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# An Exact Theory of Imaging with a Parabolic Continuously Refractive X-ray Lens

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**Abstract**—A theory is developed of image formation with an X-ray lens that consists of a large number of elements. Each element has a biconcave parabolic profile and weakly refracts an X-ray beam. Since such a lens can have a relatively large length comparable to the focal length, the thin-lens approximation is inapplicable. An exact expression for the propagator of a continuously refractive lens is derived that describes the transfer of radiation through a refractive parabolic medium. We calculate the image propagator that describes the focusing of a parallel beam and the image transfer (the focusing of a microobject), as well as the Fourier transform of the transmission function for a microobject with a lens, is calculated. The effective aperture of an X-ray lens is completely determined by the absorption of radiation and does not depend on its geometrical cross-sectional sizes. If we write the complex refractive index as $n = 1 - \delta + i\beta$, then the beam diameter at the focus is approximately a factor of $0.8\beta/\delta$ smaller than the diameter of the effective aperture, with the index depending only slightly on the wavelength. A continuously refractive lens has no aberrations in the sense that all of the rays that passed through the lens aperture are focused at a single point. The lens can focus radiation inside it and has the properties of a waveguide; i.e., it can reconstruct the beam structure for some lengths to within the absorption-caused distortions. Nonuniform X-ray absorption in the lens leads to the interesting visualization effect of transparent microobjects when their image is focused. In this case, the phase shift gradient produced by the microobject is imaged. We discuss the properties of the Fourier transform pertaining to the absorption of radiation in the lens. *© 2003 MAIK "Nauka/Interperiodica".*

## 1. INTRODUCTION

The focusing of electromagnetic radiation by refractive lenses is of great importance both in everyday practice and in scientific research. In particular, the human eye has a lens in its structure that focuses visible light at various distances. A microscope and a telescope extend the vision of the world to mini- and macrosizes, respectively. Because of its high penetrability, hard X-ray radiation with photon energies $E$ from 10 to 50 keV allows the internal structure of microobjects to be studied by nondestructive methods. Clearly, the development of effective refractive lenses for hard X-rays could significantly enhance the possibilities of scientific research into the structure of matter in many fields of science. However, this could not be done for 100 years since the discovery of X-ray radiation mainly for two reasons. First, the refractive index of X-rays is very close to unity. Second, all materials absorb X-ray radiation. For example, for aluminum and $E$ = 25 keV, the complex refractive index is $n = 1 - \delta + i\beta$, where $\delta = 8.643 \times 10^{-7}$ and $\beta = 1.747 \times 10^{-9}$.

The problem of weak refraction was first solved in 1996 [1] by using compound lenses made up of a long row of elementary lenses. Each elementary lens has a biconcave shape and a radius of curvature $R$ on the order of 1 mm that is large enough for it to be easily produced. Accordingly, the focal length of such a lens, $F_1 = R/2\delta$, is very large and can reach hundreds of meters. In this case, the focal length of a compound lens with $N$ elements is $F = F_1/N$. Therefore, the focal length can be decreased to a value acceptable for experiments by increasing the number of elements. Fortunately, $\mathrm{Re}\,n < 1$ for X-rays; as a result, the focusing lens is biconcave. In this case, the thickness of the material in the central part of the lens is small compared to the absorption length. Nevertheless, absorption does exist and causes both an overall reduction in the beam intensity and a restriction of the aperture of X-ray compound refractive lenses. The nonuniform absorption of radiation in an X-ray lens is a new property compared to lenses for visible light, which leads to interesting properties of the image, as we show below.

The relatively small aperture of an X-ray lens (fractions of a millimeter) is not a drawback because the X-ray beams generated by synchrotron radiation sources have small cross-sectional sizes and weak divergence. Thus, on third-generation (ESRF, APS, SPring-8) synchrotron radiation sources, the vertical size of the emitting region does not exceed 30 μm, while the distance from the source to the sample is more than 50 m. At present, many papers in which various methods of producing compound refractive lenses for X-rays have been published. The simplest method involves drilling a row of circular holes to obtain a lin-

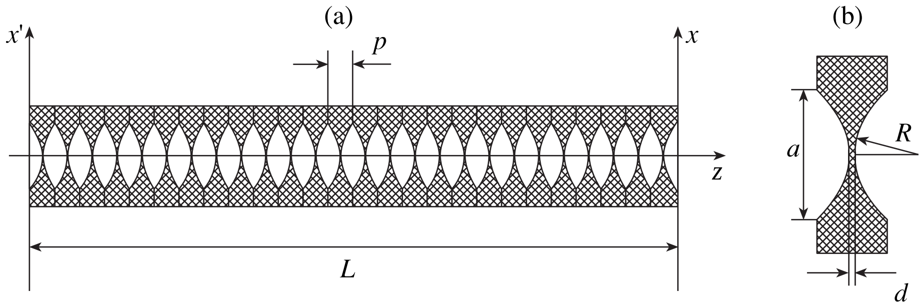


**Fig. 1.** (a) A compound refractive X-ray lens and (b) the parameters of an individual lens element.

ear focus [1] or two mutually perpendicular rows of holes to obtain a point focus [2–4]. An advantage to such lenses is their extremely low expense; drawbacks are the high surface roughness, the spherical aberrations, and the anomalously large length for a large radius of curvature of the holes. An alternative method consists in forcing air bubbles into a glue [5]. An "alligator" lens in which the parabolic phase shift profile is roughly specified by two rows of perpendicular teeth located at a small angle to the X-ray beam was suggested and tested [6, 7]. Planar lenses in which an accurate parabolic profile with a radius of curvature on the order of 1 μm is produced in a thin layer on a silicon surface were produced by using the complex deep lithography technique [8].

However, compound lenses with circular apertures and parabolic profiles are of greatest interest in terms of their imaging properties. The cross section of such a lens and the parameters of one element are shown in Fig. 1. The elements of a compound parabolic lens are produced by embossing a parabolic profile in aluminum plates [9–12] or in plates of various plastics (see, e.g., [13, 14]). The number of elements in a compound lens can be varied to obtain the required focal length. Presently, lenses with up to several hundred elements are used. A lens with 1000 elements or more can be easily produced. In this case, the length of the compound lens of $L = Np$ increases with $N$, while its focal length $F$ decreases. Clearly, as long as $L \ll F$, the focal length can be estimated using the thin-lens formula, $F \approx R/2N\delta$. Otherwise, the problem of radiation transfer through a long compound lens with allowance made for the change in the path of rays in the lens itself should be solved. This can be easily done by geometrical optics techniques [15].

