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An Existence Theorem of Solutions for the System of Generalized Vector Quasi-Variational-Like Inequalities

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ABSTRACT

In this paper, we introduce and study the system of generalized vector quasi-variational-like inequalities in Hausdorff topological vector spaces, which include the system of vector quasi-variational-like inequalities, the system of vector variational-like inequalities, the system of vector quasi-variational inequalities, and several other systems as special cases. Moreover, a number of C-diagonal quasiconvexity properties are proposed for set-valued maps, which are natural generalizations of the g-diagonal quasiconvexity for real functions. Together with an application of continuous selection and fixed-point theorems, these conditions enable us to prove unified existence results of solutions for the system of generalized vector quasi-variational-like inequalities. The results of this paper can be seen as extensions and generalizations of several known results in the literature.

Keywords: The System of Generalized Vector Quasi-Variational-Like Inequalities; Fixed Point Theorem; Open Lower Section; Upper Semicontinuous; C-Diagonal Quasiconvexity

1. Introduction and Formulation

In recent years, the system of generalized vector quasi-variational-like inequality, which is a unified model for the system of vector quasi-variational-like inequalities, the system of vector variational-like inequalities, the system of vector variational inequalities, the system of vector equilibrium problems and the system of variational inequalities etc., has been studied (see [1-18] and references therein).

In this paper, we consider the systems of four kinds of generalized vector quasi-variational-like inequalities with set-valued mappings and discuss the existence of its solutions in locally convex topological vector space (l.c.s. in short), motivated and inspired by the recent works of Peng [1] and Ansari *et al*. [2].

Throughout this paper, unless otherwise specified, assume that I be an index set. For each $i \in I$, let Z_i be a locally convex topological vector space (l.c.s., in short) and K_i be a nonempty convex subset of Hausdorff topological vector space (t.v.s., in short) E_i. Let Y_i be a subset of continuous function space $L(E_i, Z_i)$ from E_i into Z_i, where $L(E_i, Z_i)$ is equipped with a σ-topology. Let $\operatorname{int} A$ and $\operatorname{co} A$ denote the interior and convex hull of a set A respectively. Let $C_i : K \to 2^{Z_i}$ be a set-valued mapping such that $\operatorname{int} C_i(x) \neq \varnothing$ for each $x \in K$. Denote that $K = \prod_{i \in I} K_i$ and $E = \prod_{i \in I} E_i$.

For each $i \in I$, let $\eta_i : K_i \times K_i \to E_i$ be a vector-valued mapping, $G_i : L(E, Z) \to 2^{L(E_i, Z_i)}$, $S_i : K \times K \to 2^{Z_i}$, $T_i : K \to 2^{Y_i}$ and $D_i : K \to 2^{K_i}$ be four set-valued mappings. Then,

1) Strong type I system of generalized vector quasi-variational-like inequalities which is to find $(\bar{x}, \bar{t}) \in K \times Y$ such that $\bar{x}_i \in D_i(\bar{x})$, $\bar{t}_i \in T_i(\bar{x})$ and

$$\langle G_i \bar{t}_i, \eta_i(y_i, \bar{x}_i) \rangle + S_i(\bar{x}_i, y_i) \subseteq C_i(\bar{x}), \ \forall y_i \in D_i(\bar{x}), \quad (1.1)$$

2) Strong type II system of generalized vector quasi-variational-like inequalities which is to find $(\bar{x}, \bar{t}) \in K \times Y$ such that $\bar{x}_i \in D_i(\bar{x})$, $\bar{t}_i \in T_i(\bar{x})$ and

$$\left\{ \langle G_i \bar{t}_i, \eta_i(y_i, \bar{x}_i) \rangle + S_i(\bar{x}_i, y_i) \right\} \cap C_i(\bar{x}) \neq \varnothing, \ \forall y_i \in D_i(\bar{x}), \quad (1.2)$$

3) Weak type I system of generalized vector quasi-variational-like inequalities which is to find $(\bar{x}, \bar{t}) \in K \times Y$ such that $\bar{x}_i \in D_i(\bar{x})$, $\bar{t}_i \in T_i(\bar{x})$ and

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$$\left\{\left\langle G_i\bar{t}_i,\eta_i\left(y_i,\bar{x}_i\right)\right\rangle+S_i\left(\bar{x}_i,y_i\right)\right\}\bigcap -\operatorname{int}C_i\left(\bar{x}\right)\neq\varnothing,\quad \forall y_i\in D_i\left(\bar{x}\right), \tag{1.3}$$

4) Weak type II system of generalized vector quasi-variational-like inequalities which is to find $\left(\bar{x},\bar{t}\right)\in K\times Y$ such that $\bar{x}_i\in D_i\left(\bar{x}\right)$, $\bar{t}_i\in T_i\left(\bar{x}\right)$ and

$$\left\langle G_i\bar{t}_i,\eta_i\left(y_i,\bar{x}_i\right)\right\rangle+S_i\left(\bar{x}_i,y_i\right)\nsubseteq -\operatorname{int}C_i\left(\bar{x}\right),\ \forall y_i\in D_i\left(\bar{x}\right), \tag{1.4}$$

where $\langle l,x\rangle$ denotes the evaluation of $l\in L(E,Z)$ at $x\in E$. By the corollary of the Schaefer [3], $L(E,Z)$ becomes a l.c.s.. By Ding and Tarafdar [4], the bilinear map $\langle\cdot,\cdot\rangle: L(K,Z)\times K\to Z$ is continuous.

The following problems are the special cases of above four kinds of systems of generalized vector quasi-variational-like inequalities.

The above system of generalized vector quasi-variational-like inequalities encompass many models of system of variational inequalities. The following problems are the special cases of problem (1.4).

