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An exact approach for the consistent vehicle routing problem (ConVRP)

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A B S T R A C T

This paper proposes a mathematical model for the Consistent Vehicle Routing Problem (ConVRP). The ConVRP is an extension of the VRP, considering customer satisfaction through consistent service. The consistency may be based on time or on the vehicle that offers the service. This paper proposes a novel mathematical model that allows solving the ConVRP for several companies for which visits to the customers need to be from the same service provider (namely, the same vehicle and driver). The efficiency of the model is tested on structured instances by changing customer distribution (uniform or clustered), depot location, and arrival time to the customer and removing certain constraints to see if they affect the performance of the objective function. The mathematical model is flexible and could be adapted to any characteristic of instances. The model was developed in the AMPL programming language and solved with the solvers CPLEX and Gurobi. The results are promising based on the efficiency of the proposed method at solving the problem.

A R T I C L E I N F O

Keywords:
Vehicle routing problem (VRP);
Consistent vehicle routing (ConVRP);
Mathematical model;
Mixed integer linear programming model;
Optimization;
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1. Introduction

The main objective of the consistent vehicle routing problem (ConVRP) is customer satisfaction through consistent service. The consistency may be based on time or on the vehicle that offers the service. There is extensive literature on methods to solve the well-known vehicle routing problem (VRP) and its variants [1], but few studies are related to ConVRP. The VRP is critical in successful logistics execution. The emergence of technologies and information systems, allowing for seamless mobile and wireless connectivity between delivery vehicles and distribution facilities, has generated a new research field integrating real-time VR with consistent distribution management [2].

The study of logistical problems such as vehicle routing and its different approaches is gaining more interest both in academia and its application to real issues [3, 4]. Although cost minimization is traditionally only considered in a VRP problem, other objectives can be defined for the same problem (distance, time, capacity, etc.). There are also other decision-making criteria of a more significant impact, such as delays or delivery schedules. Gaining a better market position requires focusing more on the ConVRP, whose objective is to emphasize customers through a more consistent service [5].

This paper proposes a mathematical mixed-integer linear programming model to solve the ConVRP. The proposed model can determine the total travel time of a vehicle fleet for a certain number of specific days, which routes should be taken by each vehicle per day, and the vehicles' arrival time for each customer. The mathematical model was tested on structured instances by analyzing changes to aspects such as customer distribution (uniform or clustered), depot location, arrival time at the customer, and removing certain constraints that affect the performance of the objective function. The model was developed in the AMPL programming language and solved with the solvers CPLEX and Gurobi. Although a mathematical model for the ConVRP attracts growing attention due to the few studies related to the problem. The mathematical structure of the model is novel, and to the best of the author's knowledge, this is the first time a model for the ConVRP solves real and structured instances for companies that provide a consistent service over time, such as courier companies, elderly care service companies, and cleaning sectors, under realistic constraints.

The paper is organized as follows: Section 2 gives an overview of the literature on the problem of ConVRP. Section 3 presents the proposed mathematical model with its considerations and assumptions, and Section 4 presents the computational results obtained from instances. Finally, the conclusions and future work are addressed in Section 5.

2. Literature review

This study first investigates the ConVRP in known instances from the literature, through an analysis of the VRP, to apply the method subsequently to real problems that are similar to the issue analyzed. The VRP consists of determining a set of routes for a vehicle fleet that leaves one or more depots to meet the demand of various geographically dispersed customers [6–8]. Usually, the VRP consists of determining the best routes and/or assignments to deliver goods or services to geographically distributed customers. This problem involves assigning a group of customers to a group of vehicles and drivers and sequencing their visits. The objective of the VRP is to deliver a product or service with the minimum total cost involved in the routes [9].

According to [10], there are other important components when defining a route problem, such as the type of network and objectives to optimize. Concerning the kind of structure, problems are classified according to node covering or arc covering issues. It is also possible for the objective to be to minimize total cost, total time, or total distance traveled and, at the same time, maximize the quality of service or the profits from charging customers.

