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An Experimental Study of Cooperative Duopoly

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1. Introduction

At least from the time of Adam Smith, economists may be found who believe that oligopolists will collude rather than behave competitively. Some think collusion very likely, while others merely admit the possibility. Smith is among the first group:[1]

> People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices. It is impossible indeed to prevent such meetings, by any law which either could be executed, or would be consistent with liberty and justice. But though the law cannot hinder people of the same trade from sometimes assembling together, it ought to do nothing to facilitate such assemblies; much less to render them necessary.

Cournot and Wicksell recognize the possibility of collusion, but neither commits himself on its likelihood. Cournot states that if a

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1. Smith, Adam, The Wealth of Nations, Modern Library Edition, p. 128.

pair of duopolists collude, "the results ... would not differ, so far as consumers are concerned, from those obtained in treating of a monopoly."[1] Cournot appears to assume the duopolists will maximize their joint profit and then find some mutually acceptable manner in which to split the proceeds. This implication is apparent from his phrase of qualification "so far as consumers are concerned". He commits himself only to the position that the pair will jointly maximize.

Cournot's position was not markedly improved upon until the publication of the Theory of Games by von Neumann and Morgenstern. In passing it may be noted that Chamberlin believed that if the members of an oligopoly realized their fortunes were interdependent, they would behave as joint profit maximizers even if they did not explicitly collude.[2] Von Neumann and Morgenstern present as the solution to the general two person game all those outcomes which are Pareto optimal and in which each participant gets at least as large a profit as he could guarantee himself if he did not collude with the other.[3] This "solution" is not satisfying

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1. Cournot, Antoine Augustin, Researches into the Mathematical Principles of the Theory of Wealth, translated by N. T. Bacon, A. M. Kelley (New York), 1960, p. 80. See also Knut Wicksell, Lectures on Political Economy, Vol. I, Routledge and Kegan Paul, Ltd. (London) 1934, pp. 96-97.

2. Chamberlin, Edward H., The Theory of Monopolistic Competition, 7th ed., Harvard University Press (Cambridge) 1956, pp. 46-51. Chamberlin is dealing here with a Cournot-type case: identical firms producing a perfectly homogeneous product.

3. Von Neumann, John and Oskar Morgenstern, The Theory of Games and Economic Behavior, 3rd ed., Princeton University Press (Princeton, N. J.) 1953, pp. 549-550. Von Neumann and Morgenstern are dealing here with games which have transferable utility. The solution is all payoffs (α_1, α_2) for which $\alpha_1 \geqq V((1))$, $\alpha_2 \geqq V((2))$ and $\alpha_1 + \alpha_2 = V((1,2))$. $V((S))$ denotes the value of the game to the set S .

because it does not predict a specific outcome. It only limits the outcome to a specific set of possibilities. There have been several attempts at developing "arbitration schemes" designed to predict the outcome of a duopoly situation. By far the best and most interesting of these is the cooperative game solution proposed by John Nash.[1] This solution will be examined later in the paper.

Very little evidence is available to test the conjectures and theories alluded to above. Market data are generally useless for the purpose because not enough is available to determine where, on a spectrum from Pareto optimality to competitive behavior, the actions of a group of firms lie. One method which may be employed is controlled experiment in which subjects may be used to take the part of firms in a simulated duopoly situation. This technique is utilized in the research reported here. The experiment consists of a number of cooperative duopoly games. The games are cooperative in the sense that "... the two individuals are supposed to be able to discuss the situation and agree on a rational _joint plan_ of action ... ."[2]

Among the things the experiment reported here was designed to measure is the extent to which subjects could agree on a joint course of action. On the basis of economists' conjectures, one would expect agreement

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1. Nash, John F., "Two Person Cooperative Games," _Econometrica_, Vol. 21 (1953) pp. 128-140.

2. _Ibid_; p. 128.

most of the time, and Pareto optimality should characterize the agreements which are made. Clearly, if a pair can come to an explicit agreement on a joint course of action, it is unlikely they would choose an action in which each receives less then he could get if he made another agreement. Finally, among those decisions which are Pareto optimal, an attempt is made to find whether subjects choose points which afford equal profits, jointly maximal profits, or the division of profits indicated by Nash's solution for cooperative games.

This experiment is a natural extension of earlier experimental research in oligopoly by the present author and others. The quantity adjuster oligopoly experiments of Fouraker and Siegel showed subjects to be noncooperative profit maximizers in incomplete information games, and raised questions about the nature of behavior in complete information games.[1] Incomplete information obtains when a subject knows only the profit he receives as a function of the choices of all subjects in a game. Complete information obtains when a subject knows the profits to each subject as a function of the choices of all. The complete information games, both duopoly and triopoly, showed very large differences in behavior from game to game. In some games subjects maximized industry profits; in some they each appeared to maximize the difference between his own profit and that of the others in the game; and other games were arrayed between these extremes. Fouraker and Siegel conjectured that in games of complete

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1. Fouraker, Lawrence E. and Sidney Siegel, Bargaining Behavior, McGraw-Hill (New York) 1963. See Chapters 7, 8 and 9.

information the extent to which subjects are cooperative depends upon personal characteristics of the subjects. They were not able to test hypotheses about individual subjects due to a lack of data. Each subject in their experiments played in only one game.

The Fouraker-Siegel experiments led the present writer to conclude that the behavior of a subject in cooperative games might depend upon personal characteristics and the degree to which others in the game are cooperative.[1] This called for an experiment in which each subject played in many games, providing many periods of data on each subject and allowing each to be grouped with different subjects from game to game. An index of cooperativeness was defined whose value is -1 if a subject sets his prices so as to maximize the difference between his profit and that of the others, it is 0 if he sets his price to maximize his own profit, and it is +1 if his price is chosen to maximize industry profits. Of course intermediate values are possible. It was postulated that a subject's index of cooperativeness is a linear function of the indices of the others, the parameters of the function being estimated from the data. This model appeared appropriate and the hypotheses were both confirmed: behavior of a subject appeared to depend upon the behavior of others in the game, and subjects did not behave identically. It is generally true that a subject is more cooperative, the more cooperative are those in the game with him.

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1. Friedman, James W., "An Experimental Study in Oligopoly," Cowles Foundation Discussion Paper, No. 174, September 23, 1964.

The experiments of both Fouraker-Siegel and of the present writer do not permit communication between subjects. Subjects occupied separate rooms, did not see one another and were not allowed to send messages or converse, etc. The experiment described below differed in that it employed complete information games with messages. The introduction of messages allows a game to be explicitly cooperative. Subjects may discuss joint strategy and make agreements. Another important way the experiment differs from previous work is in the use of asymmetric games.[1] In symmetric games the joint profit maximum coincides with both the Pareto optimal point at which all have equal profits and the Nash cooperative game solution. Thus it is a very natural point for subjects to choose. The presence of asymmetric games, in which these three points are separated, allows for a test of whether one of them, in particular, characterizes most agreements.

Four sections follow the present one. Section 2 gives the experimental design and market model for the experiments and Section 3 discusses the experimental procedures. Section 4 contains the analysis, and Section 5, the concluding section, contains additional comments and conclusions.

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1. Symmetry will be defined below on page 7.

2. The Market Model and Experimental Design

2.1 The Market Model

The model from which the payoff matrices are derived utilizes a demand function which is linear in the prices of the two players:

(1) $$q_i = 100 - 3.5\, p_i + 2.0\, p_j \qquad i, j = 1, 2 \quad j \neq i$$

Three different total cost curve configurations are used. The first gives rise to a symmetric game:

(2) $$C_i = 820 + 10\, q_i + .1\, q_i^2 \qquad i = 1, 2$$

One may write profit functions for each player:

(3) $$\Pi_i = p_i q_i - C_i \qquad i = 1, 2$$

By substituting into equations (3) from equations (1), profit for each can be expressed as a function of the two prices:

$$\Pi_1 = \Pi_1 (p_1, p_2)$$
$$\Pi_2 = \Pi_2 (p_2, p_1)$$

The game is symmetric in the following sense: Let p^* and p^{**} be two prices, not necessarily equal. Then

$$\Pi_1(p^*, p^{**}) = \Pi_2(p^*, p^{**}) .$$

The profit received by player 1 when he charges p^* and player 2 charges p^{**} is the same as player 2 receives when player 1 charges p^{**} and he charges p^*. Of course both have equal profits when they charge the same prices.

