

# An extended standard model and its Higgs geometry from the matrix model

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based on arXiv:1401.2020  
joint work with Harold Steinacker

Bayrischzell, May 2014

- The **IKKT** [Ishibashi, Kawai, Kitazawa, Tsuchiya 97] matrix model is a candidate for a nonperturbative definition of a fundamental theory of matter and gravity.
- Solutions can be interpreted as **noncommutative branes**, embedded in  $\mathbb{R}^{10}$ , giving rise to an **emergent geometry** [Steinacker 08].
- There are some promising hints:
  - The perturbations around the Moyal plane lead to Ricci-flat geometries in the absence of matter [Rivelles 03].
  - $3 + 1$  dimensions and an expanding universe seem to be dynamically generated [Kim, Nishimura, Tsuchiya 2012].
- ? How to embed the standard model in this framework?
- Generically, fermions are in the adjoint representation of  $SU(N)$  and are not chiral. What about the Higgs?

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The IKKT model is defined by the action

$$S = \Lambda_0^4 \text{tr} \left( [X^A, X^B][X_A, X_B] + \bar{\Psi} \Gamma^A [X_A, \Psi] \right),$$

where the  $X^A \in \text{Mat}(N \times N)$ ,  $0 \leq A \leq 9$  are hermitean matrices, and the indices are raised and lowered with  $\eta = \text{diag}(1, -1, \dots, -1)$ . The fermions  $\Psi$  are  $\text{Mat}(N \times N)$ -valued Majorana-Weyl spinors of  $SO(9, 1)$ , and the  $\Gamma$ 's are the corresponding  $\gamma$  matrices.  $\Lambda_0$  is some energy scale.

The action is invariant under 10-dimensional Poincaré transformations and unitary transformations of the  $X^A, \Psi$ . There is also an  $\mathcal{N} = 2$  supersymmetry.

For  $\Psi = 0$ , the action leads to the [equation of motion](#)

$$[X^A, [X_A, X^B]] = 0.$$

A particular solution is thus the [Moyal plane](#)  $\mathbb{R}_{\theta}^{2n}$ , with commutation relations

$$[X^A, X^B] = i\lambda_{NC}^2 \theta^{AB}$$

for an antisymmetric matrix  $\theta$ .

# Noncommutative $\mathcal{N} = 4$ Super Yang-Mills

A particular solution is the 4-dimensional Moyal plane, i.e.,

$$\bar{X}^A = (\bar{X}^\mu, 0), \quad \mu \in \{0, \dots, 3\}.$$

We denote the corresponding algebra by  $\mathfrak{A}(\mathbb{R}_\theta^4)$ , represented on  $\mathcal{H}_\theta$ . A stack of  $N$  coincident planes is described by  $\mathfrak{A}(\mathbb{R}_\theta^4) \otimes \text{Mat}(N \times N)$ , represented on  $\mathcal{H}_\theta \otimes \mathbb{C}^N$ . For perturbations

$$Y^A = X^A - \bar{X}^A \otimes \text{id} = (\theta^{\mu\nu} A_\nu, \Phi^i),$$

the IKKT action reduces to a NC  $\mathcal{N} = 4$  super Yang-Mills  $U(N)$  gauge theory:

$$S = \int \sqrt{G} \left( -\frac{1}{4g_{YM}^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D^\mu \phi^a D_\mu \phi_a + \frac{g_{YM}^2}{4} [\phi^a, \phi^b][\phi_a, \phi_b] \right. \\ \left. + i\bar{\psi} \not{D} \psi + g\bar{\psi} \Gamma^a [\phi_a, \psi] \right) d^4 x.$$

Here

$$G^{\mu\nu} = \sqrt{|\theta^{-1}|} \theta^{\mu\mu'} \theta^{\nu\nu'} \eta_{\mu'\nu'}, \quad D_\mu = \partial_\mu - i[A_\mu, \cdot], \quad g_{YM} = \pi |\theta^{-1}|^{\frac{1}{4}} \Lambda_0^{-1},$$

and  $\phi, \psi$  are rescaled versions of  $\Phi, \Psi$ . All products are Moyal products. The tracial part of  $U(N)$  corresponds to dynamical gravity [Steinacker 07] and will be ignored henceforth.

