1-1-1961

An experimental analysis of some variables of minimax theory.

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AN EXPERIMENTAL ANALYSIS OF SOME VARIABLES OF MINIMAX THEORY

BRAYER 1961

AN EXPERIMENTAL ANALYSIS OF SOME VARIABLES OF MINIMAX THEORY

by

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M.A., University of Utah, 1956

Thesis Submitted in Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

University of Massachusetts, Amherst

October, 1961

ACKNOWLEDGMENTS

The author wishes to express appreciation to Drs. David W. Lewit, Jerome L. Myers, Ernest Dzendolet, and James M. Ferrigno for their suggestions, criticisms, assistance, and seemingly inexhaustible patience and guidance.

October 21, 1961

Approved by

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INTRODUCTION

In recent years there has been an increasing use of game theory in psychological research. Suppes and Atkinson (1960) have demonstrated the similarities between game theory and the Estes Stimulus Sampling model often employed for predicting and interpreting two-choice learning data. Other psychologists, including Deutsch (1958), Hoffman, Festinger, and Lawrence (1954), and Thibaut and Kelley (1959), have employed games in discussing and investigating various social structures and processes.

In their book <u>Theory of Games and Economic Behavior</u>, Von Neumann and Morgenstern (1944) mathematically proved that for certain games a strategy exists which would assure players a minimal gain or loss. This proof was based on the economist's concept of "expected utility" which may be defined as the attractiveness of the consequences of a behavior to the individual.

Below is an example of a 3 x 3 two-person game. Both players, A and B, have three alternative choices (rows and columns respectively). The combination of their choices determines the outcome of an event (cell). If player A were to choose alternative a_1 and player B, b_1, event a_1b_1 would occur and player A would win 11 utiles and player B would lose 11 utiles. Likewise, if player A were to choose alternative a_3 and player B, b_2, event a_3b_2 would occur and player

A would lose 4 utiles and player B would win 4 utiles. This is called a zero-sum game since the amount one player wins, the other player loses.

If player A were to choose alternative a_1 in the game below, he could win 1 or 2 utiles, depending on what column player B selected. Player A's poorest outcome for row a_1 is therefore a 1 utile gain. Similarly, his poorest outcome for rows 2 and 3 respectively are -12 and -4 utiles. Player B's poorest outcomes for columns b_1, b_2, and b_3, are -11, -1, and -23 utiles. Therefore player A can assure himself of winning at least 1 utile by choosing alternative a_1. This is called his maximin alternative because it maximizes his minimum gain. Player B can assure himself of losing no more than 1 utile by playing alternative b_2. This is called his minimax alternative because it minimizes his maximum loss. The combination of these choices determines cell a_1b_2 which yields the payoff of 1 utile to A and the loss of 1 utile to B. Thus this game contains a saddle point, a cell which minimizes one player's loss while at the same time maximizing the other's minimum gain. Such a game is called "Strictly Determined."

		Player B			
		b_1	b_2	b_3	Maximin Outcomes (Rows)
	a_1	2	1	1	1*
Player A	a_2	-12	-7	23	-12
	a_3	11	-4	8	-4
Minimax Outcomes (Columns)		-11	-1**	-23	

*Largest Row Minimum = Maximin
**Smallest Column Maximum = Minimax

Little research has been conducted on the above class of games. The bulk of decision making research has dealt with games which permit some degree of cooperation. Thibaut and Kelley (1959), for example, have attempted to translate characteristics of small groups such as power, dependence, and status into the reward-cost language of game theory. They have done this by structuring the reward-cost matrix in such a way as to give one person varying control over the rewards of the other. Similarly, Wilson (1960) has investigated forms of social control by varying the structure and payoffs of two-person games. Hoffman, Festinger, and Lawrence (1954) have studied the tendency of members within a group to compare their performance with that of other members in competitive bargaining situations by examining the effect of peerage, and importance of task upon the formation of coalitions in three-person games. Deutsch (1958) and more recently Solomon (1960) employed two-person games to investigate interpersonal trust.

Lieberman (1958, 1959) has studied game behavior in more competitive situations which conformed to the model outlined above. He employed both 2 x 2 and 3 x 3 strictly determined games and found that Ss did learn to play a minimax strategy. In the latter study, pairs of Ss played a single game for 200 trials. In this situation both Ss' minimax strategy was to always play choice (row, column) 3. The question arises as to whether the Ss learned the concept of a minimax solution which might be transferred to other similar games or merely learned that choosing the bottom row in this particular game yielded the greatest reward, i.e., simple discrimination or "position habit."

Other investigators (Estes, 1957; Atkinson and Suppes, 1958) have reported that Ss did not behave in accord with predictions based on the game model. However, these investigations can not be considered adequate tests of the predictive power of the model in that in neither case were the assumptions of the model strictly satisfied. In the Estes study the payoff in a 2 x 2 situation was merely an acknowledgment of whether the player was right or wrong. Furthermore, the payoffs for any combination of choices were not certain or even defined for the players, but rather a probability. That is, if player one selected choice 1 and player two selected choice 2, player one would be told he was correct 50% of the time and player two would be told he was correct 50% of the time. In the Atkinson and Suppes study the players were not aware of the payoff matrix, nor were they directly informed of the responses of the other player.

Estes (1957) and Atkinson and Suppes (1958) have argued that the game model is not a behavior theory since it does not describe how behavior is modified through the game experience of the individual. Luce and Raiffa in their book Games and Decisions (1957) have cautioned that game theory is not descriptive of behavior, but is a normative theory. It does not describe how individuals play, it suggests a method of play for the achievement of certain goals. However, the question of whether or not Ss do choose a minimax strategy or under what conditions they approximate such behavior is an empirical one, one which has not been adequately investigated in the studies cited above. The present study deals with this question.

The Model

The model under investigation in this study can be described by the following statements:[1]

1) Each player is presented with a finite number of alternatives.

2) The games are by nature zero-sum.

3) The games are strictly determined.

4) Each player knows the alternatives available both to him and to his opponent, and he knows the outcome depends upon these choices, i.e., he knows the game matric and its functions.

5) The outcomes of the games are "certain," i.e., the outcome for any combination of opponents' choices is exactly represented in the game matrix.

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1. Von Neumann and Morgenstern employed the concept of utility as cell payoffs. In this study the payoffs are poker chips which are transferable for money. Thus this study can not be considered a strict test of the Von Neumann and Morgenstern theory.

6) It is assumed that each player has a preference ordering over these outcomes, and he knows his opponent's preference pattern for the outcomes.

Variables to be Considered

A person's strategic decision is not solely a function of the game matrix. It seems reasonable to assume, and Lieberman's (1958, 1959) data would indicate that such a decision would also be a function, among other things, of 1) an opponent's strategy, 2) an individual's expectation of his opponent's strategy, and 3) the value of the game.[2] The purpose of the present study is to investigate the effects of these three variables on the use of minimax and other strategies in games consistent with the model described above.

Opponent's Strategy

When playing competitive games, a player's decision is seldom based upon the structure or nature of the game alone. Explicitly or implicitly, the player may make some assumption about how his opponent is playing.

In the Lieberman studies (1958, 1959) discussed above, two Ss were required to play against each other. Lieberman did not control either strategy to determine how persons

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2. The value of a strictly determined zero-sum game is defined as the magnitude of the payoff in the saddle point cell.

would play against specific experimentally defined strategies. In the present study, Ss play against one of two experimentally defined strategies; random or minimax. They are also provided with three distinct choices, each structured so as to provide a logical alternative strategy *a priori*; a minimax choice, a choice in which the average of the payoffs in that row is the greatest of the three alternative rows, and a row which contains the cell with the highest payoff in the matrix.

When playing against a minimax strategy Ss should learn to play their own minimax. Two factors would lead to this expectation. First, if Ss play anything but a minimax alternative against a minimax opponent they will lose. Second, if Ss are able to note what E plays they will note that he always plays his minimax and their logical counter choice is their own minimax alternative.

