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An Exact Line Integral Representation of the Physical Optics Scattered Field: The Case of a Perfectly Conducting Polyhedral Structure Illuminated by Electric Hertzian Dipoles

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***Abstract*— An exact line integral representation of the electric physical optics scattered field is presented. This representation applies to scattering configurations with perfectly electrically conducting polyhedral structures illuminated by a finite number of electric Hertzian dipoles. The positions of the source and observation points can be almost arbitrary. The line integral representation yields the exact same result as the conventional surface radiation integral; however, it is potentially less time consuming and particularly useful when the physical optics field can be augmented by a fringe wave contribution as calculated from physical theory of diffraction equivalent edge currents. The final expression for the line integral representation is lengthy but involves only simple functions and is thus suited for numerical calculation. To illustrate the exactness of the line integral representation, comparisons of numerical results obtained from the surface and the line integral representations are performed.**

I. Introduction

PHYSICAL optics (PO) is a well-tried and widely used technique for approximate analysis of the scattering of electromagnetic fields by perfectly conducting structures. The PO technique consists of two steps: First, the exact surface current on the structure is approximated by the PO surface current which is equal to $2\hat{n} \times \vec{H}^i$ on the illuminated part of the structure and zero elsewhere ($\hat{n}$ denotes the outward unit normal vector of the structure and $\vec{H}^i$ the incident magnetic field). Second, the PO scattered field is obtained by evaluating a surface radiation integral of the PO surface current. The reported applications of PO in antenna, as well as radar configurations, are numerous. Furthermore, several works have addressed various theoretical aspects of PO such as the validity of the foundation of PO [1], the accuracy of PO [2], the formulation of PO for nonperfectly conducting structures [3], asymptotic formulation of PO [4], and removal of shadow boundary contributions [5].

The purpose of this paper is to present an edge-associated line integral representation of the electric PO scattered field. The line integral representation provides some advantages in comparison with the surface radiation integral. First, its numerical calculation is potentially less time consuming than that of the surface radiation integral because fewer sample points are generally needed. The precise time reduction will depend on the scattering configuration. Second, the line integral representation is useful in cases where the PO field can be augmented by the physical theory of diffraction (PTD) field [6] calculated from incremental diffraction coefficients [7], [8], PTD equivalent edge currents [9], [10], or elementary edge waves [11] because this calculation also consists of an evaluation of a line integral along the edge of the structure. By combining the two approaches, it is thus possible to obtain an accurate approximation to the exact scattered field from the evaluation of one line integral.

The line integral representation is obtained through a mathematically exact transformation of the conventional surface radiation integral for the following type of scattering configuration: The structure must be a perfectly electrically conducting polyhedron. The incident field must be that of one or more arbitrarily oriented electric Hertzian dipoles. The Hertzian dipoles and the observation point can be positioned almost arbitrarily off the structure; a restriction is introduced, however, in the course of the derivation. There are two notes to be made concerning this configuration. First, the structure can be plane, i.e., a flat plate, and in this case the edge can be arbitrarily shaped. Second, from the use of a finite number of Hertzian dipoles, several basic radiating elements can be modeled, e.g., monopoles, dipoles, and turnstiles.

Much work addressing the transformation of surface integral representations of scattered or diffracted fields into line integral representations has been reported. For an account of the historical development and a description of the individual works, the reader is referred to Rubinowicz [12] and Asvestas [13]. Furthermore, a recent paper by the authors [14] reports an earlier stage of the work presented herein.

The present work was inspired by the three papers by Asvestas [13], [15], [16]. Asvestas [13] considered the electromagnetic diffraction by an aperture in a perfectly conducting plane screen and took as his starting point the Kirchhoff diffraction integral for electromagnetic fields as derived by Kottler [17]. Through use of a vector calculus theorem [18],

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Asvestas derived a line integral representation of the magnetic field in front of an aperture illuminated from the back by a Hertzian dipole. The key step in the derivation was the analytical evaluation of a dyadic integral to obtain an explicit dyadic expression. Asvestas also introduced the corresponding dyadic integral for the electric field but did not evaluate this. In a recent paper [14], the explicit dyadic expression associated with the electric field was reported. This expression can be used in combination with Asvestas' procedure to express the electric field in terms of a line integral for an aperture configuration. In [14] it was also employed to obtain a line integral representation of the PO scattered field from a perfectly conducting plane structure illuminated by a finite number of Hertzian dipoles. Due to the procedure employed in [14], however, the resulting line integral representation therein is subject to restrictions of a more severe nature than those found in the present work; this is discussed in more detail later. In the present work, the dyadic divergence theorem is employed and thus, this procedure does not rely on the vector calculus theorem [18] used by Asvestas.

This paper is organized as follows. In Section II, the transformation of the PO surface radiation integral into a line integral is performed for a plane structure. First, the PO scattered field $\vec{E}^{PO}$, as given by the conventional surface radiation integral, is expressed in terms of another surface integral $\vec{E}^{A}$ which expresses the field in terms of electric as well as magnetic surface currents on the structure. To accomplish this, the surface integral $\vec{E}^{A}$ must be considered not only at the observation point itself but also at the image of the observation point. The surface integral $\vec{E}^{A}$ is then transformed into the Kottler representation which comprises both surface and line integrals. The surface integral $\vec{I}_A$ in the Kottler representation expresses the flux of a dyadic function $\overline{\overline{V}}$ through the structure. Two truncated cones are then introduced. The vertices of these coincide with the observation and image points, respectively, and the generators extend from these points to the edge of the structure; consequently, the structure forms the bases of each of these cones. Employing the dyadic divergence theorem, the integral $\vec{I}_A$ can thus be expressed as the sum of a volume integral $\vec{I}_V$ of the divergence of $\overline{\overline{V}}$ over the volumes of these cones and a surface integral $\vec{I}_B$ of the flux of $\overline{\overline{V}}$ over the curved surfaces of the cones. The volume integral $\vec{I}_V$ is evaluated to yield two contributions: The first of these is the negative of the incident field if the source is inside the cone and zero otherwise. The second, which is proportional to the solid angle subtended by the structure as seen from the observation or image points, can be written as a line integral. The remaining surface integral $\vec{I}_B$ is written as a double integral; the outer integral is to be evaluated along the edge of the structure while the inner, denoted by $\overline{\overline{W}}$, is an integral of the gradient of the incident electric field along the generators of the cones. It is the analytical evaluation of this dyadic integral $\overline{\overline{W}}$ which completes the transformation of the PO surface radiation integral. Section III provides an overview of the line integral representation. In Section IV a numerical example is presented to illustrate the exactness of the line integral representation.

