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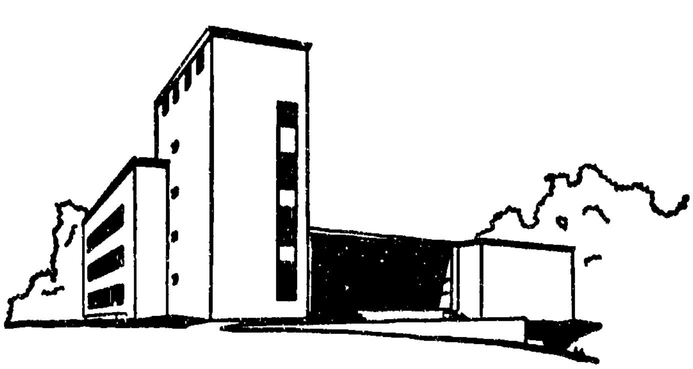

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AN EXTENDED GOAL PROGRAMMING MODEL FOR MANPOWER PLANNING

BY

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Abstract

Previous developments of manpower planning models involving uses of goal programming with embedded Markoff processes are here extended in order to (a) explicitly comprehend truncational effects--e.g., those due to retirement--and (b) allow for interperiod Markoff transition matrices which change over time.

I. Introduction

This paper reports on some further developments in a research program being undertaken in cooperation with OCMM (Office of Civilian Manpower Management). This part of the OCMM effort looks toward improving the processes of manpower planning for the U. S. Navy by means of computer assisted mathematical models. See, e.g., [4]-[8] and [11] in the bibliography, which reference earlier papers that extended the "state of the art" for modelling manpower planning by (a) combining the ideas of "goal programming" and Markoff transition processes and (b) utilizing multiple objectives along with (c) other constraints -- such as budgetary limitations -- so that these related considerations could also be simultaneously treated along with the personnel programs (and proposals) to which they might be related.[1/]

In this report, these previously developed models are further extended and refined to explicitly comprehend truncational effects (e.g. those associated with retirement) as well as the possibility of interperiod Markoff transition matrices which change over time. It is then also possible, as we shall soon see, to extend the transformational formulas that were found advantageous for simplifying and improving computations for the original model.

[1/] See [3] for further detailed development on the ideas of goal programming, multiple objectives, etc. Possibilities for policy evaluations and sensitivity analysis, as well as various byproducts of value for personnel planning are also examined in [4], [5], [6], and [7].

Furthermore, although only one particular device for handling truncations is presented here, it will be seen that others can also be used in addition to the "locked-in" truncations that we shall depict in this report. (It is important to observe this kind of possibility especially in connection with the many byproduct uses that the extended model is designed to yield.)

All of the above research is specifically designed to tie in with the automated civilian manpower information systems that the U. S. Navy is developing. These models and their related developments are, as will be seen, also applicable in other contexts as well.

II. Recapitulation of OCMM's Goal Programming Model for Manpower Planning:

For ease of reference we recapitulate and summarize relevant details from the already developed model which we have previously reported. See, e.g., [4] through [7]. Thus we write

$$\min \sum_k \sum_t \mu_{kt} \left(E_k^+(t) + E_k^-(t)\right)$$

subject to

$$(1) \qquad \sum_{i \in I_k} \sum_{\tau=1}^{t} \sum_{j \in J_o} (M^{t-\tau})_i x^j(\tau) - E_k^+(t) + E_k^-(t) = g_k(t)$$

where

$$(1.1) \qquad g_k(t) = f_k(t) - \sum_{i \in I_k} (M^t)_i \, a.$$

and $f_k(t)$ is the ceiling prescribed for the k^{th} manpower category. The subtraction from $f_k(t)$ in (1.1) yields a $g_k(t)$ value that represents the net requirement for the k^{th} manpower category after allowance for the available persons remaining in this category through the growth and attrition which occurred during the t previous periods, where

(2)

a = vector of initial inventory of personnel in all job types.

M = Markoff transition matrix depicting the transfer probabilities between job types

while $(M^t)_i$ = the i^{th} row of M^t and $M^t = MM...M$ (t times). Thus $\left[\sum_{i \in I_k} (M^t)_i \; a\right]$ represents the carry-forward of the initial states as they pertain to k^{th} manpower category comprised of the collection I_k of job types i.

The above model does not contain budgetary or other constraints and hence we can look at it for purely manpower planning aspects in a model where the objective is to minimize a weighted sum of the deviations for each manpower category via

(3)

$E_k^+(t)$, $E_k^-(t)$ = positive or negative deviation, respectively, for k^{th} manpower category.

