

# An Explicit RC-Circuit Delay Approximation Based on the First Three Moments of the Impulse Response\*

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## Abstract

*Due to its simplicity, the ubiquitous Elmore delay, or first moment of the impulse response, has been an extremely popular delay metric for analyzing RC trees and meshes. Its inaccuracy has been noted, however, and it has been demonstrated that higher order moments can be mapped to dominant pole approximations (e.g. AWE) in the general case. The first three moments can be mapped to a two-pole approximation, but stability is an issue, and even a stable model results in a transcendental equation that must be iteratively evaluated to determine the delay.*

*In this paper we describe an explicit delay approximation based on the first three moments of the impulse response. We begin with the development of a provably stable two-pole transfer function/impedance model based on the first three moments (about  $s=0$ ) of the impulse response. Then, since the model form is known, we evaluate the delay (any waveform percentage point) in terms of an analytical approximation that is consistently within a fraction of 1 percent of the "exact" solution for this model. The result is an accurate, explicit delay expression that will be an effective metric for high-speed interconnect circuit models.*

## 1 Introduction

Moments and moment-matching approximations are widely used as delay metrics and measures for RC circuit models of gates and their associated interconnect. The Elmore delay [1, 2], or first moment of the impulse response, has been the metric of choice for high-level design automation applications due to the ease of calculation and the resulting closed form expression for delay as a function of circuit parameters. For more accurate analyses, several moments are often used to generate reduced-order transfer function approximations (e.g. AWE [3]) that are efficiently applied for various back-end analyses.

As RC interconnect effects become more pronounced,

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however, the simple Elmore delay approximation is an inadequate delay measure for all nodes of a gate and interconnect path model [4]. Moreover, when the RC interconnect effects are dominant, the need for high-level delay metrics becomes even more critical. It is important that the RC interconnect delays are considered during the early phases of design in such cases, and that the high-level design metrics are correlated to the more accurate back-end analyses.

It was recently demonstrated [4] that the first three moments of the impulse response are required to predict the delay of all nodes in an RC tree with reasonable accuracy. In [5], the first few moments and the sum of the open circuit time constants were used to generate a stable two-pole model. Unfortunately, the approximation is inaccurate at times, and is subject to yielding a complex pole pair for RC circuits. Of course the first three moments can also be used to generate a two-pole model via moment matching, e.g. AWE, but higher order moments, or expansions about other frequency points are sometimes required for ensuring stable approximations.

In this paper we develop a two-pole approximation, based upon the first three moments of the impulse response, that generates stable real poles for RC circuits. While this is an advantage for high-level design approximations, it still renders a transcendental expression for the voltage waveform that must be solved iteratively to determine the delay. But with an understanding of the form of this two-pole approximation, we estimate the delay (any waveform percentage point) with extreme accuracy without nonlinear iterations. The result is an explicit expression for the delay as a function of the first three moments of the impulse response.

## 2 Background

A decade ago, the Elmore delay [1] was considered accurate enough to approximate the delay for most linear RC interconnects. For an RC tree, the Elmore delay at some node  $e$ ,  $T_{De}$ , can be computed with linear complexity:

$$T_{De} = \sum_k R_{ke} C_k \quad (1)$$

$R_{ke}$  is the common resistance path between node  $k$  to the input node and node  $e$  to the input node,  $C_k$  is the capacitance at node  $k$ , and the summation is over all nodes. Due to the uncertainty of the Elmore delay accuracy, bounds for the step response delay, based on one pole expressions with time constants determined using the circuit elements values, were developed [2]. Recently it was shown that the Elmore delay is

an upper bound on the 50% step response delay, based on the unimodal property of the RC tree impulse responses [4].

