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An Example of a Repeated Partnership Game with Discounting and with Uniformly Inefficient Equilibria

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In this note we present an example of a repeated partnership game with imperfect monitoring in which all supergame equilibria with positive discount rates are bounded away from full efficiency uniformly in the discount rate, provided the latter is strictly positive. On the other hand, if the players do not discount the future, then every efficient one-period payoff vector that dominates the one-period equilibrium payoff vector can be attained by an equilibrium of the repeated game. Thus the correspondence that maps the players' discount rate into the corresponding set of repeated-game equilibrium payoff vectors is discontinuous at the point at which the discount rate is zero.

INTRODUCTION[1]

In a repeated game in which after each repetition each player can observe the strategies used by the other players (perfect monitoring), there may be Nash equilibria of the repeated game—or supergame—that are more efficient (in the Pareto sense) than the Nash equilibria of the one-period game. In particular, if the players do not discount future utilities then, roughly speaking, the set of equilibrium payoff vectors for the supergame will be the same as the set of feasible and individually rational payoff vectors for the one-period game. (See Rubinstein (1979) for a discussion of alternative formulations of the "no discounting" case, and for further references.) If the players *do* discount future utility, then all supergame equilibria may be inefficient, although there will typically be supergame equilibria that are more efficient than any one-period equilibrium, provided the players' discount rates are not too small. However, in two-player games with perfect monitoring, the supergame equilibria will have the following Continuity Property: one can approximate any efficient, individually rational, payoff vector of the one-period game as closely as one likes with supergame equilibrium payoff vectors by taking the discount rates of the players sufficiently small but still positive.[1]

If one departs from the condition of perfect monitoring, then the situation is more complex, and has not yet been fully explored. For repeated principal-agent games with discounting, Radner (1985*a*) has shown that (under suitable regularity conditions), for every efficient behaviour that dominates a one-period Nash equilibrium, every positive epsilon, and every pair of discount factors (for the principal and agent) sufficiently close to unity, given epsilon, there exists a supergame equilibrium that is within epsilon, in normalized discounted expected utility for each player, of the target efficient behaviour. In particular, for every pair of discount factors above some critical level there exists a supergame equilibrium that is strictly more efficient than any inefficient one-period equilibrium.

In another class of supergames with imperfect monitoring, called partnership games, one can also attain efficiency with supergame equilibria, provided the players do not discount future utility, and a certain convexity condition is satisfied. (See the preceding article in this issue.) More precisely, every efficient one-period payoff vector that dominates the convex hull of the set of one-period equilibria can be attained by an equilibrium of the supergame (without discounting). On the other hand, in this note we present an example of a repeated partnership game in which all supergame equilibria with positive discount rates are bounded away from full efficiency *uniformly in the discount rates*, provided the latter are strictly positive. Fudenberg and Maskin (1985) show that this example is fully representative of the general case. Thus it would appear that, in the case of discounting, repeated partnership games do not have a Continuity Property like the one described above for principal-agent supergames. In Section 4 we attempt an informal explanation of this discrepancy. (The reader is referred to Radner (1985*b*, 1986*b*) for a comparison of these two classes of supergames in the context of a more general theory of decentralized economic organization.)

Since partnership games are described fully in the preceding article (Radner, 1986*a*), we shall not give a general definition here, but shall proceed immediately to the example that is the subject of this note.

1. THE ONE-PERIOD GAME

Two partners each contribute effort to an enterprise. The consequence of their combined effort is either success ($C=1$) or failure ($C=0$). Suppose that the partners choose their respective efforts simultaneously, and that neither partner can observe the other's effort. This lack of observability introduces an element of "moral hazard" into the situation, and motivates the assumption that the sharing of the consequence C depends only on C and not on the individual efforts. Assume that the consequence is shared equally between them (interpret the units of C as, say, thousands of dollars). Let a_1 and a_2 denote the respective (nonnegative) efforts of the two partners, and assume that

$$\begin{aligned}\text{Prob}\,(C=1) &= \min\,(a_1+a_2, 1) \\ &\equiv G(a_1, a_2),\end{aligned} \tag{1.}$$

and that the utility to partner i is

$$U_i = P\left(\frac{C}{2}\right) - qa_i^2, \tag{1.2}$$

where P is a "utility of money" and qa_i^2 represents a "disutility of effort" ($q>0$). By appropriate choice of origin and units in the measurement of utility, one can put (1.2) in the form