There are different variants of the VRP: Multi-Depot VRP [8], Multi-Depot VRP with Heterogeneous Fleet [11], Periodic VRP [12], Split-Delivery VRP [13], Split-Delivery VRP with Time Windows [14], Heterogeneous VRP with Time Windows [15], stochastic VRP [16], VRP with Pickup and Delivery [17], VRP with Backhauls [18], VRP with time windows [19], and ConVRP [20]. ConVRP is defined as an extension of periodic VRP in which minimum-cost routes are designed to deliver a service to a group of customers with known demand over time through a homogeneous fleet of qualified vehicles. Customers may receive the service either once or at a predefined frequency; however, frequent customers must receive consistent service over a planning period, such that the same vehicle consistently satisfies the time windows and service [5, 21-22].

According to [21], in the ConVRP, each customer must receive the service on a specific day of a set D. Each customer can receive one service no more than once a day from any of the K identical vehicles. When a customer receives the service, the same driver will make the delivery at roughly the same time during the planning horizon of D days, so the maximum variation of the arrival time is no more than L time units. Each vehicle has a capacity of Q units and cannot operate for more than T units of time. The objective is to develop a set of routes for the fleet that minimizes the total operating time of the vehicle over D days.

The study by [23] demonstrated that some of the solutions provided by Groër et al. (2009) are not feasible and proposed an algorithm called ConRTR that obtained viable solutions. The results showed that with a relatively small increase in total travel time, the solutions provided by ConRTR were much more consistent than those proposed by [22].

ConVRP has numerous real-life applications in which visits to the customers need to be from the same service provider (namely, the same vehicle and driver). Also, in many cases, customers need to be visited at a given frequency. Some real-life examples of this type of service factor are parcel delivery and courier services, elderly care services, and cleaning services. In these cases, the primary endeavor is to gain a competitive advantage by bonding with the customer. In [3], it is considered the problem of consistency in the transport service for persons with disabilities. The authors presented a VRP that was consistent over time and solved it with a neighborhood search heuristic. Each iteration of the heuristic required solving a complex VRP with multiple time windows and no wait time.

The research in [24] pointed out that one of the most common problems at present is the transportation of persons with mental disabilities to rehabilitation centers. Because of the patients' lack of autonomy, most of those affected do not have a vehicle and cannot use public transport. Consequently, social health centers generally use specialized transport companies to organize and carry out daily trips. Customers need to be transported regularly, but sometimes with some variations during the week. These users are particularly sensitive to changes, so it is essential to keep consistent schedules and to provide the service by the same driver to create a bond of trust and gain a competitive advantage.

The paper [25] proposed a heuristic for a ConVRP in a distribution center of a food company. The problem was characterized by a group of customers that varied from day to day, and by their demand. Besides, each customer requires orders delivered within a given time period.

In [26], it is developed as a mathematical model to address the ConVRP of multiple daily deliveries and different service-level configurations, such as time windows and release dates. A mathematical programming-based decomposition approach was proposed to solve the problem. The algorithm was tested with real data from a Portuguese pharmaceutical distribution company. [27] proposed a metaheuristic algorithm based on local search for scheduling and routing of mobile nurses. Mobile nurses visited patients regularly at home. Some patients require up to three visits per day on predefined time windows. The visits required different qualifications, and the nurses had different qualification levels. This problem involved consistency in the visits.

In [28], an exact algorithm based on column generation was proposed for the ConVRP. Each variable of this model represented the set of routes assigned to a vehicle over the planning horizon. The proposed algorithm could solve small and medium-sized instances with five planning periods and 30 customers.

The work [5] presents a solution framework for the ConVRP. In this problem, customers received service once or at a predefined frequency, but frequent customers had to receive consistent service; that is, they had to be visited by the same driver throughout the planning period. The proposed algorithm adopted a two-level decomposition scheme. The computational experiments with benchmark data sets illustrated the competitiveness of the proposed approach compared to current results.

