Exact SU(2) Symmetry and Persistent Spin Helix in a Spin-Orbit Coupled System

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Spin-orbit coupled systems generally break the spin rotation symmetry. However, for a model with equal Rashba and Dresselhauss coupling constants, and for the [110] Dresselhauss model, a new type of SU(2) spin rotation symmetry is discovered. This symmetry is robust against spin-independent disorder and interactions and is generated by operators whose wave vector depends on the coupling strength. It renders the spin lifetime infinite at this wave vector, giving rise to a persistent spin helix. We obtain the spin fluctuation dynamics at, and away from, the symmetry point and suggest experiments to observe the persistent spin helix.

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The physics of systems with spin-orbit coupling has generated great interest from both academic and practical perspectives [1]. In particular, spin-orbit coupling allows for purely electric manipulation of the electron spin [2–5] and could be of use in areas from spintronics to quantum computing. Theoretically, spin-orbit coupling reveals fundamental physics related to topological phases and their applications to the intrinsic and quantum spin Hall effect [6–10].

While strong spin-orbit interaction is useful for manipulating the electron spin by electric fields, it is also known to have the undesired effect of causing spin decoherence [11,12]. The decay of spin polarization reflects the nonconservation of the total spin operator $\vec{S}$, i.e., $[\vec{S}, \mathcal{H}] \neq 0$, where $\mathcal{H}$ is any Hamiltonian that contains spin-orbit coupling. In this Letter, we identify an exact SU(2) symmetry in a class of spin-orbit systems that renders the spin lifetime infinite. The symmetry involves components of spin at a finite wave vector and is different from the U(1) symmetry that has previously been associated with this class of models [13,14]. As a result of this symmetry, spin polarization excited at a certain "magic" wave vector will persist. If this symmetry can be realized experimentally, it may be possible to manipulate spins through spin-orbit coupling without spin-polarization decay, at length scales characteristic of today's semiconductor lithography.

We consider a two-dimensional electron gas without inversion symmetry, allowing spin-orbit coupling that is linear in the electron wave vector. The most general form of linear coupling includes both Rashba and Dresselhaus contributions:

$$\mathcal{H} = \frac{k^2}{2m} + \alpha(k_y\sigma_x - k_x\sigma_y) + \beta(k_x\sigma_x - k_y\sigma_y), \quad (1)$$

where $k_{x,y}$ is the electron momentum along the [100] and [010] directions, respectively, α and β are the strengths of the Rashba and Dresselhauss spin-orbit couplings, and m is the effective electron mass. We shall be interested in the special case of $\alpha = \beta$, which may be experimentally accessible through tuning of the Rashba coupling via externally applied electric fields [2]. When $\alpha = \beta$, the spin-orbit coupling part of the Hamiltonian reads $\alpha(k_x + k_y) \times (\sigma_x - \sigma_y)$. Rotating the spatial coordinates by introducing $k_\pm = (1/\sqrt{2})(k_y \pm k_x)$, and also performing the global spin rotation generated by $U = (1/\sqrt{2})\{1 + [(i/\sqrt{2})(\sigma_x + \sigma_y)]\}$, brings the Hamiltonian to the diagonal form:

$$\mathcal{H}_{\mathrm{ReD}} = U^\dagger \mathcal{H} U = \frac{k_+^2 + k_-^2}{2m} - 2\alpha k_+ \sigma_z. \quad (2)$$

Throughout this Letter, we shall be using both the original spin basis and the transformed spin basis for the $\mathcal{H}_{\mathrm{ReD}}$ model, depending on the context. It should always be remembered that σ_z in the transformed spin basis corresponds to $(1/\sqrt{2})(\sigma_x - \sigma_y)$ in the original spin basis. $\mathcal{H}_{\mathrm{ReD}}$ is mathematically equivalent to the Dresselhauss [110] model, describing quantum wells grown along the [110] direction, whose Hamiltonian is given by:

$$\mathcal{H}_{[110]} = \frac{k_x^2 + k_y^2}{2m} - 2\alpha k_x \sigma_z. \quad (3)$$