The complete solution of the problem must be in the form of an integral equation similar to the Kirchhoff integral. If the change in the transverse structure of the wave field in the thickness $p$ of one element of a compound lens is small, then we may average the density of the lens material over the length $p$ and treat the lens as a homogeneous parabolic medium along the beam direction. A compound lens that satisfies this requirement is called a parabolic continuously refractive (PCR) X-ray lens. The kernel of the integral equation for such a lens is a continuous function of its length and has an analytic form, as was first shown in [16].

Here, we present an exact theory of image formation with a continuously refractive X-ray lens. Apart from deriving the propagator of the lens itself and studying its properties, we calculate the image propagator and analyze the imaging properties of a PCR lens. We show that a continuously refractive lens has no aberrations in the sense that all of the rays emerging from a single point and passing through different parts of the lens aperture converge to a single point in the image. Its effective aperture is completely determined by the absorption of radiation in the lens and decreases with increasing wavelength. At the same time, the beam diameter at the focus is approximately a factor of $0.8\beta/\delta$ smaller than the diameter of the effective aperture, and the numerical coefficient depends on wavelength only slightly. Nonuniform absorption leads to the visualization of transparent microobjects when they are imaged, with the phase shift gradient produced by the microobject being imaged. We also discuss absorption-related properties of the Fourier transform of the transmission function for the object.

## 2. THE EXACT PROPAGATOR OF A PARABOLIC CONTINUOUSLY REFRACTIVE X-RAY LENS

Let us assume that the synchrotron radiation is premonochromated and has a high degree of spatial coherence. These conditions are satisfied on third-generation synchrotron radiation sources [17]. We choose the optical axis along the $z$ axis (see Fig. 1) and represent the general solution of the Maxwell equation as

$$E(x, y, z) = \exp(ikz)A_t(x, y, z),$$

where $k = \omega/c$ is the wave number in a vacuum. The function $A_t(x, y, z)$ describes the transfer of the transverse dependence of the wave field along the $z$ axis. Since the wavelength $\lambda = 2\pi/k$ is many orders of magnitude smaller than the scale length of the X-ray interaction with the material $\lambda/\delta$, we can use the paraxial approximation with a high accuracy; i.e., we can disregard the second derivative of $A_t$ with respect to $z$ compared to its first derivative. As a result, substituting this solution into the Maxwell equation yields the parabolic equation for the function $A_t(x, y, z)$

$$\frac{dA_t}{dz} = -ik\eta s(x, y, z)A_t + \frac{i}{2k}\left(\frac{d^2A_t}{dx^2} + \frac{d^2A_t}{dy^2}\right), \quad (1)$$

where $\eta = 1 - n = \delta - i\beta = \delta(1 - i\gamma)$. In the radiation transfer problem, the wave field on the entrance lens surface is assumed to be given, i.e., $A_t(x, y, 0)$ if the $z$ coordinate is measured from the entrance lens surface. Inside an actual compound lens, the function $s(x, y, z)$ is equal to 1 in the regions filled with the lens material and 0 in the voids (see Fig. 1).

Passing to the limit of a PCR lens implies that the thickness $p$ of one element tends to zero, while the number of elements $N$ increases in such a way that the total length of the lens and its aperture do not change. The radius of curvature of the surfaces also increases. In this case, instead of the actual function $s(x, y, z)$, we may use its average value, which does not depend on the longitudinal coordinate,

$$\bar{s}(x, y) = s_0 + \frac{x^2}{pR} + \frac{y^2}{pR}, \quad s_0 = \frac{d}{p}. \quad (2)$$

This dependence holds only within the geometrical lens aperture of diameter

$$a = 2[R(p-d)]^{1/2}$$

(see Fig. 1). We are interested in sufficiently long lenses where the effective working area (effective aperture) of the lens is determined by the absorption of X-rays in the lens material and has a size smaller than the geometrical aperture. In this case, we may ignore the edge effects and formally assume that dependence (2) holds in the entire region of the transverse $xy$ plane concerned.

After the substitution of $\bar{s}$ for $s$, the general solution of Eq. (1) can be written as the integral equation

$$A_t(x, y, z) = \int dx'dy'P_L^{(t)}(x, y, x', y', z)A_t(x', y', 0). \quad (3)$$

The propagator of a PCR lens, i.e., the kernel of Eq. (3), is the solution of Eq. (1) with the initial function

$$P_L^{(t)}(x, y, x', y', 0) = \delta(x - x')\delta(y - y'),$$

where $\delta(x)$ is the Dirac delta function. Given the form of the initial function, it is easy to understand that the solution can be sought by the separation of variables,

$$P_L^{(t)}(x, y, x', y', z) = \exp(-ik\eta s_0 z)P_L(x, x', z)P_L(y, y', z). \quad (4)$$

The partial propagator $P_L(x, x', z)$ satisfies the equation

$$\frac{dP_L}{dz} = -ik\frac{x^2}{2z_c^2}P_L + \frac{i}{2k}\frac{d^2P_L}{dx^2},$$
$$P_L(x, x', 0) = \delta(x - x'), \quad z_c = \left(\frac{pR}{2\eta}\right)^{1/2}. \quad (5)$$

This equation is formally identical to the Schrödinger equation for a particle in a parabolic potential if the $z$ coordinate is substituted with time. In quantum mechanics, one is usually interested in stationary states and writes the solution as a series each term of which contains the product of the functions of each individual coordinate.

In [16], we obtained a solution in the form of a simple analytic expression by using the Fourier transform and recurrent equations for the coefficients of various powers of $x$ in the argument of the exponent. Below, we derive the same solution in a more straightforward way. Taking into account the reciprocity principle, the solution should be sought in the form of a symmetric function of the $x$ and $x'$ coordinates. In addition, at small longitudinal distances, the solution must be close to the propagator in the empty space, i.e., to the Kirchhoff propagator,

$$P(x - x', z) = \frac{1}{(i\lambda z)^{1/2}}\exp\left[i\pi\frac{(x - x')^2}{\lambda z}\right], \quad (6)$$

which is significant when the lens is illuminated by a point source. On the other hand, it must contain the phase factor characteristic of a thin lens when it is illuminated by a plane wave. Taking into account these considerations, we will seek the solution in the form

$$P_L(x, x', z) = T\left(x, \frac{r}{a}\right)P(x - x', r)T\left(x', \frac{r}{a}\right),$$
$$T(x, z) = \exp\left(-i\pi\frac{x^2}{\lambda z}\right) \quad (7)$$

with the two unknown functions $r(z)$ and $a(z)$.