The paper [29] proposes a variable neighborhood search (VNS) for the ConVRP. The proposed algorithm consisted of two stages. In the first stage, VNS was applied to obtain approximate solutions. The obtained solutions might be unfeasible; if a solution had acceptable quality, the second step attempted to improve it. The proposed algorithm was tested on the benchmark ConVRP data set and compared with existing ConVRP approaches from the literature. The results showed that VNS outperformed all existing ConVRP methods regarding the quality of the solutions obtained.

The work proposed by [30] studied a variant of the traditional ConVRP considering profits. There were two sets of customers in this problem, the frequent customers that must be serviced and the infrequent potential customers with known and estimated profits, both having known demands and service requirements over a planning horizon of multiple days. The objective was to determine the vehicle routes that maximized net profit while satisfying vehicle capacity, route duration, and consistency constraints. This paper proposed a new mathematical model that captures the profit-collecting nature and other features of the problem and an adaptive tabu search that used short- and long-term memory structures to guide the search process. In [31], the generalized ConVRP (GenConVRP) is considered, where a limited number of drivers visited each customer, and variation in arrival times was penalized in the objective function. The vehicle departure times could be adjusted to obtain stable arrival times.

Additionally, customers were associated with time windows. Unlike the previous work on the ConVRP, this study used the template concept to generate route plans. The proposed approach was based on a flexible neighborhood search. Several destroy-and-repair heuristics were designed to remove customers from the routes and to reinsert them in better positions. The multi-purpose version of the GenConVRP is reviewed in [32].

3. Problem description and proposed approach

3.1 ConVRP description

Most studies in the ConVRP literature emphasize many logistics companies' current problems: low customer satisfaction with the company's performance stemming from its sole focus on total cost minimization and not caring about customer service. For this reason, companies now seek to increase service quality even if it means an increase in costs.

The ConVRP has great relevance and importance to that problem. The main objective of the ConVRP is the minimization of the travel time of vehicles on the days required. In a general context, the goal is to meet customers' individual demands in the defined planning horizon, leaving the customer satisfied and achieving consistent times and drivers. As mentioned above, it is essential that each customer receive service by the same driver each time, since this creates a bond of trust with the customer and improves customer satisfaction with the service offered. Moreover, there is a time constraint on each vehicle's arrival to the customer, since the customer should receive service simultaneously on each service day to achieve consistency concerning time [21]. The critical elements of the ConVRP are:

- Unequal hours of service delivery to the customer.
- Undefined routes for each vehicle.
- Inconsistency in the drivers who deliver service to the customers.

3.2 Proposed mathematical model

The proposed model solves real and structured instances for companies that provide a consistent service over time, such as companies in the courier, elderly care, and cleaning sectors, under realistic constraints. The proposed mixed-integer linear programming model determines customers' routes to visit, the vehicles assigned, and the arrival times to the customers.

Sets

D	Planning horizon, equivalent to the set of days the service is required.
V	Available vehicles (homogeneous fleet)
$Nodes$	Set of nodes (depot plus customers)
$Arcs$	Set of arcs between nodes (physical connections)

Parameters

T	Daily capacity in hours per vehicle
S_{id}	Service time for customer i on day d
L	Maximum arrival-time variation between two customers
Q	Maximum capacity of each vehicle
q_{id}	Demand of customer i on day d
w_{id}	1 indicates customer i requires service on day d, 0 indicates otherwise
t_{ij}	Travel time associated with arc (i, j)

Decision variables

$x_{ijkd} = \begin{cases} 1 & \text{if vehicle k visits customer } j \text{ after customer } i \text{ on day } d \\ 0 & \text{otherwise} \end{cases}$

$y_{ikd} = \begin{cases} 1 & \text{if customer i is visited by vehicle } k, \text{on day } d \\ 0 & \text{otherwise} \end{cases}$

a_{id} is arrival time to customer i on day d ($i = 0$ at the depot)

u_{id} is auxiliary variable for subtour elimination

Objective function

$$\text{Minimize} \sum_{d \in D} \sum_{k \in V} \sum_{(i,J) \in Arcs} t_{ij} x_{ijkd} \quad (1)$$

Eq. 1 seeks to minimize the total travel time of the vehicles to the customers over the whole planning horizon.

