

An Experimental Analysis of Dynamic Hypotheses About Cognitive Abilities and Achievement From Childhood to Early Adulthood

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This study examined the dynamics of cognitive abilities and academic achievement from childhood to early adulthood. Predictions about time-dependent “coupling” relations between cognition and achievement based on R. B. Cattell’s (1971, 1987) investment hypothesis were evaluated using linear dynamic models applied to longitudinal data ($N = 672$). Contrary to Cattell’s hypothesis, a first set of findings indicated that fluid and crystallized abilities, as defined by the Woodcock–Johnson Psycho-Educational Battery—Revised (WJ–R; R. W. Woodcock & M. B. Johnson, 1989–1990), were not dynamically coupled with each other over time. A second set of findings provided support for the original predictions and indicated that fluid ability was a leading indicator of changes in achievement measures (i.e., quantitative ability and general academic knowledge). The findings of this study suggest that the dynamics of cognitive abilities and academic achievement follow a more complex pattern than that specified by Cattell’s investment hypothesis.

Over a period of 30 years of research, Raymond Cattell developed his investment hypothesis of intelligence, which postulates a complex developmental process of cognitive abilities involving genetic, learning, and environmental influences. This hypothesis, part of Cattell’s theory of fluid and crystallized intelligences (Gf – Gc ; Cattell, 1941, 1943, 1957, 1963, 1971, 1987; Horn & Cattell, 1966, 1967), proposes that in the development of children there is initially—after 2 or 3 years from birth—a single, general cognitive ability that develops with the general maturation of the cortex. This broad ability is associated with genetic factors and neurological functioning and is used by the child in motor, sensory, and rote learning. Because Cattell did not believe that this early ability was linked to any specific habit or brain area, he called it “fluid” ability (Gf).

According to Cattell, a child’s learning rate in tasks requiring complex spatial, numerical, or conceptual relations largely depends on this fluid intelligence, in addition to other factors such as motivation and teaching. Cattell conjectured that through practice and experience, however, children add perceptual, discriminatory, and executive skills to their cognitive repertoire. As such complex abilities are acquired, they attach to particular perceptual and motor areas of the brain and become hardened or “crystallized”

abilities (Gc). The development of these more complex and specialized abilities enables a child to learn and improve in school activities such as reading, writing, and arithmetic. According to Cattell’s hypothesis, then, achievement in school is influenced by both Gf and Gc over time, in addition to various other external factors such as learning opportunities, interest, and motivation.

The relations among Gf , Gc , and academic achievement described by the investment hypothesis were believed to operate within a specific causal framework (Cattell, 1967b, 1987). Such a framework was not considered stable over the life span but rather was presumed to change according to factors such as the individual’s neurological development and years of schooling. Depending on these developmental and environmental factors, the theory postulates different dynamics among Gf , Gc , and academic achievement. For example, Gf and Gc would be closely related in childhood but begin to diverge during late childhood and adolescence, with differences becoming more manifest in adulthood (Horn & Cattell, 1966). Moreover, such dynamics would follow a specific time relation in which “this year’s crystallized ability . . . is a cumulative function of several years’ operation levels of g_f , but last year will be most important—in the case of growing children, but not adults” (Cattell, 1987, pp. 139–140).

Hence the investment hypothesis describes a complex set of developmental lagged relations between cognitive abilities and achievement. This sequence of accumulated influences requires a dynamic framework to evaluate its scientific validity. The purpose of the current study was to apply such a dynamic framework to study the relationship between cognitive abilities and academic achievement from childhood through early adulthood in order to test Cattell’s (1971, 1987) hypothesis.

Supportive Evidence for Gf – Gc Theory and the Investment Hypothesis

There is now abundant research supporting both the structural distinction between fluid and crystallized abilities (Carroll,

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1993, 1997; Horn, 1968, 1988; Horn & Cattell, 1966) and the different trajectories (i.e., kinematics) that these abilities follow over time (Bayley & Oden, 1955; Horn, 1991; Horn & Noll, 1997; Jones & Conrad, 1933; McArdle, Ferrer-Caja, Hamagami, & Woodcock, 2002; Schaie, 1996). Substantial support for the structural components comes from the work by Carroll (1993, 1997). Recent theorizing on cognitive abilities proposes a synthesis of Carroll's three-stratum theory with *Gf-Gc* theory to form CHC theory (for Cattell-Horn-Carroll; see McGrew, 1997). CHC theory suggests a hierarchical organization of cognitive abilities with three strata: general intelligence or *g*, broad cognitive abilities, and narrow cognitive abilities.

Support for the kinematic predictions has been provided in cross-sectional and longitudinal studies, both showing developmental differences across the life span (Baltes & Mayer, 1999; Baltes & Smith, 1997; Horn, 1991; Horn & Noll, 1997; Lindenberger & Baltes, 1997; McArdle et al., 2002; Schaie, 1996). For example, McArdle et al. (2002) described the developmental trajectories for all the cognitive abilities with a double exponential model that represented rise and decline over the life span. When individual curves were compared, large differences were evident in the growth and decline of fluid and crystallized abilities. Whereas the former reached a peak at about age 22, the latter peaked at about age 36. Furthermore, a single general intelligence as a common factor underlying the different ability curves proved to be an unreasonable hypothesis. For any comparison, such a single *g* model yielded a much worse fit than individual separate growth curves, thus representing an overly simplistic view of growth and change over time.

Evidence for the dynamic interrelations (i.e., kinetics) postulated by the investment hypothesis is almost absent. Very few studies have been conducted to investigate interrelations among specific cognitive abilities over time. Moreover, these studies have produced inconsistent findings. We still do not have precise answers to the dynamic questions posited by *Gf-Gc* theory. An early attempt to investigate the investment hypothesis was made by Schmidt and Crano (1974) using a cross-lagged correlation analysis. In line with the hypothesis, their findings indicated that *Gf* was more strongly related to *Gc* over time than *Gc* was to *Gf*. This finding, however, vanished when the researchers adjusted for differences in reliability between both measures. A more direct examination was conducted by McArdle (2001) in a study examining the relationship between the verbal and nonverbal subscales from the Wechsler Intelligence Scale for Children (WISC; Wechsler, 1974) among children measured during the first, second, fourth, and sixth grades. Using analytic models developed to test dynamic hypotheses, McArdle found that nonverbal scores had a negative effect on changes in verbal scores. In contrast, the reversed effects (i.e., from verbal to nonverbal scores) were not perceptible. These findings suggest that, for this sample of children, nonverbal scores led to declines in verbal scores. Under the questionable assumption that the WISC subscales represent *Gf* and *Gc*, these findings were opposite to Cattell's (1987) investment hypothesis.

A recent attempt at testing *Gf-Gc* dynamic hypotheses over the adult life span (ages 16 to 68) was made by McArdle, Hamagami, Meredith, and Bradway (2000). In this study, data on the Wechsler Adult Intelligence Scale (WAIS; Wechsler,

1981) from two longitudinal studies were combined (i.e., the Bradway-McArdle Longitudinal Study [Bradway, 1944; Bradway & Thompson, 1962] and the Berkeley Growth Study [Bayley, 1957]) in order to examine interrelationships among the Block Design, Vocabulary, Digit Span Forward, and Digit Symbol Substitution subtests, presumably representing fluid abilities (*Gf*), crystallized abilities (*Gc*), short-term memory (*Gsm*), and processing speed (*Gs*), respectively. The results indicated that *Gf* followed a general decline, with changes positively influenced by previous *Gs* scores. *Gc* scores increased initially and then flattened, with changes positively influenced by *Gsm*. *Gsm* also showed a general decline and had a positive, yet small, influence from *Gf*. Finally, changes in *Gs* showed a general decline, with positive influences from *Gsm* and negative influences from *Gf* and *Gc*. These results suggested a complex pattern of interrelations among the four cognitive variables and, according to McArdle et al. (2000), "rule out the simple interpretation of a single 'leading indicator of decline'" (p. 68) in this multivariate system.

These findings reinforce the notion of a complex network of interrelations among cognitive abilities that departs from the investment hypothesis. Participants in the McArdle et al. (2000) study, however, were beyond childhood, the period when *Gf* is presumably "invested" and when such an investment is theoretically most influential. It is possible that during childhood, fluid abilities are influential in the rise of other cognitive abilities but that this influence diminishes, or disappears, when *Gf* reaches its peak during early adulthood. It is possible that other functions such as memory take over then as key influences in the maintenance of crystallized abilities or the decline of fluid abilities and processing speed. Studies addressing differences in dynamic influences across different developmental periods are pertinent.

Gf-Gc Theory and Scholastic Achievement

The relationship between cognitive abilities and academic achievement is a central element of the investment hypothesis. Such a relationship is widely recognized (e.g., McArdle & Woodcock, 1998) and has a long history in which researchers have tried to identify the links between cognition and achievement. In some instances, ability and achievement are seen as part of the same dimension (Snow, 1998). According to Cattell (1987), however, fluid intelligence acts to produce school achievement. On the basis of this assumption, Cattell and Butcher (1968) studied the contribution of different ability factors to scholastic achievement. As expected, verbal scores were most predictive of verbal achievement (i.e., word meaning and paragraph meaning), whereas number scores were most predictive of arithmetic achievement (i.e., arithmetic comprehension and arithmetic reasoning). Scores from measures of spatial ability and reasoning, however, were not highly predictive of any aspect of academic achievement.

Scholastic achievement is an important component of *Gf-Gc* theory. According to this theory, the investment of fluid abilities into crystallized abilities occurs extensively during the schooling years, times when individuals acquire the complex abilities needed to learn school activities such as reading, writing, and arithmetic. Scholastic achievement, thus, is strongly related to both *Gf* and *Gc*. Cattell (1967b, 1987) stated

that scholastic achievement is influenced by three sources: (a) concurrent levels of *Gc*, (b) concurrent and historical levels of *Gf*, and (c) concurrent levels of memory and interest. This theoretical postulation did not exclude other external sources of influence such as learning opportunities, interest, and motivation.

Major components of scholastic achievement in *Gf-Gc* theory are reading, writing, and arithmetic, with emphasis also on social studies. This conceptualization is similar to Gustafsson and Balke's (1993) clustering of achievement dimensions or to the classification of achievement factors included in psychoeducational batteries such as the Woodcock-Johnson Psycho-Educational Battery—Revised (WJ-R; Woodcock & Johnson, 1989–1990). Based on *Gf-Gc* theory, the WJ-R recognizes the link between cognition and achievement, the former assumed to bring about the latter. Such a conceptualization implies that measures of cognitive abilities have a predictive value for academic achievement.

Research using WJ-R and WJ-III (Woodcock-Johnson Psycho-Educational Battery—Third Edition; Woodcock, McGrew, & Mather, 2001) data has provided evidence for the predictive value of cognitive abilities for achievement, with specific sources predicting different forms of achievement (Evans, Floyd, McGrew, & Leforgee, 2002; Floyd, Evans, & McGrew, 2003; McGrew, Werder, & Woodcock, 1991; Vanderwood, 1997; Vanderwood, McGrew, Flanagan, & Keith, 2001). For example, Floyd et al. (2003) investigated the relation of cognitive abilities to mathematics achievement for individuals from 6 to 19 years of age using WJ-III data. Their findings indicated that Math Reasoning was moderately related to crystallized abilities, especially after about age 14, but weakly related to fluid abilities. Math Calculations was also poorly related to both fluid and crystallized abilities. With regard to reading, research has indicated that this form of achievement was related to specific cognitive abilities, especially to *Gc* and *Ga* (auditory processing)—but not to a general factor *g*—with differences in such relationships across ages 5 to 18 (Vanderwood, 1997; Vanderwood et al., 2001).