The second pair of cost curves is

$$C_1 = 10\, q_1 + .1\, q_1^2$$
$$C_2 = q_2^2$$

The third set is identical to the second, except players 1 and 2 trade cost curves. These three pairs of cost functions will be denoted B, A, and C, respectively.

2.2 Experimental Design

The experiment consisted of three replications, each of which used six different subjects. Thus there were eighteen subjects in total. The description which follows indicates the experimental design which was intended for each replication.[1] Each subject received one of the following

1. Through an oversight the first replication utilized a slightly different design which is given below on page 16.

labels: A1, A2, A3, B1, B2, B3 . An "A" subject was always paired with a "B" subject in each game, and conversely. The reason for this will be apparent soon. Table 1, below, gives the three possible player pairings used in the experiment. They are denoted α, β and γ . Thus with the α pairing, A1 and B1 form a duopoly, A2 and B2 form a duopoly, as do A3 and B3.

Table 1

Player Pairings

α	A1 B1	A2 B2	A3 B3
β	A1 B2	A2 B3	A3 B1
γ	A1 B3	A2 B1	A3 B2

In each game, the subjects possessed a payoff matrix such as that shown below in Table 2. The matrices used in the experiment had thirty rows and columns. If, for example, the "A" subject chooses a price of 42 and the "B" subject 34, the profit of the "A" subject is -7.2 points and that of the "B" subject, 43.8 points. In all matrices profits are given in "points", and there is a conversion constant from points to pennies for each subject. These constants change from game to game and are not necessarily the same for both subjects. In all games a subject knows his own points/money ratio, but he knows the ratio of his competitor in only half of the games. Ignorance of one another's points/money ratios puts an

Table 2

Payoff Matrix

Prices available to "A" subject	Prices available to "B" subject: 29	31	34	38	42	45
29	5.4 5.4	8.4 9.5	12.1 8.4	16.1 -6.3	18.7 -36.0	19.8 -68.3
31	9.5 8.4	13.7 13.7	19.6 14.7	26.2 2.8	31.6 -24.3	34.7 -54.5
34	8.4 12.1	14.7 19.6	23.6 23.6	34.3 15.8	43.8 -7.2	50.0 -34.4
38	-6.3 16.1	2.8 26.2	15.8 34.3	31.9 31.9	46.8 14.4	57.1 -8.7
42	-36.0 18.7	-24.3 31.6	-7.2 43.8	14.4 46.8	34.7 34.7	49.1 15.7
45	-68.3 19.8	-54.5 34.7	-34.4 50.0	-8.7 57.1	15.7 49.1	33.2 33.2

obstacle in the way of attaining an equal profit for each. It was thought that an equal split of profits would be more likely, and the Nash solution less likely, if points/money ratios were known. Varying points/money ratios and player pairings each help to prevent a subject from discovering the identity of his competitor.

Table 3 gives the experimental design which was employed in the experiment.

Table 3

	points/money known			points/money not known		
	I	II	III	I'	II'	III'
1	Aβ	Cγ	Bα	Aβ	Cγ	Bα
2	Cα	Bβ	Aγ	Cα	Bβ	Aγ
3	Bγ	Aα	Cβ	Bγ	Aα	Cβ

This is a double Graeco-Latin square design in which A, B, and C represent the three basic models described above. The Roman and Arabic numbers refer to various points/money relationships:

1¢ = 1/3 point for 1, I and I'
1¢ = 1 point for 2, II and II'
1¢ = 3 points for 3, III and III'

The Arabic numbers denote points/money for the "A" subject; the Roman for the "B". Thus the A game corresponding to (3, III) has approximately the same money payoffs as the A game corresponding to (2, I); however, in the (3, III) game the payoff matrix shows a number of points for each player three times the money payoffs, and in the (2, I) game the "A" player's points equal his money payoffs while, for the "B" player, his points are one third of his money payoffs.

The two Graeco-Latin squares are identical. This is in order to provide two replications of the basic set of nine games under each of the two information conditions. The games represented by the first square (with column headings I, II, III) were all played with subjects knowing their own points/money relationship and knowing that of their competitors. In the games represented by the second square, subjects knew only their own points/money relationship.

It is clear from the cost functions that the A games are the transpose of the C games; hence, for an "A" subject the strategic position and possibilities in the A games are the same as for a "B" subject in the C games. It may be seen from Table 3 that A1 is paired for six games with B1 , six with B2 and six with B3 . Similarly for A2 and A3 . It should be clear now that the eighteen games in which a single player plays can be grouped into nine pairs. In each pair everything is absolutely identical -- the person with whom he is paired, his and the competitor's cost functions, his and the competitor's points/money ratios -- except for one thing. In one game the players know one another's points/money ratio, in the other they do not.

The use of the Latin square imparts balance to the design of the experiment.[1] Looking at the left-hand square in Table 3 it may be noted that the three A games are each played just once with each of the player pairings, α, β and γ, that they are played just once with each of the points/money ratios of the "A" subject, and of the "B" subject. The same holds for the B and C games.

The eighteen games were played in an order which was randomly chosen. The number of each game is given below:

Table 4

	I	II	III	I'	II'	III'
1	4	2	17	3	9	1
2	7	11	18	8	13	6
3	15	12	10	5	16	14

The nine pairs of games each had all their profits multiplied by a constant so that overall profit levels for the games would allow the subjects an opportunity to earn $2.00 to $3.00 per hour. These multiplicative factors were not identical for all A and C games and for all B games, but were varied over a modest range so that the set of money payoffs would not be quite identical among a large number of games. They are shown in Table 5.

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1. On Latin squares an experimental design see Cochran, William G. and Gertrude M. Cox, Experimental Designs, Second Ed. John Wiley and Sons (New York), 1957.

Table 5

Game Number	Profit Multiplier
1	.06
2	.013
3	.01516
4	.01516
5	.066
6	.01624
7	.01408
8	.01408
9	.013
10	.01732
11	.075
12	.0184
13	.075
14	.01732
15	.066
16	.0184
17	.06
18	.01624

3. Experimental Procedure

Subjects were, in all cases, hired from the Financial Aids Office of Yale University. As much care as possible was taken to see that the seven people hired for an experimental session lived in different buildings. They were all hired on the understanding they were to do clerical work at the going rates for such work. Hiring seven students provided flexibility in two ways. First, each subject could be given the option of "clerical work" (actually helping to run the games) and being a subject. Of twenty-one students, all opted to be subjects. Second, before the games began, each of the seven was in a monopoly game used as a rationality check. If a subject behaved unreasonably in this (did not maximize profits fairly soon) he was pulled out and used to help run the games. This had to be done only once. If all seven wanted to be subjects and passed the rationality check, one of the seven was randomly chosen to help run the games. The subjects were matched randomly with company titles, A1, B1 , etc.

Upon entering the building, each subject was escorted to a room and given a copy of the instructions to read. Shortly thereafter someone would ask each subject if there were any questions.[1] Each subject was in a room by himself in order to preserve anonymity. Each game was between 5 and 25 periods, although most were from 6 to 14. Because player pairings were changed from game to game, it was necessary to end all three games simultaneously even if this meant they all ended in different periods of play. Also, it was desirable to allow each game to run at its own natural pace -- largely to help avoid boredom.

A period consisted of each player sending two messages to the other, then each choosing his price. One player was designated to send the first message. The second would read and reply. A second round of messages would ensue after which the first would choose a price, then the second would do so. The messages were carried back and forth by a project assistant. After collecting both prices, he would inform each of the price charged by the other; then the next period would begin. If someone wanted to send no message, he was instructed to write an X. On the average, a game took about 35 minutes to play. This included about 8-10

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1. A copy of the instructions appears in the Appendix.

minutes at the beginning when the subjects could examine the new payoff matrix, before the first period began.

The first of the three experimental sessions was not run according to the design described in Section 2. This was due to some labelling errors on some of the experimental materials. Wrong points/money ratios were specified for some of the games. The error was discovered during the first evening of play and some of the unplayed games could be corrected. The design of this first set is given in Table 6. It will be noted that the games in which the players know one another's points/money relationship, and the set where they do not, still constitute two otherwise identical replications. The effect of the mistake was to unbalance the design. The affected games are underlined in the table.