Chiral fermions at the intersection of quantum planes [Chatzistavrakidis, Steinacker, Zoupanos 11]:

$$\phi^{4,5} = \begin{pmatrix} X^{1,2} & 0 \\ 0 & 0 \end{pmatrix} \quad \phi^{6,7} = \begin{pmatrix} 0 & 0 \\ 0 & Y^{1,2} \end{pmatrix} \quad \psi = \begin{pmatrix} 0 & \psi_{(12)} \\ \psi_{(21)} & 0 \end{pmatrix}$$

The internal Dirac operator

$$\not{D}_{\text{int}}\psi = \sum_{a=4}^9 \Gamma^a [\phi_a, \psi]$$

then acts on  $\psi_{(12)}$  as

$$\begin{aligned} \not{D}_{\text{int}}\psi_{(12)} &= \Gamma^4 X^1 \psi_{(12)} + \Gamma^5 X^2 \psi_{(12)} + \Gamma^6 \psi_{(12)} Y^1 + \Gamma^7 \psi_{(12)} Y^2 \\ &= \alpha a^* \psi_{(12)} + \alpha^* a \psi_{(12)} + \beta \psi_{(12)} b^* + \beta^* \psi_{(12)} b \end{aligned}$$

where

$$\begin{aligned} a &= X^1 - iX^2 & b &= Y^1 - iY^2 \\ \alpha &= \frac{1}{2}(\Gamma^4 - i\Gamma^5) & \beta &= \frac{1}{2}(\Gamma^6 - i\Gamma^7) \end{aligned}$$

Hence, there is a **zero mode** localized at the intersection:

$$\psi_{(12)} = |0, \downarrow\rangle \langle 0, \uparrow|.$$

It has a definite chirality. This is **stable** under deformations [Steinacker, Z 13].

## The fuzzy sphere

A particularly simple matrix geometry is the **fuzzy sphere**  $S_N^2$  [Hoppe; Madore]. Let  $J^i$  be the generators of the  $N$ -dimensional irreducible representation of  $\mathfrak{su}(2)$ . Then set

$$X^{i+3} = R J^i.$$

It can be seen as a quantization with  $N$  quantum cells of the sphere of radius  $R \frac{N-1}{2}$  with symplectic structure

$$\{x^i, x^j\} = \frac{2R}{N} \varepsilon^{ijk} x_k.$$

The fuzzy sphere is not a solution to the IKKT equation of motion:

$$[X^j, [X_j, X^i]] = 2R^2 X^i.$$

There are several possibilities to obtain a fuzzy sphere solution:

- Add a term  $\text{tr} \varepsilon_{456}^{ijk} X_i X_j X_k$  to the action.
- Add a term  $\text{tr} X_i X^i$  to the action.
- Let the sphere rotate, for example in the planes 4 – 7, 5 – 8, 6 – 9.

By replacing  $R$  by  $R_i$ , one obtains a **fuzzy ellipsoid**. For  $N = 2$ , consider

$$X^4 + iX^5 = \phi \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad X^6 = \frac{r}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

This can be seen as two quantum points at  $(0, 0, \pm r/2)$ , which connect to an ellipsoid upon switching on  $\phi$ .



## Chiral fermions on $S_2^2$ intersecting $\mathbb{R}_\Theta^2$

Consider the intersection of the fuzzy minimal ellipsoid with  $\mathbb{R}_\Theta^2$ :

$$\phi^i = \begin{pmatrix} \phi_{(1)}^i & 0 \\ 0 & \phi_{(2)}^i \end{pmatrix}, \quad \phi_{(1)} = \frac{1}{2} (\phi\sigma_1 \quad \phi\sigma_2 \quad r\sigma_3 \quad 0), \quad \phi_{(2)} = (0 \quad 0 \quad X^1 \quad X^2).$$

Now consider again the fermions  $\psi_{(12)}$  connecting the branes.

- For  $\phi = 0$ , there are in total 4 zero modes, located at  $x^6 = \pm r/2$ . At both locations, both chiralities occur.
- For  $\phi > 0$ , there are 2 zero modes  $\psi_0^\pm$ , located at  $x^6 = \pm r/2$ , of opposite chirality. There are **mirror fermions**  $\psi_1^\pm$ , also located at  $x^6 = \pm r/2$ , with opposite chirality of  $\psi_0^\pm$ , and

$$\mathcal{D}_{\text{int}}\psi_1^\pm = \pm\phi\psi_1^\mp.$$

On top of that, there are modes with masses of the order  $\sqrt{\theta^{-1}}$ .