The initial condition is satisfied if $r(z) \approx z$ and $a(z) \approx z^2$ for $z \longrightarrow 0$. Substituting this form of the solution into the equation and equating the coefficients of the same powers of $x$ and $x'$, we obtain the system of two ordinary differential equations

$$\frac{dr}{dz} = 1 - a, \quad \frac{da}{dz} = \frac{r}{z_c^2}, \quad (8)$$

whose solution can be easily found:

$$r(z) = z_c\sin\frac{z}{z_c}, \quad a(z) = 1 - \frac{dr}{dz} = 1 - \cos\frac{z}{z_c}. \quad (9)$$

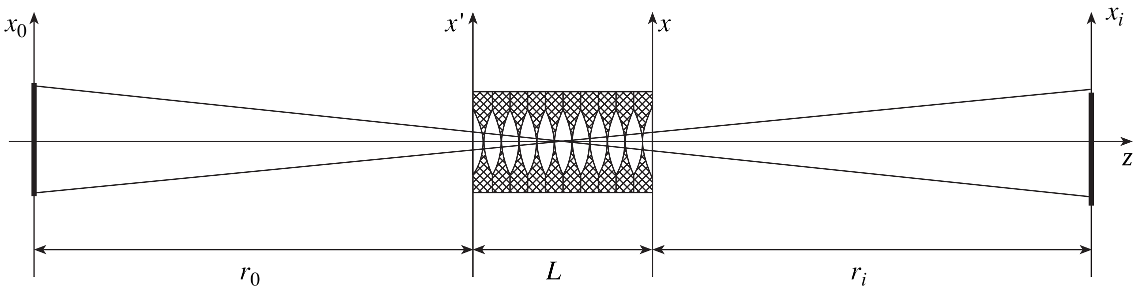


**Fig. 2.** The experimental scheme for imaging an object with an X-ray PCR lens.

As a result, the propagator of a PCR lens can be written as

$$P_L(x, x', z) = \frac{1}{(i\lambda z_c s_z)^{1/2}} \exp\left[i\pi \frac{(x^2 + x'^2)c_z - 2xx'}{\lambda z_c s_z}\right], \qquad (10)$$

where we denoted

$$s_z = \sin\frac{z}{z_c}, \quad c_z = \cos\frac{z}{z_c}.$$

This expression differs from the formula derived in [16] only by the notation. This notation better takes into account the symmetry properties of the propagator. If the PCR-lens length $L \ll \mathrm{Re} z_c$, then

$$P_L(x, x', L) \xrightarrow[L \to 0]{} \exp\left[-i\pi \frac{(x^2 + x'^2 + xx')}{3\lambda F_c}\right] P(x - x', L), \qquad (11)$$

where

$$F_c = \frac{F}{1 - i\gamma} = \frac{z_c^2}{L} = \frac{R}{2N\eta}$$

is the complex focal length of a thin PCR lens. Since we used the relation $F \gg L$ to derive this expression, we may roughly substitute $x$ for $x'$ in the exponential factor. In addition, if the second derivative of the incident wave phase with respect to the transverse coordinates is much smaller than $2\pi/\lambda L$, then the propagator of the empty space can be roughly substituted with the delta function to give

$$P_L(x, x', L) \approx T(x, F_c)\delta(x - x').$$

This expression is commonly used in the thin-lens approximation. Formula (11) gives a more universal approximation for the thin-lens propagator.

Clearly, a PCR lens for which the parameter $\gamma = \beta/\delta$ is at a minimum has the best properties. For this reason, the actual lenses are made of elements with a small atomic number $Z$ (lithium, beryllium, carbon, aluminum). In virtually all interesting cases, $\gamma < 0.005$. If we ignore the absorption, which can be done at least for the rays near the optical axis, then the waveguide properties of a PCR lens follow from the exact expression for the propagator. For

$$L = L_0 = \frac{\pi}{2}(LF)^{1/2} = \frac{\pi}{2}\left(\frac{pR}{2\delta}\right)^{1/2},$$

we obtain $s_z = 1$ and $c_z = 0$. Therefore, a PCR lens makes the Fourier transform of the incident wave. For $L = 2L_0$, the propagator is $\delta(x + x')$; accordingly, the PCR reproduces the incident wave at the exit in an inverse form. For $L = 3L_0$, the lens again makes the Fourier transform but with the opposite sign. Finally, for $L = 2L_0$, the lens faithfully reproduces the incident wave. As the PCR-lens length increases further, these phases are repeated again and again. Since X-rays are absorbed in the PCR lens, both the image and the Fourier transform are produced in a bounded region within the gradually decreasing aperture.

## 3. THE IMAGE PROPAGATOR WITH AN ARBITRARILY LONG PCR LENS

In an actual experiment, the object being studied, the lens, and the detector are at comparatively large distances from each other, as can be seen from Fig. 2, which also shows the notation for the distances and the coordinate axes. Let us consider the more complex problem of the wave field transfer from the plane immediately behind the object to the detector plane. Clearly, the propagator of this problem is also factorized in the $x$ and $y$ coordinates. Therefore, it will suffice to calculate only the partial image propagator $G(x_i, x_o, r_o, L, r_i)$. It is determined by the convolution of the propagator for a PCR lens with the Kirchhoff propagators that correspond to empty space,

$$G(x_i, x_o) = \int dx dx' P(x_i - x, r_i) P_L(x, x', L) P(x' - x_o, r_o). \qquad (12)$$

Below, to save space, we omit the longitudinal distances in the list of arguments for the image propagator.

We use the following algorithm to calculate the integrals. Let us first consider the extreme case of a thin lens where $P_L(x, x', L)$ is substituted with $T(x, F_c)\delta(x - x')$. Accordingly, the propagator is determined by the single integral

$$G_0(x_i, x_o) = \int dx P(x_i - x, r_i) T(x, F_c) P(x - x_o, r_o). \tag{13}$$

This integral reduces to the Fourier transform of the Gaussian function and is again equal to the Gaussian function. It is convenient to write the result as

$$G_0(x_i, x_o) = T\left(x_i, \frac{r_g}{a_i}\right) P(x_i - x_o, r_g) T\left(x_o, \frac{r_g}{a_o}\right), \tag{14}$$

where we introduced the parameters

$$r_g = r_o + r_i - \frac{r_o r_i}{F_c}, \quad a_i = \frac{r_o}{F_c}, \quad a_o = \frac{r_i}{F_c}. \tag{15}$$

Expression (14) for the image propagator of a thin lens is identical in form to expression (7) for the propagator of a PCR lens. It immediately follows from this expression that in the limit $\gamma = 0$ and when the condition $r_g = 0$ (the thin-lens formula) is satisfied, the propagator

$$G_0(x_i, x_o) = M^{1/2}\delta(x_i + x_o M),$$

where $M = r_i/r_o$ is the magnification factor. Thus, this expression reproduces the well-known property of a thin lens to focus the image when the lens formula holds:

$$r_o^{-1} + r_i^{-1} = F^{-1}.$$

It is also easy to see that when the condition $a_o = 1$, i.e., $r_i = F$, is satisfied, the term proportional to $x_o^2$ in the argument of the exponent vanishes and the propagator makes the Fourier transform of the wave field located in the plane immediately behind the object. However, if the object is illuminated by a point source, then the Fourier transform of the transmission function for the object takes place in the focusing plane of the point source. This property also follows from propagator (14), but further calculations are required to prove it (see below).