Table 6

	I	II	III	I'	II'	III'
1	Aβ <u>Cβ</u>	<u>Aγ</u> Cγ	Bα	Aβ <u>Cβ</u>	<u>Aγ</u> Cγ	Bα
2	<u>Aα</u>	Bβ		<u>Aα</u>	Bβ	
3	Bγ	<u>Cα</u>		Bγ	<u>Cα</u>	

4. Analysis

There are three lines of analysis which are followed. The first is concerned with the probability that a pair of subjects will agree on a pair of prices to charge, and whether the agreement obtained is invariant to the amount of past experience in the games or to certain changes in the structure and information conditions. The second line of analysis employs a limited dependent variables regression model to estimate the probability that a decision will be Pareto optimal, and the expected value of its distance from the Pareto set as a function of the presence or absence of agreement and of other variables characterizing the game. The third line of analysis is concerned only with those points which are in the Pareto set and which come from asymmetric games. The object here is to determine whether the Nash solution, the joint maximum or the point of equal profits characterizes the Pareto optimal choices.

Except when explicitly noted, all of the analysis which follows utilizes observations which are weighted according to a scheme which is explained in Section 4.1. Sections 4.2 through 4.4 contain the analysis described in the preceding paragraph.

4.1 Weighting the Data

The data are weighted because a standard number of minutes was not allotted to each period, and certain periods naturally took much less time than others. A period of agreement which followed a period of honored agreement was generally very short. This is because the negotiation necessary to making an agreement had generally been completed in an earlier period. Because these periods took less time than the rest, there are a larger proportion of them in the experiment than there would have been if all periods had been forced to a standard length. The data are weighted to compensate for the bias toward too many short periods which is inherent in the way the experiment was conducted. The following regression line was fitted for this purpose:

$$9.58 = \sum_{i=1}^{27} \alpha_i D_{it} N_t + \alpha_{28} S_t + u_t \qquad t = 1, \ldots, 162$$

N_t = the number of periods in the t-th game

S_t = the number of short periods in the t-th game

D_{it} = 1 if the i-th player pairing corresponds to the t-th game

= 0 otherwise

The first player pairing is A1 B1 of the first session, or replication. The second is A1 B2 etc. The ninth is A3 B3 of the first session and the twenty-seventh is A3 B3 of the third session. A short period is a period of agreement (whether or not the agreement is honored) which is preceded by a period of honored agreement. The dependent variable is the mean number of periods in the 162 games. The important thing to note here is that the dependent variable has the same value in each observation. The use of 9.58 causes a convenient normalization of the coefficients by which the weighted sum of periods equals the unweighted sum. The regression is more properly interpreted as a means of estimating the amount of time necessary to complete a period under the assumption that all games lasted the same number of minutes.

Thus if all games lasted 9.58 minutes, $\hat{\alpha}_5$ is the estimate of time necessary for the fifth pair of subjects to complete a (long) period $\hat{\alpha}_5 + \hat{\alpha}_{28}$ is the time they need to complete a short period. In practice, all games were allowed approximately 30 minutes each. The regression coefficients appear in Table 7.

4.2 Agreement Analysis

In this section only two aspects of a period are taken into account: whether or not the subjects agreed upon a course of action, and, if there was agreement, whether it was honored. Each period is classified as:

non-A	nonagreement
A	agreement (honored by both)
AC	agreement (with one "cheating"; i.e., not honoring it)
ACC	agreement (with both "cheating")

It is assumed that the probability of being in each of these four "states" in a given period depends only on the "state" which characterizes the preceding period. In other words a game is being viewed as a first order Markov process in which there are four possible states, non-A, A, AC and ACC. A matrix P in which each row corresponds to a "current state" and each column corresponds to a "next state", and in which an entry, p_{ij}, gives the probability that the next state will be state j, given that the present state is state i is called a "transition matrix" or "matrix of transition probabilities".

Table 7

	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$
player pairing	A1 B1 ses. 1	A1 B2 ses. 1	A1 B3 ses. 1	A2 B1 ses. 1	A2 B2 ses. 1
coefficient	1.4237	1.1098	1.2975	1.1863	1.2966
standard error	(.157)	(.121)	(.135)	(.127)	(.137)
	$\hat{\alpha}_6$	$\hat{\alpha}_7$	$\hat{\alpha}_8$	$\hat{\alpha}_9$	$\hat{\alpha}_{10}$
player pairing	A2 B3 ses.1	A3 B1 ses. 1	A3 B2 ses. 1	A3 B3 ses. 1	A1 B1 ses. 2
coefficient	1.4521	1.1402	1.2821	1.5660	1.0260
standard error	(.148)	(.119)	(.136)	(.163)	(.117)
	$\hat{\alpha}_{11}$	$\hat{\alpha}_{12}$	$\hat{\alpha}_{13}$	$\hat{\alpha}_{14}$	$\hat{\alpha}_{15}$
player pairing	A1 B2 ses. 2	A1 B3 ses.2	A2 B1 ses.2	A2 B2 ses.2	A2 B3 ses. 2
coefficient	1.2683	1.8866	1.1747	1.1175	1.4661
standard error	(.131)	(.191)	(.127)	(.116)	(.151)
	$\hat{\alpha}_{16}$	$\hat{\alpha}_{17}$	$\hat{\alpha}_{18}$	$\hat{\alpha}_{19}$	$\hat{\alpha}_{20}$
player pairing	A3 B1 ses. 2	A2 B2 ses. 2	A3 B3 ses. 2	A1 B1 ses. 3	A1 B2 ses. 3
coefficient	1.0932	1.3491	1.1203	1.4555	1.1927
standard error	(.120)	(.144)	(.128)	(.158)	(.124)
	$\hat{\alpha}_{21}$	$\hat{\alpha}_{22}$	$\hat{\alpha}_{23}$	$\hat{\alpha}_{24}$	$\hat{\alpha}_{25}$
player pairing	A1 B3 ses. 3	A2 B1 ses. 3	A2 B2 ses. 3	A2 B3 ses. 3	A3 B1 ses. 3
coefficient	1.8445	.7611	1.1998	1.6056	1.2240
standard error	(.191)	(.085)	(.129)	(.167)	(.134)
	$\hat{\alpha}_{26}$	$\hat{\alpha}_{27}$	$\hat{\alpha}_{28}$		
player pairing	A3 B2 ses. 3	A3 B3 ses. 3	ALL		
coefficient	1.0506	1.4097	-.3881		
standard error	(.113)	(.146)	(.089)		

Transition matrices were calculated with the data divided in several ways: 1) A four way split according to knowledge of points/money ratios and symmetry of the game structure. 2) A split according to which the first transition from each of the games are used to estimate one transition matrix, the second transition from each of the games are used to estimate another, etc. 3) The transitions from the first two games the subjects play are used to estimate one matrix, the transitions from the third and fourth games are used to estimate another, etc. The third of these approaches turned out to be the most interesting.

The first approach tests whether the structure of the game and the information conditions affect transition probabilities. The remaining two are species of learning hypotheses. The first of the pair tests for the presence of a learning pattern which takes place within the game and operates in the same manner within each game. The final approach tests for changes in behavior as the subjects gain experience within the experiment as a whole.

The four matrices estimated under the first approach were tested to determine whether they are significantly different from one another. Denoting by P_{KS} the matrix estimated from the transitions which occurred during symmetric games in which points/money ratios were known, the hypothesis tested is:

$$H: \quad P_{KS} = P_{K\not{S}} = P_{\not{K}S} = P_{\not{K}\not{S}}$$

This is tested by a χ^2 test with 30 degrees of freedom. The calculated value is 47.93 which is less than the critical value at the 1% level of 50.89. The conclusion is that any variation among the four matrices is due to chance.

Under the second approach, the data are divided according to transition. Let P_i $(i = 1, \ldots, 8)$ be the transition matrix estimated from the i-th transition, and let P_9 be the transition matrix estimated from all transitions subsequent to the eighth. The following hypothesis is tested:

$$H: \quad P_1 = P_2 = \cdots = P_9$$

The calculated χ^2 value is 84.9. These are 80 degrees of freedom. The value $\sqrt{2\chi^2} - \sqrt{2n-1}$ (n = degrees of freedom) is known to be approximately normally distributed with a mean of zero and variance of one. $\sqrt{2\chi^2} - \sqrt{2n-1} = .42$ which is less than the critical value of 2.33 (1% level for a one tailed test). The conclusion is that the transition probabilities do not depend on the timing of the transitions within the game.