Those Ss who play against a random opponent will attain a net gain by choosing either the minimax or the high average row. Since the latter is more profitable, it is predicted that this group will choose the high average row more often than any other. However, since there is an alternative (minimax) which offers some gains, the high average should not be as often chosen by this group as the minimax should be by Ss playing against a minimax strategist. Furthermore, Ss playing against a minimax strategist should adopt their strategy more rapidly and utilize it more consistently over trials. This prediction is based on the assumption that Ss will be influenced by the knowledge of their opponent's choices, an assumption supported by various sequential choice studies (e.g., Anderson, 1960). In the case of the Ss who play against a minimax opponent, knowledge of the opponent's choices should facilitate the development of a minimax strategy. In the case of the Ss who play against a random opponent, such knowledge should hinder the development of any consistent strategy.

Expectation of Opponent's Strategy

It is reasonable to assume that an individual's game strategy is in part a function of the strategy he thinks his opponent is employing. There is much psychological evidence that demonstrates that a person behaves in accordance with his expectations about other people. Hyman (1955), for example, indicates that responses during interviews are influenced by the interviewee's concept of the interviewer. Hovland, Janis, and Kelley (1953) have shown that the qualifications and perceived intentions of a communicator affect the acceptance of his communications. Neimark and Rosenberg (1959) have shown that when a discriminative stimulus is thought to be produced by another S in a two-choice situation, Ss' learning is retarded more than if it is thought to be merely a warning signal.

In order to test the affect of expectation upon choices in the game situation, instructions are used to induce a particular set. Ss are told either that their opponent is rational or that his choices are purely random. Combined with the two strategies actually played against him, this provides four instruction-strategy groups; rational-rational, rational-random, random-rational, and random-random.

Instructions would not be expected to influence Ss' playing against a consistent minimax opponent as much as they would Ss' playing against a random opponent. It would seem that if Ss are able to recognize a consistent minimax strategy, as suggested above, instructions would have little effect. However, when the opponent is not obvious, such as in the random conditions, instructions should have more influence on a S's game behavior.

Value

The value of Lieberman's game (Lieberman, 1959) was zero. From post-session interviews Lieberman found that some Ss claimed that they were willing to gamble on non-optimal strategies because they stood to gain nothing by playing their minimax. It seems that with low value games (e.g., Lieberman's zero-value) the utility of a S's choice of play is not determined merely by the expected payoff of that choice, but is a function of other factors such as winning an improbably high amount and the utility of gambling itself. It is therefore possible that in higher value games where the assured payoff of a minimax choice is relatively greater, the utility of the non-monetary factors become relatively less functional, thus motivating S to play his more conservative minimax choice. To test for this effect, the games used in this study will include a range of ten values.

Summary

This study is an attempt to investigate the proposition that a person's choices in a game situation are a function of his espectations of his opponent's choices and also the value of the game. The S's expectation of his opponent's choices are in turn a function of the instructions concerning the opponent and S's observation of his opponent's choices.

METHOD

Apparatus

The apparatus consisted of a 10 x 15 x 4 inch cardboard box containing a 3 x 3 display of lights which indicated game outcomes to S and E; 2 sets of 3 switches (1 set each for S and E) with which to indicate choices of S and E; 2 8 x 11 inch cardboard shields to prevent the players from seeing their opponent's selections before the play was made; and a pack of 270 game matrices on translucent tracing paper. The games were similar in structure; all being strictly determined and generated from the basic game below:

Player B (Experimenter)

		b_1	b_2	b_3
	a_1	11	-7	8
Player A	a_2	1	1*	2
	a_3	-10	-7	21

* = saddle point

The rows and columns of the basic game were permuted to yield 9 similar games. In permuting rows and columns care was taken that the saddle point fell once, and only once, in each cell of the matrix. A constant of 1 was successively added to each cell of the 9 permuted matrices until the values of the games ranged from 1 to 10. This yielded 9 x 10 or 90 similarly structured games. Each game was repeated three times during the experimental session yielding a full series of $9 \times 10 \times 3$ or 270 games.

The games were constructed so as to afford each S three types of pure strategies which on an a priori basis seemed to have high attraction for Ss: a minimax strategy; a high-average payoff strategy where the mean of the three cells of a particular row was considerably higher than the mean of either of the other two rows; and a row which contained the highest single cell value of the matrix. Since at least one cell in each of the S's rows yielded a higher payoff than a cell in the same column of the other rows, none of the S's rows could be logically eliminated on the basis that it consistently yielded lower payoffs. The three pure strategies were equally distributed over all three rows.

Procedure

Upon entering the experimental room, S was seated at a table opposite E. Between them on the table was the box containing the 3×3 display of lights. In front of each player was the panel of three toggle switches and shield. The S was then given \$1.50 worth of poker chips with which to play the games with the understanding that at the end of the session he could exchange whatever chips he had accumulated for money, minus \$1.00 of the original stake.

Instructions for the particular condition were read to

S as he followed along on his own copy.[3] The instructions for all conditions were identical except that for two conditions (rational-rational and rational-random) Ss were told that inasmuch as E had to pay them each time they won, he would play in such a way as to reduce their winnings. For the other two conditions (random-random and random-rational) Ss were told that E's choices were completely random and had been selected ahead of time.

Each of 100 Ss played E in a series of 270 games, 25 Ss under each of the four conditions outlined above. E placed a sheet of translucent tracing paper containing a game matrix on the box in front of the players. S and E then examined the game and indicated their choice of play by throwing the appropriate toggle switch. S's task was to choose one of three possible rows labeled a_1, a_2, or a_3. E's task was to choose one of three columns labeled b_1, b_2, or b_3. When both decisions had been made and the appropriate switches thrown, the light in the box corresponding to that cell in the game matrix which was the intersection of S's row and E's column lighted up. The number of poker chips indicated by the number in the activated cell was then given to or taken from S. S's response was recorded by E and the next game placed in position on the game box. This procedure was repeated through all 270 games.

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3. Instructions are presented in Appendix A.

Questionnaire[4]

After playing the games, Ss were interviewed by E. The interview was based on a questionnaire designed to attempt to ascertain what strategies S thought he used and why, what strategies he thought E had used, as well as such factors as S's understanding of the rules of the game, how much he likes gambling, and his motivation for winning.

Subjects

Subjects were 100 volunteer male undergraduate students from the University of Massachusetts.

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4. The questionnaire is presented in Appendix B.

RESULTS

The 270 games played by each S were divided into nine blocks of thirty trials. In each successive block of trials the ten value levels were repeated three times. The basic data for each of three analyses of variance (one for each of S's three possible alternatives) were the number of choices of a given alternative for each set of three trials at the same value and trial block.[5]

Tables 1, 2, and 3 present the parallel analyses of variance for minimax, high-average, and high-cell choices respectively. Examination of these tables discloses a consistent pattern of significant effects. Strategy, value, blocks, value by strategy, blocks by strategy, and value by blocks are significant in all three analyses at the .05 level or beyond. Instructions, instructions by strategy, and value by blocks by strategy are significant at the .05 level or beyond in the minimax and high-average analyses.

The strategy chosen by E accounts for the largest proportion of the total variability in all three analyses of variance. Figure 1 represents the mean number of minimax, high-average, and high-cell choices per block of three trials against a random or minimax strategy. When E played a random

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5. Although three specific analyses of variance were computed, it should be noted that they are not completely independent inasmuch as Ss only had three possible alternatives.