μ_{kt} = the weight which is applicable to k^{th} manpower category in period t.

Using $B(t)$ to represent an applicable budgetary ceiling for expenditures in period t and $c^T(t) = (c_1(t),\ldots,c_i(t),\ldots,c_m(t))$ as a vector of salary rates, we can extend the model in (1) by adjoining constraining budgetary relations as follows:

$$\min \sum_k \sum_t \mu_{kt} \left[E_k^+(t) + E_k^-(t)\right]$$

subject to

(4)

$$\sum_{i \in I_k} \sum_{\tau=1}^{t} \sum_{j \in J_o} (M^{t-\tau})_i \, x^j(\tau) - E_k^+(t) + E_k^-(t) = g_k(t)$$

$$\sum_i \sum_{\tau=1}^{t} \sum_{j \in J_o} c_i(\tau) \, (M^{t-\tau})_i \, x^j(t) \leq B(t) - c^T(t) \, M^t a$$

where J_o refers to the collection of external manpower sources and $x^j(t) \geq 0$ refers to the vector of amounts secured from source $j \in J_o$ in period t.

As was noted in [4], the basic variables $x^j(t)$ only enter in certain combinations. Thus it is possible to simplify the model in (4) by making certain transformations for which the following symbols (and related definitions) may be utilized:

$$\xi(\tau) = \sum_{j \in J_o} x^j(\tau)$$

(5)

$$\eta(t) = \sum_{\tau=1}^{t} M^{t-\tau} \xi(\tau)$$

The pertinent parts of (4) can then become simplified via the expressions in (5) --viz.,

(6) $$\sum_{i \in I_k} \sum_{\tau=1}^{t} \sum_{j \in J_o} (M^{t-\tau})_i \, x^j(\tau) = \sum_{i \in I_k} \sum_{\tau=1}^{t} (M^{t-\tau})_i \, \xi(\tau) = \sum_{i \in I_k} (\eta(t))_i .$$

Furthermore, we can utilize the definition $M^0 \equiv I$ (the identity matrix) to obtain

(7)

$$\sum_{i \in I_k} \sum_{\tau=1}^{t+1} (M^{t+1-\tau})_i \, \xi(\tau) =$$

$$= \sum_{i \in I_k} \sum_{\tau=1}^{t} M^{t+1-\tau} \xi(\tau) + M^0 \xi(t+1) =$$

$$= \sum_{i \in I_k} M \sum_{\tau=1}^{t} M^{t-\tau} \xi(\tau) + \xi(t+1).$$

Hence, we also have

(8) $$\eta(t+1) = M\eta(t) + \xi(t+1).$$

Since the $\xi(t+1)$ are vectors of decision variables and are non-negative, we can proceed as in [4] to replace them by the choice vectors $\eta(t)$ with the requirement

$$\eta(t+1) - M\,\eta(t) \geq 0$$

(9) and

$$\eta(t) \geq 0 \ .$$

Preparatory to the interpretations and elaborations undertaken in the next section, we can utilize these developments to replace the original model in (4) by the following transformed and reduced version -- viz.,

$$\min \sum_k \sum_t \mu_{kt} \left[E_k^+(t) + E_k^-(t)\right]$$

subject to:

(10)

$$\sum_{i \in I_k} \eta_i(t) - E_k^+(t) + E_k^-(t) = g_k(t)$$

$$\sum_i c_i(t)\,\eta_i(t) \leq B(t) - c^T(t)\,M^t a$$

$$-(M)_i\,\eta(t) + \eta_i(t+1) \geq 0$$

$$\eta_i(t),\ E_k^+(t),\ E_k^-(t) \geq 0.$$

III. Interpretations and Further Development:

To interpret what has been achieved above, note first that $\eta_i(t)$ is the i^{th} component of $\eta(t)$ where

(11) $\eta(t)$ = Accumulated inventory from hiring to period t.

The vector $\eta(t)$ can be related to the recruitment that will be programmed by means of the vector $\xi(t)$ and other relations as follows. First, the amount to be recruited in period 1 is given by

$$\eta(1) = \xi(1)$$

where $\xi(1)$ is the amount to be recruited for period 1. See (5). For period 2, however,

$$\eta(2) = M\xi(1) + \xi(2)$$

where $\xi(2)$ is the amount to be recruited in period 2 and M is the Markoff matrix which has as its elements the probabilities of movement into and out of the various jobs. See (2). Of course, $(M)_i$ is the i^{th} row of M and

$$(M)_i = (M_{i1},\ldots,M_{i\ell},\ldots,M_{in})$$

relates this row to the elements $M_{i\ell}$ of M. The expression which relates the inventory of hirings in period 3 to the hiring activities up through this period is

$$\eta(3) = M^2\,\xi(1) + M\,\xi(2) + \xi(3),$$

and so on.