As the Hamiltonians $\mathcal{H}_{[\mathrm{ReD}]}$ and $\mathcal{H}_{[110]}$ are already diagonal, the energy bands (for $\mathcal{H}_{[\mathrm{ReD}]}$) are simply given by $\epsilon_{\downarrow\uparrow}(\vec{k}) = (k^2/2m) \pm 2\alpha k_+$, where $\downarrow, \uparrow$ are the spin components in the new, unitary transformed, spin basis: $\vec{S} \rightarrow U^\dagger \vec{S} U$. The bands have an important *shifting property*:

$$\epsilon_\downarrow(\vec{k}) = \epsilon_\uparrow(\vec{k} + \vec{Q}), \quad (4)$$

where $Q_+ = 4m\alpha$, $Q_- = 0$ for the $\mathcal{H}_{[\mathrm{ReD}]}$ model and $Q_x = 4m\alpha$, $Q_y = 0$ for the $\mathcal{H}_{[110]}$ model. The Fermi surfaces consist of two circles shifted by the magic shifting vector $\vec{Q}$, as shown in Fig. 1.

$\mathcal{H}_{[\mathrm{ReD}]}$ and $\mathcal{H}_{[110]}$ have the previously known U(1) symmetry generated by σ_z [13,14], which ensures a long lifetime for uniform spin polarization along the z axis[15].

The exact SU(2) symmetry discovered in this work is generated by the following operators, expressed in the transformed spin basis as:

$$S_Q^- = \sum_{\vec{k}} c_{\vec{k}\downarrow}^\dagger c_{\vec{k}+\vec{Q}\uparrow}, \qquad S_Q^+ = \sum_{\vec{k}} c_{\vec{k}+\vec{Q},\uparrow}^\dagger c_{\vec{k}\downarrow},$$
$$S_0^z = \sum_{\vec{k}} c_{\vec{k}\uparrow}^\dagger c_{\vec{k}\uparrow} - c_{\vec{k}\downarrow}^\dagger c_{\vec{k}\downarrow}, \tag{5}$$

with $c_{k\uparrow,\downarrow}$ being the annihilation operators of spin-up and -down particles. These operators obey the commutation relations for angular momentum

$$[S_0^z, S_Q^\pm] = \pm 2 S_Q^\pm; \qquad [S_Q^+, S_Q^-] = S_0^z. \tag{6}$$

The shifting property Eq. (4) ensures that the operators defined in Eq. (5) commute with the Hamiltonian

$$[\mathcal{H}_{\mathrm{ReD}}, c_{\vec{k}+\vec{Q}\uparrow}^\dagger c_{\vec{k}\downarrow}] = (\epsilon_\uparrow(\vec{k}+\vec{Q}) - \epsilon_\downarrow(\vec{k})) c_{\vec{k}+\vec{Q}\uparrow}^\dagger c_{\vec{k}\downarrow} = 0 \tag{7}$$

and similarly for $c_{\vec{k}\downarrow}^\dagger c_{\vec{k}+\vec{Q}\uparrow}$, thus uncovering the SU(2) symmetry. This symmetry is robust against both spin-independent disorder and Coulomb (or other many-body) interactions as the spin operators commute with the finite-wave-vector particle density $\rho_q = \sum_k c_{\vec{k}+\vec{q}}^\dagger c_{\vec{k}}$:

$$[\rho_q, S_{\vec{Q}}^\pm] = [\rho_q, S_0^z] = 0. \tag{8}$$

As a result, single-particle potential scattering terms such as $\sum_q V_q \rho_q$, as well as many-body interaction terms such as $\sum_q V_q \rho_q^\dagger \rho_q$, all commute with the generators (5), and the SU(2) symmetry is robust against these interactions.