Analogous results hold for  $S_2^2 \times \mathbb{R}_\Theta^2$  intersecting  $\mathbb{R}_\Theta^2$  if the supplementary plane spans the 8 – 9 plane. We will replace the quantum planes by large fuzzy spheres, which locally look like the quantum plane. One then expects (and numerically confirms) **would-be zero modes**  $\psi_0^\pm$ , i.e.,

$$\mathcal{D}_{\text{int}}\psi_0^\pm = \pm\phi f_\psi \psi_0^\mp,$$

where  $f_\psi \ll 1$  in the appropriate limit.

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- We consider branes of the form

$$\mathcal{D}_i = \mathbb{R}_\theta^4 \times \mathcal{K}_i,$$

where  $\mathcal{K}_i$  is a fuzzy matrix geometry, such as  $S_{N_i}^2$ .

- On the intersections, fermionic would-be zero modes form.
- Choosing  $\mathcal{K}_i$  as a stack of  $n_i$  coincident branes,  $\mathcal{K}_i = \tilde{\mathcal{K}}_i \otimes \text{Mat}(n_i \times n_i)$ , the brane  $\mathcal{D}_i$  carries a gauge group  $U(n_i)$ .
- The fermions localized at the intersection of  $\mathcal{D}_i$  and  $\mathcal{D}_j$  are then charged in the bi-fundamental representation of  $U(n_i) \times U(n_j)$ .
- The goal is to choose the configuration such that the gauge group is broken to the standard model gauge group, with the fermionic would be zero modes appropriately charged.

# The choice of branes

An arrangement of 6 branes ( $\mathcal{D}_w, \mathcal{D}_x, \mathcal{D}_y, \mathcal{D}_z, \mathcal{D}_\ell, \mathcal{D}_B^3$ ) to achieve fermions charged as in the standard model was proposed in [Chatzistavrakidis, Steinacker, Zoupanos 11]:

$$X^a = \begin{pmatrix} X_w^a & 0 & 0 & 0 & 0 & 0 \\ & X_x^a & 0 & 0 & 0 & 0 \\ & & X_y^a & 0 & 0 & 0 \\ & & & X_z^a & 0 & 0 \\ & & & & X_\ell^a & 0 \\ & & & & & X_B^a \end{pmatrix}, \quad \Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & \nu_L & u_L \\ & 0 & 0 & 0 & e_L & d_L \\ & & 0 & 0 & e_R & d_R \\ & & & 0 & \nu_R & u_R \\ & & & & 0 & 0 \\ & & & & & 0_3 \end{pmatrix},$$

The electric charge  $Q$  and the weak hypercharge  $Y$  are realized by the adjoint action of

$$t_Q = \frac{1}{2} \text{diag}(1, -1, -1, 1, 1, -\frac{1}{3}), \quad t_Y = \text{diag}(0, 0, -1, 1, 1, -\frac{1}{3}).$$

For  $X_w = X_x$ , the gauge group is broken from  $U(N)$  to  $U(2) \times U(1)^3 \times U(3)$ .

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**Solution:** Fuse  $\mathcal{D}_w, \mathcal{D}_z$  and  $\mathcal{D}_x, \mathcal{D}_y$  into  $\mathcal{D}_u$  and  $\mathcal{D}_d$ , by turning on Higgs fields  $\phi_{u/d}$ . Concretely,  $\mathcal{D}_{d,u} = S_{N_{d,u}}^2 \times S_2^2$ , such that  $e_L, \nu_L$  are located at the south pole of  $S_2^2$  and  $e_R, \nu_R$  at the north pole, with the poles connected by  $\phi_{u/d}$ . With  $X_w = X_x$  and  $X_y = X_z$ , we have a  $SU(2)$  symmetry at both poles, spontaneously broken by  $\phi_{u,d}$ .