$$U_i = C - qa_i^2. \tag{1.3}$$

(Because of the rescaling, the parameter q in (1.3) will typically differ from the corresponding parameter in (1.2).) Partner i's expected utility is

$$W_i(a_1, a_2) = G(a_1, a_2) - qa_i^2. \tag{1.4}$$

The situation just described defines a two-person game with payoff functions W_1 and W_2, as in (1.4). The unique Nash equilibrium of this game is

$$a_1 = a_2 = a^* \equiv \frac{1}{2q}, \tag{1.5}$$

and yields each player the expected utility

$$u^* \equiv \frac{3}{4q}, \tag{1.6}$$

provided $q \geqq 1$. If $q > 1$ then the Nash equilibrium is not efficient (Pareto-optimal). For example, if $q > 2$ an efficient effort-pair that dominates the Nash equilibrium is

$$\begin{aligned} a_1 = a_2 = \hat{a} &\equiv \frac{1}{q}, \\ W_i(\hat{a}, \hat{a}) = \hat{u} &\equiv \frac{1}{q}. \end{aligned} \tag{1.7}$$

More generally, one can characterize the set $\hat{U}$ of efficient outcomes as follows. For every pair $\lambda = (\lambda_1, \lambda_2)$ of strictly positive numbers that sum to unity let $[\hat{u}_1(\lambda), \hat{u}_2(\lambda)]$ be the pair of expected utilities that maximizes $\lambda_1 u_1 + \lambda_2 u_2$ in the set of feasible nonnegative expected utility pairs; then the "efficiency frontier" $\hat{U}$ is the closure of the set of such pairs $[\hat{u}_1(\lambda), \hat{u}_2(\lambda)]$. One can verify that $\hat{U}$ is characterized parametrically (in λ) by the equations

$$\hat{u}_i(\lambda) = \frac{1}{2q\lambda_1\lambda_2} - \frac{1}{4q\lambda_i^2}, \qquad i = 1, 2, \tag{1.8}$$

with $\lambda_1\lambda_2 \geqq 1/2q$. (We omit the details.)

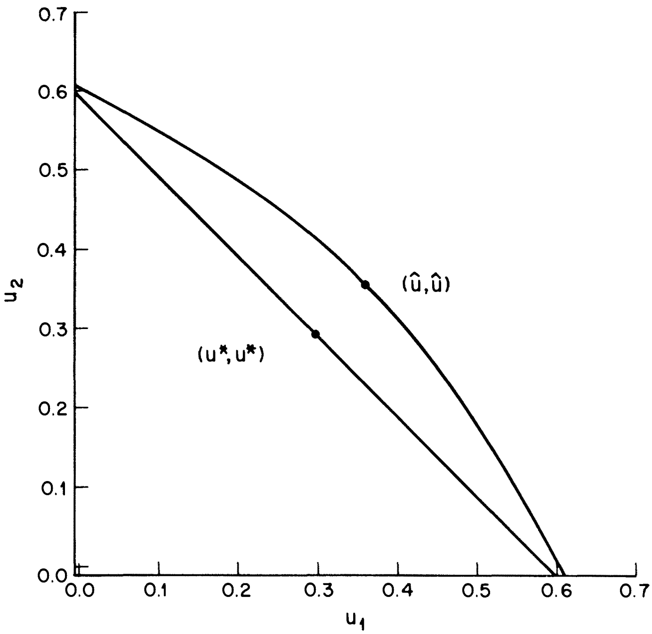
By putting in zero effort, each partner can guarantee himself an expected utility of at least zero; hence it would not be rational for a partner to accept a negative expected utility. One can verify that, if $q > \frac{9}{4}$, then the nonnegative expected-utility-pairs on the efficiency frontier are those pairs (1.8) for which $\lambda_1\lambda_2 \geqq \frac{2}{9}$. In what follows we suppose that $q > \frac{9}{4}$ and consider only the nonnegative part of the efficiency frontier, which is a curve in the (u_1, u_2)-plane, concave towards the origin (see Figure 1).