Table 1

Summary of Analysis of Variance for

Subjects' Minimax Choices

Source of Variance	Degrees of Freedom	Mean Squares	F
Total	8999		
Between Ss	99		
Instructions (I)	1	114.70	11.69**
Strategy (S)	1	10,497.60	1070.09**
I x S	1	142.90	14.56**
Ss/I x S (1)	96	9.81	
Within Ss	8900		
Value (V)	9	2.71	3.98**
Blocks (B)	8	21.38	23.23**
V x B	72	.44	1.69**
V x I	9	.72	1.05
V x S	9	1.62	2.38*
V x I x S	9	.38	.55
B x I	8	.31	.33
B x S	8	53.12	57.73**
B x I x S	8	1.11	1.20
V x B x I	72	.33	1.26
V x B x S	72	.46	1.76**
V x B x I x S	72	.30	1.15
V x Ss/I x S (2)	864	.68	
B x Ss/I x S (3)	768	.92	
V x B x Ss/I x S (4)	6912	.26	

* $p < .05$ level of significance

** $p < .01$ level of significance

(1) Error term for Instructions, Strategy, I x S

(2) Error term for Value, V x I, V x S, V x I x S

(3) Error term for Blocks, B x I, B x S, B x I x S

(4) Error term for V x B, V x B x I, V x B x S, V x B x I x S

Table 2

Summary of Analysis of Variance for

Subjects' High Average Choices

Source of Variance	Degrees of Freedom	Mean Squares	F
Total	8999		
Between Ss	99		
Instructions (I)	1	71.03	5.33*
Strategy (S)	1	5906.52	443.76**
I x S	1	75.53	5.67*
Ss/I x S (1)	96	13.31	
Within Ss	8900		
Value (V)	9	6.04	6.49**
Blocks (B)	8	20.89	21.98**
V x B	72	.68	1.94**
V x I	9	1.29	1.38
V x S	9	5.26	5.65**
V x I x S	9	.30	.32
B x I	8	.29	.30
B x S	8	29.68	31.24**
B x I x S	8	1.07	1.12
V x B x I	72	.40	1.14
V x B x S	72	.61	1.74**
V x B x I x S	72	.36	1.02
V x Ss/I x S (2)	864	.93	
B x Ss/I x S (3)	768	.95	
V x B x Ss/I x S (4)	6912	.35	

* $p < .05$ level of significance

** $p < .01$ level of significance

(1) Error term for Instructions, Strategy, I x S

(2) Error term for Value, V x I, V x S, V x I x S

(3) Error term for Blocks, B x I, B x S, B x I x S

(4) Error term for V x B, V x B x I, V x B x S, V x B x I x S

Table 3

Summary of Analysis of Variance for

Subjects' High Cell Choices

Source of Variance	Degrees of Freedom	Mean Squares	F
Total	8999		
Between Ss	99		
Instructions (I)	1	5.30	.56
Strategy (S)	1	655.60	70.26**
I x S	1	10.50	1.12
Ss/I x S (1)	96	9.33	
Within Ss	8900		
Value (V)	9	1.95	5.13**
Blocks (B)	8	4.15	7.83**
V x B	72	.30	1.36*
V x I	9	.24	.63
V x S	9	2.53	6.65**
V x I x S	9	.22	.57
B x I	8	.07	.13
B x S	8	9.07	17.11**
B x I x S	8	.11	.20
V x B x I	72	.27	1.22
V x B x S	72	.25	1.13
V x B x I x S	72	.24	1.09
V x Ss/I x S (2)	864	.38	
B x Ss/I x S (3)	768	.53	
V x B x Ss/I x S (4)	6912	.22	

* $p < .05$ level of significance

** $p < .01$ level of significance

(1) Error term for Instructions, Strategy, I x S

(2) Error term for Value, V x I, V x S, V x I x S

(3) Error term for Blocks, B x I, B x S, B x I x S

(4) Error term for V x B, V x B x I, V x B x S, V x B x I x S

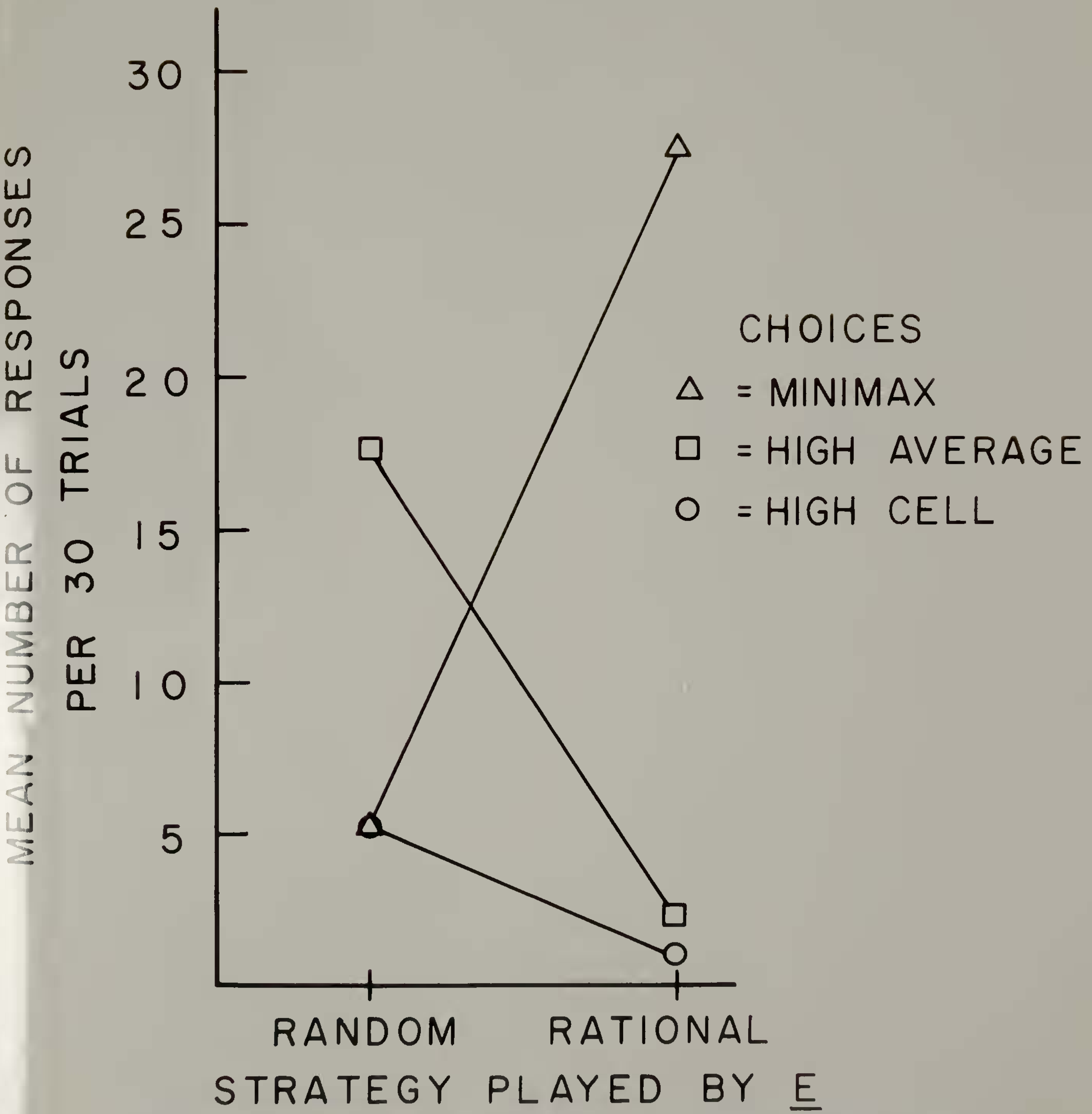

Fig. 1. Mean number of alternative choices per block of three trials against random and minimax strategies.

strategy, Ss played their minimax alternative an average of only .59 times in three games as contrasted with 2.7 times against E's minimax strategy. The mean number of high-average and high-cell alternatives played by Ss were 1.8 and .59 out of three respectively when E played randomly, but only .018 and .005 respectively for Ss against whom E played minimax.

All but one S playing against a minimax strategy learned to play their own minimax. Playing against a random strategy however, most Ss did not play any alternative consistently. Two Ss did play the high average alternative consistently, but the post-session interview disclosed that they realized from the start that this choice would yield the highest overall expected payoff against a random strategist. Thus their choices can not be attributed to a game learning process.

Against a random strategy Ss did, however, make more high average choices than either of the other two alternatives. All but one of the 25 Ss in the rational-random group, and all but five Ss in the random-random group selected more high-average than either of the other two choices. Four of these five Ss in the random-random group played more high-cell than minimax or high-average alternatives.

During the post-session interviews only two of the 50 Ss who played against a minimax-playing E described E's strategy in terms of minimizing his losses. Other Ss simply described the characteristics of E's choices. For example, they would say, "You always played the column with the low numbers.", or, "...the column with two minuses." Other Ss had difficulty verbalizing E's choices, but when presented with a series of games could indicate which choice E would make.

Four Ss in the random-rational group claimed that E had no strategy and one S wasn't sure. Upon questioning, all five made statements to the effect, "You said you were random." When questioned as to whether or not they really believed E played randomly, all replied, "Yes."

Of those Ss who played against a minimax strategy, 50% in the rational-rational and 53% in the random-rational groups responded that E should have played differently in order to minimize their winnings. Some Ss suggested that E should have played randomly. Others suggested that he should have made non-minimax choices once in a while in order to force them from their minimax play. Most Ss claimed that they played their minimax because E played his.