Constraints

$$y_{0kd} = 1 \quad \forall k \in V\ \forall d \in D \quad (2)$$

$$a_{0d} = 0 \quad \forall d \in D \quad (3)$$

$$\sum_{k \in V} y_{ikd} = w_{id} \quad \forall i \in Nodes\ \forall d \in D, i \geq 1 \quad (4)$$

$$\sum_{i \in Nodes} q_{id} * y_{ikd} \leq Q \quad \forall i \in Nodes\ \forall d \in D\ \forall k \in V, \quad i \geq 1 \quad (5)$$

$$\sum_{i \in Nodes} x_{ijkd} = y_{jkd} \quad \forall j \in Nodes\ \forall k \in V\ \forall d \in D, \quad i \geq 1 \quad (6)$$

$$\sum_{i \in Nodes} x_{jikd} = y_{jkd} \quad \forall j \in Nodes\ \forall k \in V\ \forall d \in D, \quad i \geq 1 \quad (7)$$

$$w_{id_\alpha} + w_{id_\beta} - 2 = y_{ikd_\alpha} - y_{ikd_\beta} \quad \forall d_\alpha, d_\beta \in D\ \forall i \in Nodes\ \forall k \in V, \alpha \neq \beta \quad (8)$$

$$w_{id_\alpha} + w_{id_\beta} - 2 = y_{ikd_\beta} - y_{ikd_\alpha} \quad \forall d_\alpha, d_\beta \in D\ \forall i \in Nodes\ \forall k \in V, \alpha \neq \beta \quad (9)$$

$$a_{id} + x_{ijkd}(s_{id} + t_{ij}) - (1 - x_{ijkd}) * T \leq a_{jd} \quad \forall\ i,j \in Arcos\ k \in V\ d \in D, \quad j \geq 1 \quad (10)$$

$$a_{id} + x_{ijkd}(s_{id} + t_{ij}) - (1 - x_{ijkd}) * T \geq a_{jd} \quad \forall\ i,j \in Arcos\ k \in V\ d \in D, \quad j \geq 1 \quad (11)$$

$$a_{id} + w_{id} * (s_{id} + t_{i0}) \geq 0 \quad \forall i \in Nodes\ \forall d \in D, i \geq 1 \quad (12)$$

$$a_{id} + w_{id} * (s_{id} + t_{i0}) \leq T * w_{id} \quad \forall i \in Nodes\ \forall d \in D, i \geq 1 \quad (13)$$

$$a_{id_\alpha} - a_{id_\beta} \leq L + T * (2 - w_{id_\alpha} - w_{id_\beta}) \quad \forall d_\alpha, d_\beta \in D\ \forall i \in Nodes, \alpha \neq \beta \quad (14)$$

$$a_{id_\beta} - a_{id_\alpha} \leq L + T * (2 - w_{id_\alpha} - w_{id_\beta}) \quad \forall d_\alpha, d_\beta \in D\ \forall i \in Nodes, \alpha \neq \beta \quad (15)$$

$$u_{id} + 1 \leq u_{jd} + Nodes * (1 - x_{ijkd}) \quad \forall i,j\ \in Arcos\ k \in V\ d \in D, \quad j \geq 1 \quad (16)$$

$$x_{ijkd} \in \{0,1\} \quad \forall i,j \in Nodes\ \forall k \in V\ \forall d \in D \quad (17)$$

$$y_{ikd} \in \{0,1\} \quad \forall i \in Nodes\ \forall k \in V\ \forall d \in D \quad (18)$$