For today's deep submicron technologies, however, the reduced wiring dimensions have increased the resistance of the interconnect paths and the single pole representations of the waveforms are no longer reliable. Maximum errors due to these first order models are typically over 100% for high-speed interconnect paths since the higher-order pole effects can no longer be ignored. One of the first attempts to characterize transfer functions using two poles was reported in [5]. The "charge-sharing" model proposed in [5] attempted to characterize the moments of a transfer function in terms of circuit elements. For example, the transfer function at node  $e$  was expressed as:

$$H(s) = 1 - sT_{De} + s^2 \sum_k R_{ke} C_k T_{Dk} + \dots \quad (2)$$

No higher order moments were determined, so the approximation was built using only the first two moments, corresponding to a two pole and a zero approximation of the form:

$$H(s) = \frac{k(1 + s\tau_z)}{(1 + s\tau_1)(1 + s\tau_2)} \quad (3)$$

The lack of higher order information forced the authors of [5] to use heuristics based on the sum of the reciprocal poles,  $T_P$ :

$$T_P = \sum_k R_{kk} C_k \quad (4)$$

Asymptotic Waveform Evaluation (AWE) [3] was later developed as a more general method for computing any number of transfer-function moments for general RLC circuits:

$$H(s) = m_0 + m_1 s + m_2 s^2 + m_3 s^3 + \dots, \quad (5)$$

The coefficients of the Taylor series expansion of a transfer function  $H(s)$  about  $s=0$  (shown in (5)) are expressed in the time domain as:

$$m_k = \frac{(-1)^k}{k!} \cdot \int_0^\infty t^k \cdot h(t) dt \quad (6)$$

where  $h(t)$  is the time domain form of the transfer function. It is apparent from (6) that these coefficients are related to the

moments of  $h(t)$  by the factor  $\frac{(-1)^k}{k!}$ .

AWE applied moment matching, recognized as a Padé approximation, to generate reduced order transfer function approximations. A Padé approximation can be used to transform the moment-series representation of the transfer function, shown in (5), to a rational form:

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0} \quad (7)$$

in terms of the dominant poles and zeros

Padé approximations are simple and easy to implement,

but they suffer from the drawback that they can yield positive (unstable) or complex poles for circuits with negative real poles (e.g. RC circuits). The approaches used by the various AWE techniques to overcome potential instability generally involves calculating more moments, or moments about multiple expansion points. In [6], using the fact that higher order moments contain less information about the higher order poles, moment shifting is used to converge on the low frequency pole values. The technique is efficient, but the shifting process requires the calculation of more moments. Frequency shifting [7] can be used to generate higher order approximations that cover a wider bandwidth, however, more moments, expanded about points other than  $s=0$ , are required. The latest advance is the Padé Via Lanczos (PVL) process [8] which, like the frequency shifting technique, can calculate a large number of poles for RLC circuits. PVL also has error control which makes it a robust approach for generating a transfer function model that must cover a large bandwidth of interest. Of course, all of these benefits are at a cost of increased computational complexity as compared with the linear complexity required to calculate moments of RC trees.

The objective of the work described in this paper is to generate a stable, reliable approximation in terms of just a few moments expanded about  $s=0$ . We describe the essence of the approach in the following two sections.

### 3 Calculating First Two Poles with Guaranteed Stability

It was demonstrated in [6] that the ratio of successive moments forms a series asymptotically converging to the value of the dominant pole:

$$p_1 = \lim_{j \rightarrow \infty} \left( \frac{m_j}{m_{j+1}} \right) \quad (8)$$

For a circuit with  $N$  poles, the expression of the  $j$ -th moment in terms of poles and residues is:

$$m_j = \sum_{i=1}^N \frac{k_i}{(p_i)^{j+1}} \quad (9)$$

where  $k_i$  is the  $i$ -th residue and  $p_i$  is the  $i$ -th pole. Using (9), we

can express the ratio of  $\frac{m_j}{m_{j+1}}$  as:

$$p_1 \frac{1 + \frac{k_2}{k_1} \left( \frac{p_1}{p_2} \right)^{j+1} + \frac{k_3}{k_1} \left( \frac{p_1}{p_3} \right)^{j+1} + \dots + \frac{k_N}{k_1} \left( \frac{p_1}{p_N} \right)^{j+1}}{1 + \frac{k_2}{k_1} \left( \frac{p_1}{p_2} \right)^{j+2} + \frac{k_3}{k_1} \left( \frac{p_1}{p_3} \right)^{j+2} + \dots + \frac{k_N}{k_1} \left( \frac{p_1}{p_N} \right)^{j+2}} \quad (10)$$

As stated in [6], as  $j$  increases in value,  $\frac{m_j}{m_{j+1}}$  in (10) is approaching the value of the dominant pole, since  $p_1 < p_2 < \dots < p_N$ .