1) If for each $i\in I$, let G_i be an identity mapping, $S_i\equiv 0$, problem (1.4) reduces to the system of generalized quasi-variational-like inequalities of finding $\bar{x}\in K$ such that for each $i\in I$, $\bar{x}_i\in D_i\left(\bar{x}\right)$ and

$$\forall y_i\in D_i\left(\bar{x}\right),\ \exists\bar{t}_i\in T_i\left(\bar{x}\right):\left\langle \bar{t}_i,\eta_i\left(y_i,\bar{x}_i\right)\right\rangle\notin -\operatorname{int}C_i\left(\bar{x}\right), \tag{1.5}$$

which was introduced and studied by Peng [1].

2) If for each $i\in I$, let G_i be an identity mapping, $S_i\equiv 0$ and $D_i(x)=K_i$, problem (1.5) reduces to the system of generalized variational-like inequalities of finding $\bar{x}\in K$ such that for each $i\in I$, $\bar{x}_i\in K_i$ and

$$\forall y_i\in K_i,\ \exists\bar{t}_i\in T_i\left(\bar{x}\right):\left\langle \bar{t}_i,\eta_i\left(y_i,\bar{x}_i\right)\right\rangle\notin -\operatorname{int}C_i\left(\bar{x}\right). \tag{1.6}$$

In addition, let $Z_i=\mathbb{R}$ and let $C_i(x)=\mathbb{R}^+=\{r\in\mathbb{R}\mid r\geq 0\}$ for all $x\in K$, then problem (1.5) reduces to the system of generalized vector quasi-variational inequalities studied by Ansari and Yao [5].

3) If for each $i\in I$, G_i be an identity mapping, $S_i\equiv 0$, $\eta_i\left(y_i,\bar{x}_i\right)=y_i-\bar{x}_i$ and $D_i(x)=K_i$, then problem (1.5) reduces to the system of generalized vector variational inequalities of finding $\bar{x}\in K$ such that for each $i\in I$, $\bar{x}_i\in K_i$ and

$$\forall y_i\in K_i,\ \exists\bar{t}_i\in T_i\left(\bar{x}\right):\left\langle \bar{t}_i,y_i-\bar{x}_i\right\rangle\notin -\operatorname{int}C_i\left(\bar{x}\right). \tag{1.7}$$

4) If $I=\{1\}$, problem (1.4) reduces to generalized vector quasi-variational-like inequalities of finding $\bar{x}\in K$ such that $\bar{x}\in D\left(\bar{x}\right)$ and

$$\left\langle G\bar{t},\eta\left(y,\bar{x}\right)\right\rangle+S\left(\bar{x},y\right)\nsubseteq -\operatorname{int}C\left(\bar{x}\right),\ \forall y_i\in K, \tag{1.8}$$

such type of problem studied in [6-10].

5) If $I=\{1\}$ and $\eta\left(y,\bar{x}\right)=y-\bar{x}$, T is single valued mapping, G be an identity mapping, $S\equiv 0$, and $C(x)=\mathbb{R}^+$ for all $x\in K$, then problem (1.4) reduces to classical variational inequality problem of finding $\bar{x}\in K$ such that $\bar{x}\in D\left(\bar{x}\right)$ and

$$\forall y\in D\left(\bar{x}\right),\ \exists\bar{t}\in T\left(\bar{x}\right):\left\langle T\left(\bar{x}\right),\left(y-\bar{x}\right)\right\rangle\notin -\operatorname{int}C\left(\bar{x}\right), \tag{1.9}$$

which was introduced and studied by Hartman and Stampacchia [11].

2. Preliminaries

Definition 2.1. [12] Let E and Z be two t.v.s. and K be a convex subset of t.v.s. E. Let $C:K\to 2^Z$ and $\theta:K\times K\to 2^Z$ be two set-valued mappings. Assume given any finite subset $\Lambda=\{x_1,x_2,\cdots,x_n\}$ in K, any $x=\sum_{i=1}^{n}\alpha_i x_i$, with $\alpha_i\geq 0$ for $i=1,\cdots,n$, and $\sum_{i=1}^{n}\alpha_i=1$. Then, 1) θ is said to be strong Type I C-diagonally quasiconvex (SIC-DQC, in short) in the second argument if for some $x_i\in\Lambda$,

$$\theta\left(x,x_i\right)\subseteq C(x);$$

2) θ is said to be strong Type II C-diagonally quasiconvex (SIIC-DQC, in short) in the second argument if for some $x_i\in\Lambda$,

$$\theta\left(x,x_i\right)\bigcap C(x)\neq\varnothing;$$

3) θ is said to be weak Type I C-diagonally quasiconvex (WIC-DQC, in short) in the second argument if for some $x_i\in\Lambda$,

$$\theta\left(x,x_i\right)\bigcap -\operatorname{int}C(x)\neq C(x);$$

4) θ is said to be weak Type II C-diagonally quasiconvex (WIIC-DQC, in short) in the second argument if for some $x_i\in\Lambda$,

$$\theta\left(x,x_i\right)\nsubseteq -\operatorname{int}C(x).$$

It is easy to verify that the following proposition, 1) SIC-DQC implies SIIC-DQC; 2) SIIC-DQC implies WIC-DQC; 3) WIC-DQC implies WIIC-DQC. The converse is not true. Following example shows that the con0 verse is not true.

Example 2.1. Let $E=Z=\mathbb{R}$ and $\varphi\left(x_1,x_2\right)=\operatorname{co}\{x_1,x_2\}$.

1) If $C(x)=[x+\epsilon,+\infty)$. Then φ is SIIC-DQC, but it is not SIC-DQC.

2) If $-\operatorname{int}C(x)=(-\infty,x+\epsilon)$. Then φ is WIIC-DQC, but it is not WIC-DQC.