The value effect was significant beyond the .01 level on all three analyses. The Ss showed a general tendency to play more minimax and high-cell choices for higher value games than for lower value games. They tended to play more high-average choices for lower value games than for higher value games.

A clearer picture of the value effect can be seen by looking at its interaction with the strategy played by E. The value by strategy effect was significant beyond the .01

level on the high-average and high-cell analyses, and the .05 level on the minimax analysis. Figure 2 presents the mean number of alternative choices across the 10 value levels against a minimax or random strategy. The value of a game has relatively little effect on S's choices when E consistently plays a minimax strategy. When, however, E plays randomly, the value effects mentioned above become accentuated.

The blocks and blocks by strategy effects were significant beyond the .01 level on all three analyses. Disregarding the strategy played against them, Ss selected an increasing number of minimax alternatives over blocks of trials and a decreasing number of high-average and high-cell choices. However, the strategy played against Ss dramatically effected their choices over blocks. Figure 3 presents the mean number of alternative choices per block of trials against random or minimax strategies. When E played a minimax strategy, Ss played an increasing number of minimax choices over blocks and a decreasing number of high-average and high-cell choices. When E played a random strategy, Ss played an increasing number of high-average alternatives for the first 60 games and then fewer over the remaining 210 games. The curve for the number of minimax choices played by Ss shows a fluctuating decline over the 270 games, while the high-cell choices show an initial decline over the first 90 games but an increase over the last 180.

Although the value by trials interaction proved to be

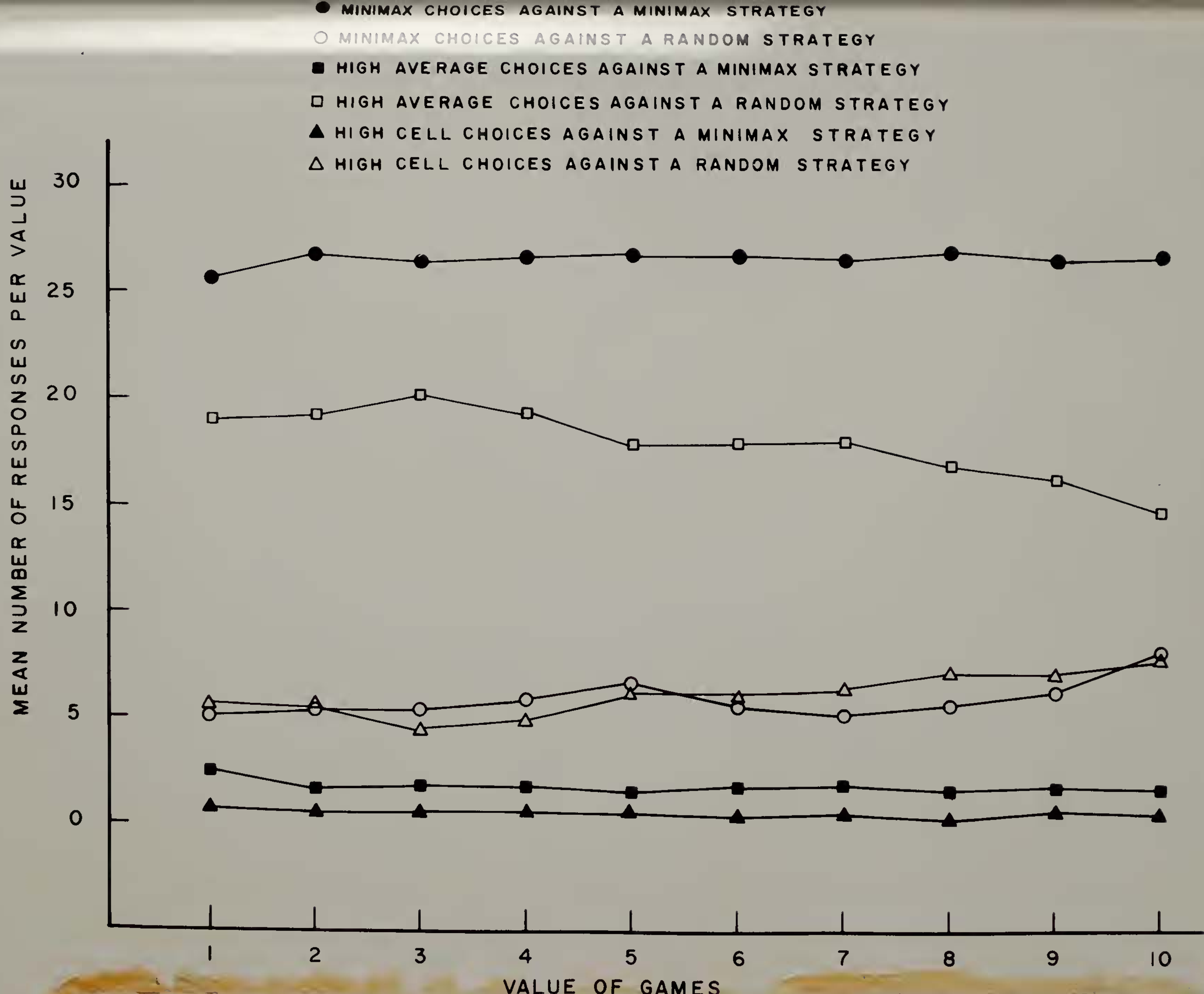

Fig. 2. Mean number of alternative choices for each of 10 value levels against random and minimax strategies.