Having concluded that transition probabilities are not affected by information conditions, game structure and timing of transitions within the game, it remains to test whether transition probabilities are affected by the amount of experience the subject has had within the experiment itself. Let $P_{i,\ i+1}$ $(i = 1, 3, 5, \ldots, 17)$ be the transition matrix estimated from the i-th and i+1 st games. The hypothesis tested is:

$$H: \quad P_{1,2} = P_{3,4} = \cdots = P_{17,18} \quad .$$

The value of χ^2 is 203.5, and the number of degrees of freedom is 80. $\sqrt{2\chi^2} - \sqrt{2n-1} = 7.56$ which exceeds the critical value of 2.33 (1% level). It must be concluded that learning takes place as subjects gain experience in the games. The contribution to the χ^2 value of 203.5 from each of the nine matrices are, respectively, 89.8, 25.0, 33.1, 5.3, 12.5, 10.0, 9.7, 13.8 and 4.2. Glancing at these numbers raises the question of whether behavior is settled and invariant from the 7 through the 18th games. This hypothesis is tested as:

$$H: \quad P_{7,8} = P_{9,10} = \ldots = P_{17,18} .$$

The calculated χ^2 value is 45.1. There are 35 degrees of freedom $\sqrt{2\chi^2} - \sqrt{2n-1} = 1.19$ which is less than the critical value of 2.33. Thus it appears the transition probabilities became stabilized by the end of the sixth game.

Table 8 shows the transition matrices $P_{1,2}$, $P_{3,4}$, $P_{5,6}$ and $P_{7, \ldots 18}$. The latter is estimated from games 7 through 18. The most striking difference between the first three and the last is the increase in the first two probabilities on the diagonal: the probability of going from non-A to non-A and from A to A.

Generally a matrix of transition probabilities uniquely determines a set of asymptotic probabilities for being in each state in the t-th period as t goes to infinity. These "equilibrium distributions" have

Table 8

Matrices of Transition Probabilities

games 1-2

.558	.331	.083	.028
.161	.803	.036	0
.327	.085	.588	0
0	0	1	0

games 3-4

.847	.131	0	.022
.052	.842	.106	0
.289	.606	0	.105
.667	0	0	.333

games 5-6

.632	.088	.250	.030
0	.903	.097	0
.527	.250	.223	0
1	0	0	0

games 7-18

.826	.128	.046	0
.007	.961	.032	0
.408	.257	.292	.043
1	0	0	0

the property that $Px = x$, where P is the matrix of transition probabilities and x is the (column) vector giving the equilibrium distribution. Also $P^t y \to x$ as $t \to \infty$ where y is an arbitrary initial distribution. Table 9 gives the equilibrium distribution which correspond to the transition matrices in Table 8. The last column of Table 9 gives the observed distribution of transitions in the sample for games 7 through 18. Games 7-18 show an equilibrium distribution in which the second state, A , has a probability of .8, much higher than the earlier games. This is not surprising given the very high transition probability of .96 for the "from

Table 9

Equilibrium Distributions

	games 1-2	games 3-4	games 5-6	games 7-18	observed, games 7-18
non-A	.301	.384	.237	.150	.190
A	.562	.537	.604	.802	.759
AC	.129	.057	.152	.046	.049
ACC	.008	.022	.007	.002	.002

A to A" [illegible]. All three remaining probabilities are substantially smaller for games 7-18. From this analysis, it appears that honored agreement will prevail in cooperative duopoly games a substantial majority of the time after the initial few games when subjects are becoming experienced. Even in the earlier games, honored agreement has an asymptotic probability in excess of .5.

4.3 Distance From the Contract Curve

The analysis in this and the following section utilizes distances measured in profit space. A pair of prices chosen by subjects determine a pair of profit levels and, hence, a unique point in a space in which the profits of the first subject are measured on one axis and the profits of the second are measured on the other. The Pareto set may also be drawn in this space, and the distance of a point from the Pareto set may be defined as the Euclidean distance from the point in question to that point of the Pareto set which is nearest the point in question. The distance concept requires some sort of normalization so that distances do not change when, for example, the unit of measurement of profit changes. Also, two games which are identical in every way, except that profit levels in one are double those of the other, would seem to present the same strategic possibilities. The unit of distance in the high profit game should be double the unit of the other in money terms so that the distance between a pair of corresponding points is the same in each game.

Figure 1 is a representation in profit space of the asymmetric games used in the experiment. The horizontal axis gives the profit of the subject whose cost curve is $C = q^2$. The cost curve of the other subject is $C = 10q + .1q^2$. Any particular asymmetric game differs only by the application of a multiplicative factor to the profits from the game illustrated in Figure 1.

The outer line, CC' is the set of Pareto optimal points for the game. Thus it is the profit frontier and only points on or southwest of this line are attainable by the two subjects. Three points are singled out. JM is the joint maximum, where total profits for the pair are maximized. NS is the Nash solution when the game is viewed as a cooperative game. ES is the point, in the Pareto set, where both receive equal profits. The other curve in the diagram is the optimal threat curve. The point on it labelled NT is the Nash threat point.[1]

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1. Both the Nash solution and Nash threat are located here under the special assumption that the marginal utility of money is constant for all subjects throughout the ranges of payoffs prevailing in the experiment.