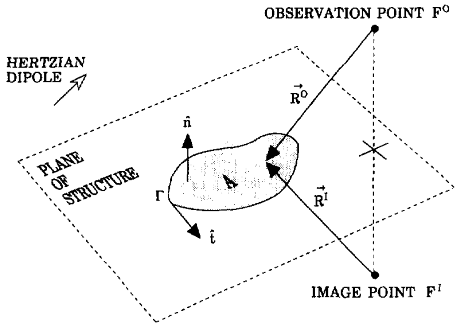

Fig. 1. A plane, perfectly conducting plate A illuminated by a Hertzian dipole. The unit normal vector $\hat{n}$ is directed into the half-space into which the Hertzian dipole is located. The observation point F^O is positioned in the same half-space as the Hertzian dipole. The edge unit tangent vector $\hat{t}$ and $\hat{n}$ are related via the right-hand rule. The image point F^I is the image of F^O with regard to the plane of the plate.

II. Derivation of the Line Integral Representation

In the following, a perfectly conducting polyhedral structure illuminated by a finite number of Hertzian dipoles is considered. The electric PO scattered field $\vec{E}^{PO}$ is calculated at an observation point F^O. Because of the superposition principle and the fact that there is no coupling between the PO currents on the facets of the polyhedral structure, it is sufficient to consider one Hertzian dipole illuminating a plane, perfectly conducting plate A. In this case, the plate can be arbitrarily shaped. The unit normal vector $\hat{n}$ of the plate is chosen to be directed into the half-space into which the Hertzian dipole is located (see Fig. 1). The half-space includes the plane of the plate. It is assumed, however, that the Hertzian dipole is not located on the plate itself. Without loss of generality, it is further assumed that the observation point F^O is located in the same half-space as the Hertzian dipole except on the plate. The field in the other half-space can be found using the symmetry relations for the field around a plane current sheet [19].

The PO scattered field is obtained through evaluation of the conventional surface radiation integral (the time factor $\exp(j\omega t)$ is suppressed)

$$\vec{E}^{PO}(F^O) = \frac{Z}{jk}\nabla^O\times\nabla^O\times\int_A 2\hat{n}\times\vec{H}^i(\vec{R}^O)G(\vec{R}^O)\,dA. \quad (1)$$

In this expression, the superscript O on the curl operators indicates that these act upon the coordinates of the observation point, $\vec{R}^O$ is the vector from the observation point F^O to the integration point (see Fig. 1), and R^O is the length of this vector. The use of the vector $\vec{R}^O$ to describe the integration point is in line with the notation of Asvestas [13] and facilitates the subsequent derivations. Furthermore, Z denotes the intrinsic impedance of the ambient medium, k is the wave number, and $\vec{H}^i$ the incident magnetic field from the Hertzian dipole. G is the free-space Green's function

$$G(\vec{R}^O) = \frac{\exp(-jkR^O)}{4\pi R^O}. \quad (2)$$

Introducing the image point F^I, i.e., the image of the observation point F^O with regard to the plane of the plate as shown in Fig. 1, the PO surface radiation integral (1) is now expressed in terms of another surface integral $\vec{E}^A$, to be defined below, as in [20]

$$\vec{E}^{PO}(F^O) = \vec{E}^A(F^O) + (\overline{\overline{I}} - 2\hat{n}\hat{n}) \cdot \vec{E}^A(F^I). \quad (3)$$

In this expression $\overline{\overline{I}}$ denotes the unit dyad. The surface integral $\vec{E}^A$ is given by

$$\vec{E}^A(F^{O,I}) = \nabla^{O,I} \times \int_A \hat{n} \times \vec{E}^i(\vec{R}^{O,I}) G(\vec{R}^{O,I})\, dA$$
$$+ \frac{Z}{jk} \nabla^{O,I} \times \nabla^{O,I} \times \int_A \hat{n} \times \vec{H}^i(\vec{R}^{O,I}) G(\vec{R}^{O,I})\, dA \quad (4)$$

where $\vec{E}^i$ denotes the incident electric field from the Hertzian dipole, $\vec{R}^I$ is the vector from the image point F^I to the integration point, and R^I is the length of this vector. The superscript I on the curl operators in (4) indicates that the operator acts upon the coordinates of the image point F^I. It is noted that the surface integral $\vec{E}^A(F^{O,I})$ is the electric field at the point $F^{O,I}$ produced by the magnetic and electric surface currents, $-\hat{n} \times \vec{E}^i$ and $\hat{n} \times \vec{H}^i$, respectively, distributed over the plate. Equation (3) states that the PO scattered field at the observation point F^O can be determined from the integral $\vec{E}^A$ evaluated both at the observation point F^O and at the image point F^I. The reason for introducing the integral $\vec{E}^A$ is its close relation to integrals occurring in the analysis of diffraction by an aperture in a plane screen. Indeed, $\vec{E}^A(F^I)$ is the negative electric field behind the complementary aperture to the plate A as determined from the Kirchhoff-approximated Huygens sources, $\hat{n} \times \vec{E}^i$ and $-\hat{n} \times \vec{H}^i$, in the aperture. Asvestas [13] considered the transformation of an integral of this type into a line integral, and some of his results can thus be used. Expression (4) above for $\vec{E}^A$ can be transformed into the Kottler representation ([13], (6))

$$\vec{E}^A(F^{O,I}) = -\int_A \Big[G(\vec{R}^{O,I})\hat{n} \cdot \nabla \vec{E}^i(\vec{R}^{O,I})$$
$$- \hat{n} \cdot \nabla G(\vec{R}^{O,I}) \vec{E}^i(\vec{R}^{O,I})\Big]\, dA$$
$$+ \int_\Gamma G(\vec{R}^{O,I}) \hat{t} \times \vec{E}^i(\vec{R}^{O,I})\, d\Gamma$$
$$+ \frac{jZ}{k} \int_\Gamma \hat{t} \cdot \vec{H}^i(\vec{R}^{O,I}) \nabla G(\vec{R}^{O,I})\, d\Gamma. \quad (5)$$

Herein, Γ denotes the edge of the plate, and $\hat{t}$ is the edge unit tangent vector. $\hat{t}$ and $\hat{n}$ are related via the right-hand rule. Note that the operator $\nabla^{O,I}$ has been expressed in terms of the operator ∇ acting upon the coordinates of the integration point on the plate.