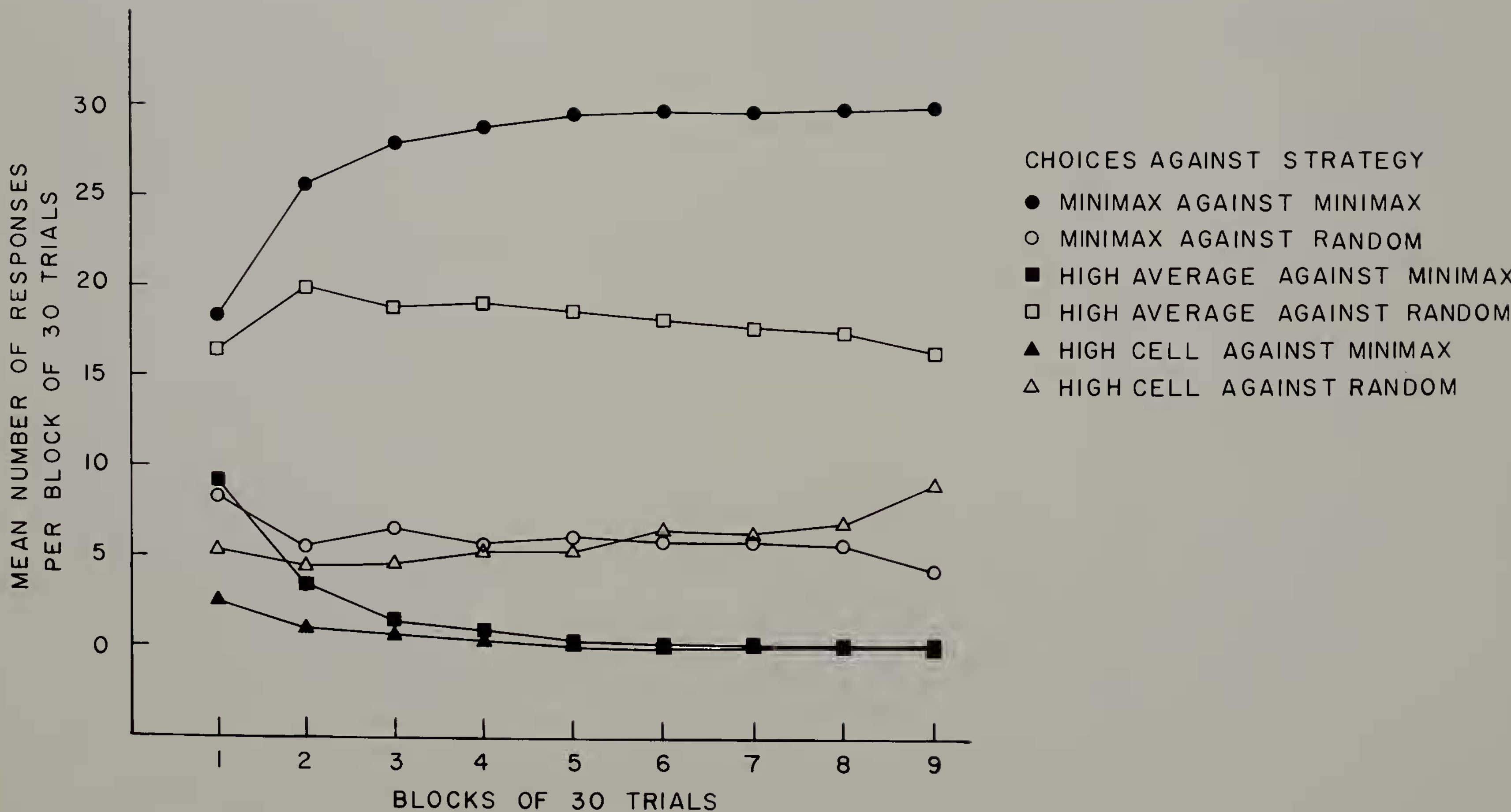

Fig. 3. Mean number of alternative choices for each of thirty trials against random and minimax strategies.

significant beyond the .05 level for high-cell choices and the .01 level for high-average and minimax choices, the value by trials by strategy interaction gives a truer picture of this effect inasmuch as the nature of the value by trials interaction is very much dependent upon the strategy played by E. The value by trials by strategy interaction was significant beyond the .01 level on both the minimax and high-average analyses, but not significant on the high-cell analysis.

Against either a minimax or random strategy, Ss initially made more minimax choices for high value games than for low values. Against a minimax strategy, however, they increased their number of minimax choices over trials, while decreasing them against a random strategy. In both cases the rate of increase or decrease is greater for higher than lower value games during the early trials, but after 60 to 120 games the differences in these rates between high and low value games is negligible.

Against either a minimax or a random strategy, Ss initially made more high average choices for lower value games than higher value ones. Against a random strategy, however, Ss increased their number of high average choices for the first 60 to 90 games and then showed a slow decrease over the remainder of the games. Against a minimax strategy, they showed an immediate and sharp decrease. In both cases the initial increase or decrease was greater for lower than higher value games. The slow decline against a random strategy was somewhat greater for higher than lower value games. After 90 to 120 games against a minimax strategy, the differences between low and high value games were negligible.

The instructions effect was significant at the .05 level on the high-average analysis and the .01 level on the minimax. Figure 4a presents the mean number of minimax and high-average choices per block of three trials for the two instruction groups. The Ss played more minimax and fewer high-average alternatives when told that their opponent was rational than when told he was playing randomly.

Figure 4b presents the mean number of minimax and high-average choices for the two instruction groups against random and minimax strategies. It is evident from Figures 4a and 4b that instructions were effective only when a random strategy was played against Ss. Inspection of Figure 4b also indicates that against a minimax strategy Ss made more minimax choices and against a random strategy, more high-average choices regardless of the instructions given them.

Questionnaire results relevant to the instructional set given to the Ss show that 13 out of 25 Ss in the rational-random group stated that they thought that E had a strategy, eight said he had no strategy, and four didn't know. Seven Ss in the random-random group stated that E had a strategy, 15 said no, and three didn't know.

Twenty-three Ss in the rational-random group mixed their

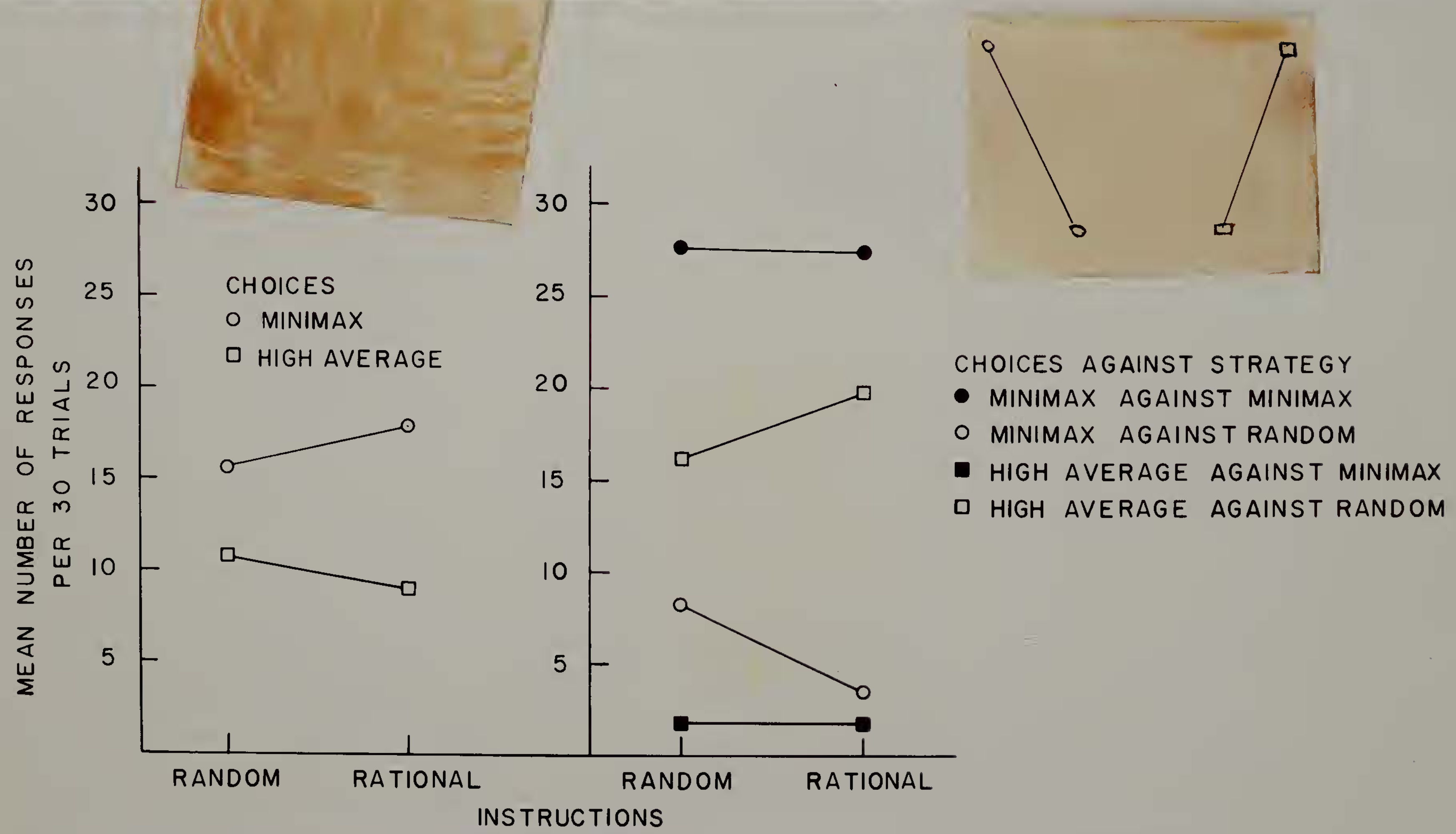

Fig. 4a. Mean number of minimax and high-average choices per block of three trials for random and rational instructions.

Fig. 4b. Mean number of minimax and high-average choices for random and rational instructions against random and minimax strategies.

choices relying on no alternative in particular. Two Ss in this group, although playing a mixture of all three alternatives, relied most heavily on high-average choices. Ten Ss in the random-random group played a mixed strategy, while 11 played a mixed strategy which relied heavily on high-average choices.

Nine Ss in the rational-random group indicated they thought E's choices depended on what they played, as opposed to none in the random-random group. Seventeen Ss in the rational-random group indicated that E tried to influence their choices by making certain choices of his own compared to three in the random-random group.

Seventeen Ss in the rational-random group claimed that their choices depended upon E's choices as contrasted to four Ss in the random-random group. Eight Ss in the rational-random group stated that they tried to influence E's playing by making certain choices of their own compared to one S in the random-random group.