Definition 2.2. [13] Let E and Z be two t.v.s. and K be a convex subset of t.v.s. E. A mapping $\theta:K\times K\to\left(2^Z\right)Z$ is called (generalized) vector 0-

diagonally convex if for any finite subset $\Lambda=\{x_1,x_2,\cdots,x_n\}$ of K and any $x=\sum_{i=1}^{n}\alpha_i x_i$ with $\alpha_i\ge 0$ for $i=1,\cdots,n$, and $\sum_{i=1}^{n}\alpha_i=1$,

$$\sum_{i=1}^{n}\alpha_i\theta(x,x_i)(\not\subseteq)\notin -\operatorname{int}C(x).$$

Definition 2.3. [14] Let X and Y be two topological spaces and $T:X\to 2^Y$ be a set-valued mapping. Then,

1) T is said to have open lower sections if the set $T^{-1}(y)=\{x\in X: y\in T(x)\}$ is open in X for every $y\in Y$;

2) T is said to be upper semicontinuous (u.s.c., in short) if for each $x_o\in X$ and each open set U in Y with $T(x_o)\subset U$, there exists an open neighborhood V of x_o in X such that $T(x)\subset U$ for each $x\in V$;

3) T is said to be lower semicontinuous (l.s.c., in short) if for each $x_o\in X$ and each open set U in Y with $T(x_o)\cap U\neq\varnothing$, there exists an open neighborhood V of x_o in X such that $T(x)\cap U\neq\varnothing$ for each $x\in V$;

4) T is said to be continuous if it is both upper and lower semicontinuous;

5) T is said to be closed if for any net $\{x^o\}$ in X such that $x^o\to x^*$ and any net $\{y^o\}$ in B such that $y^o\to y^*$ and $y^o\in T(x^o)$ for any o, we have $y^*\in T(x^*)$.

Lemma 2.1. [15] Let X and Y be two topological spaces. If $T:X\to 2^Y$ is u.s.c. set-valued mapping with closed values, then T is closed.

Lemma 2.2. [16] Let X and Y be two topological spaces and $T:X\to 2^Y$ is u.s.c. mapping with compact values. Suppose $\{x^o\}$ is a net in X such that $x^o\to x^*$. If $y^o\in T(x^o)$ for each o, then there are a $y^*\in T(x^*)$ and a subnet $\{y^n\}$ of $\{y^o\}$ such that $y^n\to y^*$.

Lemma 2.3. [17] Let X and Y be two topological spaces. Suppose that $T:X\to 2^Y$ and $K:X\to 2^Y$ are set-valued mappings having open lower sections, then

1) A set-valued mapping $F:X\to 2^Y$ defined by, for each $x\in X$, $F(x)=\operatorname{co}T(x)$ has open lower sections;

2) A set-valued mapping $J:X\to 2^Y$ defined by, for each $x\in X$, $J(x)=T(x)\cap K(x)$ has open lower sections.

For each $i\in I$, E_i a Hausdorff t.v.s. Let $\{K_i\}$ be a family of nonempty compact convex subsets with each K_i in E_i. Let $K=\Pi_{i\in I}K_i$ and $E=\Pi_{i\in I}E_i$. The following system of fixed-point theorem is needed in this paper.

Lemma 2.4. [18] For each $i\in I$, let $T_i:K\to 2^{K_i}$ be a set-valued mapping. Assume that the following conditions hold.

1) For each $i\in I$, T_i is convex set-valued mapping;

2) $K=\bigcup\{\operatorname{int}T_i^{-1}(x_i): x_i\in K_i\}$.

Then there exist $\bar{x}\in K$ such that $\bar{x}\in T(\bar{x})=\Pi_{i\in I}T_i(\bar{x})$, that is, $\bar{x}_i\in T_i(\bar{x})$ for each $i\in I$, where $\bar{x}_i$ is the projection of $\bar{x}$ onto K_i.

3. Main Results

Theorem 3.1. For each $i\in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i,Z_i)$, which is equipped with a σ-topology. For each $i\in I$, assume that the following conditions are satisfied.

1) $D_i:K\to 2^{K_i}$ and $T_i:K\to 2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For each $t_i\in Y_i$ and $x_i\in\operatorname{co}\Lambda_i$, the mapping $\langle G_i t_i,\eta_i(.,x_i)\rangle+S_i(x_i,.):K\to 2^{Z_i}$ is WIIC-DQC;

3) For each $y_i\in K_i$, the set $\{(x,t)\in K\times Y:\langle G_i t_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i)\subseteq -\operatorname{int}C_i(x)\}$ is open.

Then there exist $\bar{x}_i\in D_i(\bar{x})$ and $\bar{t}_i\in T_i(\bar{x})$ such that

$$\langle G_i\bar{t}_i,\eta_i(y_i,\bar{x}_i)\rangle+S_i(\bar{x}_i,y_i)\not\subseteq -\operatorname{int}C_i(\bar{x}),$$
$$\forall y_i\in D_i(\bar{x}).$$

Proof. Define a set-valued mapping $P_i:K\times Y\to 2^{K_i}$ by

$$P_i(x,t)=\{y_i\in K_i:\langle G_i t_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i)\subseteq -\operatorname{int}C_i(x)\},\quad \forall(x,t)\in K\times Y.$$

We first prove that $x_i\notin\operatorname{co}(P_i(x,t))$ for all $(x,t)\in K\times Y$. To see this, suppose, by way of contradiction, that there exist some $i\in I$ and some point $(\bar{x},\bar{t})\in K\times Y$ such that $\bar{x}_i\in\operatorname{co}(P_i(\bar{x},\bar{t}))$. Then, there exist finite points $y_{i_1},y_{i_2},\cdots,y_{i_n}$ in K_i and $\alpha_j\ge 0$ with $\sum_{j=1}^{n}\alpha_j=1$ such that $\bar{x}_i=\sum_{j=1}^{n}\alpha_j y_{i_j}$ and $y_{i_j}\in P_i(\bar{x},\bar{t})$ for all $j=1,\cdots,n$ such that

$$\langle G_i\bar{t}_i,\eta_i(y_{i_j},\bar{x}_i)\rangle+S_i(\bar{x}_i,y_{i_j})\subseteq -\operatorname{int}C_i(\bar{x}),\quad j=1,\cdots,n,$$

which contradicts the hypothesis 2). Hence, $x_i\notin\operatorname{co}(P_i(x,t))$.