This leads to a "triangular system" which may be developed for convenient representation and solution by means of the following definitions

$$\begin{bmatrix} I & 0 \\ -M & I \end{bmatrix} \begin{bmatrix} I & 0 \\ M & I \end{bmatrix} = \begin{bmatrix} I & 0 \\ X+M & I \end{bmatrix}$$

(12) where

I = Identity matrix

X = -M.

Thus, for the 2 period case:

$$\begin{bmatrix} I & 0 \\ M & I \end{bmatrix} \begin{bmatrix} \xi(1) \\ \xi(2) \end{bmatrix} = \begin{bmatrix} \eta(1) \\ \eta(2) \end{bmatrix}$$

and

$$\begin{bmatrix} \xi(1) \\ \xi(2) \end{bmatrix} = \begin{bmatrix} I & 0 \\ M & I \end{bmatrix}^{-1} \begin{bmatrix} \eta(1) \\ \eta(2) \end{bmatrix}$$

$$\equiv \begin{bmatrix} I & 0 \\ M & I \end{bmatrix} \begin{bmatrix} \eta(1) \\ \eta(2) \end{bmatrix}$$

since -- see (12) -- the indicated inverse exists.

Similarly for 3 periods

$$\begin{bmatrix} \xi(1) \\ \xi(2) \\ \xi(3) \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ M & I & 0 \\ M^2 & M & I \end{bmatrix}^{-1} \begin{bmatrix} \eta(1) \\ \eta(2) \\ \eta(3) \end{bmatrix} \equiv \begin{bmatrix} I & 0 & 0 \\ -M & I & 0 \\ 0 & -M & I \end{bmatrix} \begin{bmatrix} \eta(1) \\ \eta(2) \\ \eta(3) \end{bmatrix}$$

or

$$\xi(1) = \eta(1)$$
$$\xi(2) = \eta(2) - M\eta(1)$$
$$\xi(3) = \eta(3) - M\eta(2),$$

and so on.

More generally, we can write

(13) $$\xi(t) = \eta(t) - M\eta(t-1).$$

Pausing and reflecting on this structure we see that it can be significantly generalized to the situation where the Markoff matrices may differ from period to period. Thus if $\eta(t)$ is accumulating through period t we can write

(14) $$\xi(t) = \eta(t) - M\binom{t-1}{t} \eta(t-1)$$

where $M\binom{t-1}{t}$ is the probability matrix applying to the transitions from period t-1 to t. (Note that the inversion is still straightforward in any case and requires no division operations, whether M is non-singular or not.)

IV. Adjustments for Variations in Proportions Eligible for Retirement:

Part One: Estimation and Adjustment Formulas

We now develop a procedure for further extending the OCMM goal programming model--or any other such model--and its application by developing certain formulas which make better allowance for potential retirements. The nature of the approach to be suggested will be sufficiently clear, we think, if we proceed by reference to a five-period example by means of which we can provide more specific and concrete interpretations.

Those who are, or will be, eligible to retire in the next five years are represented by the vector b,

$$b = b^5 + b^4 + b^3 + b^2 + b^1 + b^0$$

where

(15)

b^5 = those first eligible to retire in period 5

b^4 = those first eligible to retire in period 4

b^3 = those first eligible to retire in period 3

b^2 = those first eligible to retire in period 2

b^1 = those first eligible to retire in period 1

b^0 = those eligible to retire in period 0, but did not do so.

Next, let $a = \bar{a} + b$ where a = total "on board" at t = 0, $\bar{a}$ = those on board not eligible to retire in the next five years and b = those on board who are eligible to retire in the next five years.

Now we introduce the following expressions and definitions:

$$\alpha \equiv C_o/B_o , \qquad \bar{\alpha} = (1 - C_o/B_o)$$

where

(16)

C_o = Personnel remaining from those eligible to retire in some base period

B_o = Total eligible to retire in some base period.