DISCUSSION

In the present study the strategy played against S was the strongest single factor determining how he played. Examination of the data suggests two possible explanations for this effect: discrimination and imitation. Against a consistent strategy, such as a minimax strategy, Ss learned to play a consistent counter strategy. The games employed in this study were such that if S played any alternative but his minimax when playing against a minimax strategist, he lost chips. Against a minimax playing opponent, then, the task might be considered a straightforward discrimination problem; play minimax and win, play anything else and lose. Such a task does not require Ss to concentrate on what their opponent plays, only on those characteristics which discriminate their own minimax row. This explanation is supported by the fact that many Ss in the rational-rational and random-rational groups either were unable to verbalize their opponent's strategy or claimed E had no strategy despite the fact that all but one of them learned to play a minimax strategy consistently. It is suggested that if games were employed in which non-minimax choices did not necessarily lead to a loss of chips, the discrimination task would not be as simple and Ss might not as easily learn the minimax solution.

In further support of the above explanation, over half of the Ss playing against a minimax strategist claimed E played unwisely. It is apparent that these Ss did not learn the rationale for a minimax solution. Perhaps if the task were less mechanical, requiring Ss to concentrate on the outcomes of the various combinations of alternatives, more of them would have learned the rationale of a minimax solution. A game situation in which two Ss play against each other would require greater concentration on the task and also provide experience of outcomes of alternatives other than minimax. Lieberman (1958, 1959) found that Ss playing each other a single game did learn to play minimax in 90 to 150 trials. However, the solution was always the same, either row 3 or column 3. A series of similar games in which the minimax alternative varied over the three rows and columns would require the Ss to generalize their strategy to other games.

Against a random strategy, Ss were not consistently rewarded or punished for choosing any particular alternative. Therefore, Ss would not be expected to learn to play any alternative consistently. Over a number of trials, however, they should have accumulated more chips by playing their high-average alternative. This might explain why Ss playing against a random strategy, in general, made more high-average than minimax or high-cell choices. Also the high-average row provided a two-thirds chance of winning a middle range value payoff, compared to a sure but low value payoff for the minimax row, and one-third chance of winning a high value payoff for choosing the high-cell alternative. It is possible that

Ss were attracted to the favorable two-thirds probability of the high-average alternative. A test for this suggestion would be to provide games in which the high-average row has only a one-third chance of winning.

The second possible explanation for the strategy effect is that Ss attempted to imitate their opponent's strategy. Evidence from two-choice situations which involve S's predicting which of two discrete events will occur in each of a series of trials, have demonstrated a strong tendency on the part of Ss to estimate closely the probability of each event's occurring. Since against the rational-rational and random-rational groups E always played his minimax, those Ss would be expected to learn to play their minimax, i.e., "match" E's play, after an initial period where they learned to discriminate the characteristics of E's choice and the fact that he never deviated from that choice. That Ss were able to discriminate E's choice, though perhaps not being able to verbalize it, was demonstrated during the interview where most Ss were able to indicate what E's choice would be on any given trial.[6] Against a randomly playing opponent Ss would not be expected to play any consistent alternative since E did not play a consistent alternative.

It is possible in these terms to consider the Ss in the

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6. This is consistent with evidence from concept formation research (e.g., Mair, 1931; Rees and Israel, 1935) which has demonstrated that Ss are often able to perform tasks adequately without being able to verbalize their solution.

present study to have imitated or "matched" E's behavior by approaching in their choices of minimax rows, the frequency with which E chose the minimax column, i.e., one hundred percent of the choices. This would apply to those 50 Ss who played against a minimax-playing E. Since E consistently chose the minimax column, no test is possible among these Ss of whether their minimax choices would increase or decrease with increases or decreases in E's minimax choices.

The data of the 50 Ss who played against a random-playing E do provide a basis for crudely testing the "matching" hypothesis, since E's choice of minimax columns did vary from one block of 30 trials to another (see Figure 5). When the Ss' mean number of minimax choices is compared with the number of times E played his minimax alternative in the preceding block, Ss generally increased their minimax choices when E had increased his minimax choices and decreased when E decreased. The "matching" effect is also suggested by similar inspection of block by block choices for the high-average alternative.

The fact that the strategy played against Ss is such a powerful determinant of their choices suggests that a systematic manipulation of various strategies might be carried out. Would Ss learn to play a consistent strategy against a consistent strategy other than a minimax, for example a consistent high-average strategy? Also, various percentages of minimax choices might be employed to ascertain at what level

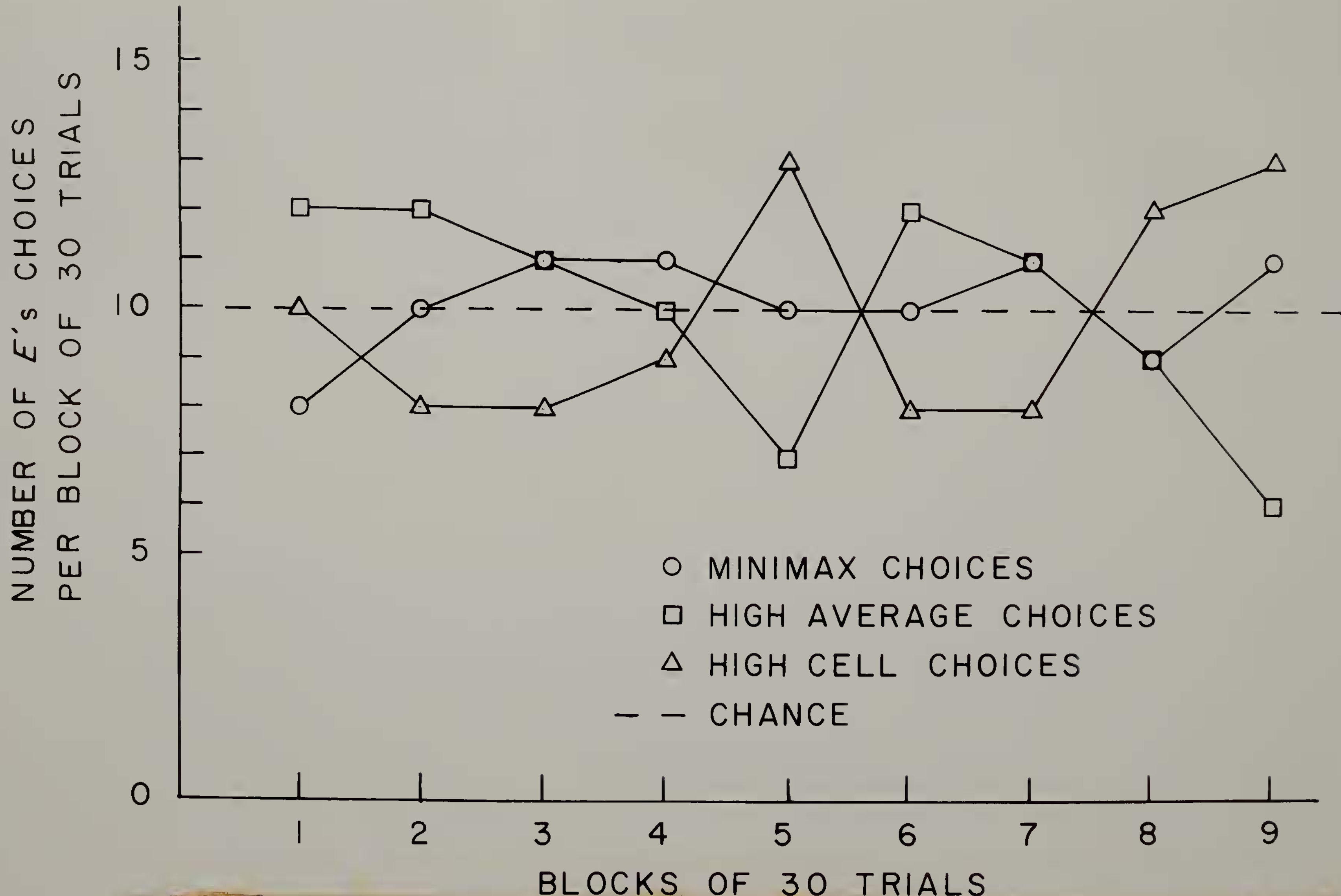

Fig. 5. Number of alternative choices played by *E* for each block of thirty trials.