By hypothesis 3), for each $i\in I$ and each $y_i\in K_i$, we known that

$$Q_i^{-1}(y_i)=\{(x,t)\in K\times Y:\langle G_i t_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i) \subseteq -\operatorname{int} C_i(x)\}$$

is open and so P_i has open lower sections.

For each $i\in I$, consider a set-valued mapping $Q_i:K\times Y\to 2^{K_i}$ defind by

$$Q_i(x,t)=\operatorname{co}(P_i(x,t))\cap D_i(x),\quad \forall (x,t)\in K\times Y.$$

Since D_i has open lower sections by hypothesis 1), we may apply Lemma 2.3 to assert that the set-valued mapping Q_i has also open lower sections. Let

$$W_i=\{(x,t)\in K\times Y:Q_i(x,t)\neq\varnothing\}\subset K\times Y.$$

There are two cases to consider. In the case $W_i=\varnothing$, we have

$$\operatorname{co}(P_i(x,t))\cap D_i(x)=\varnothing,\forall (x,t)\in K\times Y.$$

This implies that, $\forall (x,t)\in K\times Y$,

$$P_i(x,t)\cap D_i(x)=\varnothing.$$

On the other hand, by condition 1), and the fact K_i is a compact convex subset of E_i, we can apply Lemma 2.4 to assert the existence of a fixed point $x_i^*\in D_i(x^*)$. Since $T_i(x^*)\neq\varnothing$, picking $t_i^*\in T_i(x^*)$, we have

$$P_i(x^*,t^*)\cap D_i(x^*)=\varnothing.$$

This implies $\forall y_i\in D_i(x^*), y_i\notin P_i(x^*,t^*)$. Hence, in this particular case, the assertion of the theorem holds.

We now consider the case $W_i\neq\varnothing$. Define a set-valued mapping $S_i:K\times Y\to 2^{K_i}$ by

$$S_i(x,t)=\begin{cases}Q_i(x,t), & (x,t)\in W_i\\ D_i(x), & (x,t)\in K_i\times Y_i\setminus W_i.\end{cases}$$

Then, $S_i(x,t)$ is a convex set-valued mapping and for each $u\in K$, $S_i^{-1}(u)=Q_i^{-1}(u)\cup(D_i^{-1}(u)\times Y_i)$ is open. For each $i\in I$, consider the set-valued mapping $H:K\times Y\to 2^{K\times Y}$ where $H=\Pi_{i\in I}H_i$ defined by

$$H_i(x,t)=(S_i(x,t),T_i(x)).$$

By condition 1) and the properties of $S_i(x,t)$, H_i satisfies all the conditions of Lemma 2.4. Therefore, there exists $(x^*,t^*)\in K\times Y$ such that $(x_i^*,t_i^*)\in H_i(x^*,t^*)$. Suppose that $(x^*,t^*)\in W_i$, then

$$x_i^*\in\operatorname{co}(P_i(x^*,t^*))\cap D_i(x^*),$$

so that $x_i^*\in\operatorname{co}(P_i(x^*,t^*))$. This is a contradiction. Hence, $(x^*,t^*)\notin W_i$. Therefore,

$$(x_i^*,t_i^*)\in(D_i(x^*),T_i(x^*)),\ \text{and}\ Q_i(x^*,t^*)=\varnothing.$$

Thus

$$x_i^*\in D_i(x^*),t_i^*\in T_i(x^*),\quad \operatorname{co}(P_i(x^*,t^*))\cap D_i(x^*)=\varnothing.$$

This implies

$$P_i(x^*,t^*)\cap D_i(x^*)=\varnothing.$$

Consequently, the assertion of the theorem holds in this case.

Corollary 3.2. For each $i\in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i,Z_i)$, which is equipped with a σ-topology. For each $i\in I$, assume that the following conditions are satisfied.

1) $D_i:K\to 2^{K_i}$ and $T_i:K\to 2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For all $y_i\in K_i$, the mapping $\langle G_i.,\eta_i(y_i,*)\rangle+S_i(*,y_i):K\times Y\to 2^{Z_i}$ is an u.s.c. set-valued mapping;

3) $C_i:K\to 2^{Z_i}$ is a convex set-valued mapping with $\operatorname{int} C_i(x)\neq\varnothing$ for all $x\in K$;

4) $\eta_i:K_i\times K_i\to E_i$ is affine in the first argument and for all $x_i\in K_i$, $\eta_i(x_i,x_i)=0$;

5) $S_i:K\times K\to 2^{Z_i}$ is a generalized vector 0-diagonally convex set-valued mapping;

6) For a given $x_i\in K_i$, and a neighborhood U_i of x, for all $u\in U_i$, $\operatorname{int} C_i(x)=\operatorname{int} C_i(u)$.