The next step applied by Asvestas for transforming the remaining surface integral in (5) into a line integral involves the use of a vector calculus theorem [18]. This method is valid for $\vec{E}^A(F^I)$ but not for $\vec{E}^A(F^O)$ since the Hertzian dipole is located in the same half-space as the observation point F^O. In the following, the dyadic divergence theorem is invoked to transform both of the integrals $\vec{E}^A(F^I)$ and $\vec{E}^A(F^O)$ into line integrals. For the integral $\vec{E}^A(F^I)$, the result is the same as that of Asvestas; for the integral $\vec{E}^A(F^O)$, an additional term will be found.

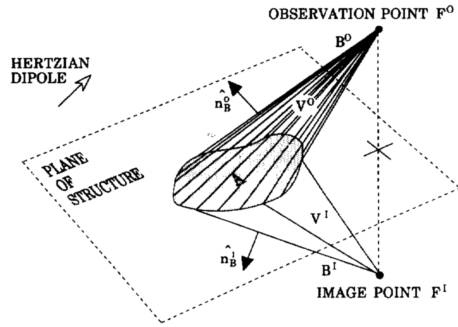

Fig. 2. Truncated cones V^O and V^I with vertices at F^O and F^I, respectively. The generators extend from F^O and F^I to the edge of the plate. For simplicity, only a few generators are shown for V^I. The curved surfaces of the cones, B^O and B^I, have the outward unit normal vectors $\hat{n}_B^O$ and $\hat{n}_B^I$.

A. Application of the Dyadic Divergence Theorem

To use the dyadic divergence theorem, two truncated cones, V^O and V^I, are introduced. These cones have vertices at the observation point F^O and the image point F^I, respectively, and the generators extend from these points to the edge of the plate A (see Fig. 2). Consequently, the plate A forms the base for each of the two truncated cones. The curved surface of the cone $V^{O,I}$ is denoted by $B^{O,I}$. The outward unit normal vector of $B^{O,I}$ is $\hat{n}_B^{O,I}$ as shown in Fig. 2.

The surface integral in the Kottler representation (5) is denoted by $\vec{I}_A^{O,I}$, i.e.,

$$\vec{I}_A^{O,I} = -\int_A \hat{n} \cdot \overline{\overline{V}}(\vec{R}^{O,I})\, dA \quad (6)$$

where the dyad $\overline{\overline{V}}(\vec{R}^{O,I})$ is

$$\overline{\overline{V}}(\vec{R}^{O,I}) = G(\vec{R}^{O,I}) \nabla \vec{E}^i(\vec{R}^{O,I}) - \nabla G(\vec{R}^{O,I}) \vec{E}^i(\vec{R}^{O,I}). \quad (7)$$

It is observed that the surface integral $\vec{I}_A^O$ in (6) expresses the flux of the dyad $\overline{\overline{V}}$ out of the base of the cone V^O, and the surface integral $\vec{I}_A^I$ in (6) is the flux of the dyad $\overline{\overline{V}}$ into the base of the cone V^I. The volume integral of the divergence of $\overline{\overline{V}}$ over the cone $V^{O,I}$ is denoted by $\vec{I}_V^{O,I}$

$$\vec{I}_V^{O,I} = \int_{V^{O,I}} \nabla \cdot \overline{\overline{V}}(\vec{R}^{O,I})\, dV^{O,I} \quad (8)$$

and the flux of the dyad $\overline{\overline{V}}$ out of the curved surface $B^{O,I}$ is denoted by $\vec{I}_B^{O,I}$

$$\vec{I}_B^{O,I} = \int_{B^{O,I}} \hat{n}_B^{O,I} \cdot \overline{\overline{V}}(\vec{R}^{O,I})\, dB^{O,I}. \quad (9)$$

Applying the dyadic divergence theorem [21], it is found that

$$\vec{I}_A^{O,I} = \pm \vec{I}_V^{O,I} \mp \vec{I}_B^{O,I}. \quad (10)$$

In the expression above, and henceforth, the upper sign applies to the superscript O and the lower sign to the superscript I. The integrals $\vec{I}_V^{O,I}$ and $\vec{I}_B^{O,I}$ are transformed into line integrals.

B. Transformation of the Volume Integral $\vec{I}_V^{O,I}$

Since the dyad $\overline{\overline{V}}(\vec{R}^{O,I})$ of (7) is divergence-free everywhere except at $F^{O,I}$ and at the location of the Hertzian dipole, it will be found that the contributions to the integral (8) stem from these points. Insertion of the expression for the divergence of the dyad $\overline{\overline{V}}(\vec{R}^{O,I})$ into volume the integral (8) yields

$$\vec{I}_V^{O,I} = \int_{V^{O,I}} \left(G(\vec{R}^{O,I})\nabla^2\vec{E}^i(\vec{R}^{O,I}) - \nabla^2 G(\vec{R}^{O,I})\vec{E}^i(\vec{R}^{O,I})\right) dV^{O,I}. \quad (11)$$