Therefore, using only the information contained in the first three moments of the impulse response, the most accurate approximation of the first pole that we can apply is:

$$p_1 = \frac{m_2}{m_3} \quad (11)$$

This asymptotic convergence is also explained by Padé tables [9], but it is important to point out that this convergence is not necessarily monotonic. In fact, we can show that based on the monotonicity of the series from (8), we can determine when a Padé (moment matching) approximation fails to yield a stable two-pole approximation.

### 3.1 Two-pole Padé instability

For a Padé approximation of the form  $H(s) = \frac{N(s)}{D(s)}$ , the coefficients of  $D(s)$  can be expressed in terms of the moments of  $H(s)$ . Expanding  $H(s)$  around  $s = 0$  we obtain:

$$H(s) = m_0 + m_1 s + m_2 s^2 + m_3 s^3 + \dots \quad (12)$$

where the series coefficients in (12) are given by (6) and will be referred to as moments for notational simplicity.

For a second order Padé approximation, it has been well documented in the literature [3] that the denominator polynomial coefficients can be obtained by first calculating the moments of  $H(s)$ , then solving the following linear set of equations:

$$\begin{bmatrix} m_0 & m_1 \\ m_1 & m_2 \end{bmatrix} \begin{bmatrix} d_2 \\ d_1 \end{bmatrix} = - \begin{bmatrix} m_2 \\ m_3 \end{bmatrix} \quad (13)$$

where the denominator polynomial  $D(s)$  is:

$$D(s) = 1 + d_1 s + d_2 s^2 \quad (14)$$

For this two pole case, the coefficient values are:

$$d_2 = \frac{m_1 m_3 - m_2^2}{m_0 m_2 - m_1^2} \quad \text{and} \quad d_1 = \frac{m_0 m_3 - m_1 m_2}{m_1 - m_0 m_2} \quad (15)$$

Because  $D(s)$  is a second order polynomial, if  $d_2$  is negative, the roots will be real and of opposite sign (one negative and one positive pole). From (15) we can write:

$$\frac{1}{d_2} = \frac{m_2 m_1 \left( \frac{m_1}{m_2} - \frac{m_0}{m_1} \right)}{m_3 m_2 \left( \frac{m_2}{m_3} - \frac{m_1}{m_2} \right)} \quad (16)$$

For a passive RC interconnect circuit, the moments of  $H(s)$  will alternate in sign. Namely,  $m_0$  and  $m_2$  are positive while  $m_1$  and  $m_3$  are negative [3]. The sign of  $d_2$  is given by

the monotonicity of the successive moments ratios:  $\frac{m_0}{m_1}, \frac{m_1}{m_2}$

and  $\frac{m_2}{m_3}$ . Therefore, it follows from (16) that a Padé approximation based on the first three moments will yield two real negative poles only when the ratio of successive moments (equation (8)) is monotonically approaching the value of the first pole.

### 3.2 Approximating a second pole

Equation (11) is a viable approximation for the first pole of our two pole model, however, the stability of a second pole approximation will depend on the convergence properties of the successive moment ratios. To explain how we arrive at a guaranteed stable approximation for the second pole, we begin with the ratio of two successive moments in (10). Using polynomial division, we can rewrite (10) as:

$$p_1 \left[ 1 + \frac{k_2 \left( \frac{p_1}{p_2} \right)^{j+1}}{k_1 \left( \frac{p_1}{p_2} \right)} \left( 1 - \frac{p_1}{p_2} \right) + \frac{k_3 \left( \frac{p_1}{p_3} \right)^{j+1}}{k_1 \left( \frac{p_1}{p_3} \right)} \left( 1 - \frac{p_1}{p_3} \right) + \dots \right] \quad (17)$$