2. THE REPEATED GAME

At each of infinitely many dates t ($= 1, 2, \ldots,$ *ad infinitum*) the two partners play the game described in Section 1. The probability of success at date t depends only on the efforts at that date. At the beginning of game t, each partner knows only the history of his own previous efforts and the previous consequences, and he can choose his effort in game t as a function of that history.

Formally, let a_{1t}, a_{2t}, and C_t denote the efforts and consequence at date t, and let

$$H_{it} = (a_{i1}, C_1, \ldots, a_{it}, C_t), \qquad i = 1, 2, \qquad H_t = (H_{1t}, H_{2t}). \tag{2.1}$$



$q = 2.8$
Curved Line is efficiency frontier.
Equation of straight line is $u_1 + u_2 = 2(1 - \frac{q}{4})$

FIGURE 1

Make the convention that $H_{10} = H_{20} = H_0$ is some constant. A *strategy* for partner i is a sequence $\alpha_i = (\alpha_{it})$ of functions determining his successive efforts as follows:

$$a_{it} = \alpha_{it}(H_{i,t-1}), \qquad t \geqq 1. \tag{2.2}$$

Given the strategies of the two partners the stochastic process of consequences is determined by

$$\begin{aligned} \text{Prob}\,(C_t = 1 | H_{t-1}, a_{1t}, a_{2t}) &= G(a_{1t}, a_{2t}) \\ &= \min\,(a_{1t} + a_{2t}, 1). \end{aligned} \tag{2.3}$$

Let u_{it} denote partner i's expected utility in game t, i.e.,

$$u_{it} = E(C_t - qa_{it}^2). \tag{2.4}$$

(Note that for $t > 1$ the efforts are random variables.) The payoff for player i in the repeated game is defined to be the normalized sum of his discounted expected utilities, i.e.

$$V_i(\alpha_1, \alpha_2) = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} u_{it}. \tag{2.5}$$

(The sum of discounted expected utilities is normalized by multiplying by $(1 - \delta)$, so that if the terms u_{it} were all equal to u_i, the corresponding payoff would also be u_i.) To maintain the symmetry of the partners, the discount factor, δ, is assumed to be the same for both partners.

Equations (2.1)-(2.5) define a game in which the players' respective strategies are α_1 and α_2, and the corresponding payoff functions V_i are given by (2.5). Following standard terminology we shall call this the *supergame*, to distinguish it from the one-period game of Section 1.

3. AN UPPER BOUND ON THE EFFICIENCY OF SUPERGAME EQUILIBRIA

Let α_1 and α_2 be an arbitrary pair of pure supergame strategies, fixed for the time being. Let $X_i(\alpha_1, \alpha_2)$ denote i's normalized discounted *conditional* expected utility from period 2 on, given that there was a *success* in period 1, and let $Y_i(\alpha_1, \alpha_2)$ denote the corresponding conditional expectation, given that there was a failure in period 1. Finally, let a_i denote i's first-period effort, and let p denote the probability of a first-period success, i.e. $p = \min(a_1 + a_2, 1)$. By the Markovian recursion formula familiar from dynamic programming,[3]

$$V_i(\alpha_1, \alpha_2) = (1-\delta)(p - qa_i^2) + \delta[pX_i(\alpha_1, \alpha_2) + (1-p)Y_i(\alpha_1, \alpha_2)] \tag{3.1}$$

(cf. equations (1.3), (2.4), and (2.5)).
Define:

$$V(\alpha_1, \alpha_2) = (\tfrac{1}{2})[V_1(\alpha_1, \alpha_2) + V_2(\alpha_1, \alpha_2)],$$
$$X(\alpha_1, \alpha_2) = (\tfrac{1}{2})[X_1(\alpha_1, \alpha_2) + X_2(\alpha_1, \alpha_2)],$$
$$Y(\alpha_1, \alpha_2) = (\tfrac{1}{2})[Y_1(\alpha_1, \alpha_2) + Y_2(\alpha_1, \alpha_2)];$$

then corresponding to (3.1) one has

$$V(\alpha_1, \alpha_2) = (1-\delta)\left[p - q\left(\frac{a_1^2 + a_2^2}{2}\right)\right] + \delta[pX(\alpha_1, \alpha_2) + (1-p)Y(\alpha_1, \alpha_2)]. \tag{3.2}$$

Finally, let $\hat{V}$ be the supremum of all $V(\alpha_1, \alpha_2)$ such that (α_1, α_2) is a subgame-perfect equilibrium of the supergame. We shall derive an upper bound on $\hat{V}$ that is independent of the discount factor δ, provided that δ is strictly less than 1.