The free-space Green's function satisfies the scalar wave equation

$$(\nabla^2 + k^2)G(\vec{R}^{O,I}) = -\delta(\vec{R}^{O,I}) \quad (12)$$

where δ is Dirac's delta function. The incident electric field $\vec{E}^i$ obeys the vectorial wave equation

$$(\nabla^2 + k^2)\vec{E}^i(\vec{R}^{O,I}) = jkZ\vec{\alpha}\cdot\left(\overline{\overline{I}} + \frac{\nabla\nabla}{k^2}\right)\delta(\vec{R}^{O,I} - \vec{R}_S^{O,I}) \quad (13)$$

where $\vec{\alpha}$ is the dipole moment of the Hertzian dipole and $\vec{R}_S^{O,I}$ is the vector from $F^{O,I}$ to the location of the Hertzian dipole. From the wave equations (12) and (13), the Laplacians of the Green's function and the incident electric field can be found and inserted into the volume integral (11) to yield

$$\vec{I}_V^{O,I} = \int_{V^{O,I}} jkZG(\vec{R}^{O,I})\vec{\alpha} \cdot\left(\overline{\overline{I}} + \frac{\nabla\nabla}{k^2}\right)\delta(\vec{R}^{O,I} - \vec{R}_S^{O,I})\, dV^{O,I} + \int_{V^{O,I}} \vec{E}^i(\vec{R}^{O,I})\delta(\vec{R}^{O,I})\, dV^{O,I}. \quad (14)$$

It is now assumed that the Hertzian dipole is located either inside or outside the cone V^O and thus not on the curved surface B^O. The implication of this restriction on numerical calculations will be discussed following (38). The first volume integral in (14) can be shown to yield

$$\int_{V^{O,I}} jkZG(\vec{R}^{O,I})\vec{\alpha}\cdot\left(\overline{\overline{I}} + \frac{\nabla\nabla}{k^2}\right)\delta(\vec{R}^{O,I} - \vec{R}_S^{O,I})\, dV^{O,I} = jkZ\vec{\alpha}\cdot\left(\overline{\overline{I}} + \frac{\nabla\nabla}{k^2}\right)G(\vec{R}_S^{O,I})\chi^{O,I} = -\vec{E}^i(F^{O,I})\chi^{O,I} \quad (15)$$

where

$$\chi^O = \begin{cases} 1 & \text{if the Hertzian dipole is inside } V^O \\ 0 & \text{if the Hertzian dipole is outside } V^O \end{cases} \quad (16)$$

and $\chi^I = 0$ as the Hertzian dipole is located in the half-space into which $\hat{n}$ is directed and thus always outside the cone V^I.

The transformation of the second volume integral in (14) into a line integral is described next. The contributions of this integral stem from the observation and the image points. It is found that these contributions can be expressed as the product of the incident electric field at the observation or image points and the relative solid angle $\Omega^{O,I}(4\pi)^{-1}$ subtended by the plate A as seen from these points

$$\int_{V^{O,I}} \vec{E}^i(\vec{R}^{O,I})\delta(\vec{R}^{O,I})\, dV^{O,I} = \frac{\vec{E}^i(F^{O,I})}{4\pi}\Omega^{O,I}. \quad (17)$$

It is noted that $\Omega^O = \Omega^I$. The solid angle $\Omega^{O,I}$ can be calculated from a line integral using the results of Asvestas [15]. Thus, the volume integral (14) is

$$\vec{I}_V^{O,I} = -\vec{E}^i(F^{O,I})\chi^{O,I} \pm \vec{E}^i(F^{O,I})\int_\Gamma \hat{t}\cdot\vec{V}(\vec{R}^{O,I})\, d\Gamma \quad (18)$$

where

$$\vec{V}(\vec{R}^{O,I}) = \frac{\hat{p}\times\hat{R}^{O,I}}{4\pi R^{O,I}(1 - \hat{R}^{O,I}\cdot\hat{p})} \quad (19)$$

with $\hat{p}$ being an arbitrary unit vector in the plane of the plate A and $\hat{R}^{O,I} = (R^{O,I})^{-1}\vec{R}^{O,I}$. In summary, the volume integral $\vec{I}_V^{O,I}$ is expressed as the sum of a constant term (which is the negative of the incident field at F^O if the source is inside V^O and zero otherwise) and a line integral along the edge of the plate.

C. Transformation of the Surface Integral $\vec{I}_B^{O,I}$

The surface integral in $\vec{I}_B^{O,I}$ (9) is written as a double integral. The inner integral is evaluated along the generators of the cones while the outer integral is evaluated along the edge of the plate. Using $\vec{R}^{O,I}$ as the vector from $F^{O,I}$ to a point of the edge of the plate and introducing $\tau \in [0; 1]$, the points along a generator of the cone $V^{O,I}$ are described by $\tau\vec{R}^{O,I}$. In the $\tau\Gamma$-system, the infinitesimal area element $dB^{O,I}$ can be written as

$$dB^{O,I} = R^{O,I}\tau|\hat{R}^{O,I}\times\hat{t}|\, d\tau\, d\Gamma. \quad (20)$$

Using the dyad $\overline{\overline{V}}(\vec{R}^{O,I})$ of (7), the fact that $\nabla G(\vec{R}^{O,I})$ is perpendicular to $\hat{n}_B^{O,I}$, and the expression

$$\hat{n}_B^{O,I} = \pm\frac{\hat{R}^{O,I}\times\hat{t}}{|\hat{R}^{O,I}\times\hat{t}|} \quad (21)$$

it is found that the surface integral $\vec{I}_B^{O,I}$ in (9) can be written as

$$\vec{I}_B^{O,I} = \pm\int_\Gamma(\hat{R}^{O,I}\times\hat{t}) \cdot\int_0^1 \nabla\vec{E}^i(\tau\vec{R}^{O,I})G(\tau\vec{R}^{O,I})R^{O,I}\tau\, d\tau\, d\Gamma = \mp\int_\Gamma \hat{t}\cdot\overline{\overline{W}}(\vec{R}^{O,I})\, d\Gamma \quad (22)$$

where the dyad $\overline{\overline{W}}(\vec{R}^{O,I})$ is

$$\overline{\overline{W}}(\vec{R}^{O,I}) = \frac{1}{4\pi}\hat{R}^{O,I} \times \int_0^1 \nabla\vec{E}^i(\tau\vec{R}^{O,I})\exp(-jk\tau R^{O,I})\,d\tau. \tag{23}$$

Equation (22) shows that a line integral representation of $\vec{I}_B^{O,I}$ exists if it is possible to derive an explicit expression for the dyad $\overline{\overline{W}}(\vec{R}^{O,I})$. The derivation of this explicit expression is the key step in obtaining the line integral representation of the electric PO scattered field for illumination by a Hertzian dipole. This derivation is performed in Appendix A.

