# An Exact Solution for the Level-Crossing Rate of Shadow Fading Processes Modelled by Using the Sum-of-Sinusoids Principle

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**Abstract** The focus of this paper is on the higher order statistics of spatial simulation models for shadowing processes. Such processes are generally assumed to follow the lognormal distribution. The proposed spatial simulation model is derived from a non-realizable lognormal reference model with given correlation properties by using Rice's sum-of-sinusoids. Both exact and approximate expressions are presented for the level-crossing rate (LCR) and the average duration of fades (ADF) of the simulation model. It is pointed out that Gudmundson's correlation model results in an infinite LCR. To avoid this problem, two alternative spatial correlation models are proposed. Illustrative examples of the dynamic behavior of shadow fading processes are presented for all three types of correlation models. Emphasis will be placed on two realistic propagation scenarios capturing the shadowing effects in suburban and urban areas.

**Keywords** Mobile fading channels  $\cdot$  Shadowing effects  $\cdot$  Spatial shadowing processes  $\cdot$  Lognormal processes  $\cdot$  Level-crossing rate

# 1 Introduction

Empirical studies in [1–4] have shown that long-term effects such as shadowing can be modelled well by lognormal processes. A good understanding of lognormal process and its parameterization is therefore important for the performance study of mobile communication systems. To study the impact of combined slow and fast fading effects, lognormal processes have been proposed as an integral part of many channel models, such as the Suzuki model [5] and the Loo model [6]. Lognormal processes also play an important role in the performance analysis of handover processes [7,8]. The problem of finding realistic parameters of

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lognormal processes was the subject of many measurement campaigns conducted in different areas. In urban areas, it has been observed [9] that the standard deviation of shadowing ranges from 6.5 to 8.2 dB, whereas in macrocells the standard deviation is between 5 and 12 dB [1–4].

The spatial correlation properties of shadowing have been studied in [10–12]. Gudmundson [10] has proposed a negative exponential correlation model, which has later been improved in [12], where it was shown that shadowing cannot decorrelate exponentially with distance in physical channels. Despite the problems stressed in [12], Gudmundson's correlation model has been adopted by many researchers and is even part of the evaluation methodology of UMTS [13].

Although a variety of issues concerning the statistics of lognormal processes have been discussed in the literature, there are still some untouched topics. For example, little is known about the higher order statistics of shadowing processes. In [14], the level-crossing rate (LCR) of a reference model for lognormal processes has been derived, but only an approximate solution has been presented for the corresponding simulation model. In this paper, we present the exact solution that gives an insight into the influence of the model parameters on the rate of fading caused by shadowing. The problem with Gudmundson's correlation model is that the LCR of the lognormal process becomes infinite if the correlation function of the underlying Gaussian process equals the negative exponential function. To avoid this problem, two alternative correlation models for shadowing in mobile radio systems are introduced. We also present a user-friendly concept for the design of a lognormal channel simulator. The flexibility and high performance of the simulation model is demonstrated by applying the proposed design concept on all three types of presented correlation models.

The paper is structured as follows. Section 2 introduces a spatial lognormal process, which is considered as reference model. From the reference model, we derive in Sect. 3 the corresponding simulation model. Its LCR and average duration of fades (ADF) are analysed in Sect. 4. In Sect. 5, we apply the proposed design procedure on three different spatial correlation models and discuss the achieved theoretical and numerical results. Finally, Sect. 6 draws the conclusion.

#### 2 The Reference Model

Here, it is assumed that the mobile station starts at the origin x = 0 and moves along the x-axis with velocity v. During the movement of the mobile station, the transmitted signal experiences fast fading and shadowing. It is widely accepted that the shadowing effects can well be modelled in the spatial domain by a lognormal process

$$\lambda(x) = 10^{(\sigma_L \nu(x) + m_L)/20} \tag{1}$$

where the parameters  $\sigma_L$  and  $m_L$  are called the shadow standard deviation and the area mean, respectively, and  $\nu(x)$  is a spatial Gaussian process with zero mean and unit variance. The distribution of  $\lambda(x)$  follows the lognormal probability density function (PDF)

$$p_{\lambda}(z) = \frac{20}{\sqrt{2\pi} \ln(10) \sigma_L z} e^{-\frac{(20 \log(z) - m_L)^2}{2\sigma_L^2}}, \quad z \ge 0.$$
 (2)

It is worth mentioning that the distribution in (2) does not provide any information on how fast the spatial lognormal process  $\lambda(x)$  changes with distance. However, this information is provided by the LCR and the ADF.