$$a_{id} \geq 0 \quad \forall i \in Nodes\ \forall d \in D \quad (19)$$

$$u_{id} \geq 0 \quad \forall i \in Nodes\ \forall d \in D \quad (20)$$

Eq. 2 determines that each vehicle must visit the depot first and then go to its respective customer. Eq. 3 corresponds to the arrival time of the vehicle at the depot, which must be equal to 0. This ensures that the vehicles visit the depot at time 0 every day. Eq. 4 ensures that customers are visited exactly once when the service is required, that is, each customer is visited once per service day. Constraints 5 limit the capacity of the vehicles to prevent them from exceeding the Q units of product the vehicles can deliver per day. Eqs. 6 and 7 ensure that each customer has only one predecessor and successor.

The following constraints achieve model consistency with the customers that will be visited in the planning horizon. Eqs. 8 and 9 ensure that each customer receives the service by the same driver each time the customer requests service. Constraints 10 and 11 determine the daily arrival times of individual customers, which is necessary to achieve consistency in the time windows. Eqs. 12 and 13 limit the time the vehicles circulate, which translates to the maximum travel time of each vehicle. Constraints 14 and 15 are those that achieve time consistency, as they indicate that the difference in arrival times to customer i between days α and β cannot be more than L time units. This ensures customers are serviced on both days at roughly the same time. The value of L is initially adjusted to be the same as T, which leads to a relaxation of the problem, since there is no consistency in the times but there is in the vehicles. Therefore, when L has that value, it indicates that the vehicle can deliver the service within the daily limit of hours in circulation. Later, the L value is adjusted to a number lower than T to achieve consistency in the service delivery time. Constraint 10 seeks to eliminate subtours with Eq. 16, which is adapted from [33]. Computational tests were conducted on removing Constraints 16 and checking the behavior of the model. Constraint 11 keeps vehicles constantly moving to meet arrival times. Finally, Eqs. 17 and 18 output binary variables, while Eqs. 19 and 20 output nonnegative variables, which means that the variables cannot be negative.

4. Results, analysis and discussion

The following section shows the different results obtained from different structural instances under the conditions explained below. Experiments were performed with test instances to verify the proper operation of the proposed algorithm. The general computational results are analyzed for different structural instances. Different solver options are also tested to analyze other results based on computation times or objectives. The proposed model was developed in AMPL language version 20180822 and solved with the solvers CPLEX 12.8.0 and Gurobi 8.0.0 in an Intel Xeon E5-2660 v2 Dual Core @ 2.20 GHz computer with 64 GB RAM and a 64-bit Red Hat Enterprise Linux 7 operating system.

4.1 Instances definition

The instances are developed taking into account the following considerations:

- Customers: A total of three groups, of 10, 15, and 20 customers, are considered, in addition to uniform and cluster distributions.
- Depot: It is located at the center of the data or at one of the ends of the region.
- Arrival time difference (L): Instances are created with different L parameters, which are $L = T$, $L = 3$, and $L = 1$.
- Constraints: A modification is made to the model by removing Constraints 16 and 11.

Certain conditions mentioned by [21] and [34] in the literature are established when the instances are created; those conditions are the following:

- The uniform distributions of customer locations are generated within a square with vertices (0,0), (10,0), (10,10), (0.10). Cluster instances are generated according to the method proposed by [34]. This procedure consists of four steps:
  1. Two clusters are distributed uniformly between the coordinates mentioned above.
  2. Then customers are generated uniformly, and the Euclidean distance between the point and the cluster is obtained using (21):

$$D = \sqrt{(C_x - x)^2 + (C_y - y)^2} \tag{21}$$

3. To verify that the point is close to the cluster, Eq. 22 is used:

$$P_1 = e^{\left\{\frac{-D(p,C_1)}{0,8}\right\}} + e^{\left\{\frac{-D(p,C_2)}{0,8}\right\}} \tag{22}$$

4. Finally, P_1 is compared with $P_2 = \cup(0,1)$. If $P_1 < P_2$ then the point is added; otherwise, the point is discarded, and the same process is repeated until all customers are captured.