The relationship between cognitive abilities, and between such abilities and achievement, can be influenced by other factors. Cattell (1987) himself described the role of motivation and environmental factors such as home and school. Similarly, other factors may be important to understanding the hypothesized relationships. Ackerman (1996, 1997; Ackerman & Heggestad, 1997), for example, related cognitive abilities (i.e., intelligence as process and intelligence as knowledge) to personality and interests. Schweizer and Koch (2002), on the other hand, proposed a revision of Cattell's theory in which learning mediates the influence of fluid ability on crystallized ability, with a cognitive basis (i.e., processing speed and capacity) underlying the influence of fluid intelligence on learning. Other lines of research favor the idea of a particular cognitive function as the driving force behind age-related changes in other cognitive abilities, including working memory (Swanson, 1999) or a general speed function (Birren, 1974; Salthouse, 1985, 1996). Although these are all pertinent hypotheses, here we focus on those abilities more directly relevant to the investment hypothesis.

Purpose and Hypotheses of the Current Study

In sum, there is a well-recognized relationship among cognitive abilities and between these and scholastic achievement. Evidence for this relationship, however, comes from cross-sectional or other correlational research but is not informative about developmental time-lagged sequences, as postulated by *Gf-Gc* theory. The current study used longitudinal data and dynamic models to examine the relationship between cognitive abilities and academic achievement and to evaluate Cattell's (1987) investment hypothesis as a developmental hypothesis involving processes that unfold over time. It was our goal in this study to identify an appropriate representation of the developmental changes of cognitive abilities and achievement and the time-lagged sequences underlying those changes.

The first question concerns the dynamics of fluid (*Gf*) and crystallized (*Gc*) abilities. We examined whether dynamic relations exist between these two cognitive abilities and, if so, what the leading and lagging indicators in such a cognitive system are and whether there are developmental discontinuities in such dynamics from childhood to early adulthood. In other words, we attempted to identify whether there were age intervals when the dynamics were most influential and others when cognitive abilities followed dynamically independent trajectories. The second question addresses the relations between cognitive abilities and scholastic achievement. We focused on the dynamics that underlie the development of scholastic achievement, the specific cognitive abilities responsible for such dynamics, and the ages when those influences are most apparent. To test Cattell's (1987) notion of developmental investment, we focused on the dynamics—or kinetics—of variables over time (e.g., *Gf* and *Gc* were treated as endogenous forces underlying their changes), not on whether or not they followed parallel trajectories.

Method

Participants

The data used in this study came from participants in the National Growth and Change Study (NGCS; John J. McArdle, principal investigator), an ongoing project focused on the study of cognitive abilities over the life span. The NGCS started from the base of data provided in the WJ-R norming study (Woodcock & Johnson, 1989–1990). From this norming pool ($N = 6,471$), a subsample of individuals ($N = 1,193$) was selected on the basis of a stratified randomized sampling following criteria such as geographic region, demographic density, gender, and ethnicity. These individuals repeated the initial WJ-R battery, with planned variability in the interval between the initial and second testings (i.e., ranging from 0.8 to 10 years, $Mdn = 1.5$ years). The analyses presented in this study involve data from a subsample ($N = 672$; age 20 years at the first measurement occasion) of the retest individuals. Table 1 gives a description of the participants' ages and demographics, in comparison with the norming sample. This table illustrates the resemblance of participants in the longitudinal sample to participants in the norming sample with regard to most demographic measures and age at the first measurement occasion. Because of the age selection, the current sample is younger and has less educational attainment than the norming sample.

Measures

Selected tests of the WJ-R battery (McGrew et al., 1991) were used to compute, as unit-weighted composites and according to WJ-R guide-

Table 1
Demographic Characteristics of the WJ-R Norming Sample and the Current Sample

Characteristic	Statistic	Norming sample (<i>N</i> = 6,471)	Current sample (<i>N</i> = 672)
Age at Time 1 (years)			
2 to 5 years	<i>Mdn</i> (<i>n</i>)	4.3 (816)	4.0 (163)
6 to 10 years	<i>Mdn</i> (<i>n</i>)	8.5 (1,484)	8.0 (188)
11 to 20 years	<i>Mdn</i> (<i>n</i>)	15.8 (2,069)	16.0 (321)
Total	<i>M</i> (<i>SD</i>)	20.3 (18.2)	10.7 (5.6)
	Minimum	1.9	2.0
	Maximum	95.6	20.0
Gender			
Males	<i>n</i> (%)	3,152 (48.7)	341 (50.7)
Females	<i>n</i> (%)	3,317 (51.3)	331 (49.3)
Ethnicity			
White non-Hispanic	<i>n</i> (%)	4,445 (68.7)	404 (60.1)
Black non-Hispanic	<i>n</i> (%)	1,054 (16.3)	149 (22.2)
American Indian	<i>n</i> (%)	71 (1.1)	13 (1.9)
Asian Pacific	<i>n</i> (%)	197 (3.0)	42 (6.3)
Hispanic	<i>n</i> (%)	592 (9.1)	64 (9.5)
Missing	<i>n</i> (%)	112 (1.7)	—
Education (years)	<i>M</i> (<i>SD</i>)	9.11 (4.9)	7.51 (4.4)
	Minimum	0	0
	Maximum	25.0	15.2
Educational attainment			
No high school	<i>n</i> (%)	3,440 (53.2)	408 (60.7)
High school	<i>n</i> (%)	519 (8.0)	15 (2.2)
No college	<i>n</i> (%)	1,183 (18.3)	105 (15.6)
College	<i>n</i> (%)	189 (2.9)	—
Beyond	<i>n</i> (%)	322 (5.0)	—
Missing	<i>n</i> (%)	818 (12.6)	144 (21.4)

Note. WJ-R = Woodcock-Johnson Psycho-Educational Battery—Revised.

lines, scales of cognitive abilities and scholastic achievement. The selected tests (and factors measured) were the following: Analysis Synthesis and Concept Formation (Fluid Reasoning, *Gf*); Oral Vocabulary and Picture Vocabulary (Comprehension-Knowledge, *Gc*); Science, Social Studies, and Humanities (Academic Knowledge, *Gak*); and Applied Math Problems and Calculation (Quantification, *Gq*). All of these scales are reported to have very high internal consistency, ranging from .94 to .95 (McArdle et al., 2002; McGrew et al., 1991). These

scales are presumed to represent broad intellectual ability factors and were used in all analyses.

Description of the Data

Descriptive information for each of the selected WJ-R scales and tests is presented in Tables 2 and 3. All WJ-R tests are measured using a Rasch-based measurement scale. Such measurement construction

Table 2
Descriptive Statistics for the Selected WJ-R Cognitive Factors

Factor	<i>Gf</i> _{<i>t</i>1}	<i>Gf</i> _{<i>t</i>2}	<i>Gc</i> _{<i>t</i>1}	<i>Gc</i> _{<i>t</i>2}	<i>Gak</i> _{<i>t</i>1}	<i>Gak</i> _{<i>t</i>2}	<i>Gq</i> _{<i>t</i>1}	<i>Gq</i> _{<i>t</i>2}
<i>Gf</i> _{<i>t</i>1}	—							
<i>Gf</i> _{<i>t</i>2}	.779	—						
<i>Gc</i> _{<i>t</i>1}	.818	.696	—					
<i>Gc</i> _{<i>t</i>2}	.725	.798	.879	—				
<i>Gak</i> _{<i>t</i>1}	.822	.761	.941	.892	—			
<i>Gak</i> _{<i>t</i>2}	.763	.826	.864	.938	.941	—		
<i>Gq</i> _{<i>t</i>1}	.830	.758	.905	.838	.961	.899	—	
<i>Gq</i> _{<i>t</i>2}	.748	.851	.809	.884	.914	.942	.921	—
<i>N</i>	528	571	530	579	672	633	671	631
<i>M</i>	501.9	507.6	507.2	513.7	492.6	502.5	484.8	498.5
<i>SD</i>	20.6	19.6	28.6	28.7	38.4	32.5	54.7	44.7
Minimum	441.5	441.5	414.0	414.0	392.0	413.0	333.0	360.0
Maximum	553.5	553.5	575.0	586.0	555.7	563.0	584.0	593.0

Note. WJ-R = Woodcock-Johnson Psycho-Educational Battery—Revised; *Gf* = Fluid Reasoning; *Gc* = Crystallized Knowledge; *Gak* = Academic Knowledge; *Gq* = Quantitative Ability; *t*₁ = first measurement occasion; *t*₂ = second measurement occasion.

Table 3
Descriptive Statistics for the Selected WJ-R Tests

Test	AS _{t1}	AS _{t2}	CF _{t1}	CF _{t2}	OV _{t1}	OV _{t2}	PV _{t1}	PV _{t2}	SC _{t1}	SC _{t2}	SS _{t1}	SS _{t2}	HU _{t1}	HU _{t2}	AP _{t1}	AP _{t2}	CA _{t1}	CA _{t2}	
AS _{t1}	—																		
AS _{t2}	.67	—																	
CF _{t1}	.78	.65	—																
CF _{t2}	.68	.79	.74	—															
OV _{t1}	.77	.64	.78	.66	—														
OV _{t2}	.68	.75	.69	.76	.88	—													
PV _{t1}	.48	.25	.59	.21	.84	.64	—												
PV _{t2}	.24	.41	.44	.26	.60	.67	.82	—											
SC _{t1}	.75	.69	.75	.73	.91	.87	.84	.68	—										
SC _{t2}	.68	.76	.67	.75	.81	.89	.61	.76	.92	—									
SS _{t1}	.76	.68	.76	.71	.92	.87	.85	.62	.96	.89	—								
SS _{t2}	.71	.75	.71	.75	.86	.92	.51	.67	.92	.94	.93	—							
HU _{t1}	.74	.68	.75	.72	.90	.86	.83	.62	.94	.88	.95	.91	—						
HU _{t2}	.66	.73	.66	.75	.78	.88	.55	.64	.87	.91	.86	.91	.90	—					
AP _{t1}	.78	.70	.78	.73	.90	.84	.77	.48	.95	.87	.95	.90	.93	.84	—				
AP _{t2}	.72	.81	.70	.80	.81	.88	.66	.62	.91	.92	.90	.93	.89	.89	.92	—			
CA _{t1}	.57	.16	.53	.04	.74	.39	.77	.49	.75	.43	.72	.12	.67	.46	.80	.69	—		
CA _{t2}	.20	.58	.22	.58	.07	.40	.12	.29	.21	.61	-.03	.56	.12	.67	.45	.93	.27	—	
M	502.4	506.9	501.4	508.2	507.2	513.6	519.0	533.8	490.1	500.0	492.3	503.7	495.5	505.6	484.7	498.5	531.0	544.4	
SD	20.45	18.46	23.14	22.84	28.63	28.66	19.19	14.76	39.48	32.88	43.39	37.91	34.34	28.85	54.79	44.71	24.08	20.07	

Note. WJ-R = Woodcock-Johnson Psycho-Educational Battery—Revised; AS = Analysis Synthesis; CF = Concept Formation; OV = Oral Vocabulary; PV = Picture Vocabulary; SC = Science; SS = Social Studies; HU = Humanities; AP = Applied Math Problems; CA = Calculations; *t*₁ = first measurement occasion; *t*₂ = second measurement occasion.

yields a common metric that allows direct comparisons of scores between variables and across ages. Figure 1 includes plots for all the selected WJ-R cognitive and achievement composites in rescaled units (WJ-R scores minus a constant 500, which represents the empirical average score at age 10). In these plots, each line represents an individual's scores across the two occasions of measurement. Circles represent those persons with a single score. Thus, the graphs depict individual trajectories (i.e., change in score from Time 1 to Time 2) as well as the general pattern of growth for the entire sample. These plots display some general similarities in the shape among the various composites, with curves that rise rapidly. But there appear to be some differences in the trajectories in these plots. For example, the curve for *Gf* shows an initial rapid rise with a quick deceleration and some descendant individual trajectories after ages 18 and 20. The curve for *Gc*, in contrast, shows a general rapid rise across all ages. Similarly, although the distribution of *Gc* scores shows large interindividual variability, *Gf* shows less dispersion.