III. Overview of the Line Integral Representation

The final expression for the line integral representation of the PO field is obtained by inserting the line integral representation of $\vec{I}_V^{O,I}$ in (18) and $\vec{I}_B^{O,I}$ in (22) into the equation for $\vec{I}_A^{O,I}$ in (10) and using this expression in the Kottler representation (5) followed by application of (3) relating the PO field and the field $\vec{E}^A$. In terms of the quantities defined in Section II, this yields

$$\begin{aligned}\vec{E}^{PO}(F^O) = \int_\Gamma \Bigg[\hat{t}\cdot\Bigg(&\overline{\overline{W}}(\vec{R}^O) + \overline{\overline{W}}(\vec{R}^I)\cdot(\overline{\overline{I}} - 2\hat{n}\hat{n}) \\ &+ \vec{V}(\vec{R}^O)(\vec{E}^i(F^O) - \vec{E}^i(F^I)\cdot(\overline{\overline{I}} - 2\hat{n}\hat{n})) \\ &+ \frac{2Z}{jk}G(\vec{R}^O)\left(jk + \frac{1}{R^O}\right)\vec{H}^i(\vec{R}^O)\hat{R}^O\Bigg) \\ &+ 2G(\vec{R}^O)(\overline{\overline{I}} - \hat{n}\hat{n})\cdot(\hat{t}\times\vec{E}^i(\vec{R}^O))\Bigg]\,d\Gamma \\ &- \vec{E}^i(F^O)\chi^O.\end{aligned} \tag{24}$$

The Hertzian dipole with dipole moment $\vec{\alpha}$ is described by the current density

$$\vec{J}_{HZ} = \vec{\alpha}\delta(\vec{R}^{O,I} - \vec{R}_S^{O,I}) \tag{25}$$

and it radiates the magnetic and electric fields

$$\vec{H}^i(\vec{\rho}) = G(\vec{\rho})\left(jk + \frac{1}{\rho}\right)\vec{\alpha}\times\hat{\rho} \tag{26}$$

$$\begin{aligned}\vec{E}^i(\vec{\rho}) = \frac{-jZG(\vec{\rho})}{k}\big[&(-k^2 + 3jk\rho^{-1} + 3\rho^{-2})\vec{\alpha}\cdot\hat{\rho}\hat{\rho} \\ &+ (k^2 - jk\rho^{-1} - \rho^{-2})\vec{\alpha}\big]\end{aligned} \tag{27}$$

where $\vec{\rho} = \vec{R}^{O,I} - \vec{R}_S^{O,I}$ is the position vector with respect to the location of the Hertzian dipole. Furthermore, ρ is the length of $\vec{\rho}$ and $\hat{\rho} = \rho^{-1}\vec{\rho}$. The dyad $\overline{\overline{W}}(\vec{R}^{O,I})$ is (see Appendix A)

$$\begin{aligned}\overline{\overline{W}}(\vec{R}^{O,I}) = \frac{jZ}{(4\pi)^2 k}\Bigg[&K_1\Big(\vec{\alpha}\cdot\vec{B}\hat{R}^{O,I}\times\overline{\overline{I}} + \hat{R}^{O,I}\times\vec{\alpha}\vec{B} \\ &+ \vec{A}\vec{\alpha}\cdot(\hat{R}^{O,I}(\hat{R}^{O,I} - jk\vec{B}) - jk\vec{B}\hat{R}^{O,I})\Big) \\ &+ K_2\vec{\alpha}\cdot\vec{B}\vec{A}\vec{B} + K_3\vec{A}\vec{\alpha} \\ &+ K_4(\vec{\alpha}\cdot\hat{R}^{O,I}\hat{R}^{O,I}\times\overline{\overline{I}} + \hat{R}^{O,I}\times\vec{\alpha}\hat{R}^{O,I} - jk\vec{\alpha} \\ &\cdot\hat{R}^{O,I}\vec{A}\hat{R}^{O,I}) \\ &+ K_5\vec{A}\vec{\alpha}\cdot(\hat{R}^{O,I}\vec{B} + \vec{B}\hat{R}^{O,I}) \\ &+ K_6\vec{\alpha}\cdot\hat{R}^{O,I}\vec{A}\hat{R}^{O,I}\Bigg]\end{aligned} \tag{28}$$

where $\vec{A} = \hat{R}^{O,I}\times\vec{R}_S^{O,I}$ and $\vec{B} = \hat{R}^{O,I}\times\vec{A}$. The functions $K_1,\cdots,K_6$ are

$$\begin{aligned}K_1 = &C_1\rho^3[jkC_3 + C_3^2(2 + \hat{R}^{O,I}\cdot\hat{\rho})] \\ &- C_2(R_S^{O,I})^2[C_4(1 + jkR_S^{O,I}) + C_4^2]\end{aligned} \tag{29}$$

$$\begin{aligned}K_2 = &-C_1\rho^2[C_3(k^2 - 3jk\rho^{-1} - 3\rho^{-2}) \\ &- C_3^2(2jk + 3\rho^{-1}) - 2C_3^3] \\ &+ C_2[C_4((kR_S^{O,I})^2 - 3(jkR_S^{O,I} + 1)) \\ &- C_4^2(2jkR_S^{O,I} + 3) - 2C_4^3]\end{aligned} \tag{30}$$

$$\begin{aligned}K_3 = &C_1\rho^3[C_3(k^2\rho - jk - \rho^{-1}) - C_3^2] \\ &- C_2(R_S^{O,I})^2[C_4((kR_S^{O,I})^2 - jkR_S^{O,I} - 1) - C_4^2]\end{aligned} \tag{31}$$