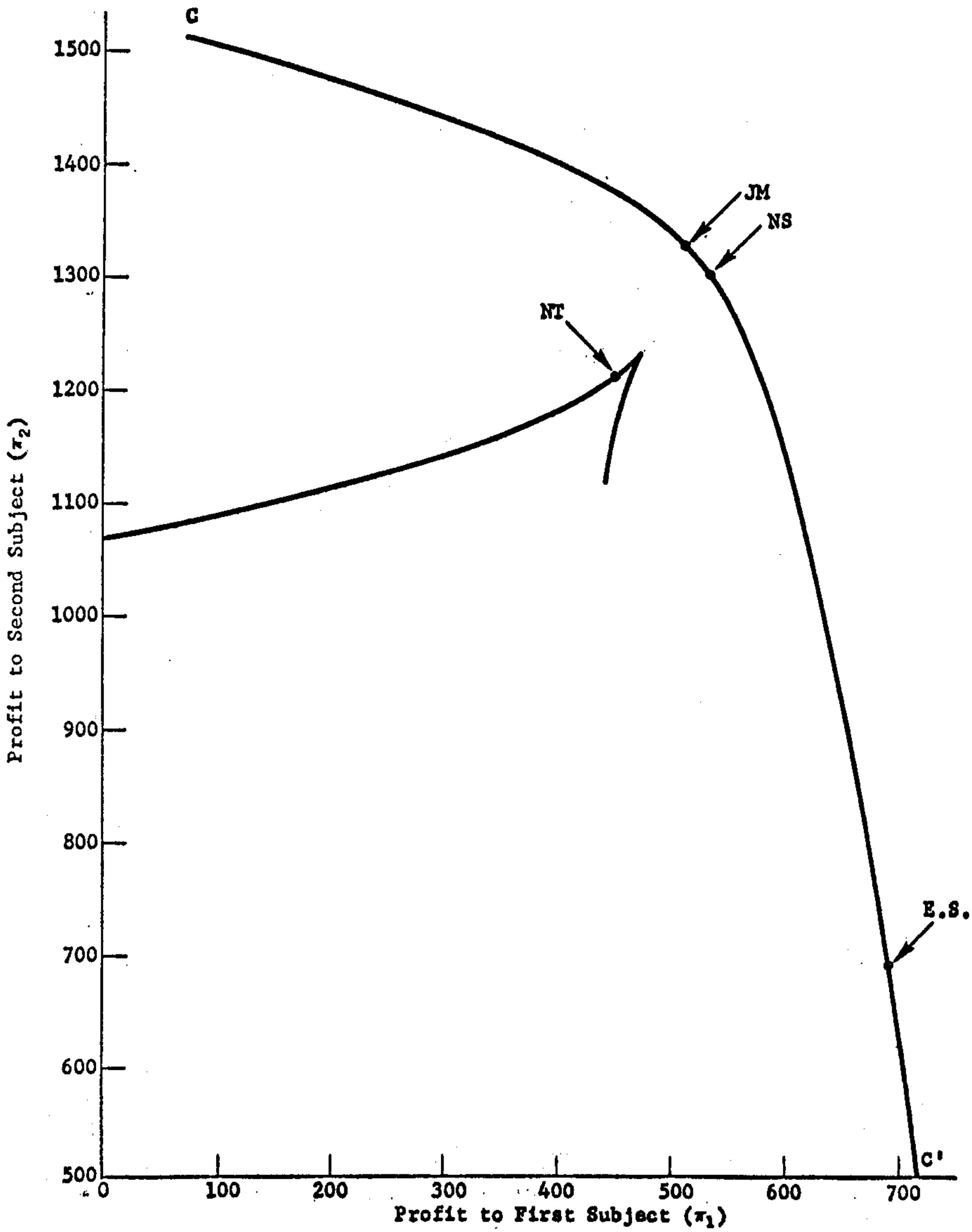

Fig. 1 -- Pareto Optimal Curve and Threat Curve in Profit Space

It may be useful to review the nature of the Nash solution before proceeding further.[1] The review which follows is intended only to outline the characteristics of the solution in terms of a duopoly model of the sort used in the present experiment. The Nash solution is the only solution which satisfies a set of axioms to be found in the original article. In the discussion which follows it is assumed that the marginal utility of money is constant for each subject and, therefore, the terms "money" and "utility" may be used interchangeably. The Nash solution is found in the following way: 1) each player chooses a threat strategy (price). A threat strategy is a strategy which the subject intends to use if he and the other subject fail to come to an agreement. 2) The pair of threat strategies uniquely determines a pair of "threat point profits" and the solution is that point which is attainable by the players which maximizes the product of their gains from cooperation. I.e., if threat point profits are Π_1^T and Π_2^T, the solution is that point at which $(\Pi_1 - \Pi_1^T)(\Pi_2 - \Pi_2^T)$ is maximized. Once a pair of players agree to the procedure of step 2), the choice of a threat strategy by a subject reduces to choosing that strategy which will make his ultimate solution point profit as large as possible.

Figure 2 is an enlargement of part of Figure 1. A point in the Pareto set may be found in either of two equivalent ways: maximize Π_1

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1. See Nash, John, "Two-Person Cooperative Games," Econometrica, Vol. 21, 1953, pp. 128-140 for a complete and formal presentation of the model. Mayberry, J. P., J. F. Nash and M. Shubik, "A Comparison of Treatments of a Duopoly Situation," Econometrica, Vol. 21, 1953, pp. 141-154, contains an application of the Nash solution to a duopoly game. A discussion of the Nash bargaining model appears in Luce, R. D. and Howard Raiffa, Games and Decisions, John Wiley and Sons (New York) 1957, pp. 124-135.

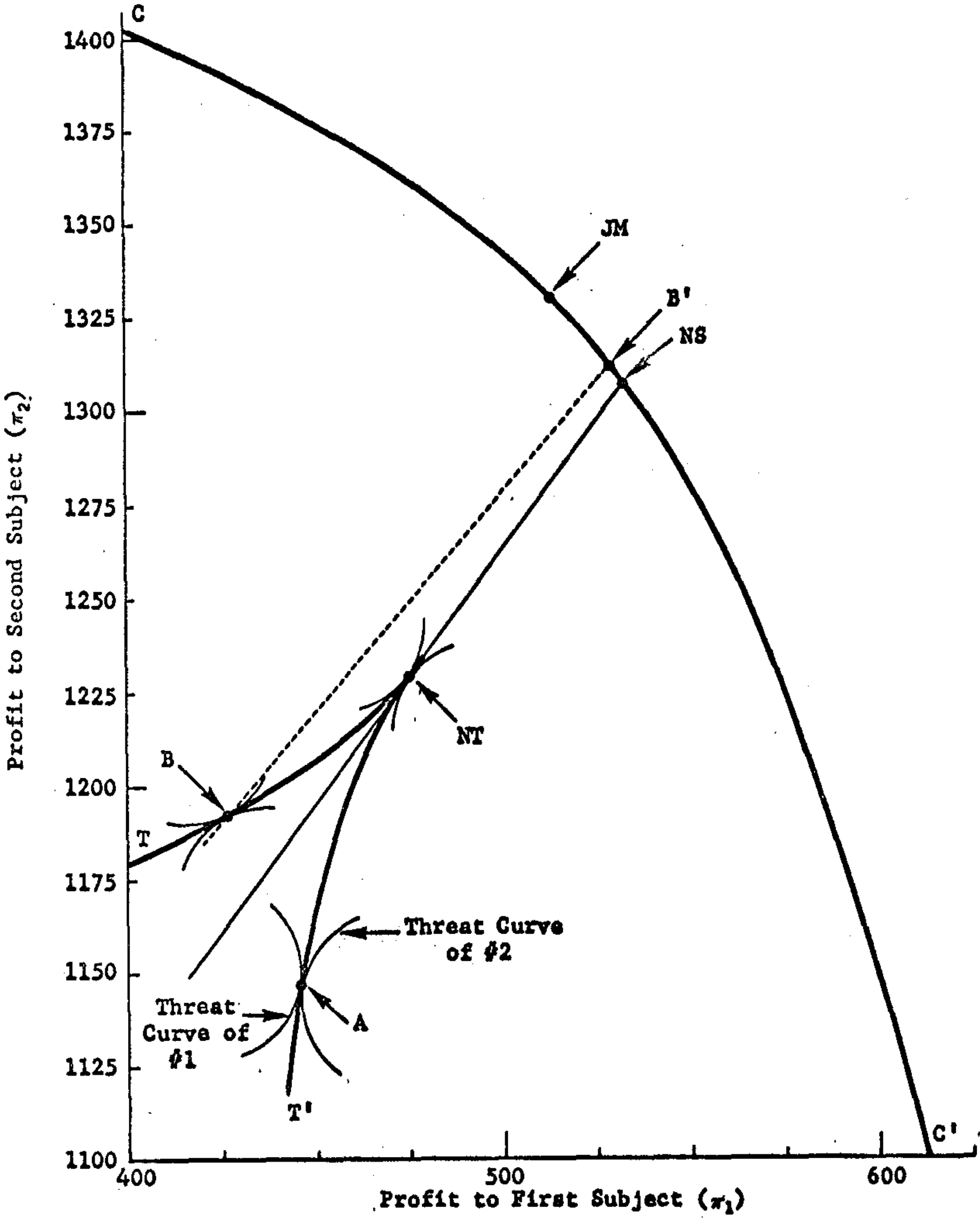

Fig. 2 -- Pareto Optimal Curve and Threat Curve in Profit Space with Illustration of the Nash Solution

with respect to p_1 and p_2, subject to the condition that $\Pi_2 = \Pi_2^*$ As Π_2^* is varied, one moves along the curve CC' in Figure 2. Alternatively, the problem could be formulated: maximize $\Pi_1 + \lambda\Pi_2$ with respect to p_1 and p_2. λ is a positive, finite constant. As λ is varied, one moves along CC'. Either formulation gives rise to a relationship which holds at every point in the Pareto set:

$$\frac{\partial\Pi_2/\partial p_1}{\partial\Pi_1/\partial p_1} = \frac{\partial\Pi_2/\partial p_2}{\partial\Pi_1/\partial p_2} = -\lambda \tag{4}$$

This condition is necessary but not sufficient for a maximum of $\Pi_1 + \lambda\Pi_2$.

The optimal threat curve, TT', may be derived in an analogous way: Minimize Π_1 with respect to p_1 and p_2 subject to the condition that $\Pi_2 = \Pi_2^*$. Again, as Π_2^* is varied, the points of TT' are traced out. A second way to derive the curve is to maximize $\Pi_1 + \lambda\Pi_2$ with respect to p_1 and p_2 where λ is a finite negative constant. The points of the optimal threat curve also satisfy equation (4) above. The second order conditions differ for the two curves. A point on the threat curve is a point of tangency between two individual threat curves. The point labelled "A" in Figure 2 will serve as an illustration. Tangent to TT' at A is a curve labelled "THREAT CURVE OF #2". This curve shows profits to both as #2 varies his price while #1 keeps his price constant. Note that in moving NE from A, #2 causes ever increasing gains for #1 per unit of gain for himself. Moving SW he gives up ever increasing amounts per unit loss caused to #1. The threat curve of #1 shows the reverse characteristics. Thus, either player finds the marginal profit trade-off getting worse if he moves from A .