Linear Dynamic Models

Examining interrelations underlying the growth and change of several variables over time requires models that can capture such a dynamic feature. One such approach is a model based on latent difference scores (LDS; Ferrer & McArdle, 2003; McArdle, 2001; McArdle & Hamagami, 2001). This LDS model combines features of latent growth curve analysis and cross-lagged regression together with factor analysis models of change and latent difference scores (McArdle & Nesselroade, 1994; Nesselroade & Cable, 1974). Starting with a classical true score model, the observed scores for a variable *Y* can be separated into true scores *y* and measurement error *e_y*, as follows:

$$Y_{[t]n} = y_{[t]n} + e_{y[t]n} \tag{1}$$

From this model, differences between scores can be written as differences between true scores. Thus, change can be expressed as a function of the current state *y*_{[*t*]n} minus the previous state *y*_{[*t*-1]n}, as in

$$\Delta y_{[t]n} = y_{[t]n} - y_{[t-1]n}, \tag{2}$$

and, alternatively, the current state can be expressed as a function of the previous state plus change, as in

$$y_{[t]n} = y_{[t-1]n} + \Delta y_{[t]n}. \tag{3}$$

From this equation, the trajectory for a variable at any time *t* can be written as its initial state plus all latent changes accumulated up to that point, as in

$$y_{[t]n} = y_{0n} + \left(\sum_{i=1}^t \Delta y_{[i]n} \right). \tag{4}$$

And a model for the latent changes can now be written. For example, the equations for change in a bivariate model can each be expressed as a function of three components, as in

$$\begin{aligned} \Delta y_{[t]n} &= \alpha_y \cdot y_{sn} + \beta_y \cdot y_{[t-1]n} + \gamma_y \cdot x_{[t-1]n} \quad \text{and} \\ \Delta x_{[t]n} &= \alpha_x \cdot x_{sn} + \beta_x \cdot x_{[t-1]n} + \gamma_x \cdot y_{[t-1]n}, \end{aligned} \tag{5}$$

where α for each variable is a coefficient associated with the slopes *y_{sn}* and *x_{sn}*, β is a self-feedback coefficient, representing the effect of itself at the previous state on the change, and γ is a coupling coefficient, representing the effect of the other variable at the previous state on the change. A path diagram of this specification is depicted in Figure 2. This figure presents a system of two variables *Y* and *X*, measured at *t* occasions (i.e., one for each year in our analyses), with latent intercepts (*y*₀ and *x*₀) and latent slopes (*y_s* and *x_s*) and covariances among them ($\sigma_{y[t], x[t]}$). According to this model, at each occasion a latent variable is modeled that represents change in the true scores for each variable ($\Delta y_{[t]}$ and $\Delta x_{[t]}$). These latent changes are then modeled as a function of three components: (a) a linear slope, α ; (b) the scores on the same variable at the previous occasion, β ; and (c) the scores

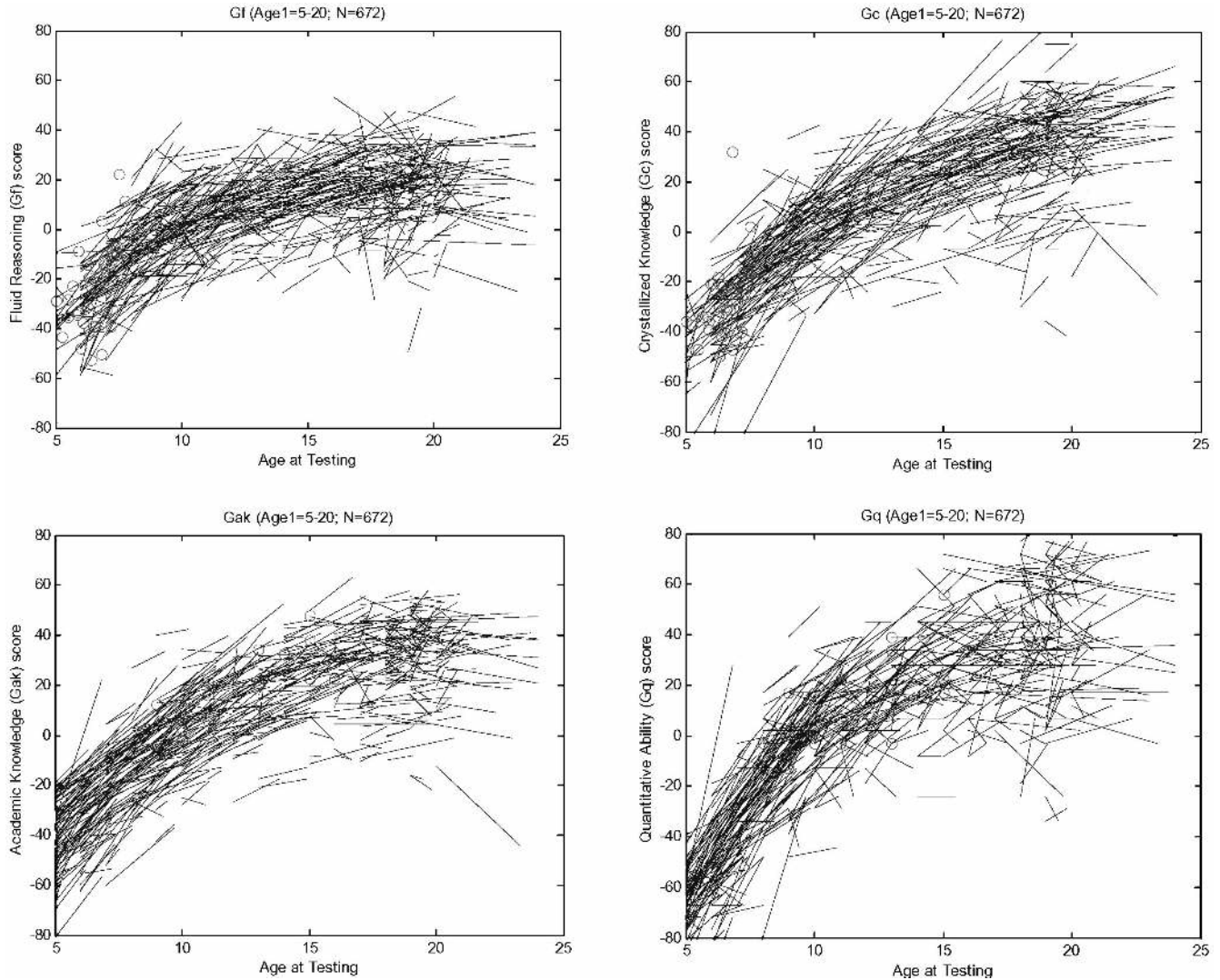


Figure 1. Individual longitudinal plots of WJ-R broad cognitive and achievement factors. All values are rescaled scores from which a constant (= 500; representing the average for a 10-year-old individual) has been subtracted. Ages are in years. WJ-R = Woodcock-Johnson Psycho-Educational Battery—Revised.

on the other variable at the previous occasion, γ .¹ This last component, the coupling parameter, represents forces from one variable that lead to changes in another variable. This is a reasonable approximation to Cattell's (1987) notion of investment, which hypothesizes influences that have specific directions and develop over time—mainly due to the previous year, with accumulation over time. Although these coupling parameters are linear, they can generate nonlinear effects, as they depend on the values of the variable that they affect, and because such values may change over time, the resulting overall trajectories can be highly nonlinear. Whereas alternative models exist that include other forms of relationships between variables (e.g., correlation of slopes; see Ferrer & McArdle, 2003), a more compelling evaluation of Cattell's theory requires models that focus on dynamics.

Linear Extrapolation of Data Segments

Given the persons' different ages and varying measurement intervals (ranging from 0.8 to 10 years), the data were rescaled in order to find a constant measurement interval among the persons and to build a longitudinal data set suited for the dynamic analyses. For this reason, a linear

¹ In order to test this model against alternative hypotheses, several assumptions need to be set, including the following: (a) The effects of the LDS model apply directly to the true scores, but the effects on the observed scores are only indirect; (b) the interval of time is discrete and constant across measurements; (c) to simplify integration, the LDS model is deterministic, without disturbances associated with the latent changes; and (d) the LDS model accounts for interindividual differences. This LDS bivariate model can be compared with competing hypotheses of bivariate change using standard structural equation modeling (SEM) techniques. Such alternatives can include, for example, a model specifying coupling effects in one direction only, such as from variable x to y ($\gamma_y \neq 0$; $\gamma_x = 0$), or a model hypothesizing that changes in the variables are independent of each other ($\gamma_y = 0$; $\gamma_x = 0$). Assuming multivariate normality, the differences in the likelihood functions among these models approximate a chi-square distribution that allows evaluating in probabilistic terms. For more technical details on the LDS models, see McArdle (2001) and McArdle and Hamagami (2001). For a comparison between the LDS model and other longitudinal models, see Ferrer and McArdle (2003).