$$\begin{aligned}K_4 = &C_1\rho^2[1 + jk\rho - C_3 jk\rho^2] \\ &- C_2(R_S^{O,I})^3[1 + jkR_S^{O,I} - C_4 jkR_S^{O,I}]\end{aligned} \tag{32}$$

$$\begin{aligned}K_5 = &-C_1[(k\rho)^2 - 3(jk\rho + 1)] \\ &+ C_2R_S^{O,I}[(kR_S^{O,I})^2 - 3(jkR_S^{O,I} + 1)]\end{aligned} \tag{33}$$

$$\begin{aligned}K_6 = &C_1(\hat{R}^{O,I}\cdot\vec{R}_S^{O,I} - R^{O,I})[(k\rho)^2 - 3(jk\rho + 1)] \\ &- C_2R_S^{O,I}\hat{R}^{O,I}\cdot\vec{R}_S^{O,I}[(kR_S^{O,I})^2 - 3(jkR_S^{O,I} + 1)]\end{aligned} \tag{34}$$

where $R_S^{O,I}$ is the length of $\vec{R}_S^{O,I}$. The quantities $C_1,\cdots,C_4$ are

$$C_1 = (R^{O,I}\rho^5)^{-1}\exp(-jk(R^{O,I} + \rho)) \tag{35}$$

$$C_2 = (R^{O,I}(R_S^{O,I})^6)^{-1}\exp(-jkR_S^{O,I}) \tag{36}$$

$$C_3 = (\rho + \hat{R}^{O,I}\cdot\vec{\rho})^{-1} \tag{37}$$

$$C_4 = (1 - \hat{R}^{O,I}\cdot\hat{R}_S^{O,I})^{-1} \tag{38}$$

with $\hat{R}_S^{O,I} = (R_S^{O,I})^{-1}\vec{R}_S^{O,I}$.

The integrand of the line integral (24) is lengthy but consists of simple functions and is thus suited for numerical evaluation. The dyad $\overline{\overline{W}}(\vec{R}^I)$ has a removable singularity if $\hat{R}^I\cdot\hat{R}_S^I = 1$ [see (63)], i.e., if the Hertzian dipole is located at the extension of the line from F^I to the edge of the plate. There is a nonremovable singularity in $\overline{\overline{W}}(\vec{R}^O)$ if $\hat{R}^O\cdot\hat{R}_S^O = 1$. This corresponds to the Hertzian dipole being on the surface of the cone V^O. This case was excluded, however, in the course of the derivation in Section II-B.

The line integral representation of the PO scattered field (24) is to be evaluated along the edge. It is important to note, however, that this fact does not imply that the PO scattered field physically originates from the edge or the edge-adjacent region of the plate. In general, there will be contributions to the PO scattered field from surface points, e.g., the reflected field from the surface stationary point, as well as from the edge, e.g., the PO-diffracted field from the edge stationary points. The line integral representation (24) includes all these contributions due to the exact derivation. The form of (24) seems unsuited, however, for interpreting the various terms appearing in the integrand in (24) physically. In particular, no single term provides the asymptotic representation. The quantity $\exp(-jk(R^{O,I} + \rho))$ describing the phase variation from the source via the edge of the plate to the observation point is contained in most of the terms in (24). Consequently, all these terms must be taken into consideration if an asymptotic expansion were to be carried out.

The restriction that the Hertzian dipole cannot be located on the surface of the cone V^O has an implication on numerical calculations as the time savings provided by the line integral representation may reduce if the Hertzian dipole is close to the surface of the cone V^O. In this case the dyad $\overline{\overline{W}}(\vec{R}^{O,I})$ of (28), and thus the integrand in (24), becomes highly peaked, and consequently, many sample points may be required in the numerical calculation of the line integral (24) to achieve a good accuracy. For observation points where the condition is very close to being satisfied, this may even result in a larger calculation time for the line integral representation than the surface radiation integral. The line integral representation will remain valid, however, even though the integrand is highly peaked. The validity of the line integral representation is violated only if the Hertzian dipole is on the surface of the cone V^O and this condition is rarely satisfied exactly in practice.

As mentioned in the Introduction, the restriction on the validity of the above-derived line integral is less severe than that of the line integral in [14]. The line integral representation in this paper is valid for source points which are not located on the surface of the cone V^O, whereas the representation in [14] is valid only for source points located outside this cone.

In this paper the source is a Hertzian dipole. It might be possible, however, to achieve a line integral representation of the electric PO field for another type of point source. In this case the incident electric and magnetic fields, $\vec{E}^i$ and $\vec{H}^i$, in (24) are the fields due to this source. The most difficult part in achieving a line integral representation for a point source other than the Hertzian dipole is the analytical calculation of the dyad $\overline{\overline{W}}(\vec{R}^{O,I})$ for this type of source.

IV. Numerical Calculations

To illustrate that the line integral representation is exact, the electric PO scattered field is calculated from both the line integral (24) and the surface integral (1), and the results are compared. For the numerical comparison the configuration under consideration is defined as follows: In a rectangular xyz-coordinate system with a spherical $r\theta\phi$-coordinate system associated in the usual manner, a perfectly conducting rectangular plate with its corners positioned at (0,0,0), (2λ,0,0), (2λ,3λ,0), and (0,3λ,0) (λ is the wavelength) is illuminated by a Hertzian dipole located at (1λ,2λ,1λ) with dipole moment $\vec{\alpha} = (1,1,1)$ Am (see Fig. 3). The observation points are located at $r = 4\lambda$, in the $\phi = 50$ degrees plane with θ ranging from 0 to 90 degrees with one degree step. For this configuration, the Hertzian dipole is located inside the cone V^O for $\theta \in [0; 57.27]$ and outside the cone for $\theta \in]57.27; 90]$. The amplitudes (for $\lambda - 1$m) of the x-, y-, and z-components of the electric PO field are shown in Fig. 4. As expected, perfect agreement is obtained between the two methods of calculation. Similar agreement is found for the phase.