As a final illustration, consider a point on the optimal threat curve TT' where profits are $\Pi_1 = \Pi_1^*$ and $\Pi_2 = \Pi_2^*$. At this point Π_1 is minimized with respect to p_2 subject to the constraint $\Pi_2 \geq \Pi_2^*$, and, simultaneously Π_2 is minimized with respect to p_1 subject to $\Pi_1 \geq \Pi_1^*$. With these properties of the threat curves in hand, the characteristics of the Nash solution may be given.

Two relationships define the Nash solution and Nash threat points. (It is assumed that the latter is one the optimal threat curve and the former is the Pareto set):

$$\left(-\frac{d\Pi_2}{d\Pi_1}\right)_{\text{at NS}} = \frac{\Pi_2^S - \Pi_2^T}{\Pi_1^S - \Pi_1^T} = \left(\frac{d\Pi_2}{d\Pi_1}\right)_{\text{at NT}} = \left(\frac{\partial\Pi_2/\partial p_i}{\partial\Pi_1/\partial p_i}\right)_{\text{at NT}} \quad i = 1, 2$$

Π_i^S = the profits to player i at the solution point

Π_i^T = the profits to player i at the threat point.

The negative of the slope of CC' at the solution point must equal the slope of the straight line from the threat point to the solution point. The latter must equal the slope of the optimal threat curve at the threat point, which, in turn, equals the slope of the individual threat curves at that point.

$$\left(\frac{d\Pi_2}{d\Pi_1}\right)_{\text{at NT}} = \left(\frac{\partial\Pi_2/\partial p_i}{\partial\Pi_1/\partial p_i}\right)_{\text{at NT}} \qquad i = 1, 2$$

is a consequence of the definition of the optimal threat curve.

$$-\left(\frac{d\Pi_2}{d\Pi_1}\right)_{\text{at NS}} = \frac{\Pi_2^S - \Pi_1^T}{\Pi_1^S - \Pi_1^T}$$

is a consequence of the condition that the solution must maximize $(\Pi_2^S - \Pi_2^T)(\Pi_1^S - \Pi_1^T)$.[1] The somewhat subtle condition is:

$$\frac{\Pi_2^S - \Pi_2^T}{\Pi_1^S - \Pi_1^T} = \left(\frac{d\Pi_2}{d\Pi_1}\right)_{\text{at NT}}$$

Consider point B in Figure 2 as a possible threat point where this equality does not hold. The broken line from B leads to B', the solution point with respect to B. From the point of view of player 1, any point below and right of the broken line is a superior threat point to B because it moves the associated solution point to the SE. Part of the threat curve of #1 is below the broken line, thus he can force the threat point in that direction

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1. This may be shown very readily. Consider the function $y = \Phi(x)$. Say it is desired to maximize $F = (y - y^*)(x - x^*) = [\Phi(x) - y^*][x - x^*]$. $\frac{dF}{dx} = \Phi(x) - y^* + (x - x^*)\ \frac{d\Phi}{dx}$. At the maximum $\frac{dF}{dx} = 0$ and $\frac{d\Phi}{dx} = -\frac{\Phi(x) - y^*}{x - x^*} = -\frac{y - y^*}{x - x^*}$, which is the statement which was to be proved. The point found is a maximum $\frac{d^2\Phi}{dx^2} < 0$.

At the point labelled NT, however, neither can move along his individual threat curve without moving the associated solution point in a direction which diminishes his final profit.

Two important characteristics of the Nash solution are that it is Pareto optimal and it takes into account the threat capabilities of the two players. In the remainder of this section the conditions under which Pareto optimal choices are made will be investigated. The next section is concerned with determining whether Pareto optimal choices are, in general, at the Nash solution.

The remainder of this section is concerned with the extent to which periods of agreement tend to be on the contract curve. The distance of points from the contract curve will be measured as a function of: 1) whether or not the period is from a symmetric game, 2) whether or not the subjects knew one another's points/money ratio and 3) whether or not the period was one of agreement. In view of the finding that transition probabilities are different in the first six games than in the remaining games, the measurement of distance will also take into account whether or not a period comes from one of the first six games. The unit of distance is taken as the length of the straight line segment from the Nash threat point to the Nash solution. This unit may not be an ideal choice; however, it has the virtue that in two games which are strategically identical, the distance between a pair of corresponding points will be the same. Consider, for example, a game A and a game B . They are identical, except the money payoffs in B are twice those of A. It is desirable that the distance from, say the equal split point to the joint maximum should be the same in each. Measured in units of profit, the distance in B would be double the distance in A ; however, using the metric defined above the distances are equal.

The estimation of distance from the Pareto set is accomplished using the technique of limited dependent variables.[1] This technique is appropriate for variables which: 1) cannot have values below (or above) a particular limit value, 2) have a positive probability, p , of being exactly equal to the limit value and 3) are distributed for values greater than the limit according to that part of the normal distribution for which $L < x < \infty$, and $\int_L^\infty f(x)\,dx = 1-p$, where

L = the limit value
x is the dependent variable in question
f(x) is the normal density function.

As the Pareto set is the frontier of attainable profit points, no actual point can lie above. Thus distance from the set can be only distance from below and left. Distance is taken as positive; hence zero is the lower limit value. Maximum likelihood estimates are calculable for the parameters of a limited dependent variables model, and hypotheses may be tested by means of likelihood ratio tests.[2]

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1. On limited dependent variables see Tobin, James, "Estimation of Relationships for Limited Dependent Variables," Econometrica, Vol. 26, (1958) pp. 24-36.

2. For any periods of agreement which involve cheating, the profits used are the agreed upon profits, rather than the observed.

Let:

K = knowledge of points/money ratios
$\not{K}$ = lack of knowledge of points/money ratios
A = agreement
$\not{A}$ = lack of agreement
S = symmetry
$\not{S}$ = non-symmetry
D = distance from the Pareto set.

Knowledge, agreement and symmetry can be combined in eight ways: KAS, KA$\not{S}$, K$\not{A}$S, ... These eight combinations are mutually exclusive and exhaust the categories into which any observation may fall. For each of these eight cells or categories the following relationship is estimated:

$$D_t = \beta_1 + \beta_2 G_t + u_t$$

where D_t is the distance of the t-th observation from the Pareto set and G_t is a dummy variable which equals 1 if the observation is from the one of the first six games and equals zero otherwise. Table 10 gives the estimates of the coefficients and their standard errors. The maximum likelihood estimate of the mean distance for each cell, the probability that an observation in a given cell will be a limit observation, the expected value of an observation in the cell, the number of limit observations in each cell, and the number of non-limit observations. The expected values exceed the maximum likelihood estimates because of the asymmetry of the distribution of the dependent variable. Clearly the symmetric games show a higher probability of limit responses. Also, it appears lack of knowledge increases the probability. The most important

Table 10

	β_1	β_2	maximum likelihood estimates of D		probability of limit response		expected value of D		number of observations	
			games 1-6	games 7-18	games 1-6	games 7-18	games 1-6	games 7-18	at limit	not at limit
KA~~S~~	-1.285 (.136)	.776 (.240)	0	0	.618	.776	.450	.218	237.26	117.76
KAS*	-.133 (.010)			0		.938		.0023	243.	17.31
K~~AS~~	.584 (.276)	1.268 (.451)	1.852	.584	.222	.405	2.162	1.286	9.58	120.16
K~~A~~S*	.625 (.111)			.625		.098		.648	1.28	35.38
~~K~~A~~S~~	-.318 (.042)	.065 (.068)	0	0	.687	.730	.104	.085	259.33	123.56
~~K~~AS	-.254 (.034)	.167 (.040)	0	0	.694	.930	.034	.0054	158.5	39.0
~~KAS~~	1.094 (.181)	-.120 (.306)	.974	1.094	.272	.248	1.242	1.331	0	127.27
~~KA~~S	.419 (.096)	-.041 (.091)	.378	.419	.023	.014	.380	.420	2.40	61.21

*For this cell there are no observations from the first six games.

variable is agreement. For games 7 through 18, the agreement periods show probabilities ranging from .73 to .94, while the non-agreement periods never exceed .41. In most of the cells the probability of limit response is substantially greater for games 7 through 18 than it is for games 1 through 6.[1]

In summary, distance from the Pareto set is determined mainly by the presence or absence of agreement. The maximum likelihood estimate of distance is zero for all cells involving agreement, and positive in the others. The symmetric games show much sharper results than the asymmetric. Symmetric agreement games have probabilities of limit response in excess of .9 (games 7-18) and symmetric non-agreement games, less than .1. This contrasts with approximately .75 and .25 to .4 for the two categories of asymmetric games. Knowledge appears to add slightly to the probability of limit response. Comparing the four pairs of cells, each K cell is seen to have a higher probability than the corresponding K̸ cell; however, the differences are less than .08 in three of the four instances.