Substituting expression (7) for the propagator of a PCR lens into integral (12) yields

$$G(x_i, x_o) = \int dx P(x_i - x, r_i) T\left(x, \frac{r_L}{a_L}\right) \times \int dx' P(x - x', r_L) T\left(x', \frac{r_L}{a_L}\right) P(x' - x_o, r_o), \tag{16}$$

where $r_L = z_c s_L$ and $a_L = 1 - c_L$. Here, the integral over $x'$ is equivalent to integral (13) but with different expressions for the parameters. Substituting solution (14) for the integral again yields an integral of type (13) with new parameters. As a result, making appropriate algebraic transformations, we derive an exact expression for the image propagator of an arbitrarily long PCR lens in a form similar to the case of a thin lens,

$$G(x_i, x_o) = T\left(x_i, \frac{\tilde{r}_g}{\tilde{a}_i}\right) P(x_i - x_o, \tilde{r}_g) T\left(x_o, \frac{\tilde{r}_g}{\tilde{a}_o}\right), \tag{17}$$

but now the parameters are

$$\tilde{r}_g = (r_o + r_i)c_L + \left(z_c - \frac{r_o r_i}{z_c}\right)s_L,$$
$$\tilde{a}_i = 1 - c_L + \frac{r_o}{z_c}s_L, \quad \tilde{a}_o = 1 - c_L + \frac{r_i}{z_c}s_L. \tag{18}$$

Interestingly, the formulas for the new parameters can also be written in a form similar to the case of a thin lens,

$$\tilde{r}_g = \tilde{r}_o + \tilde{r}_i - \frac{\tilde{r}_o \tilde{r}_i}{\tilde{F}_c}, \quad \tilde{a}_i = \frac{\tilde{r}_o}{\tilde{F}_c}, \quad \tilde{a}_o = \frac{\tilde{r}_i}{\tilde{F}_c}, \tag{19}$$

if we introduce the generalized complex distances

$$\tilde{r}_o = r_o + b_L, \quad \tilde{r}_i = r_i + b_L,$$
$$\tilde{F}_c = \frac{z_c}{s_L}, \quad b_L = \tilde{F}_c(1 - c_L). \tag{20}$$

The result is of great importance because it shows that the imaging properties of an arbitrarily long PCR lens are essentially the same as those of a thin lens. In particular, such a PCR lens has no aberrations in the sense that all ray paths converge at a single point, in contrast, for example, to a lens with a spherical profile. If there were no absorption, then the lens would focus a point source to a point. The blurring of the image point due to the absorption of radiation and the finite aperture is usually attributed to the finite lens resolution. On the other hand, the simple analytic expressions for the generalized distances allow the appropriate corrections that should be made to the experimental scheme to be easily determined.

For example, in the limit of a small lens length compared to the focal length, $L \ll F$, expanding the sine and the cosine in a power series yields

$$\tilde{r}_o = r_o + \frac{1}{2}L, \quad \tilde{r}_i = r_i + \frac{1}{2}L, \quad \tilde{F}_c = F_c + \frac{1}{6}L. \tag{21}$$

This result, which was obtained previously [18] in an approximate and complicated way, is a natural extreme case of the exact theory. It follows from this result that even when the PCR lens has an appreciable length $L$ (tens of centimeters) that satisfies the condition $L \ll F$, it can be treated as a thin lens located in the middle with a sole difference. More specifically, the focal length calculated using the thin-lens formula must be increased by one-sixth of the actual lens length.

On the other hand, at zero distances, the image propagator is identical to the propagator of a PCR lens. Note yet another obvious property of the image propagator: its convolution with the Kirchhoff propagator from a point source $P(x_o - x_s, r_s)$ is described by the same expression (17) in which $x_s$ should be substituted for $x_0$ and $r_o + r_s$ should be substituted for $r_o$. The relation

$$\tilde{a}_o + \tilde{a}_i - \tilde{a}_o\tilde{a}_i = \frac{\tilde{r}_g}{\tilde{F}_c} \quad (22)$$

that follows from definitions (19) is used to prove this property.

## 4. ESTIMATING THE APERTURE AND FOCUS SIZES FOR A PARABOLIC CONTINUOUSLY REFRACTIVE X-RAY LENS

Exact knowledge of such parameters as the size of the effective aperture of an X-ray lens and the size of the focal spot when a plane wave is focused is of considerable practical importance. In the optics of visible light, the aperture is determined by the geometrical sizes of the lens, i.e., by the area through which the rays passing then converge to a focus. For a thin absorbing X-ray lens, it will suffice to consider the intensity distribution of the radiation immediately after the lens when it is illuminated by a plane wave oriented along the optical axis. In this case, the effective aperture is determined by the absorption of radiation in the lens material. Since the total wave intensity in empty space is conserved, simple energy relationships exist between the aperture and focus sizes. For the arbitrarily long lens considered here, this approach does not work, because the incident wave can be partially or completely focused in the lens itself. Therefore, the intensity distribution of the radiation immediately behind the lens does not give us any idea of the actual lens aperture. The aperture of a long PCR lens can be defined in terms of the properties of the propagator $G(x_i, x_o)$. Let us consider a different, simpler approach based on energy considerations. By the effective lens aperture we mean the total intensity of the radiation at the focus that is equal to the intensity of the plane wave that passed through the lens without being absorbed. In this case, we disregard the parasitic absorption in the thin parts of the elementary lenses of thickness $d$ (see Fig. 1).