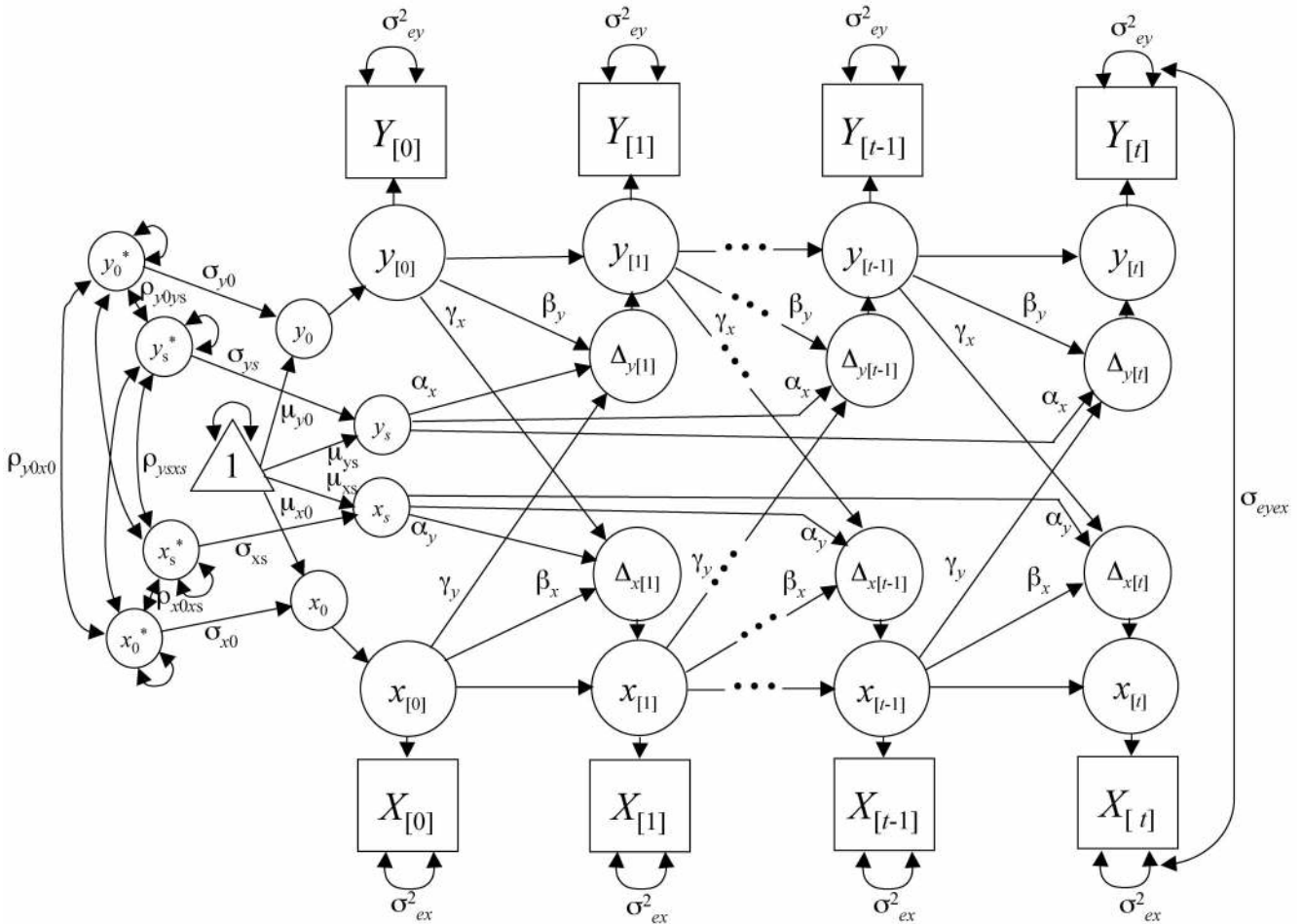


Figure 2. A path diagram of a bivariate latent difference score model. $Y_{[t]}$ and $X_{[t]}$ = observed scores at time t ; $y_{[t]}$ and $x_{[t]}$ = latent true scores at time t ; $\Delta y_{[t]}$ and $\Delta x_{[t]}$ = latent changes at time t ; y_0 and x_0 = initial level; y_s and x_s = slopes; y_0^* and x_0^* = standardized level scores; y_s^* and x_s^* = standardized slope scores; triangle = constant (= 1); α = slope parameter (= 1); β = self-feedback parameter; γ = coupling parameter; μ_{y0} and μ_{x0} = mean of level scores; μ_{ys} and μ_{xs} = mean of slope scores; σ_{y0} and σ_{x0} = deviation of level scores; σ_{ys} and σ_{xs} = deviation of slope scores; ρ_{y0x0} = correlation between the level and slope scores; ρ_{y0s} and ρ_{x0s} = correlation between the two initial levels; ρ_{ysxs} = correlation between the two slopes; σ_{ey}^2 and σ_{ex}^2 = variance of residual scores; σ_{eyex} = covariance between the residual scores. All paths without a specified parameter are fixed to 1.

extrapolation method was employed, and the individuals' scores at the first and second measurement occasions were shifted to their corresponding scores at the nearest age point. Thus, a longitudinal data set was built that included all ages from 5 to 24 years (5 to 20 years at the first occasion of measurement) with discrete intervals of 1 year.²

To handle incomplete data, we used maximum-likelihood analysis of raw data with the Mx software program (Neale, Boker, Xie, & Maes, 1999). This approach computes maximum-likelihood estimates using the available raw data. In this procedure, each observation, or vector of observations with similar structure, is treated as a group. From these groups, the program then generates means and covariance matrices and computes a raw maximum-likelihood function.

Results

Bivariate Dynamic Analyses

A first set of analyses was performed to examine the dynamics of Gf and Gc and achievement. These analyses involved three bivariate models that tested hypotheses about the investment of Gf

on each of the other variables (i.e., Gc , Gak , Gq). The results from these analyses are reported in Table 4. The first two columns present the estimates for the dynamics of Gf and Gc . These estimates include an initial mean for each variable ($\mu_{0Gf} = -38.9$;

² For each person, the data at both occasions were extrapolated using the following formula:

$$x_{it}' = x_{it} + [(age_{it}' - age_{it}) * \Delta x_{it}],$$

where x_{it}' is the rescaled score at time t for person i , x_{it} is the original score at time t , age_{it}' is the rounded age at time t , age_{it} is the original age at time t , and Δx_{it} is the yearly rate of change (i.e., for each person at his or her time span). Thus, for example, if person i had a score x at age $m = 15.4$ years, x was rescaled into the hypothetical value of x' at age $m' = 15.0$ years. This transformation facilitated the programming and interpretation of dynamic models by constructing a constant age interval of 1 year and by ensuring that the "convergence" was approximately equal across ages.

Table 4
Estimates From Bivariate Latent Difference Score Models Between Fluid Reasoning (Gf) and Crystallized Knowledge (Gc), Academic Knowledge (Gak), and Quantitative Ability (Gq)

Parameter	<i>Gf</i> - <i>Gc</i>		<i>Gf</i> - <i>Gak</i>		<i>Gf</i> - <i>Gq</i>	
Loading α	1 ^a	1 ^a	1 ^a	1 ^a	1 ^a	1 ^a
Proportion β	-0.163	-0.131	0.137	-0.406	0.302	-0.923
Coupling γ	-0.026 ^b	0.027 ^b	-0.239	0.385	-0.254	1.473
Initial mean μ_0	-38.94	-44.02	-35.88	-38.84	-35.39	-61.16
Slope mean μ_s	4.18	6.04	4.76	6.56	4.16	8.17
Initial deviation σ_0	14.89	15.69	12.28	14.68	12.61	6.87
Slope deviation σ_s	2.48	2.43	2.59	3.83	2.96	11.10
Error deviation σ_e	7.27	6.31	7.43	4.42	7.14	7.58
Correlations						
ρ_{0s}	.223	-.093	-.023	.400	-.776	-.053
$\rho_{y0:xs0} \rho_{y0:xs}$.779	-.177	.621	-.310	.465	-.738
$\rho_{ys:xs0} \rho_{ys:xs}$.304	.687	.591	.900	-.452	.900
Goodness of fit						
-2 log-likelihood	15,919		15,143		15,794	
No. of parameters	20		20		20	
χ^2/df (RMSEA ^c)						
Vs. γ only model ^d	-1/1 (.00)		-19/1 (.18)		-17/1 (.17)	
Vs. γ_{Gf} only model	-1/1 (.00)		-40/1 (.26)		-38/1 (.25)	
Vs. no γ model	-1/2 (.00)		-40/1 (.18)		-38/1 (.18)	
Vs. common factor	-1,508/10 (.51)		-1,810/10 (.56)		-1,718/10 (.55)	

Note. $N_s = 573-577$. Number of data points = 2,018-2,070. Age at Time 1 = 5-20 years. Age at Time 2 = 5-24 years. The parameters are defined at $t = 0$ when $Age_t = 5$ years and for $\Delta_t = 1$ year. All parameters are raw maximum-likelihood estimates fitted using Mx.
^a Indicates a fixed parameter. ^b Indicates a parameter whose 95% confidence intervals contain zero. ^c Root-mean-square error of approximation of the fit difference. ^d Indicates the coupling of either *Gc*, *Gak*, or *Gq*.

$\mu_{0Gc} = -44.0$), which represents the average starting point (i.e., raw score minus constant 500) at age 5 and shows substantial individual variation ($\sigma_{0Gf} = 14.9$; $\sigma_{0Gc} = 15.7$). The growth estimates indicate that changes in both variables (yearly changes from ages 5 to 24) are influenced by similar sources: a positive constant slope ($\mu_{sGf} = 4.18$; $\mu_{sGc} = 6.04$), which is larger and slightly more uniform for *Gc* ($\sigma_{sGf} = 2.48$; $\sigma_{sGc} = 2.43$), and a negative auto-proportion (or self-feedback), with larger values for *Gf* ($\beta_{Gf} = -0.163$; $\beta_{Gc} = -0.131$). The coupling parameters, however, are virtually zero for both *Gf* and *Gc* ($\gamma_{Gf} \approx 0$; $\gamma_{Gc} \approx 0$). Despite a substantial correlation between their slopes ($\rho = .69$), the couplings indicate that there are not detectable lagged interrelations (i.e., associations with a time-lagged structure) between both variables over time.

These results are estimates from a model that includes both coupling parameters. The importance of these parameters can be further examined by fitting alternative models in which such parameters are removed and the resulting likelihood functions are compared. Results from this procedure are presented at the bottom of Table 4. Fit comparisons (i.e., chi-square in relation to degrees of freedom) indicate that removing either one or both coupling parameters does not alter the fit of the model ($\chi^2/df = 1/1$; $\chi^2/df = 1/1$; $\chi^2/df = 1/2$; RMSEA [root-mean-square error of approximation] = .00), suggesting that they are not needed to account for the covariation in these data. Contrary to the investment hypothesis, there is not a perceptible influence of *Gf* on *Gc* over time.³

The estimates from the two other bivariate models suggest that *Gf* and academic achievement (i.e., *Gak* and *Gq*) follow a different pattern of dynamics than the one described for *Gf* and *Gc*. These estimates (in the third and fourth columns and in the fifth and sixth columns of Table 4, respectively) indicate non-trivial lagged influences for all the variables. The coupling parameters from *Gf* to academic achievement are both positive,

³ The fit of all the models examined here was evaluated in relative terms. That is, models were compared against each other to judge which one was more tenable in representing the structure of the data. In addition, a model could be compared against an unrestricted form of itself (i.e., with all possible covariations among the variables). This is in fact the baseline model against which most programs compare the fit of any given model before providing a so-called "absolute" measure of fit. Because of the use of individual raw data, Mx does not compute this model and, instead, gives a likelihood function. Such a baseline model, however, could be computed by fitting an unrestricted model to each group of individuals with the same pattern of incomplete data and then adding all the resulting likelihood functions. This approach, nonetheless, will fail when the number of variables exceeds the number of subjects within each group, as the sample size is too small to estimate a positive definite observed covariance matrix. In the bivariate models, there were 98 groups (i.e., patterns of incomplete data), and only 49 of these included 3 or more subjects. In the multivariate models with four variables, there were 127 groups, and only 39 had 5 or more subjects. Under these conditions, thus, unrestricted baseline models could not be computed using such an approach.

with moderate values for Gak ($\gamma_{Gak} = 0.385$) and large values for Gq ($\gamma_{Gq} = 1.47$). The coupling parameters on Gf , in contrast, are both smaller and negative ($\gamma_{Gf} = -0.239$, -0.254). As previously, the fit of these models involving both coupling effects can be compared with more restrictive hypotheses. For both sets of analyses, removing either one or both parameters degrades the fit considerably, suggesting that all the effects are contributing to the time-based covariation of these data. Following previous terminology (Ferrer & McArdle, 2003; McArdle, 2001; McArdle & Hamagami, 2001), we can characterize the dynamic relationship among these variables by saying that Gf and academic achievement (i.e., Gak and Gq) are both leading and lagging indicators of their interrelated changes. In line with the investment hypothesis, there is a positive influence from Gf to the changes in academic achievement.