It is not surprising, however, that periods of agreement are not invariable Pareto optimal. Choosing a point in the Pareto set is less safe

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1. A likelihood ratio test of the hypothesis H: $\beta_2 = 0$ for each cell independently leads to acceptance at the 1% level for K̸AS̸, K̸A̸S̸ and K̸A̸S. It is the other three cells, for which the hypothesis is rejected, which show the most striking differences between the probabilities of limit response. A joint test of the hypothesis H: $\beta_2 = 0$ for the six cells with observations from the first six games indicates rejection of the hypothesis at the 1% level. The six values of $-2 \ln \lambda$ (where λ is the likelihood ratio) are, respectively: 10.34, 7.91, 1.17, 19.58, 2.74, and .2. The critical values for 1 and 6 degrees of freedom are: 6.6 and 16.8.

than one some distance away in the sense that if one subject cheats on the agreement the loss in profit to the other will be smaller.

4.4 Location of Points in the Pareto Set

The lack of an obvious point of agreement in the Pareto set in asymmetric games makes it very important to attempt an analysis of the points chosen in order that any tendencies might be discovered. The following analysis utilizes 542 periods which involved asymmetric games and Pareto optimal choices. The Pareto set can be represented as a straight line, with an arbitrary origin chosen at the Nash solution, and the same unit of distance as was employed previously. This is displayed in Figure 3. Any point in the Pareto set may be uniquely described by a number which may be positive or negative, and which is the distance of that point from the Nash solution.

Figure 3

Figure 4 shows a histogram for the 542 observations on distance from the Nash solution. From the histogram, it looks as if there may be two separate distributions. The larger of the two, with nearly 500 observations, runs from -6.4 to, say, 2.0. The smaller, with approximately 50 observations, goes from 2.4 to 8.6. The mean of the first distribution is in the

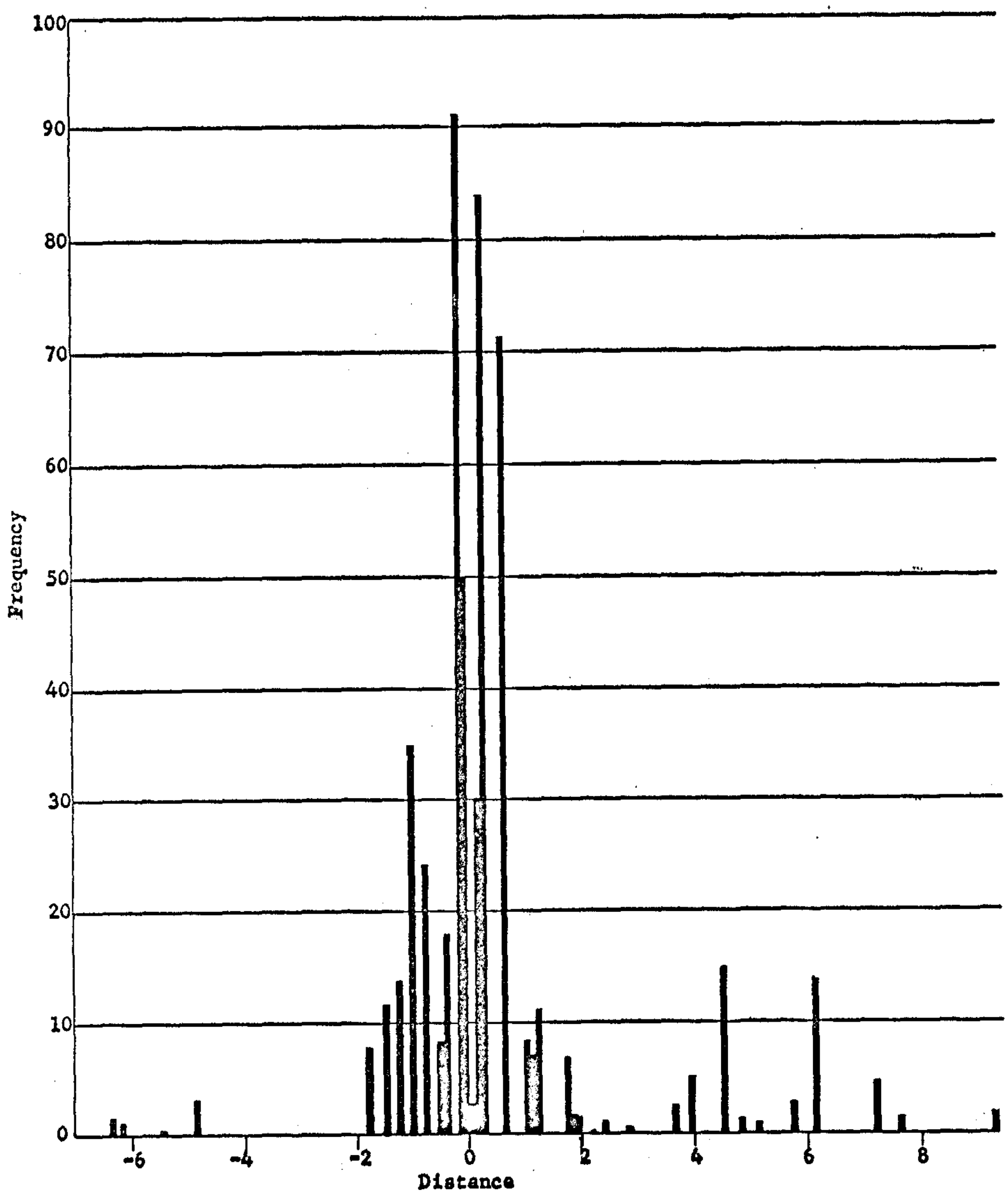

Fig. 4 -- Distances from Nash Solution for Pareto Optimal Points from Asymmetric Games

neighborhood of the joint maximum and the Nash solution, while that of the second is around 5.5 -- relatively close to the equal split point. Due to the appearance of this histogram, a dummy variable(S) was added to the regression which equals 1 when distance is greater than 2.4, and equals zero otherwise. From a glance at Figure 4, it is obvious this variable will prove highly significant statistically. The addition of this variable has two intuitive, but not theoretical, justifications: a) even in an asymmmetric game, the equal split point retains some attraction as a "fair" division of profits and b) some mysterious notion of symmetry encouraged the subjects to choose equal prices. The equation whose parameters were estimated is:

$$D_t = \beta_0 + \beta_1\, KA\not{C}_t + \beta_2\, KAC_t + \beta_3\, \not{K}A\not{C}_t + \beta_4\, \not{K}AC_t + \beta_5\, S_t + u_t$$

$KA\not{C}$ = knowledge, agreement, no cheating
KAC = knowledge, agreement, cheating
$\not{K}A\not{C}$ = no knowledge, agreement, no cheating
$\not{K}AC$ = no knowledge, agreement, cheating
$S = D > 2.4$

As with the limited dependent variables analysis, the distance measure for a period which involves cheating is distance from the point agreed upon, not distance from the point actually chosen. Results are in Table 11.