The LCR  $N_{\lambda}(r)$  of the spatial lognormal process  $\lambda(x)$  describes how often  $\lambda(x)$  crosses a given level r from up to down (or from down to up) within a unit of length (usually 1 m). In [14] it was shown that the LCR  $N_{\lambda}(r)$  of the reference model described by  $\lambda(x)$  [see (1)] is given by

$$N_{\lambda}(r) = \frac{\sqrt{\gamma}}{2\pi} e^{-\frac{(20\log(r) - m_L)^2}{2\sigma_L^2}}, \quad r \ge 0$$
(3)

where the quantity  $\gamma$  is related to the ACF  $r_{\nu\nu}(\Delta x)$  of the underlying spatial Gaussian process  $\nu(x)$  according to  $\gamma = -\frac{\mathrm{d}^2}{\mathrm{d}\Delta x^2} r_{\nu\nu}(\Delta x)|_{\Delta x=0} = -\ddot{r}_{\nu\nu}(0)$ . The ADF  $T_{\lambda_-}(r)$  is the expected value for the length of the spatial interval in which the

The ADF  $T_{\lambda_{-}}(r)$  is the expected value for the length of the spatial interval in which the lognormal process  $\lambda(x)$  is below a given level r. If the LCR  $N_{\lambda}(r)$  is known, then the ADF  $T_{\lambda_{-}}(r)$  can readily be obtained from [15]

$$T_{\lambda_{-}}(r) = \frac{F_{\lambda_{-}}(r)}{N_{\lambda}(r)} \tag{4}$$

where  $F_{\lambda_{-}}(r)$  designates the cumulative distribution function of the spatial lognormal process  $\lambda(x)$  being the probability that  $\lambda(x)$  is less than or equal to the level r, i.e.,  $F_{\lambda_{-}}(r) = Pr(\lambda(x) \le r) = \int_0^r p_{\lambda}(z) dz$ .

# 3 The Spatial Shadowing Simulator

By applying the sum-of-sinusoids principle [16], a stochastic spatial simulation model for lognormal processes  $\lambda(x)$  can be obtained from the reference model by substituting the spatial Gaussian process  $\nu(x)$  in (1) by a sum-of-sinusoids

$$\hat{\nu}(x) = \sum_{n=1}^{N} c_n \cos(2\pi \alpha_n x + \Theta_n)$$
 (5)

with constant gains  $c_n$ , constant spatial frequencies  $\alpha_n$ , and i.i.d. random phases  $\Theta_n$ , each having a uniform distribution over the interval  $[0, 2\pi)$ . The spatial autocorrelation function (ACF) of  $\hat{v}(x)$ , defined by  $\hat{r}_{vv}(\Delta x) := E\{\hat{v}(x)\hat{v}(x+\Delta x)\}$ , can be expressed as

$$\hat{r}_{\nu\nu}(\Delta x) = \sum_{n=1}^{N} \frac{c_n^2}{2} \cos(2\pi \alpha_n \Delta x). \tag{6}$$

Furthermore, the PDF of the sum-of-sinusoids  $\hat{v}(x)$  in (5) is given by [17]

$$\hat{p}_{\nu}(y) = \begin{cases} 2 \int_{0}^{\infty} \left[ \prod_{n=1}^{N} J_0(2\pi c_n z) \right] \cos(2\pi y z) \, \mathrm{d}z, & |y| \le \hat{\nu}_{\text{max}} \\ 0, & \text{otherwise} \end{cases}$$
 (7)

where  $\hat{v}_{\text{max}} = \sum_{n=1}^{N} |c_n|$  and  $J_0(\cdot)$  denotes the 0th-order Bessel function of the first kind. Our proposed stochastic simulation model for spatial lognormal processes is mathematically described by the expression

$$\hat{\lambda}(x) = 10^{(\sigma_L \hat{\nu}(x) + m_L)/20} \tag{8}$$

from which the structure shown in Fig. 1 can easily be derived.