First, if (α_1, α_2) is an equilibrium of the supergame, then for each i, a_i maximizes V_i in (3.1), given $X_i(\alpha_1, \alpha_2)$ and $Y_i(\alpha_1, \alpha_2)$ and given a_j $(j \neq i)$; this is, of course, only a necessary condition for an equilibrium.

Second, if $(\hat{\alpha}_1, \hat{\alpha}_2)$ is a supergame equilibrium that achieves $\hat{V}$, and $0 < p < 1$ (cf. (3.1)), then

$$X(\hat{\alpha}_1, \hat{\alpha}_2) \leqq \hat{V},$$
$$Y(\hat{\alpha}_1, \hat{\alpha}_2) \leqq \hat{V}. \tag{3.3}$$

Suppose to the contrary that, e.g., $X(\hat{\alpha}_1, \hat{\alpha}_2) > \hat{V}$. The continuation of $(\hat{\alpha}_1, \hat{\alpha}_2)$ from period 2 on, given a success in period 1, is also an equilibrium of the original supergame (from period 1 on), since $p > 0$. Call this continuation (β_1, β_2); then[4]

$$V(\beta_1, \beta_2) = X(\hat{\alpha}_1, \hat{\alpha}_2) > \hat{V},$$

which contradicts the supposition that $(\hat{\alpha}_1, \hat{\alpha}_2)$ is a supergame equilibrium that maximizes $V(\alpha_1, \alpha_2)$. Thus we have established (3.3).

These two considerations motivate the specification of the following constrained maximization problem, whose variables are v_i, x_i, y_i, a_i $(i = 1, 2)$, v, x, y, and p: $\hat{v} \equiv$ the maximum of v subject to

$$v=\frac{v_1+v_2}{v}, \qquad x=\frac{x_1+x_2}{2}, \qquad y=\frac{y_1+y_2}{v}, \tag{3.4}$$

$$v_i=(1-\delta)(p-qa_i^2)+\delta[px_i+(1-p)y_i], \qquad i=1,2, \tag{3.5}$$

$$a_1\geqq 0, \qquad a_2\geqq 0, \qquad p\equiv a_1+a_2\leqq 1, \tag{3.6}$$

a_i maximizes v_i, given x_i, y_i and a_j, subject to (3.6), for $i=1,2$ and $j\neq i$, (3.7)

$$x\leqq v, \qquad y\leqq v. \tag{3.8}$$

The foregoing discussion shows that for any pure-strategy supergame equilibrium (α_1, α_2), the quantities $V_i(\alpha_1, \alpha_2)$, $X_i(\alpha_1, \alpha_2)$, $Y_i(\alpha_1, \alpha_2)$, a_i $(i=1,2)$, $V(\alpha_1, \alpha_2)$, etc., satisfy the constraints (3.4)-(3.7); furthermore, if $(\hat{\alpha}_1, \hat{\alpha}_2)$ is a supergame equilibrium that achieves $\hat{V}$ then $V(\hat{\alpha}_1, \hat{\alpha}_2)$, $X(\hat{\alpha}_1, \hat{\alpha}_2)$, and $Y(\hat{\alpha}_1, \hat{\alpha}_2)$ satisfy (3.8). Hence $\hat{v}$ is an upper bound for $\hat{V}$, i.e.

$$\hat{V}\leqq \hat{v}. \tag{3.9}$$

We shall show in the Appendix that

$$\hat{v}=\max\left(1-\frac{q}{4}, \frac{3}{4q}\right). \tag{3.10}$$

Note that this upper bound does not depend on the discount factor. Note, too, that $(1-q/4)$ is each partner's one-period expected utility when each partner's effort is $\frac{1}{2}$ (in which case the probability of success is one); recall that $u^*=(3/4q)$ is the one-period equilibrium expected utility for each player. Note that if $2<q<3$, then $\hat{v}=1-(q/4)$, and $\hat{v}<\hat{u}=1/q$.