Table 11

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$
coefficients	-1.97	1.99	1.66	1.88	1.49	5.50
standard error	(.297)	(.302)	(.388)	(.301)	(.386)	(.136)

An F test which tests the hypothesis that this model is not significantly different from one in which $\beta_1 = \beta_2 = \beta_3 = \beta_4$ yields an F ratio of 1.92. The critical value of $F_{3,536}$ for the 1% level is 3.83. It must be concluded they are not significantly different. The second equation is:

$$D_t = \beta_0' + \beta_1' \not{A}_t + \beta_2' S_t + u_t$$

$$\not{A} = \text{non-agreement}$$

(KAC̸, KAC, K̸AC̸, K̸AC and A̸ are a mutually exclusive and exhaustive set of variables).

Table 12 summarizes the results.

Table 12

	$\hat{\beta}_0'$	$\hat{\beta}_1'$	$\hat{\beta}_2'$
coefficients	-.055	-1.91	5.50
standard error	(.043)	(.299)	(.136)

Thus the agreement periods with distances less than 2.4 have a predicted value nearly zero (nearly the Nash solution point), and these account for some 480 of the 542 observations. The non-agreement periods with distances less than 2.4 have a predicted value much smaller than the joint maximum. The joint maximum is at -.33, while these periods have a predicted value of -1.97. The remaining points, for which distance exceeds 2.4, are all agreement points, except for two. The agreement points over 2.4 have a predicted value of 5.44. This is significantly distant from the equal split value of 6.72 (t = 9.41), and also significantly distant from the point at which prices are equal, 4.95 (t = 3.60). It remains a mystery whether any meaning may be read into the coefficient β_2', particularly the interpretations suggested here.

The conclusion is that the Nash solution in asymmetric games appears to have a strong appeal. There remains a subgroup consisting of 10% of the sample whose values were far closer to the equal split than any other point of interest. These leave the suggestion that the equal split point or the equal price point may retain a secondary attraction as a point on which to agree.

5. Conclusions

The cooperative duopoly games reported here have some special characteristics in comparison with firms in real markets. First, all of the power and authority with a "firm" in the experiment is held by one person. He has at his disposal all relevant information about his own firm, and all about his competitors (except for the points/money trade-off). Authority in real firms is more dispersed, perhaps because the relevant information possessed by a firm is dispersed among a large number of people. And such information as a firm has is neither so complete nor so precise. Finally, real firms have more than one decision variable.

The cooperative duopoly experiment reported here is an ideal case, and therefore an appropriate starting point for the study of cooperative oligopoly games. It embodies the assumptions found in theoretical discussions and will give an indication of the appropriateness of these assumptions as they are relaxed in further experimental research and the results compared with earlier work.

The results support the Adam Smith hypothesis that, given the opportunity to discuss, "People of the same trade...[will form a]...conspiracy against the public..."[1] Over 3/4 of all the periods were periods of agreement. These were honored in nine out of ten cases. Among the agreements, better than 3/4 involved Pareto optimal points, which is in keeping with von Neumann and Morgenstern.

The Nash hypothesis received quite strong, but qualified support. If the observations more than 2.4 units to the right (see Figure 3) of the Nash solution, which seem to form a separate distribution, are left out of account, the remaining agreement periods have a mean almost exactly at the Nash solution. These periods account for 85% of the Pareto optimal periods of the asymmetric games. The points above 2.4 have a mean significantly distant from the equal split point; however, it is much closer to the equal split point than the Nash solution or the joint maximum. Perhaps the frequency with which the equal split point is chosen as a point of agreement varies inversely with its distance from the Nash solution.

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1. Wealth of Nations, Modern Library Edition, page 128.

Appendix - Instructions to Subjects

Instructions for Participants

In Research Project on Economic Decision Making

I. General Description of the Project

In this project you will be able to earn an appreciable amount of money. The amount you earn will depend upon the decisions you make and you will be able to keep all the money you earn.

You will be placed in several decision making situations. The situations all have certain things in common. In each situation you will be randomly paired with one other person. You will not see this person or speak with him at any time. You will never know his identity, nor will he ever know yours. You will be able to send written messages to him, and he to you.

You and he are the sole producers and sellers of some fictitious commodity. Each decision making situation will last for many time periods and both you and the other seller will make one decision in each period. Your decision will be to choose the price at which you sell your commodity during the period. The other seller chooses the price at which he sells. Before making your decision for a period you will be told the decision made by the other seller for the preceding period. You and he each will have time to send two messages if you wish to do so.

You will have a profit table on which the prices you may choose are listed down the left hand margin. The prices the other seller may choose are listed across the top of the table. The body of the table gives the profit to you and the profit to the other seller which correspond to any pair of prices you and he might choose. Your profit appears above and slightly to the left of his. For example, look at the sample profit table which appears below. If you charge 34 and the other seller charges 38, your profit is 34.3 points and his is 15.8 points.

The number of "points" which equal $1.00 of earnings to you will be different from one decision making situation to another; however, it will always be the same for all periods of a situation. You will always know how many points equal $1.00 for yourself. Sometimes you will not know how many points equal $1.00 for the other seller, and at those times he will not know how many points equal $1.00 for you. Your earnings for participating in the project will be the money you make in the decision making situations.

SAMPLE PROFIT TABLE

	29	31	34	38	42	45
29	5.4 5.4	8.4 9.5	12.1 8.4	16.1 -6.3	18.7 -36.0	19.8 -68.3
31	9.5 8.4	13.7 13.7	19.6 14.7	26.2 2.8	31.6 -24.3	34.7 -54.5
34	8.4 12.1	14.7 19.6	23.6 23.6	34.3 15.8	43.8 -7.2	50.0 -34.4
38	-6.3 16.1	2.8 26.2	15.8 34.3	31.9 31.9	46.8 14.4	57.1 -8.7
42	-36.0 18.7	-24.3 31.6	-7.2 43.8	14.4 46.8	34.7 34.7	49.1 15.7
45	-68.3 19.8	-54.5 34.7	-34.4 50.0	-8.7 57.1	15.7 49.1	33.2 33.2

II. Detailed Description of How the Project Will Proceed

A. Materials which you will use:

a) Profit Table

b) Price-Profit Sheet

c) Message Record

You will retain one Profit Table and one Price-Profit Sheet throughout each Decision-Making Situation. The Message Record, on which you and the other seller will write your messages, will be carried back and forth between you and the other seller by a project assistant.

B. Messages

In order to avoid biasing the results of this project, it is extremely important to adhere to a few simple rules regarding messages;

a) A message may consist of, at most, two short sentences.

b) Messages should be printed in capital letters.

c) Messages may not contain your name or other information which will identify you. They may not divulge how many points equal $1.00 for you when this information has not been given to the other seller. In your messages, you are free, however, to discuss the prices you may want to charge.

C. Procedure

A "period" is completed when both you and the other seller have each chosen your prices. A Decision-Making Situation will consist of many periods, and you will not be told in advance how many periods, in a given Situation. During a Decision-Making Situation, you will retain a Profit Table and be Paired with a certain other seller. When one Situation ends and another begins you will, in general, receive a new Profit Table and be paired with some new other seller.

During a single period both you and the other seller will send two messages each, and then you will each choose your price for that period. If there is nothing you wish to say in a message, you may put a large X across the appropriate message box.

Each Profit Table has either a large "A" or a large "B" in the lower right-hand corner. If yours has an "A", the other seller's has a "B". In the description of procedure below, the two sellers will be referred to as A and B.

A period begins when the project assistant enters the office of one seller, say A, and gives him the Message Record. He then prints a message in the first box on the side marked A (or puts an X in the box), then the project assistant takes the Message Record to B who reads A's message and sends one in reply (or puts an X in the appropriate box). The project assistant then goes to A for his second message and back to B for his second. Finally the project assistant goes to A for his price decision, and then to B. A and B write their prices on their Price-Profit Sheets, and the project assistant copies them down.

D. Final Comments.

A period will ordinarily be completed in less than five minutes; however, at the beginning of each Decision-Making Situation you will be given about ten minutes so you can familiarize yourself with the new Profit Table, before the first period begins.

At the end of the time for which you have been hired, you will be paid. Remember, your total earnings for participating in the project will be the profits which you make.

If you have any questions as the project progresses, simply inform the project assistant and he will bring someone to answer them.

The first Decision-Making Situation will be different from all that come after it, and will differ from the above description in two ways: There will be no messages and you will not know anything about profit for the other seller. All Decision-Making situations after the first will be exactly as described in these instructions.

Please do not discuss this project with anyone tomorrow or in the future, because such discussion could bias the results of future projects of this type.