Then there exists $\overline{x}_i\in D_i(\overline{x})$ and $\overline{t}_i\in T_i(\overline{x})$ such that

$$\langle G_i\overline{t}_i,\eta_i(y_i,\overline{x}_i)\rangle+S_i(\overline{x}_i,y_i)\nsubseteq-\operatorname{int} C_i(\overline{x}),\ \forall y_i\in D_i(\overline{x}).$$

Proof. Define a set-valued mapping $P_i:K\times Y\to 2^{K_i}$ by

$$P_i(x,t)=\{y_i\in K_i:\langle G_i t_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i) \subseteq-\operatorname{int} C_i(x)\},\quad \forall (x,t)\in K\times Y.$$

We first prove that $x_i\notin\operatorname{co}(P_i(x,t))$ for all $(x,t)\in K\times Y$. By contradiction, for each $i\in I$, suppose there exists some point $(\overline{x},\overline{t})\in K\times Y$ such that $\overline{x}_i\in\operatorname{co}(P_i(\overline{x},\overline{t}))$. Then, there exist finite points $y_{i_1},y_{i_2},\cdots,y_{i_n}$ in K_i, such that

$$\langle G_i\overline{t}_i,\eta_i(y_i,\overline{x}_i)\rangle+S_i(\overline{x}_i,y_i)\subseteq-\operatorname{int} C_i(\overline{x}),\ i=1,2,\cdots,n.$$

Since $\eta_i(.,x_i)$ is affine and $\operatorname{int} C_i(\overline{x})$ is convex, for $\alpha_j\ge 0$ with $\sum_{j=1}^n\alpha_j=1$ such that $\overline{x}_i=\sum_{j=1}^n\alpha_j y_{i_j}$ and $y_{i_j}\in P_i(\overline{x},\overline{t})$ for all $j=1,\cdots,n$ such that

$$\left\langle G_i\overline{t}_i,\eta_i\left(\sum_{j=1}^n\alpha_j y_{i_j},\overline{x}_i\right)\right\rangle+\sum_{j=1}^n\alpha_j S_i(\overline{x}_i,y_{i_j})\subseteq-\operatorname{int} C_i(\overline{x}),$$

$j=1,\cdots,n.$

Since $\eta_i(x_i, x_i) = 0$ for all $x_i \in K_i$

$$\sum_{j=1}^{n} \alpha_j S_i(\bar{x}_i, y_i) \subseteq -\operatorname{int} C_i(\bar{x})$$

which contradicts the hypothesis 5). Therefore $x_i \notin \operatorname{co}(P_i(x,t))$.

We now prove that for each

$$y_i \in K_i, P_i^{-1}(y_i)$$
$$= \{(x,t) \in K \times Y : \langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i)$$
$$\subseteq -\operatorname{int} C_i(x)\}$$

is open. Indeed, let $(\bar{x}, \bar{t}) \in P_i^{-1}(y_i)$, that is $\langle G_i \bar{t}_i, \eta_i(y_i, \bar{x}_i)\rangle + S_i(\bar{x}_i, y_i) \subseteq -\operatorname{int} C_i(\bar{x})$. Since $\langle G_i t_i, \eta_i(y_i, *)\rangle + S_i(*, y_i) : K \times Y \to 2^{Z_i}$ is an u.s.c. set-valued mapping, there exists a neighborhood U_i of $(\bar{x}, \bar{t})$ such that

$$\langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i) \subseteq -\operatorname{int} C_i(\bar{x}),\ \forall (x,t) \in U_i.$$

By 6),

$$\langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i) \subseteq -\operatorname{int} C_i(x),\ \forall (x,t) \in U_i.$$

Hence, $U_i \subset P_i^{-1}(y_i)$. This implies, $P_i^{-1}(y_i)$ is open for each $y_i \in K_i$, and so P_i have open lower sections. For the remainder of the proof, we can just follow that of Theorem 3.1. This completes the proof.

Corollary 3.3. For each $i \in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i, Z_i)$, which is equipped with a σ-topology. For each $i \in I$, assume that the following conditions are satisfied.

1) $D_i : K \to 2^{K_i}$ and $T_i : K \to 2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For all $y_i \in K_i$, the mapping $\langle G_i \cdot, \eta_i(y_i, *)\rangle + S_i(*, y_i) : K \times Y \to 2^{Z_i}$ is an u.s.c. set-valued mapping;

3) $C_i : K \to 2^{Z_i}$ is a convex set-valued mapping such that for each $x \in K$, $C_i(x) = C_i$ is a convex cone with $\operatorname{int} C_i(x) \neq \varnothing$;

4) $\eta_i : K_i \times K_i \to E_i$ is affine in the first argument and for all $x_i \in K_i$, $\eta_i(x_i, x_i) = 0$;

5) $S_i : K \times K \to 2^{Z_i}$ is a generalized vector 0-diagonally convex set-valued mapping;

6) For a given $x_i \in K_i$, and a neighborhood U_i of x, for all $u \in U_i$, $\operatorname{int} C_i(x) = \operatorname{int} C_i(u)$.

Then there exist $\bar{x}_i \in D_i(\bar{x})$ and $\bar{t}_i \in T_i(\bar{x})$ such that

$$\langle G_i \bar{t}_i, \eta_i(y_i, \bar{x}_i)\rangle + S_i(\bar{x}_i, y_i) \not\subseteq -\operatorname{int} C_i,\ \forall y_i \in D_i(\bar{x}).$$

Proof. By hypothesis 3), the condition 4) in Corollary 3.2 is satisfied. Hence, all the conditions are satisfied as in Corollary 3.2.

Corollary 3.4. For each $i \in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i, Z_i)$, which is equipped with a σ-topology. For each $i \in I$, assume that S_i and G_i are single valued mappings and the following conditions are satisfied.