- The planning horizon is set to 3 days.
- The probability of customers requiring service every day is 70 %.
- For customers that require service on a certain day, demand is uniformly distributed in (1,3).
- All service times are set to one unit, which varies depending on the total number of customers.
- Vehicle capacity is homogeneous, and the maximum capacity is 15 units.

A total of 108 instances are generated, divided into four types.

4.2 Structured instances results

All instances are evaluated with a maximum calculation time of 3600 seconds (1 hour) since that time is considered reasonable to find an optimal solution. For the notations on the tables, the term "CPU Time" corresponds to the time in seconds taken to find the result, "Gap" is the percentage difference between the best number obtained and the best integer solution found by the model, and "Z" is the value of the objective function.

Customer distribution

This section compares the results obtained when the model is applied to instances focused on uniform and clustered distribution of the customers for a maximum of 20 customers. Table 1 shows the results of the instances for uniformly distributed customers.

The number of vehicles used in each instance achieves balanced routes for each vehicle since the probability of the daily demand for each customer is 70 %. In particular, route balance is not achieved if the number of vehicles is high. Table 2 shows the results of the model for instances generated in clusters.

The analysis of the results obtained with customers distributed uniformly and in clusters shows that this latter model finds better solutions in less computation time. A clustered distribution of customers results in better model behavior and reduced computation times, which is reflected in a more significant number of optimal solutions. Finally, the Gurobi software delivered better results in computation time and Gap percentage, but these results were not more relevant than those obtained with CPLEX.

Table 1 Performance of the model with uniformly distributed customers

		CPLEX			Gurobi		
Number of customers	Vehicles used	Gap (%)	CPU Time (s)	Z (time)	Gap (%)	CPU Time (s)	Z (time)
10	2	0.00	0.25	136.90	0.00	0.56	136.90
15	2	11.68	3600.00	149.60	4.99	3600.00	149.60
20	3	27.16	3600.00	192.25	15.86	3600.00	192.25

Table 2 Performance of the model with customers distributed in clusters

		CPLEX			Gurobi		
Number of customers	Vehicles used	Gap (%)	CPU Time (s)	Z (time)	Gap (%)	CPU Time (s)	Z (time)
10	2	0.00	143.88	135.08	0.00	368.54	135.08
15	2	0.00	13.77	126.32	0.00	12.48	126.32
20	3	7.44	3600.00	151.71	4.13	3600.00	151.71

Depot location

This section shows the results obtained for different depot locations, considering both customer distributions, uniforms, and clustered. In particular, the depot is located at the corner or the center of the customers. In the corner option, the depot is put on one of the vertices that delimit customers' locations. Table 3 shows the results obtained when the model is applied to the instances.

The model finds an optimal solution for 50 % of the instances studied from the data obtained with the solver, in computation time of fewer than 143 seconds. In contrast, the other 50 % of the instances fail to solve optimally, but feasible solutions are still found, of which two are from customers with uniform distribution and, at the same time, have the highest Gap. The other option puts the depot precisely in the center of the coordinates of the customers. Table 4 shows the results obtained when the model is applied to the generated instances.

As shown in Table 4, customers' travel time with the clustered distribution is considerably lower than with the uniform distribution. The model performs better when the depot is located at the center than at a corner. Indeed, the model found optimal solutions for all instances of this type. Additionally, an analysis of the solver results shows that the computation times of Gurobi are shorter in most cases, though in 4 out of 12 instances, the computation time was shorter with CPLEX, which indicates a lower performance for these instances. For this reason, the arrival time and relaxation of constraints cases were not analyzed with the Gurobi solver.