The results from these bivariate analyses can be translated into change equations. These equations, specified here for each dynamic pair, can be written as follows:

$$\begin{aligned} \Delta Gf_{[t]} &= 4.2 \pm [2.5] - .16 Gf_{[t-1]} - .03 Gc_{[t-1]} \\ \Delta Gc_{[t]} &= 6.0 \pm [2.4] - .13 Gc_{[t-1]} + .03 Gf_{[t-1]}, \\ \Delta Gf_{[t]} &= 4.8 \pm [2.6] + .14 Gf_{[t-1]} - .24 Gak_{[t-1]} \\ \Delta Gak_{[t]} &= 6.6 \pm [3.8] - .41 Gak_{[t-1]} + .39 Gf_{[t-1]}, \text{ and} \\ \Delta Gf_{[t]} &= 4.2 \pm [3.0] + .30 Gf_{[t-1]} - .25 Gq_{[t-1]} \\ \Delta Gq_{[t]} &= 8.2 \pm [11.1] - .92 Gq_{[t-1]} + 1.5 Gf_{[t-1]}. \end{aligned}$$

These equations represent yearly changes in each variable (from 5 to 24 years) as a function of itself and the other variable. Such changes are based on all the components in the equation. For example, Gak is expected to increase by 6.6 (± 3.8) units per year. These increases, however, are slowed down by the Gak scores of the previous year (i.e., by a constant coefficient of $-.41$) and accelerated by the Gf scores of the previous year (i.e., by a constant coefficient of $.39$).

Plotting Dynamic Trajectories

As a result of the combined influences over time, each variable will describe a trajectory that is a direct function of the slope, auto-proportion, and coupling effects. Because each variable in the bivariate system changes over time, the resulting trajectories will be different depending on the initial conditions and the changes of both variables. This interplay between the variables' initial point and the resulting trajectories is depicted in Figures 3, 4, and 5. Here, the expected mean for each variable is plotted as a function of three initial values of itself (i.e., 1.96 standard deviations above the mean, the overall mean, and 1.96 standard deviations below the mean). Moreover, for each of these conditions, three lines are included within each panel that represent the trajectory of the variable as a function of different initial values for the other variable (i.e., $+1.96 SD$, mean, and $-1.96 SD$ correspond to the upper, middle, and lower lines, respectively). This plot illustrates how the initial conditions alter the expected trajectory for a variable in each bivariate system.

The expected trajectories for Gc , displayed in Figure 3, appear to be slightly steeper during childhood, when the initial scores are lower, but average out during adolescence. Given the lack of

coupling between Gf and Gc , these trajectories are not affected by initial scores in Gf . For Gak (see Figure 4), the dependence of the trajectories on initial scores is more pronounced. The scores reach slightly higher values under conditions of lower initial scores on Gak and higher initial scores on Gf . For Gq (see Figure 5), the dependence on initial values of Gf is more evident. Regardless of its initial score, the trajectory for Gq is steeper and reaches higher values for higher initial scores of Gf .

An alternative way to display these dynamic relations is a vector field. This approach, long used in physics and dynamics, has recently been applied to psychology as a way to visualize dynamic relationships (Boker & McArdle, 1995, in press). Figure 6 depicts vector fields for each bivariate system. These fields represent the projections in time (i.e., yearly changes) for different combinations of scores in both variables. That is, for a given pair of scores, the arrow indicates the expected changes at the next occasion. Such expected changes will be positive, negative, or neutral for each variable depending on the direction of the arrow in relation to the y - and x -axes. To interpret the dynamics within the range of the data, an ellipsoid is included that encompasses 95% of the data. Because all the variables are scaled in Rasch units, the units in the axes represent similar distances and the projected changes are directly comparable across ages and variables.

For the Gf - Gc system, the vector field suggests that changes depend on the current scores. For low and medium scores, both variables are expected to change positively and rapidly. These changes will become weaker as scores increase. For high scores in both variables, Gc is expected to increase, whereas Gf is expected to remain flat. For very high scores in both variables, Gc will remain constant, whereas Gf will decrease. The vector fields for Gf - Gak and Gf - Gq are different from the Gf - Gc plot and similar to each other. Here, the most apparent influences are for conditions of high Gf scores (i.e., close to the upper line of the ellipsoid). Under these conditions, positive changes are expected for all variables. But such changes dissipate for large current scores. These predictions reproduce the results from the numerical analyses. Given the obtained parameter estimates, changes will be positive and large when previous scores are high for Gf (they will exert a large positive effect on themselves and on the other variable) and low for Gak and Gq (they will exert a small negative effect on themselves and on Gf). And this pattern of dynamics between Gf and achievement is expected to be more pronounced for Gq than for Gak .

Developmental Discontinuities in Dynamics

The findings from these analyses pertain to the full age span from 5 to 24 years. One important question is whether the dynamics found in these analyses are invariant across this age span or whether there are age periods when such dynamics express a different pattern. This question is of particular relevance here because the investment hypothesis highlights the school years as the period with the strongest investment from fluid abilities into crystallized abilities and achievement. One possible approach for identifying periods with such discontinuities is to relax some of the constraints of the LDS models

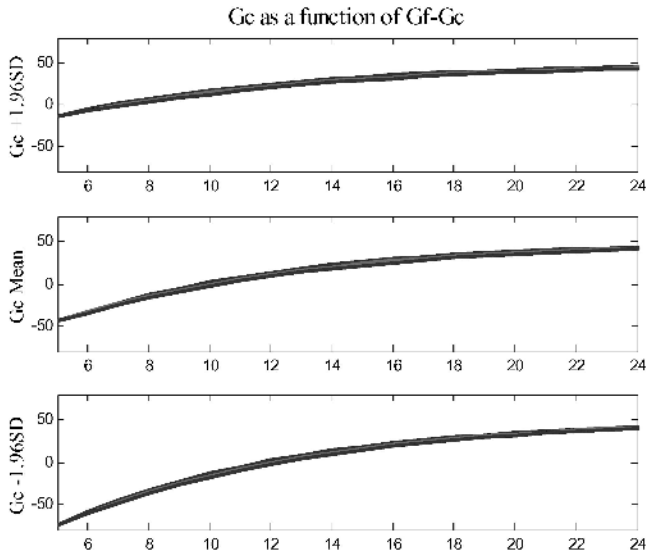


Figure 3. Expected latent trajectories for G_c as a function of the G_f - G_c system. G_c = Crystallized Knowledge; G_f = Fluid Reasoning.

(i.e., auto-proportion, coupling parameters over time).⁴ In a new set of analyses, we allowed the dynamic parameters to take different values across three age segments. To be consistent with our previous research (McArdle et al., 2002), we selected three age periods: 5–10 years, 11–15 years, and 16–24 years. For each bivariate system, a spline model was then examined in which the auto-proportion and coupling parameters were relaxed across the three segments. This approach allowed modeling age trajectories continuously through the 5- to 24-year age span but with different age functions across the three

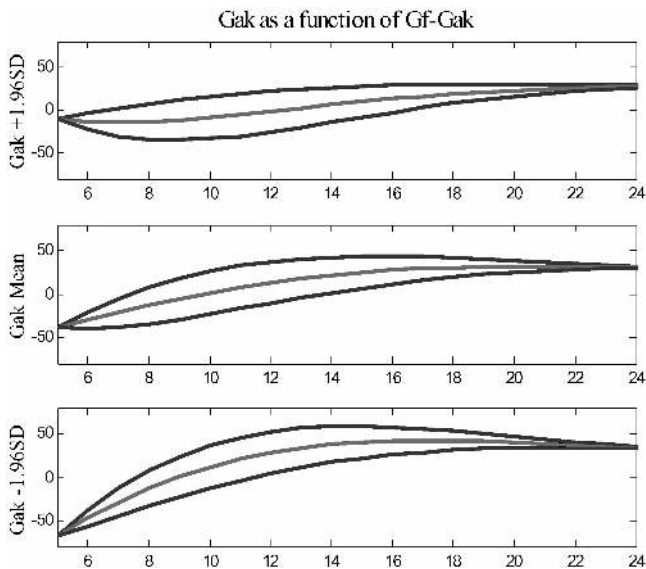


Figure 4. Expected latent trajectories for G_{ak} as a function of the G_f - G_{ak} system. Within each panel, the middle line represents the trajectory for G_{ak} as a function of average starting points in G_f , the upper line represents the trajectory for G_{ak} as a function of G_f starting points of 1.96 standard deviations above the mean, and the lower line represents the trajectory for G_{ak} as a function of G_f starting points of 1.96 standard deviations below the mean. G_{ak} = Academic Knowledge; G_f = Fluid Reasoning.

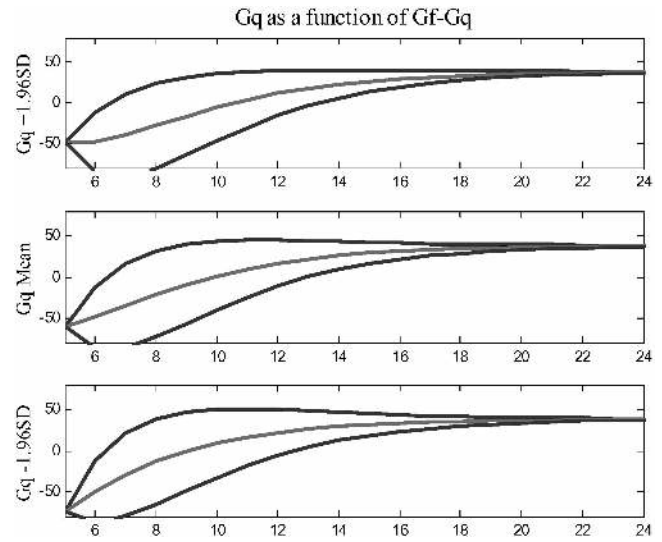


Figure 5. Expected latent trajectories for G_q as a function of the G_f - G_q system. Within each panel, the middle line represents the trajectory for G_q as a function of average starting points in G_f , the upper line represents the trajectory for G_q as a function of G_f starting points of 1.96 standard deviations above the mean, and the lower line represents the trajectory for G_q as a function of G_f starting points of 1.96 standard deviations below the mean. G_q = Quantitative Ability; G_f = Fluid Reasoning.

age segments. This model yielded an overall intercept and slope (with individual differences) and different dynamic parameters across the segments. Thus, it allowed a nonlinear modeling via three piecewise linear age segments. Such a modification was intended to capture nonlinearities in the dynamics of the variables over time.

