demography 34:67-81.
Long, J.F. 1995. "Complexity, Accuracy, and Utility of Official Population Projections." Mathematical Population Studies 5:203-16.
Long, J.F. and D.B. McMillen. 1987. "A Survey of Census Bureau Population Projection Methods." Climatic Change 11:141-77.
Lutz, W., W.C. Sanderson, and S. Scherbov. 1998. "Expert-Based Probabilistic Population Projections." Population and Development Review 24(Suppl.):139-55.
Nakosteen, R.A. 1989. "Detailed Population Projections for Small Areas: The Massachusetts Experience." Socio-Economic Planning Sciences 23:125-38.
Plane, D. 1993. "Demographic Influences on Migration." Regional Studies 27:375-83.
Rives, B. 1994. "Strategic Financial Planning for Hospitals: Demographic Considerations." Pp. 251-64 in Demographics: A Casebook for Business and Government, edited by H.J. Kintner, T.W. Merrick, P.A. Morrison, and P.R. Voss. Boulder: Westview Press.

Robinson, J.G., K. West, and A. Adlakha. 2002. "Coverage of the Population in Census 2000: Results From Demographic Analysis." Population Research and Policy Review 21:19-38.
Rogers, A. 1995. "Population Forecasting: Do Simple Models Outperform Complex Models?" Mathematical Population Studies 5:187-202.
Smith, S.K. 1986. "Accounting for Migration in Cohort-Component Projections of State and Local Populations." Demography 23:127-35.
Smith, S.K. and B. Ahmed. 1986. "Population Estimates and Projections by Age and Sex: Florida and Its Counties, 1985-2000." Florida Population Studies, Bulletin 77. Gainesville: Bureau of Economic and Business Research, University of Florida.
-_. 1990. "A Demographic Analysis of the Population Growth of States, 1950-1980." Journal of Regional Science 30:209-27.
Smith, S.K. and M. Shahidullah. 1995. "An Evaluation of Population Projection Errors for Census Tracts." Journal of the American Statistical Association 90:64-71.
Smith, S.K., M. Shahidullah, and R. Bayya. 1992. "Population Projections by Age, Sex and Race for Florida and Its Counties, 1991-2010." Florida Population Studies, Bulletin 101-102. Gainesville: Bureau of Economic and Business Research, University of Florida.
Smith, S.K. and T. Sincich. 1988. "Stability Over Time in the Distribution of Population Forecast Errors." Demography 25:461-74.
1992. "Evaluating the Forecast Accuracy and Bias of Alternative Population Projections for States." International Journal of Forecasting 8:495-508.
Smith, S.K., J. Tayman, and D.A. Swanson. 2001. State and Local Population Projections: Methodology and Analysis. New York: Kluwer Academic.
Stoto, M. 1983. "The Accuracy of Population Projections." Journal of the American Statistical Association 78:13-20.
Tayman, J. and D.A. Swanson. 1996. "On the Utility of Population Forecasts." Demography 33:523-28.
——. 1999. "On the Validity of MAPE as a Measure of Population Forecast Accuracy." Population Research and Policy Review 18:299-322.
U.S. Census Bureau. 1953. "Illustrative Projections of the Population of the United States, by Age and Sex: 1955 to 1975." Current Population Reports, Series P-25, No. 78. Washington, DC: U.S. Government Printing Office.
——. 1962a. "Interim Revised Projections of the Population of the United States, by Age and Sex: 1965 to 1970." Current Population Reports, Series P-25, No. 241. Washington, DC: U.S. Government Printing Office.
—_. 1962b. "Interim Revised Projections of the Population of the United States, by Age and Sex: 1975 to 1980." Current Population Reports, Series P-25, No. 251. Washington, DC: U.S. Government Printing Office.
1972. "Projections of the Population of the United States, by Age and Sex: 1972 to 2020." Current Population Reports, Series P-25, No. 493. Washington, DC: U.S. Government Printing Office.
1979. "Illustrative Projections of State Populations by Age, Race, and Sex: 1975 to 2000." Current Population Reports, Series P-25, No. 796. Washington, DC: U.S. Government Printing Office.
1983. "Provisional Projections of the Population of States, by Age and Sex: 1980 to 2000." Current Population Reports, Series P-25, No. 937. Washington, DC: U.S. Government Printing Office.
2003. "Technical Assessment of A.C.E. Revision II." Available on-line at http:// www.census.gov/dmd/www/pdf/ACETechAssess.pdf
White, H. 1954. "Empirical Study of the Accuracy of Selected Methods of Projecting State Populations." Journal of the American Statistical Association 49:480-98.


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    1. Demographers often draw a distinction between projections and forecasts. Projections are the outcomes of a particular set of assumptions regarding future population values, whereas forecasts are the specific projections that are expected to provide the most accurate predictions of those values. In this article, we use these terms interchangeably because we are evaluating the accuracy of projections that are used as forecasts of the future population.
[^1]:    2. Uncertainty can also be expressed by producing a range of projections that are based on different sets of assumptions. This approach has been widely used by practitioners, but does not provide an explicit measure of uncertainty.
[^2]:    3. We analyzed separate projections for males and females, but do not report the results here because of space limitations. For state and national projections, errors for males and females were similar within each age group. For counties in Florida, errors were generally a bit larger for males than for females, especially in the young-adult age groups. For ages 65+, however, females had slightly larger errors than males.
    4. We did not attempt to adjust the results for changes in census coverage over time. Nationally, the net census undercount has declined slowly but steadily since 1950, except for a small increase between 1980 and 1990. In 2000, both demographic analysis and postenumeration surveys showed a slight net overcount at the national level (Robinson, West, and Adlakha 2002; U.S. Census Bureau 2003). The results varied by age, sex, and racial group, with some showing an overcount and others showing an undercount. In some instances (e.g., the population aged $0-9$ ), estimates that were based on demographic analysis had opposite signs to estimates that were based on survey data.

    For states and counties, no estimates of undercount (or overcount) from demographic analysis are available. Instead, estimates must be based on survey data that were collected for census regions and divisions. To our knowledge, no estimates of census errors by age group at the state or county level have been produced. Although changes in census coverage undoubtedly had some effect on the results reported here, we believe that these effects were relatively small.
    5. We also evaluated the accuracy of forecasts using the median absolute percentage error as a measure of precision and the proportion of positive errors as a measure of bias. The results were similar to those shown here for MAPE and MALPE, respectively. Median absolute percentage errors were uniformly smaller than MAPEs, reflecting asymmetry in the distribution of absolute percentage errors and the influence of outliers on MAPEs (Tayman and Swanson 1999).

[^3]:    6. Most of the projections evaluated in this study covered horizons of approximately 10,20 , or 30 years. The only exceptions were S1 and C1, which covered horizons that were a few years shorter. For simplicity of exposition, we refer to all projections as covering $10-$, 20 -, or 30 -year horizons.
[^4]:    Note: $\mathrm{HP}=$ the Hamilton-Perry method, and USCB $=$ the U.S. Census Bureau.

[^5]:    Note: HP = the Hamilton-Perry method, and BEBR = the Bureau of Economic and Business Research.

[^6]:    9. The results reported here covered only 10 - and 20 -year projection horizons. The accuracy of HP projections for longer horizons is a topic that requires further study.