The SU(2) conservation laws imply that the expectation values of S^z, S_Q, and $S_Q^\dagger$ have an infinite lifetime. Since S^z is a conserved quantity, a fluctuation of the z component of spin polarization with wave vector q can only decay by diffusion, which takes time $\tau_s = 1/D_s q^2$, where D_s is the spin diffusion constant. Notice that the Hermitian operators $S_x(Q) = \frac{1}{2}(S_Q^\dagger + S_Q)$ and $S_y(Q) = \frac{1}{2i}(S_Q^\dagger - S_Q)$ create a helical spin density wave in which the direction of the spin polarization rotates in the x, y plane in the transformed spin basis. Another manifestation of this symmetry is the vanishing of spin-dependent quantum interference (or weak antilocalization) at the SU(2) point [16].

The infinite lifetime of the S_x, S_y spin helix with wave vector $\vec{Q}$ results from the combined effects of diffusion and precession in the spin-orbit effective field. A physical picture for the origin of the persistent spin helix (PSH) is sketched in Fig. 2, expressed in the transformed spin basis. Consider a particle propagating in the plane with momentum $\vec{k}$. In time t, it travels a distance along the x_+ direction $L_+ = k_+ t/m$, while its spin precesses about the z axis by an angle $\theta = 4\alpha k_+ t/\hbar$. Eliminating t from the latter equation yields $\theta = 4\alpha m L_+/\hbar$, demonstrating that the net spin precession in the x, y plane depends only on the net displacement in the x_+ direction and is independent of any other property of the electron's path. Electrons starting with parallel spin orientation and the same value of x_+ will return exactly to the original orientation after propagating $L_+ = 2\pi\hbar/Q_+$.

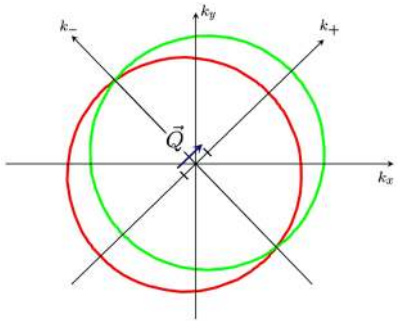

FIG. 1 (color online). Fermi surfaces of the model consist of two circles shifted by the wave vector $\vec{Q} = (Q_+, Q_-) = (4m\alpha, 0)$.

For the sake of clarity, we have depicted the PSH in Fig. 3 in the original basis. For a range of values in different materials from weak to strong spin-orbit splitting $\alpha = 10^2 \rightarrow 10^4$(m/s), the characteristic wavelength of the PSH is 10 μm $\rightarrow$ 100 nm. In GaAs, the typical value is $L_+ =$ 1 μm. This characteristic wave length is on the scale of typical lithographic dimensions used in today's semiconductor industry.

Mathematically, the PSH is a direct manifestation of a "non-Abelian" flux in the ground state of the $\mathcal{H}_{\mathrm{ReD}}$ and the $\mathcal{H}_{[110]}$ models. Following Ref. [17], we express $\mathcal{H}_{\mathrm{ReD}}$ in the form of a background non-Abelian gauge potential

$$\mathcal{H}_{\mathrm{ReD}} = \frac{k_-^2}{2m} + \frac{1}{2m}(k_+ - 2m\alpha\sigma_z)^2 + \text{const.} \tag{9}$$

In contrast to the general case where the non-Abelian gauge potential leads to a finite non-Abelian field strength, the field strength vanishes identically for $\alpha = \beta$. Therefore, we can eliminate the vector potential by a non-Abelian gauge transformation: $\Psi_\uparrow(x_+, x_-) \rightarrow \exp(i2m\alpha x_+)\Psi_\uparrow(x_+, x_-)$, $\Psi_\downarrow(x_+, x_-) \rightarrow \exp(-i2m\alpha x_+) \times \Psi_\downarrow(x_+, x_-)$. Under this transformation, the spin-orbit

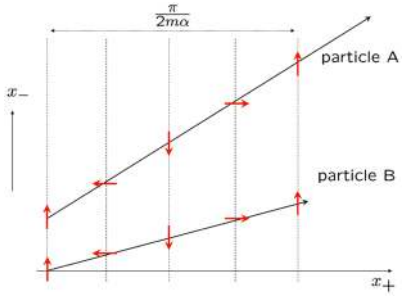

FIG. 2 (color online). Spin configurations for two particles A and B depend only on the distance traveled along the x_+ axis and not on their initial momenta. For the same x_+ distance traveled, the spin precesses by exactly the same angle. After a length $L_+ = 2\pi\hbar/Q_+$, the spins all return exactly to the original configuration.