For G_f and G_c , this model did not improve the fit of the original model ($\chi^2/df = 15/8$), suggesting that the dynamics of G_f and G_c are imperceptible and invariant across the 5- to 24-year age span. The spline models for G_f and G_{ak} , however, yielded different results. In this case, allowing different auto-proportions and couplings across groups improved the fit by a nontrivial amount ($\chi^2/df = 23/8$). The estimates of this model included positive coupling effects on G_{ak} that alternated in intensity across age groups ($\gamma_{G_{ak}} = .31, .43, \text{ and } .20$) and negative but decreasing coupling effects on G_f ($\gamma_{G_f} = -.38, -.29, \text{ and } -.14$). Similarly,

⁴ Another approach would be to create different age groups and examine the dynamics for each group separately. This approach was initially explored and then disregarded for yielding unstable and inconsistent results. Three groups were created based on the participants' age at the first measurement occasion: (a) 5–10 years, (b) 11–15 years, and (c) 16–20 years. The results revealed inconsistency of the dynamics across age groups. Such discrepancies could be due to at least two sources. First, it is possible that there were indeed different dynamics within each of the groups. Alternatively, it is possible that the groups covered a short age span in which dynamics were hard to capture. For example, if for a particular age segment the trajectory of a variable is flat, as is the case for G_f in the last group (16–25 years), identifying dynamics in that period may be difficult. Moreover, any projection based on the results for that group may be at best misleading or simply wrong. Results will be more accurate when the analyses cover the whole age span to which inferences are made, even when using very low data density. For example, Hamagami and McArdle (2001) found that it is possible to recover population parameters representing an age span of 20 years even with only two random data points, as long as these points cover the periods in which dynamics take place.

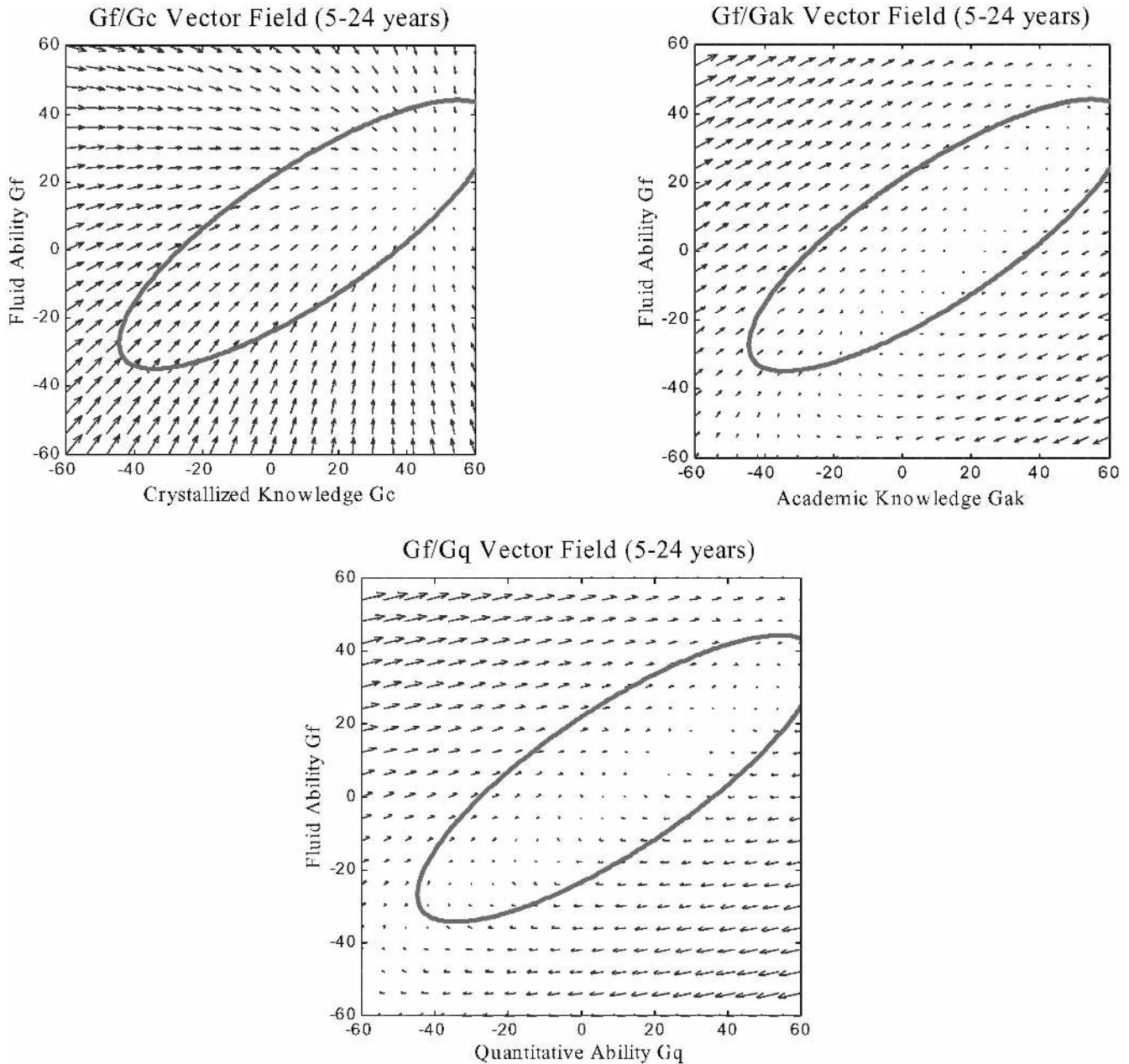


Figure 6. Vector fields for each bivariate dynamic system. The ellipsoids encompass 95% of the data.

the spline model for G_f and G_q also improved the fit of the original model ($\chi^2/df = 24/8$). The parameter estimates of this model indicated positive coupling effects on G_q that decreased in value over the age periods ($\gamma_{G_q} = 1.71, 1.20,$ and 1.08) and negative coupling effects on G_f that also decreased in value across the age segments ($\gamma_{G_f} = -.41, -.19,$ and $-.18$). The results from these spline models confirm and add to the findings from the bivariate models. Contrary to the investment hypothesis, no influences from G_f on G_c are perceptible from 5 to 24 years. In line with the hypothesis, however, a positive influence from G_f to the changes in academic achievement is apparent that is strongest during childhood and early adolescence.

Multivariate Analyses

The last set of analyses included all four variables and represented a closer examination of the investment hypothesis. As laid out by Cattell (1987), the investment hypothesis is a multivariate hypothesis that involves time-ordered relations between cognitive abilities (i.e., G_f and G_c) and achievement. To examine this multivariate prediction, we fitted various models that tested different sets of hypotheses, the idea being to approximate the overall multivariate hypotheses.

The results from these analyses are presented in Table 5. The first model includes all dynamic relations among the four variables

Table 5
Goodness-of-Fit Indices of the Multivariate Dynamic Hypotheses Models

Model	-2 log-likelihood	Parameters	χ^2/df^a	RMSEA ^b
Full dynamics (All $\gamma \neq 0$)	29,887	64		
No dynamics (All $\gamma = 0$)	30,046	52	159/12	.26
Common factor	30,666	16	779/48	.17
Extreme hypotheses (1 variable as the only "leading force")				
Model A ₁ (γ^*Gf)	29,994	55	107/9	.14
Model A ₂ (γ^*Gc)	30,005	55	118/9	.18
Model A ₃ (γ^*Gak)	30,046	55	159/9	.17
Model A ₄ (γ^*Gq)	30,043	55	156/9	.17
Leading hypotheses (1 variable is not "leading")				
Model B ₁ ($\gamma^*Gf = 0$)	29,947	61	60/3	.35
Model B ₂ ($\gamma^*Gc = 0$)	29,955	61	68/3	.19
Model B ₃ ($\gamma^*Gak = 0$)	29,969	61	82/3	.22
Model B ₄ ($\gamma^*Gq = 0$)	29,907	61	20/3	.10
Lagging hypotheses (1 variable is not "lagging")				
Model C ₁ ($\gamma Gf = 0$)	29,949	61	62/3	.19
Model C ₂ ($\gamma Gc = 0$)	29,926	61	39/3	.15
Model C ₃ ($\gamma Gak = 0$)	29,993	61	106/3	.25
Model C ₄ ($\gamma Gq = 0$)	29,978	61	91/3	.23
"Gf as Mediating" hypothesis				
Model D ₁ ($Gf \rightarrow Gc \rightarrow Gak + Gq$)	29,989	57	100/7	.15

Note. $N = 573$. Number of data points = 4,036. Age at Time 1 = 5–20 years. Age at Time 2 = 5–24 years. All likelihood functions are estimated using raw maximum likelihood in Mx. Gf = Fluid Ability; Gc = Crystallized Knowledge; Gak = Academic Knowledge; Gq = Quantitative Ability.

^aAll fit comparisons are in relation to the full model with all coupling parameters estimated. ^bRoot-mean-square error of approximation of the fit difference.

and represents a very relaxed hypothesis of dynamics. This full dynamics model was then compared with more restrictive hypotheses to determine whether there is an evident pattern of relations in this system of abilities over time and whether such a pattern is in line with Cattell's (1987) investment hypothesis. The first two more restrictive models include a model without couplings (i.e., no dynamics exist in the system) and a common factor model (i.e., all changes are structured through a general cognitive g factor). Fit comparison among these models (i.e., chi-square in relation to degrees of freedom) reveals that the latter two models are not tenable ($\chi^2/df = 159/12$ and $779/48$, respectively). The covariation in these data is best explained by a model that treats the four variables as separate entities but with lagged interrelations over time. Although these results are in line with the investment hypothesis, they suggest a complex pattern of dynamics among cognitive abilities and achievement. To identify a more specific pattern of relations among the variables, we tested more restrictive hypotheses.

In the models labeled "extreme hypotheses," one variable is the only leading force in the system (i.e., all other couplings are fixed to zero). These models test whether changes in all variables are influenced by only one variable. An extreme interpretation of the investment hypothesis is that fluid ability alone would underlie the changes in crystallized abilities and achievement. Fit comparison indicates that none of these models fits better than the full model but that the loss in fit is smallest for Gf . That is, although these

hypotheses do not appear very plausible, in line with Cattell's (1987) predictions, the hypothesis of Gf as the only leading indicator is the most plausible. The "leading hypotheses" examine whether one variable is not a leading indicator in this system. Each of these models also leads to fit loss, which is smallest for Gq and largest for Gak . That is, among these not very tenable models, the hypothesis of Gq as not being a leading force in this system is the most tenable of the four. Finally, the "lagging hypotheses" test whether one variable is not a lagging indicator in this system (i.e., no influences from other variables). All these models yield a loss in fit, which appears to be largest for Gak and Gq , suggesting that, in line with Cattell's predictions, these are the most dynamically dependent variables in this system. Finally, the last model in Table 5 tests the hypothesis of Gc mediating the relation between Gf and achievement. This model is a representation of Cattell's hypothesis expressing influences from Gf to Gc that, in turn, lead to changes in Gak and Gq . This model also results in a substantial loss of fit ($\chi^2/df = 100/7$), indicating that it is not the most parsimonious account for these data. In sum, the results from these analyses do not reject the investment hypothesis but suggest a complex pattern of interrelations between cognitive abilities and achievement. This pattern is best described by four separate entities with lagged interrelations over time.