Table 3 Performance of the model for the depot located on a corner

		CPLEX			Gurobi		
Customers	Distribution	Gap (%)	CPU Time (s)	Z (time)	Gap (%)	CPU Time (s)	Z (time)
10	Uniform	0.00	0.25	136.90	0.00	0.56	136.90
15	Uniform	11.68	3600.00	149.60	4.99	3600.00	149.60
20	Uniform	27.16	3600.00	192.25	15.86	3600.00	192.25
10	Cluster	0.00	143.88	135.08	0.00	368.54	135.08
15	Cluster	0.00	13.77	126.32	0.00	12.48	126.32
20	Cluster	7.44	3600.00	151.71	4.13	3600.00	151.71

Table 4 Performance of the model for a depot located at the center

		CPLEX			Gurobi		
Customers	Distribution	Gap (%)	CPU Time (s)	Z (time)	Gap (%)	CPU Time (s)	Z (time)
10	Uniform	0.00	1.15	81.29	0.00	0.86	81.29
15	Uniform	0.00	5.28	112.02	0.00	2.52	112.02
20	Uniform	0.00	1289.33	129.76	0.00	1971.33	129.76
10	Cluster	0.00	1.99	54.47	0.00	2.60	54.47
15	Cluster	0.00	22.62	64.96	0.00	15.59	64.96
20	Cluster	0.00	141.38	79.68	0.00	266.74	79.68

Arrival time differences (L)

The arrival time difference is the parameter that limits how long a vehicle can take to provide service to a customer on different days. We are interested in reducing that time as much as possible to achieve more consistent arrival times. Table 5 shows the results of applying the model with an L value equal to T, and Table 6 shows the results for L = 3, considering the depot located at one end and at the center, and with a clustered distribution of customers. Finally, Table 7 shows the results of the instances with L = 1.

Table 5 Performance of the model with $L = T$

				CPLEX		
Customers	Depot	T	L	Gap (%)	CPU Time (s)	Z (time)
10	Corner	30	30	0.00	143.88	135.08
15	Corner	38	38	0.00	13.77	126.32
20	Corner	40	40	7.44	3600.00	151.71
10	Center	30	30	0.00	1.99	54.47
15	Center	38	38	0.00	22.62	64.96
20	Center	40	40	0.00	141.38	79.68

Table 6 Performance of the model with $L = 3$

				CPLEX		
Customers	Depot	T	L	Gap (%)	CPU Time (s)	Z (time)
10	Corner	30	3	0.00	25.75	135.08
15	Corner	38	3	0.00	60.81	126.81
20	Corner	40	3	10.50	3600.00	154.26
10	Center	30	3	0.00	2.68	54.47
15	Center	38	3	0.00	82.52	66.72
20	Center	40	3	11.57	3600.00	82.42

Table 7 Performance of the model with $L = 1$

				CPLEX		
Customers	Depot	T	L	Gap (%)	CPU Time (s)	Z (time)
10	Corner	30	1	Unfeasible	13.87	Unfeasible
15	Corner	38	1	16.57	3600.00	146.47
20	Corner	40	1	18.27	3600.00	165.47
10	Center	30	1	0.00	8.94	61.69
15	Center	38	1	27.24	3600.00	82.56
20	Center	40	1	28.81	3600.00	98.93

The results showed that as the L parameter decreases, that is, as the difference in arrival time approaches 0, the model struggles to find solutions. The travel times obtained with the model increase as the difference in arrival time decreases; this increase in the value of the objective function is most clearly seen when L = 1. An optimal solution is not achieved for most of these results, but if the computation time had been increased, it would have been possible to obtain results closer to those obtained in the previous instances. In addition, for those cases in which feasible solutions were obtained, the time the solver took to obtain a solution was no more than roughly 3 minutes.

Relaxation of constraints

In the model developed, there are two constraints that need to be analyzed since these can significantly affect the results that will be obtained. In particular, tests were conducted without Constraints 11 and 16, and the analysis was repeated. In the first test, Constraint 16, whose function is to remove the subtour, was eliminated since Restriction 10 can fulfill the same function. By doing this, the relaxed problem can be solved with the instances consisting of clustered customers, both depot locations, and L = 3. Tables 8 and 9 show the results for each model.