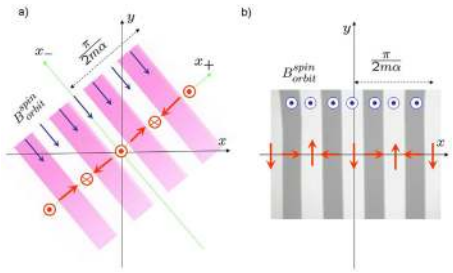

FIG. 3 (color online). (a) PSH for the $\mathcal{H}_{\mathrm{ReD}}$ model. The spin-orbit magnetic field is in-plane (blue), whereas the spin helix for the choice of the relative signs in Eq. (1) is in the (x_+, z) plane. (b) PSH for the $\mathcal{H}_{[110]}$ model. The spin-orbit magnetic field $B^{\mathrm{spin}}_{\mathrm{orbit}}$, in blue, is out-of-plane, whereas the spin helix, in red, is in-plane.

coupled Hamiltonian is mapped to that of the free Fermi gas. The cost of the transformation is twofold. First, the new wave function $\Psi(x_+, x_-)$ satisfies twisted spin boundary conditions [18]. However, these have no physical effect, since a metallic system without off-diagonal-long-range order is insensitive to a change of boundary conditions in the thermodynamic limit. Second, while diagonal operators such as the charge n and S_z remain unchanged, off-diagonal operators such as $S^-(\vec{x}) = \psi^\dagger_\downarrow(\vec{x})\psi_\uparrow(\vec{x})$ and $S^+(\vec{x}) = \psi^\dagger_\uparrow(\vec{x})\psi_\downarrow(\vec{x})$ are transformed: $S^-(\vec{x}) \to \exp(-i4m\alpha x_+)S^-(\vec{x})$, $S^+(\vec{x}) \to \exp(i4m\alpha x_+)S^+(\vec{x})$. In the transformed basis, all three components of the spin obey the simple diffusion equation, as the Hamiltonian is just that for free electrons without spin-orbit coupling. Hence, $S_x = \text{const}$ is a solution to the diffusion equation. Transforming back demonstrates that this solution corresponds to $(S_x, S_y) = (\cos(4m\alpha x_+), \sin(4m\alpha x_+)) = \text{const}$, which is the PSH.

While spin is conserved at the SU(2) point, dynamics emerge when the conditions $\vec{q} = \vec{Q}$ and/or $\alpha = \beta$ are not met. Solving for the coupled charge and spin dynamics requires Boltzmann transport equations, which we obtain below using the Keldysh formalism [19,20]. Assuming isotropic scattering with momentum lifetime τ, the retarded and advanced Green's functions are $G^{R,A}(k, \epsilon) = (\epsilon - \mathcal{H} \pm \frac{i}{2\tau})^{-1}$. We introduce a momentum-, energy-, and position-dependent charge-spin density which is a 2×2 matrix $g(k, r, t)$. Summing over momentum

$$\rho(r, t) \equiv \int \frac{d^2k}{(2\pi)^3\nu} g(k, r, t), \tag{10}$$

gives the real-space spin-charge density $\rho(r, t) = n(r, t) + S^i(r, t)\sigma_i$, where $n(r, t)$ and $S^i(r, t)$ are the charge and spin density, respectively, and $\nu = m/2\pi$ is the density of states in two dimensions. $\rho(r, t)$ and $g(k, r, t)$ satisfy a Boltzmann-type equation [19,20]:

$$\frac{\partial g}{\partial t} + \frac{1}{2}\left\{\frac{\partial \mathcal{H}}{\partial k_i}, \frac{\partial g}{\partial r_i}\right\} + i[\mathcal{H}, g] = -\frac{g}{\tau} + \frac{i}{\tau}(G^R\rho - \rho G^A) \tag{11}$$