The parameter estimates from the full model are presented in Table 6. For each variable, estimates are obtained for its initial mean and deviation, slope and deviation, auto-proportion, and

Table 6
Estimates From a Full Multivariate Latent Difference Score Model for Fluid Reasoning (Gf), Crystallized Knowledge (Gc), Academic Knowledge (Gak), and Quantitative Ability (Gq)

Parameter	Gf	Gc	Gak	Gq		
Loading α	1 ^a	1 ^a	1 ^a	1 ^a		
Proportion β	.259	-.131	.422	-.898		
Coupling γ						
Gf →	—	-0.061 (11)	0.304 (35)	1.180 (110)		
Gc →	-0.097 (18)	—	-0.376 (44)	0.125 (37)		
Gak →	0.220 (55)	0.514 (19)	—	0.046 (4)		
Gq →	-0.315 (8)	-0.111 (19)	-0.260 (22)	—		
Initial mean μ_0	-34.94	-41.79	-38.62	-62.12		
Slope mean μ_s	3.60	4.96	4.81	7.46		
Initial deviation σ_0	13.83	16.69	13.96	12.29		
Slope deviation σ_s	3.00	3.36	3.39	9.40		
Error deviation σ_e	7.15	5.91	3.94	7.57		
Correlations						
ρ_{0s}	.223	-.093	-.023	.400	-.776	-.053
$\rho_{y0,x0} \rho_{y0,xs}$.779	-.177	.621	-.310	.465	-.738
$\rho_{ys,x0} \rho_{ys,xs}$.304	.687	.591	.900	-.452	.900
Goodness of fit						
-2 log-likelihood					29,887	
No. of parameters					64	

Note. $N = 573$. Number of data points = 4,036. Age at Time 1 = 5–20 years. Age at Time 2 = 5–24 years. The parameters are defined at $t = 0$, when Age_{*t*} = 5 and for $\Delta_t = 1$ year. All parameters are raw maximum-likelihood estimates fitted using Mx. Numbers in parentheses indicate the change in misfit when removing the parameter. All correlations among levels and slopes are estimated but not shown here.

^a Indicates a fixed parameter.

couplings from each of the other variables. The resulting trajectory for each variable, thus, will be a function of these combined forces at each moment. To determine the accuracy of the coupling effects, we removed each parameter one at a time and evaluated the resulting loss of fit (given in parentheses in Table 6). Although all these effects are perceptible, the influences from the cognitive variables on achievement appear more critical. For example, the coupling of *Gf* to *Gc* is very small ($\gamma = -.061$), and its removal yields a minor decrease in fit ($\chi^2/df = 11/1$). In contrast, the coupling from *Gf* to *Gq* is large ($\gamma = 1.18$), and its removal results in a substantial misfit ($\chi^2/df = 110/1$).

The estimates from this full model are also displayed in Figure 7 (following McArdle et al., 2000). This figure is a depiction of the dynamics underlying these cognitive and achievement variables. This depiction conveys the complexity of such dynamics, with multiple forces operating simultaneously. One possible way to extract information from this system is to focus on the changes in one particular variable. For example, *Gq* scores start at -62.1 at age 5, with 68% of individuals starting between -74.4 and -49.8 (-62.1 ± 12.3). Each year, individuals increase their *Gq* scores by about 7.5 units, although with large variability (± 9.4). Furthermore, such changes are influenced by *Gq* scores the year before ($\beta_{Gq} = -.90$) and also by the previous scores on all other variables ($\gamma = 1.18, 0.13, \text{ and } 0.05$, from *Gf*, *Gc*, and *Gak*, respectively). Another way to extract information is to focus on the forces that a particular variable exerts in the system. For example, *Gf* has negligible influences on *Gc* ($\gamma_{Gc} = -0.06$), moderate and positive

influences on *Gak* ($\gamma_{Gc} = 0.30$), and strong influences on *Gq* ($\gamma_{Gc} = 1.18$).

Discussion

Summary of Findings

In this study we examined the relationship between cognitive abilities and academic achievement in the context of Cattell's (1987) investment theory. We applied dynamic models to longitudinal data from individuals measured at various ages from childhood to early adulthood. The findings from these analyses indicate that some of Cattell's predictions are supported, whereas other predictions appear less tenable. In general, the results suggest that the dynamics of cognitive abilities and academic achievement follow a more complex pattern than the one specified by a simplistic interpretation of the investment hypothesis.

In particular, our findings indicate that (a) there are no detectable coupling relations between fluid and crystallized abilities across the selected age range; (b) fluid ability is a positive leading indicator of changes in academic achievement, with stronger influences on quantitative abilities than on academic knowledge; (c) age differences exist in the dynamics of *Gf* and achievement, the effects being stronger during childhood and early adolescence; and (d) structuring the changes of these cognitive and achievement functions as part of a common factor is unreasonable because such changes are best described as following different trajectories, although being related in their sequential dynamics.

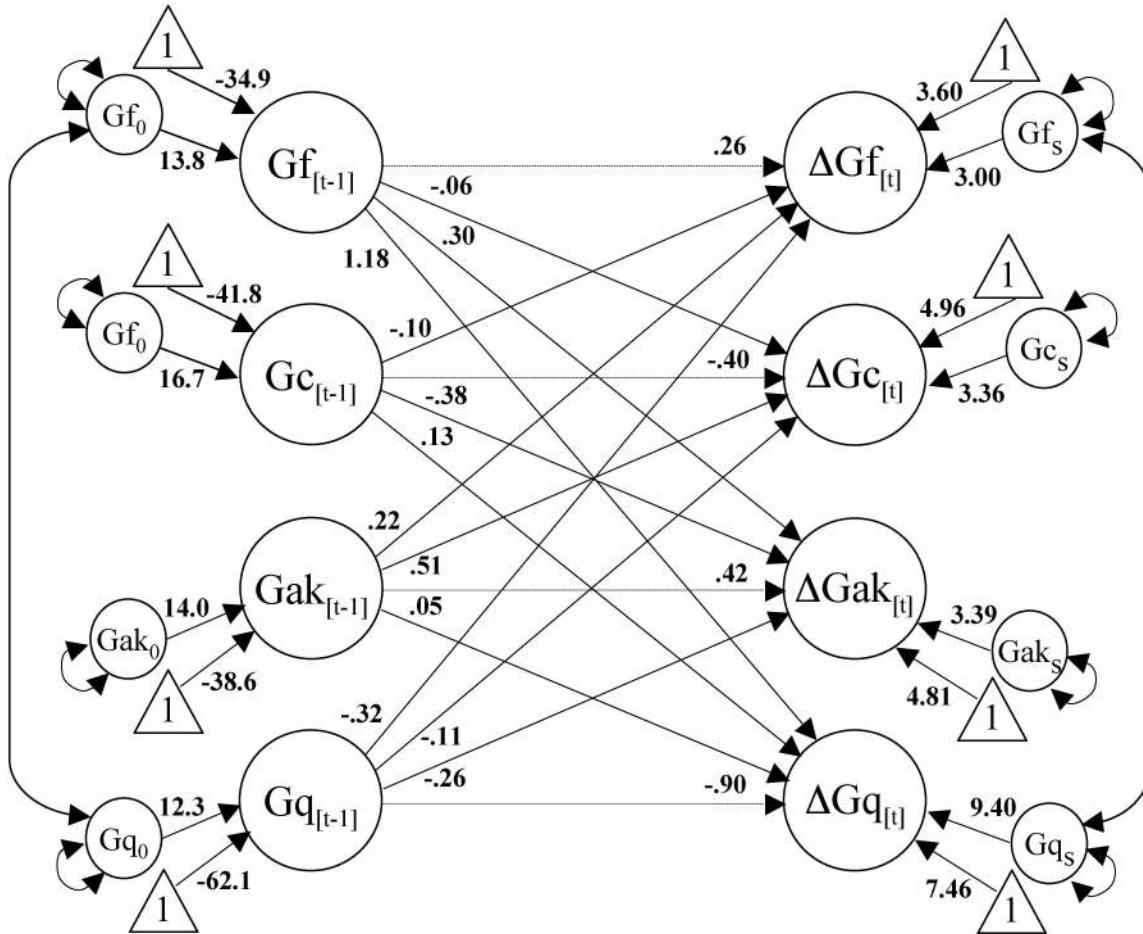


Figure 7. Dynamics of a multivariate cognitive-achievement system. The figure displays many constants for graphic simplicity. The model and programming of it require only one constant. $Gf_{[t-1]}$ = scores at time $t - 1$; $\Delta Gf_{[t]}$ = changes at time t ; triangle = constant (= 1); path from constant to score $[t - 1]$ = initial level; path from constant to change $[t]$ = slope; Gf_0 = deviation of initial level scores; Gf_s = deviation of slope scores. Gf = Fluid Ability; Gc = Crystallized Knowledge; Gak = Academic Knowledge; Gq = Quantitative Ability.

Discussion of Major Hypotheses

Two major predictions from Cattell’s (1987) investment hypothesis were examined in this study, namely, the influence of fluid abilities on crystallized abilities and the interplay between cognitive abilities and academic achievement. According to Cattell’s hypothesis, fluid abilities are invested in the development of crystallized abilities, particularly during the school years. The acquisition of these more complex abilities, in turn, enables the individual to learn and improve in school activities and to enhance his or her academic achievement. Findings from this study deviate from the former prediction but are in line with the latter one. Across ages 5 to 24 years, there were no time-dependent relations between fluid and crystallized abilities as measured by the WJ-R. Changes in crystallized abilities over time were independent of fluid ability levels at previous years.

The lack of a time-dependent interrelation between Gf and Gc is a departure from the investment hypothesis, which places this relationship at the center of its premises. There are various ways to interpret this finding. First, it is possible that the Gf and Gc factors in this study, as measured by the WJ-R, do not exactly match

Cattell’s and Horn’s theoretical constructs. In the studies leading up to the investment hypothesis, Cattell and Horn (Cattell, 1963, 1967a, 1967b; Horn, 1965; Horn & Cattell, 1966, 1967) found that both the fluid and crystallized abilities factors contained several abilities, and these may not have been completely represented by the data in this study. It is also possible that the relation between fluid and crystallized abilities is mediated by other factors (e.g., interest, personality; see Ackerman, 1996, 1997; Ackerman & Heggestad, 1997), and these were not part of this study.

Another potential way to interpret the lack of interrelations between Gf and Gc over time is to refer to what might be called “coercion to the biosocial norm.” Cattell (1971, 1987; Cattell & Butcher, 1968) mentioned this possibility when he suggested that individuals with high levels of fluid abilities do not fully develop their potential in school. On the contrary, he asserted, their creativity is hindered by the need to accommodate the low demands of the educational system. It is possible that individuals with high levels of Gf in this study perceived that they did not need to work hard to get by in school. For example, a student might know how to earn rewards and good grades in school without working very hard, thus not fully developing his or

her potential. This possibility, however, is a speculation that cannot be tested with the available data.