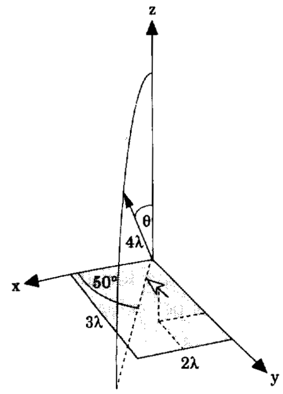

Fig. 3. Hertzian dipole illuminating a rectangular plate of dimensions 2λ by 3λ (λ being the wavelength). The Hertzian dipole is positioned at (1λ,2λ,1λ) and possesses the dipole moment (1,1,1) Am. The observation points are located at $r = 4\lambda$ in the $\phi = 50$ degrees plane.

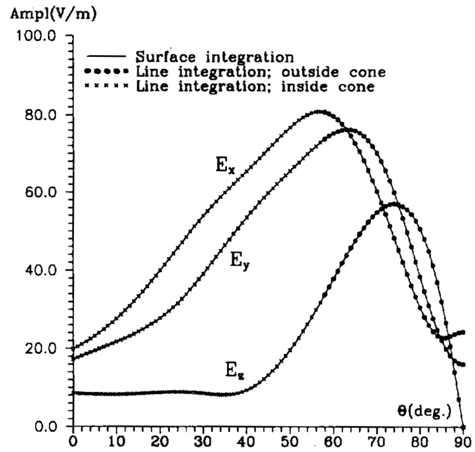

Fig. 4. The amplitudes of the x-, y-, and z-components of the electric PO field versus the observation angle θ for the configuration shown in Fig. 3.

V. Conclusions

An exact line integral representation of the physical optics scattered field has been derived. This was accomplished through a transformation of the conventional surface radiation integral. Employing an image point technique, the surface radiation integral was first expressed in terms of the radiation integral occurring in the analysis of aperture diffraction. Use of Asvestas' results next led to the Kottler-representation. Invoking the dyadic divergence theorem, a volume integral and a surface integral were then obtained. The volume integral was evaluated to yield simple terms involving the incident field and the relative solid angle subtended by the scattering structure. The surface integral led to an integral of the gradient of the incident electric field. The analytical evaluation of this dyadic integral, which is the key step in the transformation, was finally performed.

The numerical evaluation of the line integral representation is potentially less time consuming than that of the surface radiation integral. For polyhedral structures that are large in terms of wavelengths, the time savings can be significant. For some locations of source and observation points, however, the time savings may vanish due to a highly peaked behavior of the integrand. Future work will therefore address this problem.

Appendix A
Calculation of the Dyad $\overline{\overline{W}}(\vec{R}^{O,I})$

The gradient of the electric field (27) is

$$\nabla\vec{E}^i(\vec{\rho}) = \frac{-jZG(\vec{\rho})}{k} \cdot \Big[\big(-k^2\rho^{-1} + 3jk\rho^{-2} + 3\rho^{-3}\big)(\vec{\alpha}\cdot\hat{\rho}\overline{\overline{I}} + \vec{\alpha}\hat{\rho}) + \big(jk^3 + 6k^2\rho^{-1} - 15jk\rho^{-2} - 15\rho^{-3}\big)\vec{\alpha}\cdot\hat{\rho}\hat{\rho}\hat{\rho} + \big(-jk^3 - 2k^2\rho^{-1} + 3jk\rho^{-2} + 3\rho^{-3}\big)\hat{\rho}\vec{\alpha}\Big]. \quad (39)$$

Insertion of (39) into the equation for $\overline{\overline{W}}(\vec{R}^{O,I})$ of (23) yields

$$\overline{\overline{W}}(\vec{R}^{O,I}) = \frac{jZ}{(4\pi)^2 k} \cdot \Big[\Big(\vec{\alpha}\cdot\vec{R}^{O,I}\hat{R}^{O,I}\times\overline{\overline{I}} + \hat{R}^{O,I}\times\vec{\alpha}\vec{R}^{O,I}\Big)I_1 - \Big(\vec{\alpha}\cdot\vec{R}_S^{O,I}\hat{R}^{O,I}\times\overline{\overline{I}} + \hat{R}^{O,I}\times\vec{\alpha}\vec{R}_S^{O,I}\Big)I_2 - \Big(\vec{\alpha}\cdot\vec{R}^{O,I}\hat{R}^{O,I}\times\vec{R}_S^{O,I}\vec{R}^{O,I}\Big)I_3 + \Big(\vec{\alpha}\cdot\vec{R}^{O,I}\hat{R}^{O,I} \times\vec{R}_S^{O,I}\vec{R}_S^{O,I} + \vec{\alpha}\cdot\vec{R}_S^{O,I}\hat{R}^{O,I}\times\vec{R}_S^{O,I}\vec{R}^{O,I}\Big)I_4 - \vec{\alpha}\cdot\vec{R}_S^{O,I}\hat{R}^{O,I}\times\vec{R}_S^{O,I}\vec{R}_S^{O,I}I_5 + \hat{R}^{O,I}\times\vec{R}_S^{O,I}\vec{\alpha}I_6\Big] \quad (40)$$

where the integrals $I_1,\cdots,I_6$ are

$$I_1 = \int_0^1 h\big(k^2f^{-3} - 3jkf^{-4} - 3f^{-5}\big)\tau\,d\tau \quad (41)$$

$$I_2 = \int_0^1 h\big(k^2f^{-3} - 3jkf^{-4} - 3f^{-5}\big)\,d\tau \quad (42)$$

$$I_3 = \int_0^1 h\big(-jk^3f^{-4} - 6k^2f^{-5} + 15jkf^{-6} + 15f^{-7}\big)\tau^2\,d\tau \quad (43)$$

$$I_4 = \int_0^1 h\big(-jk^3f^{-4} - 6k^2f^{-5} + 15jkf^{-6} + 15f^{-7}\big)\tau\,d\tau \quad (44)$$

$$I_5 = \int_0^1 h\big(-jk^3f^{-4} - 6k^2f^{-5} + 15jkf^{-6} + 15f^{-7}\big)\,d\tau \quad (45)$$