Below, we restrict our analysis to the case where the lens length is

$$L \le L_0 = \frac{\pi}{2}L_c, \quad L_c = (LF)^{1/2} = \left(\frac{pR}{2\delta}\right)^{1/2}.$$

In this range of lengths, the lens focuses the incident wave in space at a distance $r_i > 0$. The intensity distribution of the radiation in the space behind the lens when it is illuminated by a plane wave can be analyzed to determine the focus sizes. The two transverse coordinates are again factorized, and it will suffice to consider the distribution along the $x$ axis. Clearly, the wave amplitude can be obtained by calculating the convolution of propagator (17) with a coordinate-independent unit function, i.e., by integrating over the $x_o$ coordinate. The integral reduces to the Fourier transform of the Gaussian function, and it can be calculated exactly. As a result, using relation (22) we obtain

$$\begin{aligned} A(x_i) &= \int dx_o G(x_i, x_o) \\ &= \left(\frac{\tilde{F}_c}{\tilde{F}_c - \tilde{r}_i}\right)^{1/2} \exp\left[-i\pi\frac{x_i^2}{\lambda(\tilde{F}_c - \tilde{r}_i)}\right] \\ &= \left(\frac{z_c}{z_c c_L - r_i s_L}\right)^{1/2} \exp\left[-i\pi\frac{x_i^2 s_L}{\lambda(z_c c_L - r_i s_L)}\right]. \end{aligned} \quad (23)$$

Note that the result does not depend on the distance $r_o$. From a physical point of view, it is clear that this expression can also be derived directly from the image propagator (17) by considering a point near the optical axis at an infinite distance $r_o$ and dividing it by the amplitude of the Kirchhoff propagator at the same distance, because a point source at an infinite distance gives a plane wave in front of the lens aperture.

As follows from (23), a plane wave in front of the lens transforms into a Gaussian wave behind the lens at all distances from the lens. For an arbitrarily long PCR X-ray lens, all parameters are complex. Since the absorption parameter $\gamma \ll 1$, we use the linear (in $\gamma$) approximation for qualitative estimation. With the adopted constraint on the lens length, we obtain the relations

$$\begin{aligned} s_L &\approx S_L - i\frac{\gamma}{2}u_L C_L, \quad c_L \approx C_L + i\frac{\gamma}{2}u_L S_L, \\ z_c &= L_c\left(1 + i\frac{\gamma}{2}\right), \end{aligned} \quad (24)$$

where we introduced the real functions

$$S_L = \sin u_L, \quad C_L = \cos u_L, \quad u_L = L/L_c.$$

The intensity reaches a maximum at the distance $r_i$ behind the lens that satisfies the condition

$$\mathrm{Re}(z_c c_L - r_i s_L) \approx L_c C_L - r_i S_L = 0,$$

i.e., $r_i = F_L C_L$. In this case,

$$|A(x_i)|^2 = \frac{1}{\gamma\alpha_L}\exp\left(-\frac{2\pi}{\gamma\lambda F_L \alpha_L}x_i^2\right),$$
$$\alpha_L = \frac{1}{2}\left(C_L + \frac{u_L}{S_L}\right), \quad F_L = \frac{L_c}{S_L}. \tag{25}$$

Using the derived expression, we obtain

$$A_\gamma = \int dx_i |A(x_i)|^2 = \left(\frac{\lambda F_L}{2\gamma\alpha_L}\right)^{1/2} \tag{26}$$

for the total intensity of the focus (effective aperture) and

$$s_\gamma = 0.664(\gamma\lambda F_L \alpha_L)^{1/2} = 0.47\frac{\lambda F_L}{A_\gamma} \tag{27}$$

for the full width at half maximum (halfwidth) of the intensity peak at the focus.

For a thin lens, $u_L \ll 1$, it follows from these relations that

$$r_i = F, \quad F = \frac{L_c^2}{L}, \quad A_\gamma = 0.707\left(\frac{\lambda F}{\gamma}\right)^{1/2},$$
$$s_\gamma = 0.664(\gamma\lambda F)^{1/2} = 0.47\frac{\lambda F}{A_\gamma}. \tag{28}$$

In the other extreme case, $u_L = \pi/2$, we obtain

$$r_i = 0, \quad A_\gamma = 0.798\left(\frac{\lambda L_c}{\gamma}\right)^{1/2},$$
$$s_\gamma = 0.47(\gamma\lambda L)^{1/2} = 0.47\frac{\lambda L_c}{A_\gamma}. \tag{29}$$

Thus, defining the effective aperture in terms of the total intensity of the focus does not lead to any contradictions. For a self-focusing lens, the generalized focal length is by a factor of 1.57 smaller than the length of the lens itself. Clearly, this is the minimum focal length that can be obtained for a lens with a specified radius of curvature $R$, thickness of the elementary lens $p$, and decrement of the refractive index $\delta$. It follows from the derived relations that the linear size of the focus is approximately by a factor of $0.8\gamma$ smaller than the linear size of the effective aperture for all lens lengths, with the numerical coefficient depending only slightly on the lens length. Consequently, as the lens length increases, the sizes of the focus and the effective aperture decrease proportionally to each other and the degree of beam compression depends only on the absorption factor $\gamma$.

To conclude this section, we give several numerical values for the parameters of Lengeler aluminum lenses [9–12] at a photon energy of 25 keV. These lenses have the following parameters: $p = 1$ mm, $R = 0.2$ mm, $\delta = 8.643 \times 10^{-7}$, $\gamma = 2.02 \times 10^{-3}$, and $L_c = 34$ cm. For the lens of 100 elements that was actually used in experiments, $L = 10$ cm, $u_L = 0.294$, $F_L = 117.4$ cm, and $\alpha_L = 0.986$. In this case, the effective aperture is $A_\gamma = 120$ μm and the lens resolution is $s_\gamma = 0.23$ μm. These parameters smoothly decrease with the increasing number of elements. Thus, for a lens of 300 elements, it can be easily calculated that $L = 30$ cm, $u_L = 0.882$, $F_L = 44$ cm, and $\alpha_L = 0.889$. In this case, the effective aperture is $A_\gamma = 78$ μm and the lens resolution is $s_\gamma = 0.13$ μm. Although the aperture of a long lens decreases, it has a better resolution and can be useful in imaging small objects or their fragments.