that we now solve for the Hamiltonian of Eq. (1) for arbitrary α, β. To obtain the spin-charge transport equations, we follow the sequence: Time-Fourier transform the above equation; find a general solution for $g(k, r, t)$ involving $\rho(r, t)$ and the k-dependent spin-orbit coupling; perform a gradient expansion of that solution (assuming $\partial_r \ll k_F$ where k_F is the Fermi wave vector) to second order; and, finally, integrate over the momentum. The anisotropic nature of the Fermi surfaces when both α and β are nonzero introduces coupling of charge and spin degrees of freedom beyond what was found with only Rashba coupling [21]. The final result, expressed in the $i = (x_+, x_-, z)$ spatial coordinates and the original spin basis, is

$$\partial_t n = D\partial_i^2 n + B_1\partial_{x_+}S_{x_-} - B_2\partial_{x_-}S_{x_+}, \tag{12}$$

$$\partial_t S_{x_-} = D\partial_i^2 S_{x_-} + B_1\partial_{x_+}n - C_2\partial_{x_-}S_z - T_2 S_{x_-}, \tag{13}$$

$$\partial_t S_{x_+} = D\partial_i^2 S_{x_+} - B_2\partial_{x_-}n - C_1\partial_{x_+}S_z - T_1 S_{x_+}, \tag{14}$$

$$\partial_t S_z = D\partial_i^2 S_z + C_2\partial_{x_-}S_{x_-} + C_1\partial_{x_+}S_{x_+} - (T_1 + T_2)S_z. \tag{15}$$

With an effective k_F defined as $\sqrt{2mE_F}$, the constants in the above equations are

$$\begin{aligned} B_1 &= 2(\alpha - \beta)^2(\alpha + \beta)k_F^2\tau^2, \\ B_2 &= 2(\alpha + \beta)^2(\alpha - \beta)k_F^2\tau^2, \\ C_1 &= 2\frac{1}{m}(\alpha + \beta)k_F^2\tau, \qquad C_2 = 2\frac{1}{m}(\alpha - \beta)k_F^2\tau, \\ T_1 &= 2(\alpha + \beta)^2k_F^2\tau, \qquad T_2 = 2(\alpha - \beta)^2k_F^2\tau. \end{aligned} \tag{16}$$

The diffusion constant $D = v_F^2\tau/2$. Our results for the coupling coefficients are valid in the diffusive limit $\alpha k_F\tau \ll 1$, $\beta k_F\tau \ll 1$ and reduce to the appropriate limits in the cases $\beta \to 0$ or $\alpha \to 0$ [20,21]. We observe that, for a general direction of propagation in the $x_\pm$ plane, the three components of spin, and the charge, are all coupled.

Motivated by experiments that probe spin dynamics optically, we focus on the behavior of the out-of-plane component of the spin, which is S_z in the original basis. We assume a space-time dependence proportional to $\exp[i(\omega t - \vec{q}\cdot\vec{r})]$ and compare $\vec{q}$ parallel to $[1\bar{1}0]$ and $\vec{q}$ parallel to $[110]$.

The spin-polarization lifetime for $\vec{q}$ parallel to $[1\bar{1}0]$ is not enhanced, as no shifting property exists in this direction. For this orientation, the four equations (12)–(15) separate into two coupled pairs, with n being coupled to S_{x_+} and S_z coupled to S_{x_-}. The optical probe will detect the characteristic frequencies that are solutions to the equation that couples S_z with S_{x_-}:

$$i\omega_{1,2} = -Dq^2 - \tfrac{1}{2}(2T_2 + T_1 \pm \sqrt{T_1^2 + 4q^2C_2^2}). \tag{17}$$

At $\alpha = \beta$, we have $T_2 = C_2 = 0$ and the characteristic frequencies become $i\omega_1 = -Dq^2 - T_1$, $i\omega_2 = -Dq^2$.