For those who are eligible to retire in the next five periods, we shall assume that either they will retire or that they will remain in the same job. Then we develop our formulas for what we shall call a five-period "stretch". This is done by period, as follows, where the formula for estimating those who will <u>not</u> retire is shown first in each of the five periods:

Period 1:

$$\bar{\alpha}\ (b^1 + b^0)$$

$$\alpha\ (b^1 + b^0)$$

Period 2:

$$\bar{\alpha}b^2 + \bar{\alpha}^2(b^1 + b\ )$$

$$\alpha b^2 + \alpha\bar{\alpha}(b^1 + b^0)$$

(17) Period 3:

$$\bar{\alpha}b^3 + \bar{\alpha}\ [\bar{\alpha}b^2 + \bar{\alpha}^2(b^1 + b^0)]$$

$$\alpha b^3 + \alpha\ [\bar{\alpha}b^2 + \bar{\alpha}^2(b^1 + b^0)]$$

Period 4:

$$\bar{\alpha}b^4 + \bar{\alpha}\ [\bar{\alpha}b^3 + \bar{\alpha}^2b^2 + \bar{\alpha}^3(b^1 + b^0)]$$

$$\alpha b^4 + \alpha\ [\bar{\alpha}b^3 + \bar{\alpha}^2b^2 + \bar{\alpha}^3(b^1 + b^0)]$$

Period 5:

$$\bar{\alpha}b^5 + \bar{\alpha}\ [\bar{\alpha}b^4 + \bar{\alpha}^2b^3 + \bar{\alpha}^3b^2 + \bar{\alpha}^4(b^1 + b^0)].$$

$$\alpha b^5 + \alpha[\bar{\alpha}b^4 + \bar{\alpha}^2b^3 + \bar{\alpha}^3b^2 + \bar{\alpha}^4\ (b^1 + b^0)]$$

Finally, it also seems fair to assume that those who are recruited during this five-period stretch will not be eligible to retire during this stretch.

Part Two: Goal Adjustments

An extended model may be secured from (10) by using the preceding developments and assumptions to adjust for the indicated retirements. These "retirement adjustment" corrections will be reflected, of course, in the budget constraints as well as in the goals and objectives.

As was observed in section III, the original model utilized the matrix, M, to project all of the initial manpower plus manpower hired during the "stretch".[1/] We now propose to utilize our preceding development, however, to encompass the situations involving retirement projections.

Thus, if we use $\hat{\eta}(t)$ to represent the cumulative manpower vector through period t, then we can represent as a sum of two vectors -- viz.,

$$\hat{\eta}(t) = \eta^1(t) + \eta^2(t) \tag{18}$$

where $\eta^1(t)$ represents the manpower cumulation not affected by retirement and $\eta^2(t)$ represents the cumulation affected by retirement. Furthermore, by an evident extension of (13) joined to the development in Part One of this section,

$$\begin{aligned} \eta^1(t) &= M\eta^1(t-1) + \xi(t) \\ \eta^2(t) &= \bar{\alpha}\eta^2(t-1) + \bar{\alpha}b^t. \end{aligned} \tag{19}$$

Specifically,

$$\begin{aligned} \eta^2(1) &= \bar{\alpha}\ (b^1 + b^0) \\ \eta^2(2) &= \bar{\alpha}b^2 + \bar{\alpha}\eta^2(1) \\ &= \bar{\alpha}b^2 + \bar{\alpha}^2(b^1 + b^0) \end{aligned} \tag{19.1}$$

as required for period (2) -- see (17) -- and so on.

Now we let

$$a_i = \text{those in job } i \text{ initially (at } t = 0) \tag{20}$$

1/ E.g., the five-period stretch which we are using.

and

(21) $$a_i = \bar{a}_i + b_i,$$

so that, as in the development which immediately follows (15), we have $\bar{a}_i$ representing those not eligible for retirement in the five-period stretch covered by[1/]

(22) $$b = b^0 + b^1 + \ldots + b^5.$$

The vector of personnel in period 1 coming from within is

(23) $$M\,\bar{a} + \bar{\alpha}\,(b^0 + b^1) = M\,\bar{a} + \eta^2(1),$$

via the first expression in (19.1). Similarly, the vector of personnel in period 2 coming from within is

(24) $$M^2\,\bar{a} + \bar{\alpha}\,b^2 + \bar{\alpha}^2\,(b^0 + b^1) = M^2\,\bar{a} + \eta^2(2),$$

via the second expression in (19.1).

Now the total on board in period 1 equals

(25) $$M\,\bar{a} + \eta^1(1) + \eta^2(1) + \xi(1)$$

where $\xi(1)$ represents hiring from outside while $M\,\bar{a}$ provides the transitions from the state given by $\bar{a}$, the remnant from $t = 0$. The vector of personnel from within at period two is

(26.1) $$M^2\,\bar{a} + M\,\xi(1) + \eta^2(2)$$

which, in turn, forms a part of the total period-two personnel

(26.2) $$M^2\bar{a} + M\xi(1) + \eta^2(2) + \xi(2).$$

[1/] See (15).