Before proving (3.10) we shall now show that it implies that supergame equilibria are "uniformly inefficient" for a nondegenerate interval of values of the parameter q. The average expected utility in the one-period game, $(\frac{1}{2})(u_1+u_2)$, attains its minimum value in the efficiency frontier $\hat{U}$ at the end-points of $\hat{U}$; cf. equation (1.8). Thus, if $q>\frac{9}{4}$, this minimum value is $(27/32q)$, and is attained at the two points $(27/16q, 0)$ and $(0, 27/16q)$. On the other hand, if

$$2+(\tfrac{5}{8})^{1/2}<q<3, \tag{3.11}$$

then

$$\hat{V}\leqq\hat{v}=1-\frac{q}{4}<\frac{27}{32q}. \tag{3.12}$$

In summary, if (3.11) is satisfied, then for any discount factor δ strictly less than 1, and any pure-strategy supergame equilibrium (α_1, α_2),

$$(\tfrac{1}{2})[V_1(\alpha_1,, \alpha_2)+V_2(\alpha_1, \alpha_2)]\leqq 1-\frac{q}{4}<\frac{27}{32q}, \tag{3.13}$$

whereas for any efficient expected utility pair $(\hat{u}_1, \hat{u}_2)$ in the one-period game,

$$(\tfrac{1}{2})(\hat{u}_1+\hat{u}_2)\geqq\frac{27}{32q}. \tag{3.14}$$

In other words, *there is a line of the form* $u_1+u_2=c$ *that strictly separates the set of supergame equilibrium discounted expected utility pairs (for* $\delta<1$*) from the set of efficient one-period expected utility pairs.* The situation is depicted in Figure 1.

We note that the upper bound $(1-q/4)$ can actually be attained by a supergame equilibrium, with strategies of the following form. Each player puts in effort $\frac{1}{2}$ until the first failure, and then puts in effort a^* thereafter, forever. One can show that this pair of strategies is a subgame perfect supergame equilibrium, provided the discount factor is sufficiently close to one.

Finally, it remains to extend the analysis to supergame equilibria in mixed strategies. This can be done, but the argument is lengthy, and is omitted here.

Abreu, Pearce, and Stacchetti (1984) have also analyzed optimal equilibria in repeated games with discounting and imperfect monitoring using techniques similar to those used here.

4. ONE-SIDED VERSUS TWO-SIDED IMPERFECT MONITORING

As we have noted, in repeated principal-agent games efficient outcomes can be closely approximated by equilibria for low discount rates, whereas in repeated partnership games, they cannot be, in general. The essential difference between the two classes is that in the former there is imperfect monitoring only on one side, by contrast with the latter, where neither player can be perfectly monitored.

Roughly speaking (for the precise argument, see Fudenberg and Maskin (1985)), this distinction makes a difference because players who are imperfectly monitored cannot simply be "punished" when the outcome in a given period is bad (as in the perfect monitoring case), but must also be "rewarded" when the outcome is good. Now if we are interested in sustaining the pair (V_1, V_2) as average equilibrium payoffs of the repeated game, these rewards and punishments must average out to (V_1, V_2). Thus, if player i cannot be perfectly monitored, it must be feasible to assign him more than V_i say V_i+X_i, in periods where he is rewarded. If neither player can be perfectly monitored, therefore, the point (V_1+X_1, V_2+X_2) must be feasible, which means that (V_1, V_2) must be bounded away from the frontier of the feasible set (it can be shown that X_i does not depend on the discount factor). If, on the other hand, only one player, say 1, is imperfectly monitored, the need to reward and punish him imposes no such bound; in periods where player 1 must be rewarded with V_1+X_1, we can simply reduce player 2's payoff appropriately to remain in the feasible set.

APPENDIX

In this Appendix we derive (3.10), the solution of the constrained maximization problem, (3.4)-(3.8). It is convenient to introduce the variables

$$z_1 = x_1 - y_1, \qquad z_2 = x_2 - y_2, \qquad z = x - y; \tag{A.1}$$

then from (3.4) and (3.5)

$$v = (1-\delta)\left[p - \frac{q(a_1^2 + a_2^2)}{2}\right] + \delta(pz + y). \tag{A.2}$$

For the convenience of the reader we repeat the constraints:

1. $a_1 \geqq 0, \qquad a_2 \geqq 0, \qquad p \equiv a_1 + a_2 \leqq 1.$ (A.3)

2. For each $i = 1, 2$, a_i maximizes

$$v_i \equiv (1-\delta)(p - qa_i^2) + \delta(pz_i + y_i), \tag{A.4}$$

given x_i, y_i, and a_j $(j \neq i)$, subject to (A.3).