Next we consider the following ratio of terms:

$$\frac{\left( \frac{m_j}{m_{j+1}} - \frac{m_{j+1}}{m_{j+2}} \right)}{\left( \frac{m_{j+1}}{m_{j+2}} - \frac{m_{j+2}}{m_{j+3}} \right)} \quad (18)$$

Using (17), the ratio in (18) becomes:

$$\frac{\frac{k_2 \left( \frac{p_1}{p_2} \right)^{j+1} \left( 1 - \frac{p_1}{p_2} \right)^2 + \frac{k_3 \left( \frac{p_1}{p_3} \right)^{j+1} \left( 1 - \frac{p_1}{p_3} \right)^2 + \dots}{k_1 \left( \frac{p_1}{p_2} \right)}}{\frac{k_2 \left( \frac{p_1}{p_2} \right)^{j+2} \left( 1 - \frac{p_1}{p_2} \right)^2 + \frac{k_3 \left( \frac{p_1}{p_3} \right)^{j+2} \left( 1 - \frac{p_1}{p_3} \right)^2 + \dots}{k_1 \left( \frac{p_1}{p_2} \right)}} \quad (19)$$

Note that for  $p_1 < p_2 < \dots < p_N$ , the ratio in (19) is an approximation of the ratio of the first two poles for large values of  $j$ . Due to the uncertainty of the monotonicity of the moment ratios, we do not know the sign of (19), but we can still consider convergence to the *exact* magnitude of the ratio of the first two poles. Rewriting (19) and considering just the magnitude of the ratio in (18), we obtain:

$$\left| \frac{\frac{m_j}{m_{j+1}} - \frac{m_{j+1}}{m_{j+2}}}{\frac{m_{j+1}}{m_{j+2}} - \frac{m_{j+2}}{m_{j+3}}} \right| = \frac{p_2}{p_1} \left| \frac{1 + \frac{k_3 \left( \frac{p_2}{p_3} \right)^{j+1} \left( 1 - \frac{p_1}{p_3} \right)^2}{k_2 \left( \frac{p_2}{p_3} \right)} \left( 1 - \frac{p_1}{p_2} \right)^2 + \dots}{1 + \frac{k_3 \left( \frac{p_2}{p_3} \right)^{j+2} \left( 1 - \frac{p_1}{p_3} \right)^2}{k_2 \left( \frac{p_2}{p_3} \right)} \left( 1 - \frac{p_1}{p_2} \right)^2 + \dots} \right| \quad (20)$$

As  $j$  increases, the right hand side of (20) is approaching the magnitude of the ratio of the first two poles:

$$\lim_{j \rightarrow \infty} \frac{\left( \frac{m_j}{m_{j+1}} - \frac{m_{j+1}}{m_{j+2}} \right)}{\left( \frac{m_{j+1}}{m_{j+2}} - \frac{m_{j+2}}{m_{j+3}} \right)} = \frac{p_2}{p_1} \quad (21)$$

Therefore, based on the first three moments, our most accurate approximation for the second pole is:

$$p_2 = p_1 \frac{\left( \frac{m_0}{m_1} - \frac{m_1}{m_2} \right)}{\left( \frac{m_1}{m_2} - \frac{m_2}{m_3} \right)} \quad (22)$$

This procedure can be used to obtain higher order poles too, but it is not very convenient since the required number of moments is increasing as  $2^q$ , to obtain the  $q$ -th pole. In contrast for a moment-matching approximation such as AWE, where the required number of moments increases linearly with the approximation order.

To demonstrate the accuracy of this pole convergence, we consider the 10-pole RC tree shown in Fig. 1. Note that this circuit is stiff (widely varying time constants), thereby making the pole convergence more difficult to achieve since the moment ratios converge to the poles non-uniformly. The first and second pole approximations based on (11) and (22) and normalized to their exact values are shown in Fig. 2 for higher and higher order moments. The moments used to demonstrate this convergence were those at capacitor C4.