The spin-polarization dynamics are qualitatively different in the [110] direction, which is the direction along

which the Fermi surfaces are shifted. In this case, the four equations again decouple into two pairs, with n coupled to S_{x_-} and S_z coupled to S_{x_+}. The characteristic frequencies are

$$i\omega_{1,2} = -Dq^2 - \tfrac{1}{2}(2T_1 + T_2 \pm \sqrt{T_2^2 + 4q^2C_1^2}). \quad (18)$$

We note that $i\omega_2(q)$ has a minimum at a wave vector that depends only on α, β, and m, i.e., is independent of k_F and τ. When $\alpha = \beta$, the decay rates become $i\omega_{1,2} = -Dq^2 - T_1 \mp C_1 q$, and $i\omega_2(q)$ is zero (corresponding to the PSH) at the shifting wave vector $4m\alpha$. We have checked that this relation continues to hold as we include higher order corrections to the transport coefficients. Earlier calculations based on the pure Rashba model [21,22] also predict an increased spin lifetime at a finite wave vector $(\sqrt{15}/2)\alpha m$, but the lifetime is enhanced relative to $q = 0$ only by the factor $16/7$.

Transient spin-grating experiments [23,24] are particularly well-suited to testing our theoretical predictions [Eqs. (17) and (18)] and discovering the PSH, as they inject finite wave-vector spin distributions. If at $t = 0$ an S_z polarization proportional to $\cos(qx_+)$ is excited, we predict a time evolution

$$S_z(q, t) = A_1(q)e^{i\omega_1(q)t} + A_2(q)e^{i\omega_2(q)t}, \quad (19)$$

where

$$A_{1,2}(q) = \frac{1}{2}\left[1 \pm \frac{T_2}{\sqrt{T_2^2 + 4q^2C_1^2}}\right]. \quad (20)$$

Thus, according to theory, the photoinjected spin-polarization wave will decay as a double exponential, with characteristic decay rates $i\omega_{1,2}(q)$. As $\alpha \to \beta$, the weight factors of the two exponentials become equal and the decay rate of the slower component $\to 0$ at the magic wave vector. If transient grating measurements verify these predictions, they will be able to provide rapid and accurate determination of the spin-orbit Hamiltonian, enabling tuning of sample parameters to achieve $\alpha = \beta$.

In real materials, other mechanisms not considered previously will lead to decay of the PSH. At the point $\alpha = \beta$, the most important contribution to spin relaxation will come from the cubic term of the bulk Dresselhaus coupling $\gamma(k_x k_y^2 \sigma_x - k_y k_x^2 \sigma_y)$. This term is often neglected in narrow and/or lightly doped quantum wells, for which $k_F^2 \ll \langle k_z^2 \rangle$. We compute the Dyakonov-Perel (DP) relaxation time τ_s from this term to be $4\hbar^2/\gamma^2 k_F^6 \tau$ [25]. For GaAs, $\gamma = 20$ eV Å^3 [25], and for $k_F = 10^8$ m^{-1}, $\tau = 10^{-12}$ s, we find $\tau_s \approx 10$ ns. Since DP relaxation from the cubic term is $\sim 1/k_F^6$, lowering the carrier density rapidly stabilizes the PSH. The situation is even better for the [100] Dresselhaus model, for which no cubic spin-flip terms exist. In this case, the relevant mechanisms for spin-decay are of the Elliott-Yafet type [26,27] and, for GaAs, lead to $\tau_s \approx 20\tau(E_g/kT)^2 \sim 100$ ns at room temperature [25].

In conclusion, we have discovered a new type of spin SU(2) symmetry in a class of spin-orbit coupled models including the model with equal Rashba and Dresselhauss coupling constant and the Dresselhaus [110] model. Based on this symmetry, we predict the existence of a persistent spin helix. The lifetime of the PSH is infinite within these models, and we present an estimate of other relaxation mechanisms which would lead to its eventual decay. We obtained the transport equations for arbitrary strength of Rashba and Dresselhauss couplings, and these equations provide the basis to analyze experiments in search of the PSH.

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