1) $D_i : K \to 2^{K_i}$ and $T_i : K \to 2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For all $y_i \in K_i$, the mapping $\langle G_i \cdot, \eta_i(y_i, *)\rangle + S_i(*, y_i) : K_i \times Y_i \to Z_i$ is continuous;

3) $C_i : K \to 2^{Z_i}$ is a convex set-valued mapping with $\operatorname{int} C_i(x) \neq \varnothing$ for all $x \in K$;

4) $\eta_i : K_i \times K_i \to E_i$ is affine in the first argument and for all $x_i \in K_i$, $\eta_i(x_i, x_i) = 0$;

5) $S_i : K \times K \to Z_i$ is a vector 0-diagonally convex mapping;

6) $Z_i \setminus \{-\operatorname{int} C_i(x)\}$ is an u.s.c. set-valued mapping.

Then there exist $\bar{x}_i \in D_i(\bar{x})$ and $\bar{t}_i \in T_i(\bar{x})$ such that

$$\langle G_i \bar{t}_i, \eta_i(y_i, \bar{x}_i)\rangle + S_i(\bar{x}_i, y_i) \notin -\operatorname{int} C_i(\bar{x}),\ \forall y_i \in D_i(\bar{x}).$$

Proof. Define a set-valued mapping $P_i : K \times Y \to 2^{K_i}$ by

$$P_i(x,t) = \{y_i \in K_i : \langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i)$$
$$\in -\operatorname{int} C_i(x)\},$$
$$\forall (x,t) \in K \times Y.$$

We now prove that for each

$$y_i \in K_i, P_i^{-1}(y_i)$$
$$= \{(x,t) \in K \times Y : \langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i)$$
$$\in -\operatorname{int} C_i(x)\}$$

is open, that is, the set

$$\{(x,t) \in K \times Y : \langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i)$$
$$\in Z_i \setminus \{-\operatorname{int} C_i(x)\}\}$$

is closed. Indeed, let $\{(x^o, t^o)\}$ be a net in $K \times Y$ such that $(x^o, t^o) \to (x^*, t^*)$ and

$$\langle G_i t_i^o, \eta_i(y_i, x_i^o)\rangle \cap S_i(x_i^o, y_i) \in Z_i \setminus \{-\operatorname{int} C_i(x^o)\}.$$

Since $\langle G_i \cdot, \eta_i(y_i, *)\rangle + S_i(*, y_i) : K \times Y \to 2^{Z_i}$ is continuous, hence

$$\langle G_i t_i^o, \eta_i(y_i, x_i^o)\rangle \cap S_i(x_i^o, y_i)$$
$$\to \langle G_i t_i^*, \eta_i(y_i, x_i^*)\rangle + S_i(x_i^*, y_i).$$

Since $Z_i \setminus \{-\operatorname{int} C_i(x)\}$ is an u.s.c. set-valued mapping with closed values, by Lemma 2.1, we have

$$\langle G_i t_i^*, \eta_i(y_i, x_i^*)\rangle + S_i(x_i^*, y_i) \in Z_i \setminus \{-\operatorname{int} C_i(x^*)\},$$

and hence (x^*, t^*) in the set

$$\{(x,t) \in K \times Y : \langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i) \in -\operatorname{int} C_i(x)\}.$$

This implies $P_i^{-1}(y_i)$ is open for each $y_i \in K_i$ and so P_i has open lower sections. For the remainder of the proof, we can just follow that of Theorem 3.1 and Corollary 3.2. This completes the proof.

Theorem 3.5. For each $i \in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i, Z_i)$, which is equipped with a σ-topology. For each $i \in I$, assume that the following conditions are satisfied.

1) $D_i : K \to 2^{K_i}$ and $T_i : K \to 2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For each $t_i \in Y_i$ and $x_i \in \mathrm{co}\Lambda_i$, the mapping $\langle G_i t_i, \eta_i(., x_i)\rangle + S_i(x_i, .) : K \to 2^{Z_i}$ is WIC-DQC;

3) for each $y_i \in K_i$, the set

$$\{(x,t) \in K \times Y : \{\langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i)\} \cap -\operatorname{int} C_i(x) \neq \varnothing\}$$

is open.

Then there exist $\bar{x}_i \in D_i(\bar{x})$ and $\bar{t}_i \in T_i(\bar{x})$ such that

$$\{\langle G_i \bar{t}_i, \eta_i(y_i, \bar{x}_i)\rangle + S_i(\bar{x}_i, y_i)\} \cap -\operatorname{int} C_i(\bar{x}) = \varnothing,$$
$$\forall y_i \in D_i(\bar{x}).$$

Proof. Define a set-valued mapping $P_i : K \times Y \to 2^{K_i}$ by

$$P_i(x,t) = \{y_i \in K_i : \{\langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i)\} \cap -\operatorname{int} C_i(x) \neq \varnothing\},$$
$$\forall (x,t) \in K \times Y.$$

For the remainder proof, we just follow that of Theorem 3.1.

Corollary 3.6. For each $i \in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i, Z_i)$, which is equipped with a σ-topology. For each $i \in I$, assume that the following conditions are satisfied.

1) $D_i : K \to 2^{K_i}$ and $T_i : K \to 2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For each $t_i \in Y_i$ and $x_i \in \mathrm{co}\Lambda_i$, the mapping $\langle G_i t_i, \eta_i(., x_i)\rangle + S_i(x_i, .) : K \to 2^{Z_i}$ is WIC-DQC;

3) $Z_i \setminus \{-\operatorname{int} C_i(x)\}$ is an u.s.c. set-valued mapping.