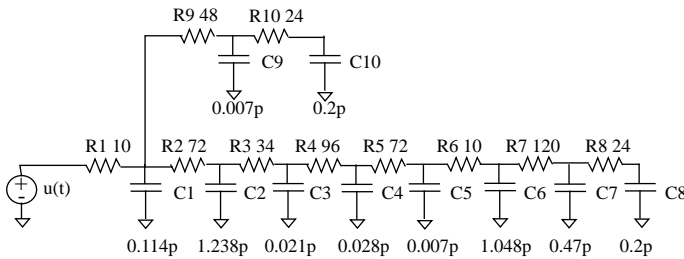


Figure 1: RC circuit with widely varying time constants.

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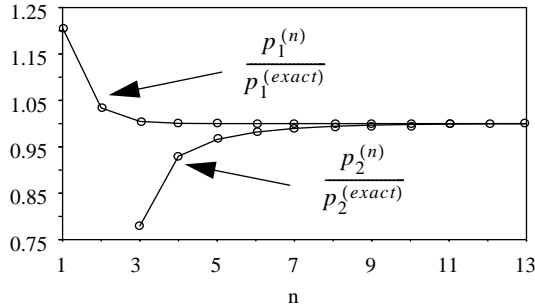


Figure 2: The first two pole approximations based on (11) and (22) and normalized to their exact values as a function of  $n$ , the highest order moment.

#### 4 Stable Two-Pole Approximation Based on First Three Moments

With a pair of stable poles that approximates the first two poles of the actual circuit, what remains is to match an impulse response of the form

$$f(t) = k_1 e^{-p_1 t} + k_2 e^{-p_2 t} \quad (23)$$

to the first two moments of the actual transfer function. To simplify terminology, we will use the above expression for the transfer function which assumes that the  $p$ -terms are positive (for a stable response). The moment matching equations are:

$$\frac{k_1}{p_1} + \frac{k_2}{p_2} = m_0 = 1 \quad (24)$$

$$\frac{k_1}{p_1^2} + \frac{k_2}{p_2^2} = -m_1 \quad (25)$$

Where the pole-terms are given by:

$$p_1 = -\frac{m_2}{m_3} \quad \text{and} \quad p_2 = p_1 \frac{\left( \frac{1}{m_1} - \frac{m_1}{m_2} \right)}{\left( \frac{m_1}{m_2} - \frac{m_2}{m_3} \right)} \quad (26)$$

From (24) and (25) the residue expressions are:

$$k_1 = \frac{1 + m_1 p_2}{p_1 - p_2} p_1^2 \quad (27)$$

$$k_2 = -\frac{1 + m_1 p_1}{p_1 - p_2} p_2^2 \quad (28)$$

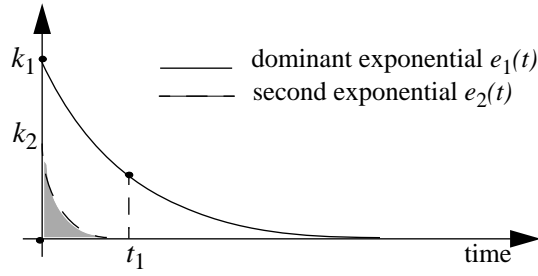
#### 5 Explicit Approximation of the Delay Point

For our two-pole approximation we can assume that two conditions are satisfied: one pole is dominant such that  $p_1 < p_2$ , and the residue of the dominant pole is always positive (since the area of the impulse response is positive). Therefore, we can assume that the exponential term corresponding to the dominant pole is also dominant. These conditions will allow us to approximate the delay in terms of an explicit expression.

##### 5.1 Delay points for the step responses

In Fig. 3 we consider our two-pole impulse response with its pair of exponential functions. First, we seek the median point of this sum of exponentials to approximate the 50% delay point of the step response (because the step response is the time integral of the impulse response). To first order, we might consider that the second exponential is rapidly decreasing to zero such that the 50% delay point ( $t_1$  in Fig. 3) occurs at a point where the value of the second exponential is practically zero. The first order estimate considers the contribution of the second exponential (the cross-hatched area) as a constant factor to the step response delay.