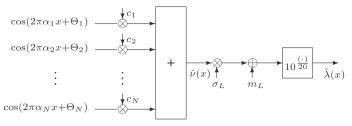


Fig. 1 Structure of the spatial shadowing simulator

# 4 LCR and ADF of the Shadowing Simulator

This subsection is concerned with the derivation of the LCR  $\hat{N}_{\lambda}(r)$  and the ADF  $\hat{T}_{\lambda_{-}}(r)$  of the stochastic simulation model for spatial lognormal processes  $\hat{\lambda}(x)$  defined in (8). First, recall that the LCR  $\hat{N}_{\lambda}(r)$  can be computed by using [18,19]

$$\hat{N}_{\lambda}(r) = \int_{0}^{\infty} \dot{z} \hat{p}_{\lambda\dot{\lambda}}(r, \dot{z}) d\dot{z}, \quad r \ge 0$$
(9)

where  $\hat{p}_{\lambda\dot{\lambda}}(z,\dot{z})$  denotes the joint PDF of  $\hat{\lambda}(x)$  and its spatial derivative  $\dot{\hat{\lambda}}(x) = d\hat{\lambda}(x)/dx$  at the same point on the *x*-axis. Our starting point for finding the exact solution of  $\hat{N}_{\lambda}(r)$  is the stochastic process  $\hat{\nu}(x)$  in (5) and its spatial derivative  $\dot{\hat{\nu}}(x)$ , which is given by

$$\dot{\hat{v}}(x) = -2\pi \sum_{n=1}^{N} \alpha_n c_n \sin(2\pi \alpha_n x + \Theta_n). \tag{10}$$

The PDF  $\hat{p}_{\dot{v}}(\dot{y})$  of  $\dot{\hat{v}}(x)$  can be expressed as

$$\hat{p}_{\dot{v}}(\dot{y}) = \begin{cases} \int_{0}^{\infty} \left[ \prod_{n=1}^{N} J_0(4\pi^2 \alpha_n c_n z) \right] \cos(2\pi \dot{y}z) dz, & |\dot{y}| \leq \dot{\hat{v}}_{\text{max}} \\ 0, & \text{otherwise} \end{cases}$$
(11)

where  $\dot{\hat{v}}_{\max} = 2\pi \sum_{n=1}^{N} |\alpha_n c_n|$ . The computation of the cross-correlation function (CCF) of  $\hat{v}(x)$  and  $\dot{\hat{v}}(x)$ , which is defined by  $\hat{r}_{\nu\dot{\nu}}(\Delta x) := E\{\hat{v}(x)\dot{\hat{v}}(x+\Delta x)\}$ , gives

$$\hat{r}_{\nu\dot{\nu}}(\Delta x) = -\pi \sum_{n=1}^{N} \alpha_n c_n^2 \sin(2\pi\alpha_n \Delta x). \tag{12}$$

This result shows that  $\hat{v}(x)$  and  $\hat{v}(x)$  are in general correlated. However, for the computation of the LCR, we can restrict our investigations to the behavior of  $\hat{v}(x_1)$  and  $\hat{v}(x_2)$  at the same point in space  $x = x_1 = x_2$ , i.e.,  $\Delta x = x_2 - x_1 = 0$ . Hence, we observe from (12) that  $\hat{r}_{v\dot{v}}(\Delta x) = 0$  holds if  $\Delta x = 0$ , i.e.,  $\hat{v}(x)$  and  $\hat{v}(x)$  are uncorrelated at the same point in space. In case of Gaussian random processes, it follows that uncorrelatedness is equivalent to independence [20]. Since the PDFs  $\hat{p}_v(y)$  and  $\hat{p}_{\dot{v}}(\dot{y})$  in (7) and (11), respectively, are both very close to the Gaussian distribution if  $N \geq 7$ , we may assume that  $\hat{v}(x)$  and  $\hat{v}(x)$  are also independent at the same point on the x-axis. This allows us to express the joint PDF  $\hat{p}_{v\dot{v}}(y,\dot{y})$  of  $\hat{v}(x)$  and  $\hat{v}(x)$  as

$$\hat{p}_{\nu\dot{\nu}}(y,\dot{y}) = \hat{p}_{\nu}(y) \cdot \hat{p}_{\dot{\nu}}(\dot{y}). \tag{13}$$