Ss no longer played a consistent minimax solution. Whether Ss would attempt to match the percentage of minimax choices played against them as suggested by the evidence from two-choice studies, or if such a situation would provide enough experience with the consequences of non-minimax alternatives that they learn the rationale for minimax play is yet to be empirically determined.

Against a random-playing opponent the decline of the high-average choices and increases of high-cell choices over the later blocks might be attributed, in part to the possibility that Ss initially played high-average alternatives in order to accumulate a store of chips. Once they were secure in the amount of chips accumulated, they could afford to play the high-paying, high-risk, high-cell alternative instead of the high-average.

It is apparent that when Ss are sure of their opponent's strategy factors such as the value of the game and instructions have very little effect. When, however, an opponent's strategy is highly ambiguous (e.g., random) such factors do effect Ss choices.

Figure 6 shows the number of times E played each of his alternatives at each value level. A comparison of Figures 2 and 6 indicates that Ss had some success in predicting E's minimax choices, but little in predicting his high-average or high-cell choices.

The similarity between Ss' and E's minimax choice

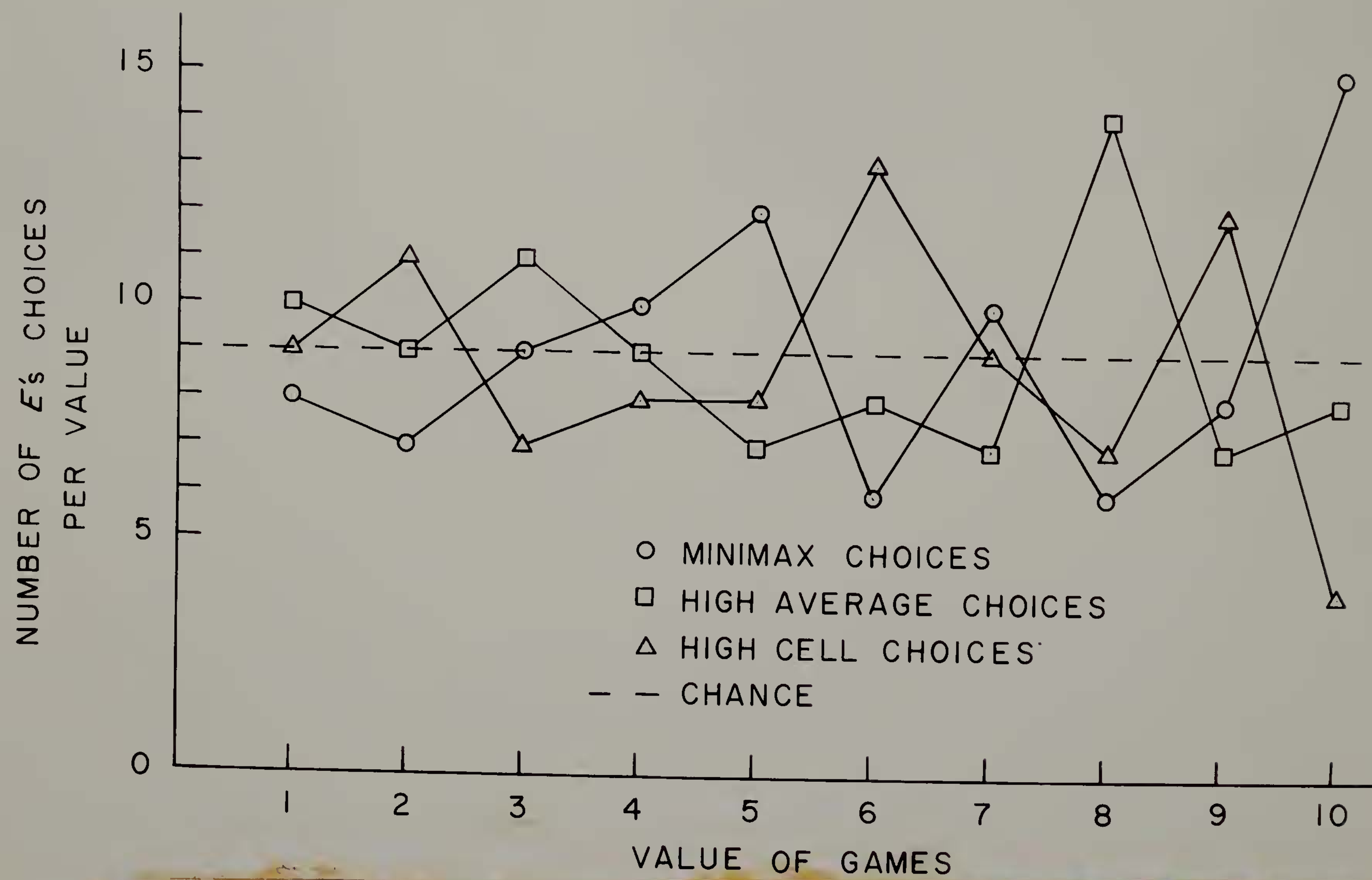

Fig. 6. Number of alternative choices played by E for each of ten value levels.

patterns raises some doubt as to whether Ss played minimax more often for higher value games because higher values were more attractive or because they were able to predict E's choices. A closer examination of the results would indicate that the former is generally true, but the latter is true for later games.

The value by blocks by strategy interaction showed that Ss initially chose more minimax alternatives for higher value games than for lower value games. This would support the contention that when an opponent's strategy was ambiguous, for example in the early games before Ss had a chance to estimate how often E played his minimax, Ss played more minimax for higher value games than for lower value games. In later games, after they had a chance to observe E's behavior, they apparently attempted to predict his minimax choices.

For higher value games Ss played more conservatively (minimax) and at the same time took more risks (high cell) than they did for lower value games. Apparently the magnitude of the high-cell and the assured positive payoffs of the minimax choices became relatively more attractive compared to the negative-to-middle range payoffs offered by the high-average choice. It appears that for low value games where Ss had to risk too much in making high-cell choices and where the assured payoff of the minimax choice was negligible, Ss preferred to make high-average choices which afforded a two-thirds chance of winning. For higher value games the probability of a middle range payoff became relatively less attractive compared to the relatively high, sure win of the minimax and the relatively low-risk, high-payoff, high-cell.

Suppes and Atkinson's (1960) demonstration that Ss in a two-choice situation tend more toward an optimal play for games involving relatively higher payoffs is consistent with the findings of this study that Ss play more minimax alternatives for higher value games. The results of this study concerning the affect of relative values on high-cell choices indicates the need for further research involving games in which the relative risk of the high-cell alternative increases or remains constant as the value of the payoff increases.

Instructions also proved to be influential on Ss' choices only when E played randomly. Questionnaire results indicated that under this condition different instructions induced somewhat different concepts of Ss' opponent. Since E's choices against the rational-random and random-random groups were identical, the S's choice differences in these two groups must be attributed to the instructions given them. The fact that Ss in the random-random group relied more heavily on high-average choices than did Ss in the rational-random group may, in part, be explained by the evidence from the questionnaire which indicated that the Ss in the former group were less prone to be influenced by and to look for patterns in E's choices than were Ss in the latter group. It is possible that the former S̲s were more attracted to the high-average alternative because it provided a two-thirds chance of winning a middle range payoff as opposed to a two-thirds chance of losing by playing a high-cell alternative, or winning a small payoff by playing a minimax alternative. On the other hand, S̲s who expected E̲ to be playing some sort of strategy might not be as attracted to the high-average alternative because they were more concerned with looking for a pattern in E̲'s choices.

The payoffs in this study were in terms of poker chips which in turn were transferable for money. Game theory, however, as presented by Von Neumann and Morgenstern (1944), defines payoffs in terms of utilities. Thus poker chips in this study are used as approximations of utilities. To the degree that these approximations are adequate, the results of this study are not restricted to monetary payoffs, but may extend to any situation, be it economic, military, political, or social, where decisions can be translated into game theory language and payoffs into some approximation of utilities. For example in military parlance, cell payoffs may be defined in terms of number of troops and equipment lost or destroyed, winning or losing of battles, the amount of time needed for delaying tactics, etc. In political situations utilities may be in terms of war or not war, making friends of foreign countries, loss or gain of prestige, etc. With regard to social situations, utilities may be in terms of gain or loss of status, opportunity to meet new or influential people, satisfaction derived from various social interactions, etc.