In the first comparison made with the removal of Constraint 16, no significant changes are seen in the solver; in fact, the results obtained are very similar, both in Gap value and computation times. Therefore, removing this constraint is no longer considered since it does not significantly improve the original model.

Table 8 Performance of the original model

			CPLEX		
Customers	Depot	L	Gap (%)	CPU Time (s)	Z (time)
10	Corner	3	0.00	25.75	135.08
15	Corner	3	0.00	60.81	126.81
20	Corner	3	10.50	3600.00	154.26
10	Center	3	0.00	2.68	54.47
15	Center	3	0.00	82.52	66.72
20	Center	3	11.57	3600.00	82.42

Table 9 Performance of the model without Constraint 16

			CPLEX		
Customers	Depot	L	Gap (%)	CPU Time (s)	Z (time)
10	Corner	3	0.00	47.09	135.08
15	Corner	3	0.00	44.36	126.81
20	Corner	3	10.23	3600.00	154.26
10	Center	3	0.00	2.98	54.47
15	Center	3	0.00	144.68	66.72
20	Center	3	12.83	3600.00	82.42

The second test was to eliminate constraint 11, which was intended to keep the vehicle in constant motion along the entire route; therefore, if this constraint is eliminated, then the vehicle will wait in its place before the next customer is visited. As a result, the instances of customers organized in clusters, both depot locations, and $L = 1$ were considered. Tables 10 and 11 show the original model results and the model without Constraint 11.

This comparison shows that Constraint 11 can be eliminated if the driver is to wait before traveling to the next customer, to provide consistent arrival times in less travel time. The results are better than those of the original model since the objective function's values are lower, which indicates a lower total travel time and, thus, a lower cost. The results obtained for vehicles' daily travel times show that the model performs better with a clustered distribution of customers and a centrally located depot since total travel times are the lowest out of all experiments. Besides, total travel times rise as the difference in arrival times to customers approaches 0.

Table 10 Performance of the original model

			CPLEX		
Customers	Depot	L	Gap (%)	CPU Time (s)	Z (time)
10	Corner	1	Unfeasible	13.87	Unfeasible
15	Corner	1	16.57	3600.00	146.47
20	Corner	1	18.27	3600.00	165.47
10	Center	1	0.00	8.94	61.69
15	Center	1	27.24	3600.00	82.56
20	Center	1	28.81	3600.00	98.93

Table 11 Performance of the model without Constraint 11

			CPLEX		
Customers	Depot	L	Gap (%)	CPU Time (s)	Z (time)
10	Corner	1	0.00	11.12	135.08
15	Corner	1	0.00	6.61	126.32
20	Corner	1	8.15	3600.00	151.91
10	Center	1	0.00	2.06	54.47
15	Center	1	0.00	29.01	65.30
20	Center	1	0.00	118.48	79.68

5. Conclusion

This article proposes a mathematical model for the ConVRP. The ConVRP is a variant of the capacitated VRP where customers can receive service either once or at a predefined frequency, receiving consistent service over a planning period that results in time windows and service consistently provided by the same vehicle. The proposed model can reduce the travel times of vehicles in the planning horizon and achieve consistency in arrival times to customers and achieve consistency in the vehicles that visit the customers.

The mathematical model was tested on structured instances to analyse how it would handle some changes to aspects such as customer distribution (uniform or clustered), depot location, and arrival times to customers and whether the removal of certain constraints would affect the performance of the objective function. The model was developed in the AMPL programming language and solved with solvers CPLEX and Gurobi. The results are outstanding for the problem considered in this study.

For future work, we suggest examining the multi-objective problem [35] to minimize both vehicle travel time and the variation in arrival time to customers, extending the model to consider a heterogeneous fleet and real constraints (rich vehicle routing problems), and developing heuristics for medium- and large-scale instances. Future research uncertainty into the model and a stochastic approach such as Sample Average Approximation must be considered to solve the considered problem. Finally, heuristic and metaheuristic algorithms based on works proposed by [36-40] could be implemented to solve large instances of issues.

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