Now, after applying the method of transformation of random variables [20], we can express the joint PDF  $\hat{p}_{\lambda\dot{\lambda}}(z,\dot{z})$  of  $\hat{\lambda}(x)$  and  $\dot{\hat{\lambda}}(x)$  as

$$\hat{p}_{\lambda\dot{\lambda}}(z,\dot{z}) = |J|^{-1} \hat{p}_{\nu\dot{\nu}} \left( \frac{20\log(z) - m_L}{\sigma_L}, \frac{20\dot{z}}{\sigma_L \ln(10)z} \right)$$

$$= |J|^{-1} \hat{p}_{\nu} \left( \frac{20\log(z) - m_L}{\sigma_L} \right) \hat{p}_{\dot{\nu}} \left( \frac{20\dot{z}}{\sigma_L \ln(10)z} \right)$$
(14)

where |J| is the absolute value of the Jacobian determinant, which is given here by  $|J| = (\sigma_L z \ln(10)/20)^2$ . Finally, after substituting (14) in (9) and using (11), we find the following exact solution for the LCR  $\hat{N}_{\lambda}(r)$  of  $\hat{\lambda}(x)$ 

$$\hat{N}_{\lambda}(r) = \hat{p}_{\nu} \left( \frac{20 \log(r) - m_L}{\sigma_L} \right) \int_{0}^{\infty} \frac{\left[ \prod_{n=1}^{N} J_0(4\pi^2 c_n \alpha_n y) \right]}{2(\pi y)^2} \cdot \left[ \cos(2\pi y \dot{\hat{\nu}}_{\text{max}}) - 1 + 2\pi y \dot{\hat{\nu}}_{\text{max}} \sin(2\pi y \dot{\hat{\nu}}_{\text{max}}) \right] dy$$
(15)

where  $\dot{\hat{\nu}}_{\max} = 2\pi \sum_{n=1}^{N} |\alpha_n c_n|$ . We mention without proof that  $\hat{N}_{\lambda}(r) \to N_{\lambda}(r)$  as  $N \to \infty$ . In [14], it was shown that the LCR  $\hat{N}_{\lambda}(r)$  of the spatial shadowing simulator can be approximated by

$$\hat{N}_{\lambda}(r) \approx \frac{\sqrt{\hat{\gamma}}}{2\pi} e^{-\frac{(20\log(r) - m_L)^2}{2\sigma_L^2}}, \quad r \ge 0$$
(16)

where

$$\hat{\gamma} = -\frac{\mathrm{d}^2}{\mathrm{d}\Delta x^2} \hat{r}_{\nu\nu}(\Delta x)|_{\Delta x = 0}$$

$$= 2\pi^2 \sum_{n=1}^{N} (\alpha_n c_n)^2. \tag{17}$$

By analogy to (4), the ADF  $\hat{T}_{\lambda_{-}}(r)$  of the stochastic simulation model can be obtained from  $\hat{T}_{\lambda_{-}}(r) = \hat{F}_{\lambda_{-}}(r)/\hat{N}_{\lambda}(r)$ , where  $\hat{N}_{\lambda}(r)$  is given by (15) and  $\hat{F}_{\lambda_{-}}(r)$  denotes the cumulative distribution function of  $\hat{\lambda}(x)$ , which can easily be computed from the PDF  $\hat{p}_{\lambda}(z)$  of  $\hat{\lambda}(x)$  derived in [14] via  $\hat{F}_{\lambda_{-}}(r) = \int_{0}^{r} \hat{p}_{\lambda}(z) dz$ .