3. $y+z \leqq v, \qquad y \leqq v.$ (A.5)

Recall also that

$$v=\frac{v_1+v_2}{2}, \qquad y=\frac{y_1+y_2}{2}, \qquad z=\frac{z_1+z_2}{2}. \tag{A.6}$$

We define $\hat{v}$ to be the maximum of v subject to constraints 1-3.

Because of the many constraints, it is convenient to examine the behaviour of v in a number of regions defined by inequalities on z_1, z_2, and z.

Case I. $$0 \leqq \frac{1-\delta+\delta z}{q(1-\delta)} \leqq 1.$$

*Case I.*1. $$1-\delta+\delta z_i \geqq 0, \qquad i=1,2.$$

Given z_1, z_2, y_1, and y_2, constraint 2 implies that for $i=1,2$,

$$a_i=\frac{1-\delta+\delta z_i}{2q(1-\delta)}, \tag{A.7}$$

$$p=\frac{1-\delta+\delta z}{q(1-\delta)}. \tag{A.8}$$

Given z and y, p is also determined by (A.8), and v is maximized with respect to z_1 and z_2 when $(a_1^2+a_2^2)$ is minimized subject to

$$a_1+a_2=\frac{1-\delta+\delta z}{q(1-\delta)},$$

or when $z_1=z_2=z$. One can verify that v is then given by

$$v=\frac{3(1-\delta+\delta z)^2}{4(1-\delta)q}+\delta y,$$

or, from (1.6), since $u^*=3/4q$,

$$v=(1-\delta)u^*\left(\frac{1-\delta+\delta z}{1-\delta}\right)^2+\delta y. \tag{A.9}$$

Note that, in (A.9), v is increasing in y, given z.

If $z \leqq 0$, then the binding constraint in (A.5) is $y \leqq v$, so v is maximized in y (given z) when $y=v$, which with (A.9) implies

$$v=u^*\left(\frac{1-\delta+\delta z}{1-\delta}\right)^2. \tag{A.10}$$

In Case I, $1-\delta+\delta z \geqq 0$, so (A.10) reaches a maximum in z subject to

$$-\frac{1-\delta}{\delta} \leqq z \leqq 0 \tag{A.11}$$

at $z=0$, in which case

$$v=u^*. \tag{A.12}$$

If $z \geqq 0$, then the binding constraint in (A.5) is $y+z \leqq v$, so v is maximized in y, given z, when $y = v - z$. In this case

$$v = u^*\left(\frac{1-\delta+\delta z}{1-\delta}\right)^2 - \frac{\delta z}{1-\delta}. \tag{A.13}$$

This last expression is a convex function of z, and so attains its maximum at either $z = 0$, when $v = u^*$, or at $z = (q-1)(1-\delta)/\delta$, when $v = 1-(q/4)$. Hence the maximum of v in the region delimited by Case I.1 is

$$\max v = \begin{cases} 1 - \frac{q}{4}, & \text{if } 2 < q \leqq 3 \\ u^*, & \text{if } q \geqq 3. \end{cases} \tag{A.14}$$

As we shall see, this is in fact the case in which the overall maximum of v is attained.

Case I.2.

$$1-\delta+\delta z_1 < 0,$$

$$0 \leqq \frac{1-\delta+\delta z_2}{2q(1-\delta)} \leqq 1.$$

In this case constraint 2 implies

$$a_1 = 0,$$

$$a_2 = \frac{1-\delta+\delta z_2}{2q(1-\delta)},$$

$$p = a_2,$$

$$v = (1-\delta)\left(a_2 - \frac{qa_2^2}{2}\right) + \delta(a_2 z + y).$$

Given z and y,

$$\frac{dv}{da_2} = (1-\delta)(1-qa_2) + \delta z$$

$$= (1-\delta)\left(1 - \frac{1-\delta+\delta z_2}{2(1-\delta)}\right) + \delta z,$$

and one can verify that this last expression is negative if and only if $1-\delta+\delta z_1$ is negative, which is so in the present case. Hence, given z, v is increased by decreasing z_2 and increasing z_1 correspondingly, until $1-\delta+\delta z_1 = 0$. At this point we are in Case I.1