$$I_6 = \int_0^1 h\big(-jk^3f^{-2} - 2k^2f^{-3} + 3jkf^{-4} + 3f^{-5}\big)\,d\tau \quad (46)$$

with

$$h = h(\tau) = \exp(-jk(\tau R^{O,I} + f)) \quad (47)$$

and

$$f = f(\tau) = |\tau\vec{R}^{O,I} - \vec{R}_S^{O,I}|. \quad (48)$$

The integrals I_2, I_5, and I_6 are calculated using the substitution

$$t = \tau R^{O,I} + f \quad \Leftrightarrow \quad \tau = \frac{t^2 - (R_S^{O,I})^2}{2(R^{O,I}t - \vec{R}^{O,I}\cdot\vec{R}_S^{O,I})} \quad (49)$$

and

$$\frac{dt}{d\tau} = \frac{R^{O,I}t - \vec{R}^{O,I}\cdot\vec{R}_S^{O,I}}{t - \tau R^{O,I}} \quad (50)$$

followed by integration by parts. The results are, using $C_1,\cdots,C_4$ defined in (35)–(38)

$$I_2 = C_1\rho^3\Big[jkC_3 + C_3^2(2 + \hat{R}^{O,I}\cdot\hat{\rho})\Big] - C_2(R_S^{O,I})^2\Big[C_4(1 + jkR_S^{O,I}) + C_4^2\Big] \quad (51)$$

$$I_5 = C_1\rho^2\big[C_3(k^2 - 3jk\rho^{-1} - 3\rho^{-2}) - C_3^2(2jk + 3\rho^{-1}) - 2C_3^3\big] - C_2\Big[C_4\Big((kR_S^{O,I})^2 - 3(jkR_S^{O,I} + 1)\Big) - C_4^2(2jkR_S^{O,I} + 3) - 2C_4^3\Big] \quad (52)$$

$$I_6 = C_1\rho^3\big[C_3(k^2\rho - jk\rho^{-1} - \rho^{-2}) - C_3^2\big] - C_2(R_S^{O,I})^2 \cdot\Big[C_4\Big((kR_S^{O,I})^2 - jkR_S^{O,I} - 1\Big) - C_4^2\Big]. \quad (53)$$

To perform the calculations of the integrals I_1, I_3, and I_4, the equations

$$\tau = \frac{ff' + \vec{R}^{O,I}\cdot\vec{R}_S^{O,I}}{(R^{O,I})^2} \quad (54)$$

and

$$f'' = \frac{(R^{O,I})^2 - (f')^2}{f} \quad (55)$$

are used together with integration by parts and the substitution (49). The results are

$$I_1 = (R^{O,I})^{-2}I_7 + (R^{O,I})^{-1}\hat{R}^{O,I}\cdot\vec{R}_S^{O,I}I_2 \quad (56)$$

$$I_3 = (R^{O,I})^{-4}I_{10} + ((R^{O,I})^{-1}\hat{R}^{O,I}\cdot\vec{R}_S^{O,I})^2I_5 + 2(R^{O,I})^{-3}\hat{R}^{O,I}\cdot\vec{R}_S^{O,I}I_8 \quad (57)$$

$$I_4 = (R^{O,I})^{-2}I_8 + (R^{O,I})^{-1}\hat{R}^{O,I}\cdot\vec{R}_S^{O,I}I_5 \quad (58)$$

with

$$I_7 = C_1\rho^5R^{O,I}(jk\rho^{-2} + \rho^{-3}) - C_2R^{O,I}(R_S^{O,I})^6 \cdot\Big(jk(R_S^{O,I})^{-2} + (R_S^{O,I})^{-3}\Big) - jkR^{O,I}I_9 \quad (59)$$

$$I_8 = C_1R^{O,I}\rho^2(k^2 - 3jk\rho^{-1} - 3\rho^{-2}) - C_2R^{O,I}(R_S^{O,I})^3\Big(k^2 - 3jk(R_S^{O,I})^{-1} - 3(R_S^{O,I})^{-2}\Big) + jkR^{O,I}I_2 \quad (60)$$

$$I_9 = C_1\rho^4C_3 - C_2(R_S^{O,I})^4C_4 \quad (61)$$

$$I_{10} = C_1\rho^2R^{O,I}\Big((R^{O,I})^2 - \vec{R}^{O,I}\cdot\vec{R}_S^{O,I}\Big) \cdot(k^2 - 3jk\rho^{-1} - 3\rho^{-2}) + C_2R^{O,I}(R_S^{O,I})^3\vec{R}^{O,I}\cdot\vec{R}_S^{O,I} \cdot\Big(k^2 - 3jk(R_S^{O,I})^{-1} - 3(R_S^{O,I})^{-2}\Big). \quad (62)$$

Finally, the results for $I_1, \cdots, I_6$, using the integrals $I_7, \cdots, I_{10}$, are inserted into (40). After subsequent reduction, the result given in (28) is obtained.

The dyad $\overline{\overline{W}}(\vec{R}^I)$ has a removable singularity for $\hat{R}^I \cdot \hat{R}^I_S = 1$. In this case

$$\overline{\overline{W}}(\vec{R}^I) = \frac{jZ}{(4\pi)^2 k}\left(\vec{\alpha}\cdot\hat{R}^I\hat{R}^I \times \overline{\overline{I}} + \hat{R}^I \times \vec{\alpha}\hat{R}^I\right) \cdot \Big[C_1(jk\rho^3+\rho^2) - \exp(-jkR^I_S)\rho^{-2}(R^I)^{-1} \cdot (-0.5jk + k^2\rho) + C_2(R^I_S)^3((R^I_S)^2k^2 - 1.5jkR^I_S - 1)\Big]. \quad (63)$$

For $\hat{R}^O \cdot \hat{R}^O_S = 1$, the dyad $\overline{\overline{W}}(\vec{R}^O)$ has a nonremovable singularity.

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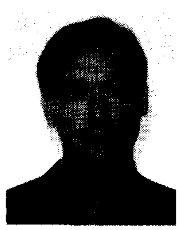

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