### 5 Correlation Models and Results

To illustrate the correctness and usefulness of the exact solution found for the LCR  $\hat{N}_{\lambda}(r)$  presented in (15), we develop a simulation model for a spatial shadowing channel in typical suburban and urban areas. Altogether three spatial correlation models are considered. For each of them, it will be shown how the parameters of the simulation model can be determined.

## 5.1 The Gudmundson Model

Supported by empirical studies in [10], Gudmundson has proposed to model the correlation properties of v(x) by the following negative exponential function

$$r_{\nu\nu}(\Delta x) = e^{-|\Delta x|/D} \tag{18}$$



**Table 1** Model parameters of the reference model (from [10])

Shadowing area	D (m)	$\Delta x_{\text{max}}$ (m)	$\sigma_L$ (dB)	$m_L$
Suburban	503.9	2500	7.5	0
Urban	8.3058	40	4.3	0

where the parameter D > 0 is called the decorrelation distance. Typical values for D and the other parameters describing the reference model are listed in Table 1.

In case of Gudmundson's model, the parameters of the simulation model can be computed by using the method of equal areas (MEA) [16], which results in the following closed-form expressions:

$$c_n = \sqrt{2/N} \tag{19}$$

$$\alpha_n = \frac{1}{2\pi D} \tan\left(\frac{\pi (n - 1/2)}{2N}\right) \tag{20}$$

for  $n=1,2,\ldots,N$ . Choosing N=25 and substituting (19) and (20) in (6), results in the spatial ACFs  $\hat{r}_{\nu\nu}(\Delta x)$  illustrated in Figs. 2 and 3 for suburban and urban areas, respectively. The problem with Gudmundson's model is that the LCR  $N_{\lambda}(r)$  of the reference model is infinite. This is because the second derivative of the spatial ACF  $r_{\nu\nu}(\Delta x)$  in (18) is indefinite at the origin, which was first observed in [12]. However, the LCR  $\hat{N}_{\lambda}(r)$  of the simulation model is limited for finite values of N, but  $\hat{N}_{\lambda}(r)$  approaches infinity as  $N \to \infty$ . We consider N=25 as an appropriate choice and substitute (19) and (20) in (15), which gives the exact result of the simulation model's LCR  $\hat{N}_{\lambda}(r)$  shown in Fig. 4. This figure also illustrates the simulation results of the LCR obtained from the simulation of (8) and averaging over 100 trials. It can be observed that the simulation results confirm the correctness of the exact solution (15), whereas the approximate solution (16) becomes less accurate at high (and low) signal levels r.

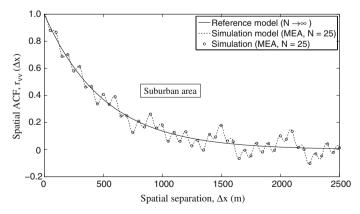
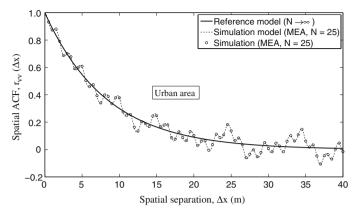
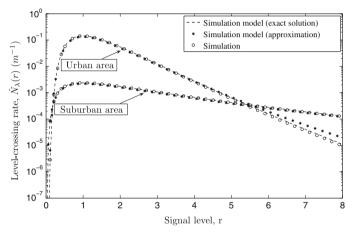


Fig. 2 Spatial ACFs  $r_{\nu\nu}(\Delta x)$  (reference model) and  $\hat{r}_{\nu\nu}(\Delta x)$  (simulation model) for suburban areas (Gudmundson's model, MEA with N=25)





**Fig. 3** Spatial ACFs  $r_{\nu\nu}(\Delta x)$  (reference model) and  $\hat{r}_{\nu\nu}(\Delta x)$  (simulation model) for urban areas (Gudmundson's model, MEA with N=25)



**Fig. 4** Comparison of the exact and approximate solutions for the LCR  $\hat{N}_{\lambda}(r)$  of the simulation model for shadow fading processes in suburban and urban areas (Gudmundson's model, MEA with N=25)

#### 5.2 The Gaussian Model

As an alternative to Gudmundson's model, we propose here the Gaussian model. In this case, the spatial correlation properties of v(x) are described by

$$r_{\nu\nu}(\Delta x) = e^{-(\Delta x/D)^2} \tag{21}$$

where D > 0. Since the above spatial correlation function has a definite second derivative in the origin, it follows that the LCR  $N_{\lambda}(r)$  of the reference model exists. The quantity  $\gamma$  is given by  $\gamma = -\ddot{r}_{\nu\nu}(0) = 2/D^2$ , so that the behavior of  $N_{\lambda}(r)$  can easily be analysed using (3).