Alternatively, the departure of these findings from the investment hypothesis can be explained by questioning the validity of the theoretical predictions. The development of the investment hypothesis, and of some current examinations of it (Schweizer & Koch, 2002), was based on cross-sectional studies and relied on correlation coefficients to examine concurrent relations among variables. Thus, despite its conceptual appeal and its language invoking time-ordered relations, the investment theory has not undergone tests that parallel its time-ordered predictions. Although studies have shown *Gf*–*Gc* correlations in line with Cattell's (1987) predictions, such concurrent correlations do not capture the time-lagged sequences among variables that the theory proposes. Furthermore, the few studies that focused on such time-lagged sequences produced findings not in line with the theory. For example, McArdle and colleagues found a small negative coupling of *Gf* on *Gc* with WISC data from children 6 to 11 years of age (McArdle, 2001) and negligible couplings with WAIS data from adults 16 to 68 years of age (McArdle et al., 2000). Using different measures, this study found support for the need for additional tests of Cattell's hypothesis.

As predicted by the second hypothesis of this study, fluid ability had a positive influence on academic achievement, with stronger effects on quantitative ability than on academic knowledge. Higher levels of fluid ability were related to larger increases in social studies, science, the humanities and, especially, applied mathematics and calculation problems scores. This positive influence was reflected in larger positive changes on achievement over time. As postulated by Cattell (1987), these findings can be said to represent an investment of *Gf* on achievement. As also predicted by Cattell, these findings indicate that such an influence was stronger during the school years. For both academic achievement and quantitative ability, the strongest investment appeared during childhood and early adolescence. Developmental differences in the relation between fluid and crystallized abilities were also found by Li et al. (2004), who reported stronger correlations in childhood and adulthood using cross-sectional data.

In contrast to this unequivocal relation, the role of crystallized abilities in academic achievement appeared more ambiguous. *Gc* had a small and positive influence on quantitative ability changes but a negative and larger influence on academic knowledge changes. The former influence was in the direction of Cattell's (1987) hypothesis; the latter influence departs from it. In our analyses, *Gc* led to positive changes in math calculations and applied problems. But it was also related to declines in social studies, science, and the humanities. The coupling from *Gc* to quantitative ability aligns with research showing moderate relations between *Gc* and the variables comprising *Gq*. For example, using the WJ–III norming sample, Floyd et al. (2003) found that *Gc* had a small predictive value for math calculations and a moderate predictive value for math reasoning (the role of *Gf* was small for math reasoning and very small for math calculations). The negative coupling from *Gc* to academic achievement is counterintuitive, and a clear explanation is not obvious.

Theoretical Implications

The results from this study have theoretical implications that deserve discussion. Certain findings depart from the investment

hypothesis and question the validity of Cattell's (1987) theory. Is fluid ability invested in crystallized abilities? From our analyses, the answer seems to be negative. Considering the difficulty in measuring the relevant constructs properly, however, and the not always clear differences between abilities and achievement (e.g., Snow, 1998)—ambiguity that Cattell himself recognized—the answer is less unequivocal. For example, one could argue that our indicators of academic achievement are indeed measures of crystallized abilities. After all, they represent what one knows. At times, Cattell (1987) described crystallized abilities as a cluster of vocabulary ability, numerical ability, and other abilities that are part of the school curriculum. If so, our vocabulary measure was merely one indicator of a broader crystallized abilities factor, and fluid ability was invested in other domains of this factor (i.e., academic knowledge and quantitative ability). It seems reasonable to think that different people invest their abilities in different areas (areas that they like, think they are better at, are more exposed to, etc.).

The results from our analyses, however, indicate that *Gf* relates differently to *Gc* than to *Gak* or *Gq*, suggesting nontrivial differences between *Gc* and the achievement measures. In addition, a test conducted with *Gc*, *Gak*, and *Gq* as indicators of a single achievement construct yielded a positive moderate coupling from *Gf* to achievement. Although positive, this single estimate obscured possible differences among the three achievement measures, and the model fit was not better than the fit from the separate analysis. A parallel argument can be made about the covariance typically found between *Gq* and *Gf* (e.g., *Gq* sometimes loads on the second-stratum *Gf*, in the CHC theory's framework). Again, the analyses reported here indicate that considering both measures as part of the same factor is substantially less tenable than treating both measures as separate—yet dynamically related—entities. Similarly, it has been suggested that *Gf* may not be entirely distinguishable from *g*. Here we examined *g* as a common factor underlying the changes in all constructs. Our analyses suggest that this is not a reasonable model, but further examination may be needed, especially to clarify the relationship between *Gf* and *g*.

One explicit premise in Cattell's (1987) original hypothesis is the role of *Gf* as the leader source of investment, particularly during the schooling years, and the role of crystallized and achievement measures as outcomes of investment. Our findings are in line with this theoretical claim. Not only does *Gf* influence academic knowledge and quantitative abilities, but a model without that influence is not tenable. Furthermore, *Gf* is the most plausible single leading indicator (i.e., exerting influences) and a very unlikely lagging indicator (i.e., receiving influences) in this system of variables. On the contrary, academic knowledge and quantitative ability appear to be the least likely leaders and the most likely lagging indicators. These relations, together with the discontinuities identified in our analyses, suggest, at least for the age period examined here, a developmental process in which *Gf* leads to positive changes in achievement the following year, and this influence is stronger during childhood and early adolescence. A relevant implication of this process concerns the neurophysiological mechanisms that underlie the development of fluid ability and, possibly, its dynamics with other cognitive abilities. Such mechanisms might be related to myelination, changes in brain volume, neuron firing, or the density of dopamine receptors.

Methodological Issues

Several methodological issues in this article deserve clarification. One concerns the density of the observations. The data in this study contained two scores from individuals who ranged in age from 5 to 24 years. Each person was assessed twice, yet inferences were made as if everybody had been measured every year from 5 to 24 years. This can be described as a severe case of incomplete data. However, the age and lag interval overlaps allowed the examination of these data using an accelerated design approach (Bell, 1953; McArdle et al., 2002).

Few software programs and algorithms are available that optimize functions under conditions of data incompleteness similar to the ones in this study. Here, parameters were estimated using raw maximum likelihood for continuous data, available in the Mx program (Neale et al., 1999). This procedure is an extension of a multigroup approach but computes twice the negative log-likelihood of the data for each observation. Thus, each observation (or, rather, each vector of observations with similar structure) is treated as a group from which means and covariance matrices are created. To converge and estimate parameters accurately, however, this method needed the data from individuals at all ages. When the models were fitted to specific age groups, optimization problems emerged, and in some cases, confidence boundaries could not be estimated. It seems that the current models work—and accurately estimate parameters of dynamics—when the covered age period is long enough and, thus, likely to contain dynamics. Similar findings were reported in a simulation study addressing this issue (Hamagami & McArdle, 2001). In that study, two random data points per person were enough to recover parameters from a population with an age span of 20 years, as long as the model covered an age range in which dynamics were observable.⁵

As developed and commonly applied, the dynamic models fitted here assume invariance over time (i.e., the same parameter estimates apply through all occasions). To capture possible developmental differences, an approach based on spline dynamics was proposed and applied to the data. This approach allowed trajectories to be piecewise linear over different age segments and to vary across segments. Such a modification was intended to capture nonlinearities in the couplings between variables over time. For some variables, this method revealed distinct dynamics operating at different ages. These findings suggest that this specification may be useful when attempting to identify discontinuities in the dynamics. The use of this method brings its own difficulties associated with the estimation and interpretation of the parameters. For example, it would be important to know how many time intervals per segment are needed to estimate new parameters accurately. Similarly, it seems critical to ask how tenable it is to assume that transition across segments is invariant across subjects (as in McArdle et al., 2002). Finally, other approaches, such as the use of a dampening parameter, could be used to detect a diminishing effect in the couplings between variables in a more parsimonious way (e.g., Boker, 2001; Boker & Nesselroade, 2002). Although appropriate, these questions do not depart from the typical issues addressed in standard spline regression models. In any case, the preliminary application of these spline dynamic models seems promising and deserves further investigation.

Future Research Directions

There are different ways in which this study can be extended to further explore the questions raised. One important aspect not addressed here is the learning context, both at school and at home. For example, exposure to books and other didactic material at home can help a child develop vocabulary and reading skills even prior to schooling. We intended to examine this contribution, already discussed by Cattell (1971, 1987), using measures of maternal education. Unfortunately, the amount of available data on this measure was too small (< 20%), and hence these analyses were limited.

This study used cognitive abilities as the only predictors of achievement. Although cognitive abilities are the central components in Cattell's (1987) theory, he also considered physiological, genetic, developmental, and social factors as mechanisms underlying the investment hypothesis. According to Cattell, motivation plays a key role in the acquisition and development of academic achievement (Cattell, 1971, 1987; Cattell & Butcher, 1968). In addition to having high levels of fluid abilities, one needs to be interested in learning in order for the cognitive abilities to become invested in, and enhance, achievement. Similarly, Ackerman's work (1996, 1997; Ackerman & Heggestad, 1997) highlights the relationship between cognitive abilities, personality, and interest. Such a relationship could also be investigated in relation to gender. There seem to be some gender differences in the development of quantitative abilities that justify a formal comparison of the ability-achievement dynamics across gender.

Another consideration for future research concerns the density of the data. More observations are needed to accurately capture dynamics and to identify developmental discontinuities in such dynamics. But these observations need to come at the ages when the dynamics take place. According to Cattell (1987), the investment of fluid abilities occurs primarily during the school years. This was true in this study for the achievement variables but not for crystallized abilities. It is possible that the investment of fluid abilities in crystallized abilities happened before the initial age considered here (i.e., 5 years). In Cattell's view, fluid ability emerges at about 2 or 3 years of age. Perhaps it is before schooling occurs that fluid ability's investment into crystallized abilities is strongest. This issue could be addressed in future studies by increasing observations during the early years. Such a consideration, however, is suggested not without reservation, given the difficulty of assessing fluid ability among very young children.

⁵ To evaluate the precision of the original results, we conducted bootstrap analyses through resampling from the data. The bootstrap analyses consisted of the following steps: (a) resample of 120 random samples of $N = 573$ with replacement; (b) analysis of the resulting 120 samples using LDS models for $Gf-Gc$, $Gf-Gak$, and $Gf-Gq$; (c) accumulation of the different parameter estimates from the repeated analyses; and (d) construction of empirical confidence intervals for the different parameters. The same sequence with a resample of 150 random samples was followed for a bootstrap analysis of the full multivariate LDS model. The bootstrap analyses yielded estimates that were remarkably similar to the estimates from the original analyses and had very low standard errors. For example, the differences in the beta and gamma parameters for the bivariate analyses appeared only in the second or third decimal places, and the differences in the mean slopes were negligible. In sum, systematic differences between the estimates from the two analyses were not evident, and thus these results give confidence in the precision of the findings with the original sample.

In sum, in this study we examined the relationship between cognitive abilities and academic achievement described in Cattell's (1987) investment theory. We used a dynamic modeling approach to test hypotheses about time-dependent relationships. Our findings depict an intricate configuration of relationships between cognitive abilities and achievement, with discontinuities over time but with some patterns of relations in line with theoretical predictions. We believe the application of dynamic modeling is critical to empirically evaluating complex developmental theories such as Cattell's. We believe our analyses represent a first step in that direction.

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