The general form of the impulse response is given by:



**Figure 3: The components of a two-pole impulse response: a dominant exponential  $e_1(t)$  and a second exponential  $e_2(t)$ .**

$$f(t) = k_1 e^{-p_1 t} + k_2 e^{-p_2 t} \quad (29)$$

so the cross-hatched area is:

$$\frac{k_2}{p_2} \quad (30)$$

with the appropriate sign given by the sign of the second pole residue  $k_2$  (shown in the figure as positive).

For our first order estimate then, we seek a point  $t_1$  which satisfies the equation:

$$\int_0^{t_1} k_1 e^{-p_1 t} dt + \frac{k_2}{p_2} = 0.5 \quad (31)$$

The solution of (31), using the constraint from (24), is:

$$t_1 = \frac{1}{p_1} \ln \left( \frac{2k_1}{p_1} \right) \quad (32)$$

The first order delay estimate,  $t_1$ , can be used as an initial guess to find the “exact” delay via a Newton-Raphson (N-R) iteration. However, since it is a good initial guess, we can also consider a single N-R iteration step as an improved explicit approximation. Using  $t_1$  as an initial guess, one N-R iteration yields a delay,  $t_{NR}$  of:

$$t_{NR} = t_1 + \frac{-0.5 + \frac{k_1}{p_1} e^{-p_1 t_1} + \frac{k_2}{p_2} e^{-p_2 t_1}}{k_1 e^{-p_1 t_1} + k_2 e^{-p_2 t_1}} \quad (33)$$

The same algorithm can be applied for any percentage delay point. In the general case, for  $100 \cdot \alpha$  % delay point, the first order approximation is:

$$t_1 = \frac{1}{p_1} \ln \left( \frac{k_1}{(1-\alpha)p_1} \right) \quad (34)$$

Using (34) as an initial guess, the explicit single-step N-R approximation becomes:

$$t_{NR} = t_1 + \frac{(\alpha-1) + \frac{k_1}{p_1} e^{-p_1 t_1} + \frac{k_2}{p_2} e^{-p_2 t_1}}{k_1 e^{-p_1 t_1} + k_2 e^{-p_2 t_1}} \quad (35)$$

## 5.2 Delay points for ramp responses

The general form of a two pole system response to a saturated ramp is given by:

$$y(t) = \begin{cases} \frac{t - B + \frac{k_1}{p_1^2} e^{-p_1 t} + \frac{k_2}{p_2^2} e^{-p_2 t}}{t_t} & \text{if } t \leq t_t \\ \frac{t_t - \frac{k_1}{p_1^2} e^{-p_1 t} (e^{p_1 t_t} - 1) - \frac{k_2}{p_2^2} e^{-p_2 t} (e^{p_2 t_t} - 1)}{t_t} & \text{if } t > t_t \end{cases} \quad (36)$$

where  $B = k_1/p_1^2 + k_2/p_2^2$  and  $t_t$  is the saturated ramp transition time.

To find the  $100 \cdot \alpha$  % delay point on this response we first have to determine the region where it belongs by computing the response at  $t = t_t$  and comparing it with  $100 \cdot \alpha$ . Then we compute the first order delay estimate,  $t_1$ , based on neglecting the second pole exponential term. We observe that, for  $t \leq t_t$ ,  $t_1$  is the solution of the following equation:

$$a e^{p_1 t_1} = t_1' \quad (37)$$

where  $a = \frac{k_1}{p_1^2} e^{-p_1 t_0}$ ,  $t_0 = \alpha t_t + B$  and  $t_1' = t_0 - t_1$ . The solutions of (37) for various  $a$ 's and  $p_1$ 's can be pre-computed and stored as a two-dimensional table.