For the computation of the parameters of the simulation model, we again apply the MEA [16]. This method provides the same expression for the gains  $c_n$  as in (19), whereas the spatial frequencies  $\alpha_n$  have to be computed by using



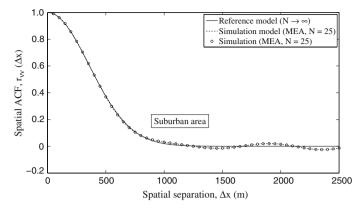


Fig. 5 Spatial ACFs  $r_{\nu\nu}(\Delta x)$  (reference model) and  $\hat{r}_{\nu\nu}(\Delta x)$  (simulation model) for suburban areas (Gaussian model, MEA with N=25)

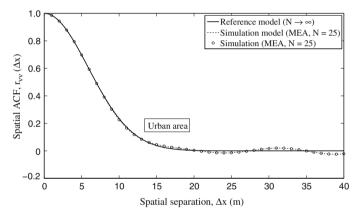


Fig. 6 Spatial ACFs  $r_{\nu\nu}(\Delta x)$  (reference model) and  $\hat{r}_{\nu\nu}(\Delta x)$  (simulation model) for urban areas (Gaussian model, MEA with N=25)

$$\alpha_n = \frac{1}{\pi D} \operatorname{erf}^{-1} \left( \frac{n - 1/2}{N} \right) \tag{22}$$

for  $n=1,2,\ldots,N$ , where  $\operatorname{erf}^{-1}(\cdot)$  denotes the inverse error function. Choosing again N=25 and substituting (19) and (22) in (6) gives the spatial ACF  $\hat{r}_{\nu\nu}(\Delta x)$  of the simulation model shown in Figs. 5 and 6. Figure 7 illustrates both the exact solution (15) and the approximate solution (16) for the LCR  $\hat{N}_{\lambda}(r)$  of the simulation model. For comparative purposes, the LCR  $N_{\lambda}(r)$  of the reference model using (3) is also shown.

#### 5.3 The Butterworth Model

As a second alternative to Gudmundson's model, we propose to describe the power spectral density (PSD)  $S_{\nu\nu}(\alpha)$  of the stochastic process  $\nu(x)$  by a function having the shape of the kth-order Butterworth filter

$$S_{\nu\nu}(\alpha) = \frac{A_k}{1 + (\alpha D_k)^{2k}} \tag{23}$$



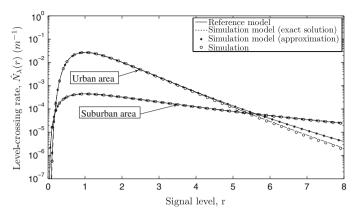


Fig. 7 Comparison of the exact and approximate solutions for the LCR  $\hat{N}_{\lambda}(r)$  of the simulation model for shadow fading processes in suburban and urban areas (Gaussian model, MEA with N=25)

where  $A_k$  and  $D_k$  are positive constants. The constant  $A_k$  is to be chosen such that the mean power of  $\nu(x)$  equals unity, i.e.,  $\int_{-\infty}^{\infty} S_{\nu\nu}(\alpha) \mathrm{d}\alpha = 1$ . Recall that the ACF  $r_{\nu\nu}(\Delta x)$  is obtained from the inverse Fourier transform of the PSD  $S_{\nu\nu}(\alpha)$ . From the Fourier transform relationship, it follows that the Gudmundson model is included in the Butterworth model as a special case, namely if k=1,  $A_1=2D$ , and  $D_1=2\pi D$ . Let us restrict our investigations to the 2nd-order Butterworth model, i.e., k=2. In this case,  $A_2$  equals  $\sqrt{2}D_2/\pi$  and the spatial correlation properties of  $\nu(x)$  are described by

$$r_{\nu\nu}(\Delta x) = \sqrt{2} e^{-\pi\sqrt{2}|\Delta x|/D_2} \sin\left(\pi\sqrt{2}\frac{|\Delta x|}{D_2} + \frac{\pi}{4}\right).$$
 (24)