## 5. THE IMAGE OF A POINT SOURCE

Let us consider the imaging properties of an X-ray PCR lens that follow from propagator (17). In the linear (in small parameter $\gamma$) approximation, the image of a point source displaced by $x_o$ from the optical axis is focused at the distances that satisfy the condition

$$\mathrm{Re}(\tilde{r}_g) = (r_i + r_o)C_L + \left(L_c - \frac{r_i r_o}{L_c}\right)S_L$$
$$= r_{iL} + r_{oL} - \frac{r_{iL} r_{oL}}{F_L} = 0, \tag{30}$$

where $r_{oL} = r_o + B_L$, $r_{iL} = r_i + B_L$, $B_L = F_L(1 - C_L)$, and $F_L$ is defined in (25). Using the generalized real distances, this condition can be written as the thin-lens formula

$$r_{iL}^{-1} + r_{oL}^{-1} = F_L^{-1}.$$

When condition (30) is satisfied, the parameter $\tilde{r}_g$ becomes purely imaginary, and for the parameters $\tilde{a}_i$ and $\tilde{a}_o$, it will suffice to use only the real part, i.e., to set $\gamma = 0$. In this approximation, the propagator takes the form

$$G(x_i, x_o) = -iM^{1/2}\delta_\sigma(x_i + x_o M), \tag{31}$$

where

$$\delta_\sigma(x) = \frac{1}{\sigma(2\pi)^{1/2}}\exp\left(-\frac{x^2}{2\sigma^2}\right),$$
$$\sigma = \left(\frac{\lambda M}{2\pi}\mathrm{Im}\tilde{r}_g\right)^{1/2}, \quad M = \frac{r_{iL}}{r_{oL}}. \tag{32}$$

Thus, the propagator is a Gaussian function with the maximum centered at $x_i = -x_o M$. In the general case of a long X-ray PCR lens, the magnification factor can differ markedly from the ordinary magnification factor of a thin lens proportional to the ratio $r_i/r_o$. Accordingly, the image intensity peak is described by the function

$$I_{ps}(x_i, x_o) = |G(x_i, x_o)|^2 = \frac{M}{2\pi\sigma^2}\exp\left[-\frac{(x_i + x_o M)^2}{\sigma^2}\right] \quad (33)$$

for which $s_\gamma = 1.665\sigma$. The expression for $\mathrm{Im}\tilde{r}_g$ under the image focusing conditions is simplest in terms of the ordinary distances:

$$\mathrm{Im}\tilde{r}_g = \gamma[L_c S_L + (r_i + r_o)\alpha_L].$$

In terms of the generalized distances, the peak width parameter is

$$\sigma = \left(\gamma\frac{\lambda\alpha_L r_{iL}^2}{2\pi F_L}K_L\right)^{1/2} = \frac{\lambda r_{iL}}{A_\gamma}\left(\frac{K_L}{4\pi}\right)^{1/2},$$
$$K_L = 1 - \frac{2B_L F_L}{r_{iL} r_{oL}}\left(1 - \frac{1 + C_L}{2\alpha_L}\right), \quad (34)$$

where $A_\gamma$ is the lens aperture defined in (26).

Let us again consider the extreme cases. For a thin lens where $L \ll F$, the focusing condition is $r_i^{-1} + r_o^{-1} = F^{-1}$, the magnification factor is $M = r_i/r_o$, and $s_\gamma = 0.47\lambda r_i/A_\gamma$. For a self-focusing lens with a length $L = (\pi/2)L_c$, the focusing condition, the magnification, factor, and the halfwidth are given by the expressions

$$r_o = \frac{L_c^2}{r_i}, \quad M = \frac{1 + h_i}{1 + h_i^{-1}},$$
$$s_\gamma = 0.47\frac{\lambda L_c}{A_\gamma}(1 + h_i)\left(1 - \frac{0.726}{2 + h_i + h_i^{-1}}\right), \quad (35)$$
$$h_i = \frac{r_i}{L_c}.$$

In the limit $r_o \longrightarrow \infty$, we again obtain (28). On the other hand, in the limit $r_i \longrightarrow \infty$, both the magnification factor and the image halfwidth indefinitely increase in the two cases, but the ratio of the size of the image of a point to the size of the entire pattern is virtually constant.

Note the following interesting feature of long lenses. As was shown above, a self-imaging lens has a length $L = \pi L_c = 3.14L_c$. On the other hand, a self-focusing lens is by a factor of 2 shorter, while the length of the experimental scheme for imaging without magnification is $(2 + \pi/2)L_c = 3.57L_c$, which is only slightly longer. For this reason, there is little point in using extremely long lenses.

## 6. THE FOCUSED IMAGE OF AN OBJECT

In an actual X-ray experiment, a relatively thin object illuminated by the wave emitted by a point source located at a distance $r_s$ from the object and having transverse coordinates $x_s$ and $y_s$ is imaged with the lens. In general, the source has finite sizes, but different points of the source are incoherent. Therefore, the intensity should be integrated over the source's coordinates at the final stage of the calculation. In this section, we restrict our analysis to the case where the distance from the source to the object is large and the angular sizes of the source in the object's plane do not exceed the object's characteristic scattering angles. In other words, the coherent image conditions are satisfied. The wave field in the object's image plane referred to the amplitude of the wave incident on the object can be calculated by using the integral

$$A^{(t)}(x_i, y_i) = i\lambda r_s\int dx_0 dy_0 G_s^{(t)}(x_i, y_i, x_o, y_o)T(x_o, y_o), \quad (36)$$

where

$$G_s^{(t)}(x_i, y_i, x_o, y_o) = \exp(-ik\eta s_0 L)G_s(x, x_o)G_s(y_i, y_o), \quad (37)$$

the function

$$G_s(x_i, x_o) = G(x_i, x_o)P(x_o - x_s, r_s)$$

is the partial image propagator for the object, and the function

$$T(x_o, y_o) = \exp[i\psi(x_o, y_o)]$$

describes the transfer of radiation through the object, i.e., the object's function.

As we have shown above, under the image focusing conditions for the object's points (30), the modulus of $G_s^{(t)}(x_i, y_i, x_o, y_o)$, which is considered as a function of the $x_o$ and $y_o$ coordinates at a given point $(x_i, y_i)$ on the image plane, has a sharp maximum at the point with $(x_{oi}, y_{oi})$ coordinates, where $x_{oi} = -x_i/M$ and $y_{oi} = -y_i/M$. Here, as in the preceding section, $M = r_{iL}/r_{oL}$. Let us assume that the complex phase of the object's function is a smooth function within the region of the propagator maximum. In the effective domain of integration, the object's function can then be approximated by the expression

$$T(x_o, y_o) = \exp\left[i\psi_i + i\frac{2\pi}{\lambda}[\xi_{ix}(x_o - x_{oi}) + \xi_{iy}(y_o - y_{oi})]\right] \quad (38)$$

with the complex parameters

$$\psi_i = \psi(x_{oi}, y_{oi}), \quad \xi_{ix} = \frac{\lambda}{2\pi}\frac{d\psi(x_{oi}, y_{oi})}{dx},$$

$$\xi_{iy} = \frac{\lambda}{2\pi}\frac{d\psi(x_{oi}, y_{oi})}{dy},$$

which depend on the coordinates of the image point.