For  $t > t_t$   $t_1$  has an explicit solution equivalent with (34), that is:

$$t_1 = \frac{1}{p_1} \ln \left( \frac{k_1 (e^{p_1 t_t} - 1)}{(1-\alpha) p_1^2 t_t} \right) \quad (38)$$

In both cases,  $t_{NR}$  can be explicitly written as a function of  $t_1$  with the one step Newton-Raphson formula:

$$t_{NR} = t_1 - \frac{y(t_1) - \alpha}{y'(t_1)} \quad (39)$$

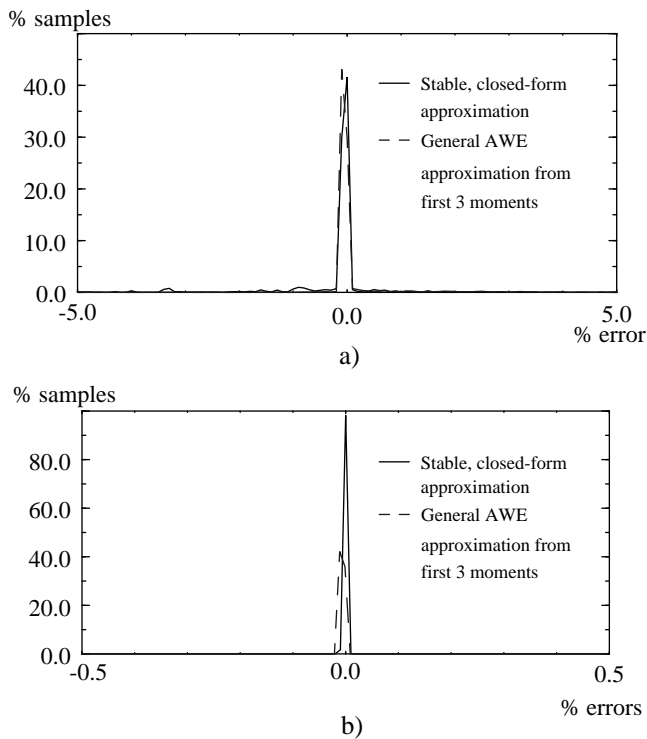
We must mention here that the general formulas are going to be more precise for  $1 > \alpha > 0.5$ , and less precise for small percentages,  $0.5 > \alpha > 0$ , since the accuracy of  $t_1$  depends on the decay of the area  $e_2(t)$  prior to time  $t_1$ .

The formulas presented above can be improved by taking into account the error given by the assumption that, for  $t_1$ , the second exponential is practically zero. Checking and considering these errors significantly complicates the delay formula without providing much improvement in the approximation. This is especially true for the saturated ramp cases.

## 6 Results

The proposed model has been tested over a large number of RC interconnect examples from the 620 PowerPC chip. The following are distributions of the model error as compared to a straightforward AWE approximation (no moment shifting or frequency shifting) and a 4 pole AWE approximation (with moment shifting) that we will consider exact for this experiment. The responses were for a sample of 10,000 nodes from thousands of RC examples.

Fig. 4a displays the modeling error for these RC interconnects when they are driven by an ideal voltage source (no driver/gate impedance considered). This represents a worst-case for any two pole model since the poles are most clustered without a driver resistance. Even so, the plot shows that the proposed model, with an explicit approximation as described in Section 5, matches the 4-th order AWE approximations very well and maintains a relatively small error in all cases.



**Figure 4:** Distribution of the 50% delay point errors for a) insignificant b) significant driver equivalent resistance. The delay approximation is in terms of equation (33) (Distribution based on 10,000 samples).

Next we added gate and transistor impedances of 100ohms, to consider a more realistic analysis problem. The same set of nodes were analyzed. The error distributions for this case, with a driver impedance, and hence, more low frequency pole dominance, is shown in Fig. 4b.

## 7 Conclusions

The first three moments of the transfer function can be used to determine stable approximations of the first two dominant poles. By matching their residues to the first two moments

we can obtain a stable time domain expression of the transfer function. A simple solution scheme is proposed that permits an explicit formula for the delay as a function of the first three moments of the impulse response from this model. The explicit formula relies on the dominant effect of the first pole and uses a single Newton-Raphson iteration to accurately predict the solution.

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