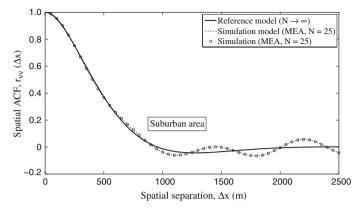
Using the above result, the quantity  $\gamma = -\ddot{r}_{\nu\nu}(0)$  can be expressed as  $\gamma = (2\pi/D_2)^2$ . Notice that the 2nd-order Butterworth model and the Gaussian model result in the same LCR  $N_{\lambda}(r)$  if we choose  $D_2 = \pi\sqrt{2}D$ . In the following, however, our choice falls on the relation  $D_2 = \pi\sqrt{2}D/1.2396$ , which guarantees that the spatial ACFs  $r_{\nu\nu}(\Delta x)$  in (18) and (24) are identical at  $\Delta x = D$ .

When using the Butterworth model of order 2, the parameters of the simulation model can also be computed by using the MEA [16]. For the gains  $c_n$ , we obtain again the same expression as in (19), whereas the spatial frequencies  $\alpha_n$  have to be determined for all n = 1, 2, ..., N from the following equation by means of numerical integration and root-finding techniques

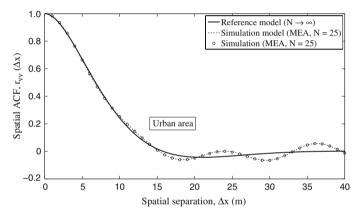
$$\int_{0}^{\alpha_n} \frac{D_2}{1 + (\alpha D_2)^4} \, d\alpha - \frac{(n - \frac{1}{2})\pi}{2\sqrt{2}N} = 0.$$
 (25)

After computing the parameters  $c_n$  and  $\alpha_n$ , the ACF  $\hat{r}_{\nu\nu}(\Delta x)$  and the LCR  $\hat{N}_{\lambda}(r)$  of the simulation model can be analysed by using the general expressions (6) and (15), respectively. The results obtained for the spatial ACF  $\hat{r}_{\nu\nu}(\Delta x)$  are illustrated in Figs. 8 and 9. In these figures, the corresponding curves found for the reference model are also plotted for comparative purposes. Finally, we mention that the results obtained for the LCR  $\hat{N}_{\lambda}(r)$  are very similar to those shown in Fig. 7.





**Fig. 8** Spatial ACFs  $r_{\nu\nu}(\Delta x)$  (reference model) and  $\hat{r}_{\nu\nu}(\Delta x)$  (simulation model) for suburban areas (2nd-order Butterworth model, MEA with N=25)



**Fig. 9** Spatial ACFs  $r_{\nu\nu}(\Delta x)$  (reference model) and  $\hat{r}_{\nu\nu}(\Delta x)$  (simulation model) for urban areas (2nd-order Butterworth model, MEA with N=25)

# 6 Conclusion

In this paper, we have studied the higher order statistics of spatial simulation models for lognormal processes. The concept of Rice's sum-of-sinusoids has been applied to derive the lognormal channel simulator from a non-realizable reference model with given spatial correlation properties. The exact solution for the LCR of the simulation model has been derived. By comparing the exact solution to an approximation, it was shown that the approximate solution is less accurate at high and low signal levels. Furthermore, it was confirmed that Gudmundson's model is not a reasonable correlation model, since the LCR of the reference model is infinite. To avoid this problem, two alternative correlation models have been introduced, which were called the Gaussian model and the Butterworth model. Illustrative examples of the dynamic behavior of shadow fading processes were given for all three types of correlation models. The proposed concept enables a quick design of a simulation model that is capable to emulate efficiently the shadowing effects in suburban and urban areas with high precision.



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