By an evident extension we can now write

(27) $$M^t \bar{a} + \sum_{\tau=1}^{t} M^{t-\tau} \xi(\tau) + \eta^2(t)$$

as the expression for the vector of total personnel in period t. By virtue of the definition of $\eta^1(t)$ in (18),

(28) $$\eta^1(t) = \sum_{\tau=1}^{t} M^{t-\tau}\xi(\tau) \equiv \xi(t) + \sum_{\tau=1}^{t-1} M^{t-\tau} \xi(\tau).$$

But,

(29) $$\sum_{\tau=1}^{t-1} M^{t-\tau} \xi(\tau) = M \sum_{\tau=1}^{t-1} M^{(t-1)-\tau} \xi(\tau) \equiv \eta^1(t-1).$$

Thus, via (28) and (29),

(30) $$\eta^1(t) = \xi(t) + \eta^1(t-1).$$

V. Adjustment and Extension of the Model:

Now turning to the model, (10), the objective function we wish to minimize is

$$\sum_k \sum_t \mu_{kt} \quad (E_k^+(t) + E_k^-(t))$$

subject to goal (e.g., manpower cei.ings) and budget constraints,

where μ_{kt} = weights associated with the k^{th} manpower ceiling in period t

$E_k^+(t)$ = excess above k^{th} goal in period t

$E_k^-(t)$ = deficiency below k^{th} goal in period t.

Both $E_k^+(t)$ and $E_k^-(t)$ will not appear in the solution at the same time since a given goal discrepancy can only be in one direction at any given time (e.g., if $E_k^+(t)$ is in the solution then $E_k^-(t)$ will equal zero).

Developing the goal constraints, in which these $E_k^+(t)$ and $E_k^-(t)$ variables appear, the k^{th} goal is defined as

$$\sum_{i \in I_k} [\text{Total personnel in period } t]_i - E_k^+(t) + E_k^-(t) = f_k(t)$$

where

$f_k(t) = k^{th}$ manpower goal (or ceiling) in period t.

Substituting the expression for total personnel in period t we have

$$\sum_{i \in I_k} [(M^t \bar{a})_i + \eta_i^1(t) + \eta_i^2(t)] - E_k^+(t) + E_k^-(t) = f_k(t)$$

Rearranging, the goal constraint is

$$\sum_{i \in I_k} \eta_i^1(t) - E_k^+(t) + E_k^-(t) = f_k(t) - \sum_{i \in I_k} (M^t \bar{a})_i - \sum_{i \in I_k} \eta_i^2(t) = \hat{g}_k(t)$$

where $\hat{g}_k(t)$ is the k^{th} manpower requirement obtained from the k^{th} goal, net after allowing for the available persons remaining in this category from the initial population.

The budget constraints are defined as

$$[\text{Cost vector in period } t]^T [\text{Total personnel vector in period } t] \leq B(t)$$

where

$B(t)$ = total dollar budget for personnel in period t and the T superscript stands for "transfers", as in (4). Substituting the expressions for the cost vector in period t and for the total personnel vector in period t we obtain

$$c^T(t)[M^{t}\bar{a} + \eta^1(t) + \eta^2(t)] \leq B(t)$$

where

$$c^T(t) \equiv [c_1(t),\ldots,c_n(t)].$$

Rearranging, we obtain the budget constraint

$$c^T(t)\ \eta^1(t) \leq B(t) - c^T(t)\ M^{t}\bar{a} - c^T(t)\ \eta^2(t).$$

Thus, in conclusion, the model (10) can now be replaced by

$$\min \sum_k \sum_t (E_k^+(t) + E_k^-(t)]$$

Subject to:

$$\sum_{i \in I_k} \eta_i^1(t) - E_k^+(t) + E_k^-(t) = \hat{g}_k(t)$$

(30)
$$c^T(t)\ \eta^1(t) \leq B(t) - c^T(t)\ M^{t}\bar{a} - c^T(t)\ \eta^2(t)$$

$$\eta^1(t) \geq M\ \eta^1(t-1)$$

$$E_k^-,\ E_k^+ \geq 0.$$

Should one wish to go still further, e.g., to also permit the transition matrix to change from period to period, all that would change in the above expressions would be to replace M^t by $M\binom{t-1}{t}\ M\binom{t-2}{t-1} \ldots M\binom{0}{1}$ and M by $M\binom{t-1}{t}$. See (14) ff.

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[11.2] ________________, R. J. Niehaus, and D. Sholtz. Memo of November 26, 1968. Rough notes only covering meeting to expand model to include the effects of retirements.

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13. ABSTRACT

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