In this case, the wave field of the image is again factorized,

$$A^{(t)}(x_i, y_i) = \exp(-ik\eta s_0 L + i\psi_i) \times A^{(x)}(x_i, y_i)A^{(y)}(x_i, y_i), \tag{39}$$

and, for example,

$$A^{(x)}(x_i, y_i) = (i\lambda r_s)^{1/2}\int dx_o G(x_i, x_o)P(x_o - x_s, r_s) \times \exp\left[i\frac{2\pi}{\lambda}\xi_{ix}(x_o - x_{oi})\right]. \tag{40}$$

The function $A^{(x)}(x_i, y_i)$ depends on $y_i$ parametrically via $\xi_{ix}$. Approximation (31) cannot be directly used to calculate the integral, because the expansion in terms of powers of $\gamma$ has already been made in it and, therefore, it is not accurate enough. The expansion in terms of powers of $\gamma$ can be made only in the final expressions. On the other hand, an exact result can be easily obtained from the following considerations.

The integral differs only by the phase factor from the convolution of the image propagator with the Kirchhoff propagator if we substitute $\tilde{x}_s = x_s - r_s\xi_{ix}$ for the true coordinate of the source $x_s$ in the latter. As was noted above, the convolution of the image propagator with the Kirchhoff propagator is again equal to the image propagator in which we should substitute $r_o + r_s$ for the distance $r_o$ and $\tilde{x}_s$ for the coordinate $x_o$. In the expression derived, we should use the image focusing condition

$$r_{iL}^{-1} + r_{oL}^{-1} = F_L^{-1}$$

and we can set $\gamma = 0$ in the preexponential term. As regards the exponent, it will suffice to include the terms of the zero and first powers of $\gamma$. Although the algorithm is simple, the calculations are cumbersome because they contain many combinations of many parameters. To obtain an unequivocal result, it is convenient to choose the coordinate of the imaged point on the object, $x_{oi}$, and the angle of ray exit from the object at this point, $\theta_o$, calculated via the phase gradient as independent transverse coordinates. A useful parameter is also the coordinate of ray entrance into the lens, $x_0$. These quantities are defined as

$$x_{oi} = -\frac{x_i}{M}, \quad \theta_o = \frac{x_{oi} - \tilde{x}_s}{r_s} = \xi_{ix} + \frac{x_{oi} - x_s}{r_s}, \quad x_0 = x_{oi} + r_o\theta_o. \tag{41}$$

It is convenient to choose $r_o$ and $L_c$ as independent longitudinal distances. The remaining parameters are expressed in terms of them as

$$r_i = Mu, \quad M = \frac{L_c}{v}, \quad u = r_oC_L + L_cS_L, \quad v = r_oS_L - L_cC_L, \quad F_L = \frac{L_c}{S_L}. \tag{42}$$

Without giving the intermediate calculations, we immediately write out the result in the linear (in $\gamma$) approximation

$$A^{(x)}(x_i, y_i) = \frac{1}{iM^{1/2}}\exp\left[i\Phi_x(x_i) - \frac{1}{2}\mu_x(x_i, y_i)\right],$$

$$\Phi_x(x_i) = \frac{\pi}{\lambda}\left[\frac{(x_{oi} - x_s)^2}{r_s} + \frac{x_{oi}^2}{r_{oL}} + \frac{x_i^2}{r_{iL}}\right], \tag{43}$$

$$\mu_x(x_i, y_i) = \gamma\frac{2\pi}{\lambda F_L} \times [\alpha_L x_0^2 + (\alpha_L - C_L)(L_c\theta_o)^2 + S_L x_0 L_c\theta_o].$$

When making the expansion in terms of $\gamma$, we assumed the conditions $\lambda r_s \gg 2\pi\sigma_o^2$, where $\sigma_o = \sigma/M$ is the width of the maximum of propagator (31) relative to the integration variable $x_o$, to be satisfied. In other words, we assumed that the phase of the incident spherical wave also smoothly varied in the domain of integration. We can easily consider the general case, but it is of no practical interest. As follows from the derived expression, the phase gradient of the object's function directly affects the image intensity. To qualitatively analyze this effect, let us consider the extreme case of an incident plane wave ($r_s \longrightarrow \infty$) and assume that the object is transparent. If the sample is homogeneous, then $\theta_o = 0$ at all points and the intensity of the radiation in the image plane is described by the Gaussian function

$$I_{im}(x_i) = \frac{1}{M}\exp\left(-\gamma\frac{2\pi}{\lambda F_L}\frac{\alpha_L}{M^2}x_i^2\right). \tag{44}$$

If, however, a phase gradient exists at some points of the sample, then $\theta_o = \xi_{ix}$ at these points and the intensity will deviate from law (44); depending on different conditions, the intensity can be lower and higher than the background intensity. Thus, a unique opportunity to visualize transparent objects and to obtain the phase contrast through the nonuniform absorption of radiation in the lens emerges. Unlike the ordinary phase contrast observed on synchrotron radiation sources (see, e.g., [19–21]), this contrast has no parasitic oscillations associated with the interference of various rays and directly allows the local phase gradient produced by the object to be determined. Abrupt changes in intensity related to a sharp phase gradient have recently been observed experimentally in the image of a sample with a profiled surface [22].

The physical nature of the visualization of the phase gradient in a sample with an X-ray PCR lens is easiest to understand in terms of geometrical optics. The ray from a source with a coordinate $x_s$ comes to the sample at a point with a coordinate $x_{oi}$. Since the lens formula holds, all of the rays that emerge from this point at different angles reach a point with a coordinate $x_i$ in the image plane after their passage through the lens. Actually, however, only one path is realized and the ray leaves the sample, making an angle $\theta_o$ with the optical ($z$) axis. Accordingly, the ray in front of the lens has the coordinate $x_0$ and angle $\theta_o$. The ray path $x = x_t(z)$ in the lens satisfies the condition under which at each point of the path its tangent makes an angle $\theta = (\lambda/2\pi)(d\varphi/dx)$ with the $z$ axis, where $\varphi(x, z)$ is the phase of the wave field in the lens. This phase is equal to the phase of the image propagator (17) if we set $\gamma = 0$, $r_i = 0$, and $x_i = x_t(z)$ in the latter and substitute $z$ for $L$. On the other hand, $\theta = dx_t/dz$. The equation for the path is particularly simple if we write it via the derivative of $\theta$. As a result,

$$\frac{d\theta}{dz} = \frac{d^2x_t}{dz^2} = -\frac{x_t(z)}{L_c^2}, \quad x_t(z) = x_0C_z + \theta_oL_cS_z, \quad (45)$$

where $x_0$ and $\theta_o$ are the initial point and initial derivative on the path. The initial coordinate and initial angle of the ray as it enters the lens correspond to the parameters defined by (41). Calculating the absorption coefficient integrated over the ray path in the lens,

$$\mu(L) = \gamma\frac{2\pi}{\lambda L_c^2}\int_0^L dz\left(x_0\cos\frac{z}{L_c} + \theta_oL_c\sin\frac{z}{L_c}\right)^2, \quad (46)$$

we obtain a result that matches formula (43). Thus, the contrast is actually related to the change in the ray path and the nonuniform absorption in a PCR lens.