SUMMARY

Although there has been considerable interest in decision making and the application of game theory models to certain psychological problems in recent years, there has been little investigation of the variables influencing game behavior. This study was an attempt to investigate some of these variables affecting the playing of two-person strictly determined games. The major questions asked were:

1. When playing a series of similarly structured, strictly determined games, do intelligent persons learn to make strategy choices consistent with deductions from a game theoretic model?

2. How does the strategy played against an individual influence the strategy he employs?

3. How does an individual's expectation of his opponent's strategy influence his own strategy?

4. Do intelligent persons play high value games differently than they play low value games?

5. How do intelligent persons modify their strategy with experience?

One hundred Ss were assigned to one of four conditions: rational-rational, rational-random, random-rational, or random-random. Each S was told in everyday language either that his opponent would play randomly or in such a way as to minimize S's winnings. Either a minimax or a random strategy

was played against him. Each S played a series of 270 similarly structured 3 x 3 strictly determined games against E. The series contained 10 value levels from one through 10. Ss were provided with three types of choices: a minimax in which the row contained the saddle point of the game, a high average in which the average of values in the cells of that row was higher than of any other row, and a high cell in which that row contained the highest valued cell of the game matrix. The spatial ordering of these choices in the matrix was varied unsystematically from game to game. The particular choice made by each S for each game was recorded. Three analyses of variance were computed, one for each class of Ss' choices.

Results from this study are generally consistent with deductions from the game theoretic model employed. The strategy played against Ss proved to be the most potent factor affecting their choices. When playing against a consistently minimax-playing opponent, Ss learned quickly to play a minimax strategy themselves. When playing against a random strategy, Ss did not learn to play any one strategy consistently, but relied most heavily on high average choices which produced the highest monetary yields. Against a random opponent, Ss also were affected by other factors such as the value of the game and their expectations of their opponent as presented to them through instructions.

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APPENDIX A

INSTRUCTIONS

We are going to play a series of games in which, if you play intelligently, you can win some money. The amount you win will depend upon how well you play the games. In each of the games you will be playing against me.

You are player "A" and I am player "B". In each game we will have a game matrix on the box in front of us similar to game matrices I and II below.

game matrix I

YOUR CHOICES

	b_1	b_2	b_3
a_1	11	-7	8
a_2	1	1	2
a_3	-10	-7	21

Each of the game matrices has different numbers in the cells. For example, the next game may look like this:

game matrix II

YOUR CHOICES

	b_1	b_2	b_3
a_1	-7	8	3
a_2	25	-6	0
a_3	2	1	2

The numbers in the cells represent how many chips you can win or lose on that particular game. You <u>win</u> the number of chips in a particular cell which has <u>no</u> minus sign in front of it. You <u>lose</u> the number of chips in a particular cell which does have a minus sign in front of it.

Inasmuch as you are player "A", your task in each game is to select choice (i.e., row) a_1, a_2, or a_3. You then indicate your choice by throwing switch a_1, a_2, or a_3, forward. Inasmuch as I am player "B", I will select column b_1, b_2, or b_3. Neither you nor I will be able to see what the other is choosing before the play is made. The particular combination of "a" and "b" that you and I choose will determine which cell in the matrix will light up on the box in front of us. You will win or lose the number of chips indicated in the corresponding cell of the game matrix.

As an example, in game matrix I above (turn back and look at game matrix I), if you choose row a_1, and I choose column b_1, you would win 11 chips. If, on the other hand, you choose a_3, and I choose b_2, you would lose seven chips.

When we have played a game, I will tell you to return your switch to the "off" position (back -- toward you). I will then give to you or take from you the number of chips indicated on that play. We will then go on to the next game matrix and you will make another choice of a_1, a_2, or a_3. I will also make another choice of b_1, b_2, or b_3.

ARE THERE ANY QUESTIONS SO FAR?

Instructions For Rational Groups

Remember your task is to win as many chips as you can. You are playing against me. Inasmuch as I have to pay you each time you win, I will try to play in such a way as to reduce your winnings.

Instructions For Random Groups

Remember, your task is to win as many chips as you can. You are playing against me. However, I am not really competing with you. My choices for each game have already been decided before hand. No matter what your choices are, I will not change my choices which have already been selected. I am a purely chance or random opponent. That is, in making my choices ahead of time I did not consider the nature of the games. When I made my choices I simply reached into a hat blindly and pulled out a number from 1 to 3 to determine what choice I would play for any particular game. In every game choices b_1, b_2, or b_3 all have an equal chance of being played by me.

ANY QUESTIONS ABOUT WHAT I MEAN WHEN I SAY THAT I AM A PURELY CHANCE OR RANDOM OPPONENT?

ANY QUESTIONS SO FAR?

In front of you is $1.50 worth of poker chips.

1 white chip = 1/10 of a cent = 1 point in the game matrix
1 red chip = 10 white chips = 1 cent
1 blue chip = 10 red chips = 100 white chips = 10 cents

At the end of the games you will give back to me $1.00 worth of chips and I will give you money for the rest of the chips which you have accumulated. Thus, if you just break even in the games you will receive $.50 for showing up (the $.50 you did not give back to me). If you should lose at the games you may lose all or any part of that $.50. You can not, of course, lose any of your own money. You can however <u>win</u> more than $.50. This experiment is being sponsored by a research foundation so don't be afraid to win as much as you can.

ARE THERE ANY QUESTIONS?

<u>Remember</u>, in each game you choose row a_1, a_2, or a_3. Then throw switch a_1, a_2, or a_3 corresponding to your choice. I will similarly choose column b_1, b_2, or b_3. The combination of our choices will be indicated by one of the lights on the box in front of us. You will <u>win</u> or <u>lose</u> the number of chips indicated in the corresponding cell of your game matrix. We will then go on to the next game matrix and make our choices for that game.

APPENDIX B

QUESTIONNAIRE

1. Do you like to gamble?

   yes___ no___

2. Do you know anything about the theory of games?

   yes___ no___

   If yes, where did you learn what you know?

   If yes, do you think it helped you in playing the games?

   yes___ no___

3. Do you know anything about probability theory?

   yes___ no___

   If yes, do you think it helped you in playing the games?

   yes___ no___

4. Were the instructions concerning what you were to do clear to you?

   yes___ no___

   If no, what was unclear?

5. Were the instructions concerning what I was going to do clear to you?

   yes___ no___

   If no, what was unclear?

6. Were there anything in the instructions that were not clear?

   yes___ no___

   If yes, what?

7. Did the instructions concerning how I was going to play the games help or hinder you in deciding how to play the games?

   help___ hinder___ no___ ?___

8. If I had told you that I was going to play (rationally, randomly) would you have played differently?

   yes___ no___ ?___

   If yes, how?

9. In making a choice of play was your choice based primarily on how many chips you could win if you played intelligently (as opposed to the desire to gamble for example)

   yes___ no___ mostly___

   Any other factors involved? If so, what?

10. How did you play the games? What strategies did you use?

    mm___ X___ HC___ mixed___

11. Did your strategy change at all during the games, i.e., did you use more than one strategy?

    yes___ no___

    If yes, why?

    If yes, describe those strategies not already described, and when you used one in preference to another?

12. Did your choices depend upon what choices I was making?

    yes___ no___

13. During the games did you try to influence my choices by making certain choices of your own?

    yes___ no___

    If yes, describe what you did?

14. Now that the games are over do you think there was any best strategy?

    yes___ no___ ?___

    If yes, what was it?

15. Did I have a strategy? How did I play? What strategy(ies) did I use?

    yes___ no___

16. Did my strategy change at all during the games?

    yes___ no___ ?___

17. Did my choices depend upon what choices you were making?

    yes___ no___ ?___

18. During the games did I try to influence your choices by making certain choices of my own?

    yes___ no___ ?___

19. Do you feel that the games were fair?

    yes___ no___

    If no, in what way were they not fair?

20. Do you think that I played fairly with you?

    yes___ no___

    If no, in what way do you think I played unfairly?

    If no, did this effect the way you played the games?

21. Are you satisfied with the way you played the games?

    yes___ no___

    If no, in what way are you not satisfied?

22. Did you enjoy playing the games?

    yes___ no___

    Why? (Why not?)

23. Would you be willing to suggest participating in this experiment to your friends?

    yes___ no___

    If no, why not?

Thank you very much for your time.