Case I.3.

$$1-\delta+\delta z_1 < 0$$

$$\frac{1-\delta+\delta z_2}{2q(1-\delta)} > 1.$$

In this case, constraint 2 implies that, given z and y,

$$a_1 = 0, \qquad a_2 = 1, \qquad p = 1,$$

$$v = (1-\delta)\left(1 - \frac{q}{2}\right) + \delta(z+y).$$

If we decrease z_2 and increase z_1 correspondingly, until we reach either Case I.1 or Case I.2, this last expression remains constant.

The remaining subcases of Case I are obtained by interchanging z_1 and z_2 in Cases I.2 and I.3, with corresponding analyses.

In summary, the maximum of v under Case I is given by (A.14).

Case II.
$$\frac{1+\delta+\delta z}{q(1-\delta)}>1.$$

Case II.1.
$$1-\delta+\delta z_i \geqq 0, \qquad i=1,2.$$

Here constraint 2 is satisfied if and only if $a_1+a_2=1$, in which case

$$v=(1-\delta)\left[1-\frac{q(a_1^2+a_2^2)}{2}\right]+\delta(z+y).$$

Given z and y, this last is maximized (subject to $a_1+a_2=1$) when $z_1=z_2$ and $a_2=a_2=\frac{1}{2}$, which yields

$$v=(1-\delta)\left(1-\frac{q}{4}\right)+\delta(z+y).$$

One easily verifies that this attains a maximum subject to constraint 3 when $z+y=v$, so that

$$v=1-\frac{q}{4}.$$

Case II.2
$$1-\delta+\delta z_1 \leqq 0.$$

A fortiori,

$$\frac{1-\delta+\delta z_2}{q(1-\delta)}>1,$$

so that constraint 2 implies

$$a_1=0, \qquad a_2=1,$$

$$v=(1-\delta)\left(1-\frac{q}{2}\right)+(z+y),$$

which attains a maximum when $z+y=v$, which yields

$$v=1-\frac{q}{2}.$$

Hence v can be increased by moving to Case II.1.

The remaining subcase of Case II is obtained by interchanging z_1 and z_2 in Case II.2, and is symmetric to it.

Case III.
$$1-\delta+\delta z<0.$$

A fortiori, in this case $z<0$.

*Case III.*1. $1-\delta+\delta z_i \leqq 0, \quad i=1,2.$

Here constraint 2 implies

$$a_1=a_2=p=0,$$
$$v=\delta y.$$

In order for y not to exceed v, y must be nonpositive, in which case the maximum value of v is 0, which is less than in Case I.

*Case III.*2.

$$1-\delta+\delta z_1<0,$$
$$1-\delta+\delta z_2>0.$$

Here constraint 2 implies

$$a_1=0, \qquad a_2=\min\left\{\frac{1-\delta+\delta z_2}{2q(1-\delta)}, 1\right\}$$
$$v=(1-\delta)\left(a_2-\frac{qa_2^2}{2}\right)+\delta(a_2 z+y).$$

Hence, given z, v is increased by decreasing z_2 and correspondingly increasing z_1, as in Case I.2. This can be done until $1-\delta+\delta z_2 \leqq 0$ (because $1-\delta+\delta z<0$), at which point we are in Case III.1.

This completes the proof of (3.10).

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NOTES

1. The views expressed here are those of the authors, and not necessarily those of AT&T Bell Laboratories.
2. See Fudenberg and Maskin (1983). The Continuity Property does not always hold in games of more than two players. All counterexamples, however, are "degenerate" in the sense that the convex hull of the one-period feasible payoff vectors has an empty interior.
3. It might be thought that X_i and Y_i should be conditioned on the actual actions taken at the first period, and not just on the occurrence of success or failure. However, for pure strategies it is not necessary to do this, because neither player can directly observe the other player's action, and hence the first-period actions can influence the future evolution of the game only through the first-period outcome (success or failure).
4. To be precise, $\beta_i(H_{it})$ is defined to be equal to $\alpha_{i,t+1}(a_{i1}, 1, H_{it})$.

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