Naturally, the overall image of an object contains both the absorptive and phase contrast and depends on the two components of the total phase gradient along the two coordinate axes. Unfortunately, in solving the inverse problem, the change in intensity alone is not enough to restore the absorption coefficient and the two components of the phase gradient. Additional information can be obtained by moving the object relative to the lens, because different portions of the lens absorb differently.

## 7. THE FOURIER TRANSFORM OF AN OBJECT WITH A PARABOLIC CONTINUOUSLY REFRACTIVE X-RAY LENS

A thin parabolic lens is known from classical optics to perform the Fourier transform of the function of an object illuminated by a spherical wave from a point source in the focusing plane of the point source. Let us consider this phenomenon for an X-ray PCR lens. We write the Fourier transform of the object's function as

$$T(x_o, y_o) = \int\frac{dqdp}{(2\pi)^2}\tilde{T}(q, p)\exp(iqx_o + ipy_o). \quad (47)$$

In this case, the amplitude of the wave field in the image plane is

$$A^{(t)}(x_i, y_i) = \exp(-ik\eta s_0L) \times\int\frac{dqdp}{(2\pi)^2}\tilde{T}(q, p)G_f(q, x_s, x_i)G_f(p, y_s, y_i), \quad (48)$$

where

$$G_f(q, x_s, x_i) = \int dx_oG(x_i, x_o)P(x_o - x_s, r_s)\exp(iqx_o) \quad (49)$$

is the partial image propagator for the separate component of the Fourier transform of the object's function. We are interested in the conditions when this propagator is closest to the delta function.

As was noted in the preceding section, the integral can be expressed in terms of the image propagator in which $\tilde{x}_s = x_s - x_q$, $x_q = q\lambda r_s/2\pi$ is substituted for $x_s$ and $\tilde{r}_s = r_o + r_s$ is substituted for $r_o$. As a result,

$$G_f(q, x_s, x_i) = \frac{1}{(i\lambda\hat{r}_g)^{1/2}}\exp\left[iq\left(x_s - \frac{1}{2}x_q\right)\right] \times\exp\left[i\frac{\pi}{\lambda\hat{r}_g}(g_ix_i^2 - 2x_i\tilde{x}_s + g_s\tilde{x}_s^2)\right], \quad (50)$$

where

$$\hat{r}_g = (\tilde{r}_s + r_i)c_L + \left(z_c - \frac{\tilde{r}_sr_i}{z_c}\right)s_L, \quad g_i = c_L - \frac{\tilde{r}_s}{z_c}s_L, \quad g_s = c_L - \frac{r_i}{z_c}s_L. \quad (51)$$

Thus, the condition for the Fourier transform is identical to the focusing condition for a point source illuminating the object, i.e., $\mathrm{Re}\hat{r}_g = 0$. However, approximate expression (31) cannot be used directly, because the propagator is in the integrand and, hence, the omitted phase factor can play a significant role.

To derive an approximate expression, we represent the complex coefficients as a series in powers of $i\gamma$:

$$\hat{r}_g = R_0 + i\gamma R_1 + (i\gamma)^2 R_2, \quad g_i = Q_{i0} + i\gamma Q_{i1}, \quad g_s = Q_{s0} + i\gamma Q_{s1}. \tag{52}$$

The focusing condition for a point source then has the approximate form $R_0 = 0$. In this case, the following relations hold:

$$r_{iL}^{-1} + r_{sL}^{-1} = F_L^{-1}, \quad Q_{i0} = -\frac{1}{M_s}, \quad Q_{s0} = M_s,$$

where

$$r_{sL} = \tilde{r}_s + B_L = r_{oL} + r_s, \quad M_s = \frac{r_{iL}}{r_{sL}},$$

and the propagator is defined as

$$G_f(q, x_s, x_i) = \frac{1}{i(\gamma\lambda R_1)^{1/2}} \exp[i\varphi(q) - \mu(q)], \tag{53}$$

where

$$\mu(q) = \frac{\pi(x_i + M_s\tilde{x}_s)^2}{\gamma\lambda R_1 M_s},$$
$$\varphi(q) = q\left(x_s - \frac{1}{2}x_q\right) + \mu(q)\gamma\frac{R_2}{R_1} + \frac{Q_{i1}x_i^2 + Q_{s1}\tilde{x}_s^2}{\lambda R_1}. \tag{54}$$

Thus, each point $x_i$ in the source's image plane can have an appreciable intensity. It maps a region in the $q$ space of the sample's function centered at point $q = q_i$, where

$$q_i = \frac{2\pi}{\lambda r_s M_s}(x_i - x_s M_s). \tag{55}$$

In addition, because of the absorption in the lens, the Fourier transform of the sample's function is modified by the phase factor; i.e., the convolution of the Fourier transform of the sample's function with some function that depends on the parameters of the lens and the experimental scheme is actually imaged. Nevertheless, if the sample's function is periodic and, hence, has a discrete series of Fourier harmonics whose separation exceeds the width of the propagator maximum, then the image is a system of spots that correspond to individual harmonics of the Fourier transform of the source's function and the size of each spot closely corresponds to the projected size of the source. The separation between the spots depends on the distances used in the experiment and is equal to

$$\Delta x_i = \Delta q \frac{\lambda r_s M_s}{2\pi}.$$

A homogeneous sample is a special case for which only the zero Fourier harmonic exists.

An alternative approach to this problem consists in analyzing expression (36). The Fourier transform is obtained if the propagator $G(x_i, x_o)P(x_o - x_s, r_s)$ does not contain the term proportional to $x_o^2$ in the phase. This condition can be written as $\mathrm{Re}C = 0$, where

$$C = \frac{1 - \tilde{a}_o}{\tilde{r}_g} + \frac{1}{r_s} = 0.$$

It is easy to verify that in the zeroth (in $\gamma$) approximation, this condition is equivalent to the focusing condition for a point source written above. In this case, however, a damped exponential of the type $\exp(-\mathrm{Im}(C)x_o^2)$ remains under the integral. In addition, since $\tilde{r}_g$ is a complex quantity, the wave vector of the Fourier transform has a small imaginary part, which depends on the separation between the object and the lens. The manifestation of these features depends on specific conditions and analysis of them is a problem in itself.

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