Then there exist $\bar{x}_i \in D_i(\bar{x})$ and $\bar{t}_i \in T_i(\bar{x})$ such that

$$\{\langle G_i \bar{t}_i, \eta_i(y_i, \bar{x}_i)\rangle + S_i(\bar{x}_i, y_i)\} \cap -\operatorname{int} C_i(\bar{x}) = \varnothing,$$
$$\forall y_i \in D_i(\bar{x}).$$

Proof. Let $P_i : K \times Y \to 2^{K_i}$ be a set-valued mapping define in Theorem 3.5. We just prove that for each

$$y_i \in K_i, P_i^{-1}(y_i) = \{(x,t) \in K \times Y : \{\langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i)\} \cap -\operatorname{int} C_i(x) \neq \varnothing\}$$

is open, that is, the set

$$\{(x,t) \in K \times Y : \{\langle G_i t_i, \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i)\} \cap -\operatorname{int} C_i(x) \neq \varnothing\}$$

is closed. Indeed, let $\{(x^o, t^o)\}$ be a net in $K \times Y$ such that $(x^o, t^o) \to (x^*, t^*)$ and

$$\{\langle G_i t_i^o, \eta_i(y_i, x_i^o)\rangle + S_i(x_i^o, y_i)\} \cap -\operatorname{int} C_i(x^o) = \varnothing.$$

This implies

$$\{\langle G_i t_i^o, \eta_i(y_i, x_i^o)\rangle + S_i(x_i^o, y_i)\} \subseteq Z_i \setminus \{-\operatorname{int} C_i(x^o)\}.$$

We now prove that

$$\{\langle G_i t_i^*, \eta_i(y_i, x_i^*)\rangle + S_i(x_i^*, y_i)\} \subseteq Z_i \setminus \{-\operatorname{int} C_i(x^*)\}.$$

If it is not true, then there exists a $w^* \in \langle G_i t_i^*, \eta_i(y_i, x_i^*)\rangle + S_i(x_i^*, y_i)$ such that $w^* \notin Z_i \setminus \{-\operatorname{int} C_i(x^*)\}$. Since Z_i is Hausdorff t.v.s. (l.c.s. is Hausdorff space) and $Z_i \setminus \{-\operatorname{int} C_i(x^*)\}$ is closed, there exists two open sets $U_i, V_i \subset Z_i$ such that $w^* \in U_i, Z_i \setminus \{-\operatorname{int} C_i(x^*)\} \subset V_i$ and $U_i \cap V_i = \varnothing$. Since $\langle G_i ., \eta_i(y_i, *)\rangle \cap S_i(*, y_i) : P \times Y \to 2^{Z_i}$ is an l.s.c. set-valued mapping and $Z_i \setminus \{-\operatorname{int} C_i(x)\}$ is an u.s.c. set-valued mapping, there exists a neighborhood $U_i(x^*, y^*)$ of (x^*, y^*) such that

$$\{\langle G_i ., \eta_i(y_i, x_i)\rangle + S_i(x_i, y_i)\} \cap U_i \neq \varnothing,$$
$$\forall (x,t) \in U_i(x^*, y^*)$$

and a neighborhood $U_i(x^*)$ of x^* such that $Z_i \setminus \{-\operatorname{int} C_i(x)\} \subset V_i, \forall x \in U_i(x^*)$.

Hence, for all $(x^o,t^o)\in U_i(x^*,y^*)\cap\{U_i(x^*)\times Y_i\}$, there exists $w^o\in\{\langle G_it_i^o,\eta_i(y_i,x_i^o)\rangle+S_i(x_i^o,y_i)\}$ such that $w^o\notin Z_i\setminus\{-\operatorname{int}C_i(x^o)\}$, which is contradiction. Therefore, the set

$$\{(x,t)\in K\times Y:\{\langle G_it_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i)\}\cap-\operatorname{int}C_i(x)=\varnothing\}$$

is closed. Hence, all the conditions of Theorem 3.5 satisfied. Consequently, the assertion of the theorem holds.

Theorem 3.7. For each $i\in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i,Z_i)$, which is equipped with a σ-topology. For each $i\in I$, assume that the following conditions are satisfied.

1) $D_i:K\to2^{K_i}$ and $T_i:K\to2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For each $t_i\in Y_i$ and $x_i\in\text{co}\Lambda_i$, the mapping $\langle G_it_i,\eta_i(.,x_i)\rangle+S_i(x_i,.):K\to2^{Z_i}$ is SIIC-DQC;

3) for each $y_i\in K_i$, the set

$$\{(x,t)\in K\times Y:\{\langle G_it_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i)\}\cap C_i(x)=\varnothing\}$$

is open.

Then there exist $\bar{x}_i\in D_i(\bar{x})$ and $\bar{t}_i\in T_i(\bar{x})$ such that

$$\{\langle G_i\bar{t}_i,\eta_i(y_i,\bar{x}_i)\rangle+S_i(\bar{x}_i,y_i)\}\cap C_i(\bar{x})\neq\varnothing,\ \forall y_i\in D_i(\bar{x}).$$

Proof. Define a set-valued mapping $P_i:K\times Y\to2^{K_i}$ by

$$P_i(x,t)=\{y_i\in K_i:\{\langle G_it_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i)\}\cap C_i(x)=\varnothing\},$$

$$\forall(x,t)\in K\times Y.$$

For the remainder proof, we just follow that of Theorem 3.1.

Corollary 3.8. For each $i\in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i,Z_i)$, which is equipped with a σ-topology. For each $i\in I$, assume that the following conditions are satisfied.

1) $D_i:K\to2^{K_i}$ and $T_i:K\to2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For each $t_i\in Y_i$ and $x_i\in\text{co}\Lambda_i$, the mapping

$\langle G_it_i,\eta_i(.,x_i)\rangle+S_i(x_i,.):K\to2^{Z_i}$ is SIIC-DQC;

3) For all $x\in K$, $C_i(x)$ is closed convex cone C_i.

Then there exist $\bar{x}_i\in D_i(\bar{x})$ and $\bar{t}_i\in T_i(\bar{x})$ such that

$$\{\langle G_i\bar{t}_i,\eta_i(y_i,\bar{x}_i)\rangle+S_i(\bar{x}_i,y_i)\}\cap C_i=\varnothing,\ \forall y_i\in D_i(\bar{x}).$$

Proof. Let $P_i:K\times Y\to2^{K_i}$ be a set-valued mapping defined in Theorem 3.7. We prove that for each

$$y_i\in K_i,P_i^{-1}(y_i)=\{(x,t)\in K\times Y:\{\langle G_it_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i)\}\cap C_i=\varnothing\}$$

is open, that is, the set

$$\{(x,t)\in K\times Y:\{\langle G_it_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i)\}\subset Z_i\setminus C_i\}$$

is open. If $(\bar{x},\bar{t})\in P_i^{-1}(y_i)$, since $Z_i\setminus C_i$ is open set and for all $y_i\in K_i,\langle G_i.,\eta_i(y_i,*)\rangle+S_i(*,y_i):K\times Y\to2^{Z_i}$, an u.s.c. set-valued mapping, there exists a neighborhood U_i of $(\bar{x},\bar{t})$, for all $(x,t)\in U_i$,

$$\{\langle G_it_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i)\}\subset Z_i\setminus C_i.$$

This implies $P_i^{-1}(y_i)$ is open for each $y_i\in K_i$. Therefore, all the conditions of Theorem 3.7 are satisfied. Consequently the assertion of the theorem holds.

Theorem 3.9. For each $i\in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i,Z_i)$, which is equipped with a σ-topology. For each $i\in I$, assume that the following conditions are satisfied.

1) $D_i:K\to2^{K_i}$ and $T_i:K\to2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For each $t_i\in Y_i$ and $x_i\in\text{co}\Lambda_i$, the mapping $\langle G_it_i,\eta_i(.,x_i)\rangle+S_i(x_i,.):K\to2^{Z_i}$ is SIC-DQC;

3) for each $y_i\in K_i$, the set $\{(x,t)\in K\times Y:\langle G_it_i,\eta_i(y_i,x_i)\rangle\cap S_i(x_i,y_i)\nsubseteq C_i(x)\}$ is open.

Then there exist $\bar{x}_i\in D_i(\bar{x})$ and $\bar{t}_i\in T_i(\bar{x})$ such that

$$\langle G_i\bar{t}_i,\eta_i(y_i,\bar{x}_i)\rangle+S_i(\bar{x}_i,y_i)\nsubseteq C_i(\bar{x}),\ \forall y_i\in D_i(\bar{x}).$$

Proof. Define a set-valued mapping $P_i:K\times Y\to2^{K_i}$ by

$$P_i(x,t)=\{y_i\in K_i:\langle G_it_i,\eta_i(y_i,x_i)\rangle+S_i(x_i,y_i)\nsubseteq C_i(x)\},$$

$$\forall(x,t)\in K\times Y.$$

The rest of the proof is similar to that of Theorem 3.1.

Corollary 3.10. For each $i\in I$, let Z_i be a l.c.s., K_i a nonempty compact convex subset of Hausdorff t.v.s. E_i, Y_i a nonempty compact convex subset of $L(E_i,Z_i)$, which is equipped with a σ-topology. For each $i\in I$, assume that the following conditions are satisfied.

1) $D_i:K\to2^{K_i}$ and $T_i:K\to2^{Y_i}$ are two nonempty convex set-valued mappings and have open lower sections;

2) For each $t_i\in Y_i$ and $x_i\in\text{co}\Lambda_i$, the mapping

$\langle G_i t_i, \eta_i(., x_i)\rangle + S_i(x_i,.) : K \to 2^{Z_i}$ is SIC-DQC;

3) $C_i(x)$ is an u.s.c. mapping with closed values.

Then there exist $\bar{x}_i \in D_i(\bar{x})$ and $\bar{t}_i \in T_i(\bar{x})$ such that

$$\langle G_i \bar{t}_i, \eta_i(y_i, \bar{x}_i)\rangle + S_i(\bar{x}_i, y_i) \subseteq C_i(\bar{x}),\ \forall y_i \in D_i(\bar{x}).$$

Proof. Let $P_i : K \times Y \to 2^{K_i}$ a set-valued mapping defined in Theorem 3.9. We prove that for each $y_i \in K_i$, the set $\{(x,t) \in K \times Y : \langle G_i t_i, \eta_i(y_i, x_i)\rangle \cap S_i(x_i, y_i) \nsubseteq C_i(x)\}$ is open, that is, the set $\{(x,t) \in K \times Y : \langle G_i t_i, \eta_i(y_i, x_i)\rangle \cap S_i(x_i, y_i) \subseteq C_i(x)\}$ is closed. Indeed, let $\{(x^o, t^o)\}$ be a net in $K \times Y$ such that $(x^o, t^o) \to (x^*, t^*)$ and

$$\langle G_i t_i^o, \eta_i(y_i, x_i^o)\rangle \cap S_i(x_i^o, y_i) \subseteq C_i(x^o).$$

We claim that

$$\langle G_i t_i^*, \eta_i(y_i, x_i^*)\rangle \cap S_i(x_i^*, y_i) \subseteq C_i(x^*).$$

To prove this assertion, we can just follow that of Corollary 3.6. Hence, the set $\{(x,t) \in K \times Y : \langle G_i t_i, \eta_i(y_i, x_i)\rangle \cap S_i(x_i, y_i) \nsubseteq C_i(x)\}$ is open. Therefore, all the conditions of Theorem 3.9 are satisfied. Consequently, the assertion of the corollary hold.

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