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We want to find such a configuration of intersecting branes, which is a solution to a **modified** IKKT model. Quasi-massless fermionic modes should be present, which are quasi-localized on the correct pole  $X_w/x/y/z$ . There is a close analogy between branes in the IKKT model and in **supergravity**, so typically one expects an attractive interaction generated by quantum effects. We model this by adding to the IKKT action a term,

$$S = S_{IKKT} - f(\text{tr} \sum_{i=4}^9 X_i X^i),$$

where  $f$  should have a nontrivial minimum. Hence, the equation of motion becomes

$$[X^\mu, [X_\mu, X^j]] = -cf'(\text{tr} \sum_{i=4}^9 X_i X^i) X^j$$

# Intersecting brane solutions

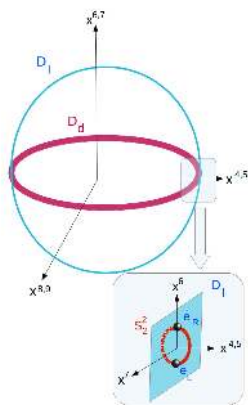
$$X_{d(u)} = \begin{pmatrix} R'_d L_3 \otimes \mathbb{1}_2 + \phi_d \mathbb{1}_{N_d} \otimes \sigma'_1 \\ \phi_d \mathbb{1}_{N_d} \otimes \sigma'_2 \\ r_d \mathbb{1}_{N_d} \otimes \sigma'_3 \\ 0 \\ R_d L_1 \otimes \mathbb{1}_2 \\ R_d L_2 \otimes \mathbb{1}_2 \end{pmatrix}, \quad X_\ell = \begin{pmatrix} R'_\ell K_3 \\ 0 \\ R_\ell K_1 \\ R_\ell K_2 \\ 0 \\ 0 \end{pmatrix}.$$

Here  $L_i, K_i$  are the generators of the  $N_{d/\ell}$ -dimensional irrep of  $\mathfrak{su}(2)$ , and  $\sigma'_i = \frac{1}{2}\sigma_i$ . This is a solution provided that

$$R_d^2 = R'_d{}^2 = R_\ell^2 = R'_\ell{}^2 = r_d^2 = \phi_d^2 = -c'f'.$$

There are two intersection regions for  $N_\ell \sim N_d \gg 1$ , locally looking like the intersection of  $\mathbb{R}_\Theta^2 \times S^2$  and  $\mathbb{R}_\Theta^2$ . We expect (quasi-) massless fermionic modes of (quasi-) definite chirality, (quasi-) localized at the poles of  $S^2$ . The lowest eigenvalues can be estimated ( $R = R_{d,\ell} = R'_{d,\ell}$ ,  $N = N_{d,\ell}$ ):

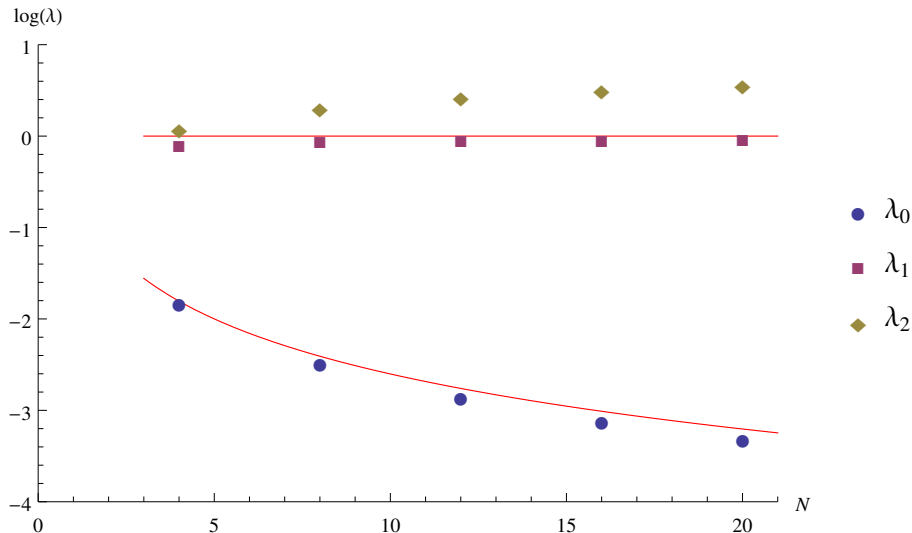
$$\mathcal{D}_{\text{int}} \psi_i^\pm = \pm \lambda_i \psi_i^\mp, \quad \lambda_0 \sim \frac{\phi r^2}{4N^2 R^2}, \quad \lambda_1 \sim \phi.$$





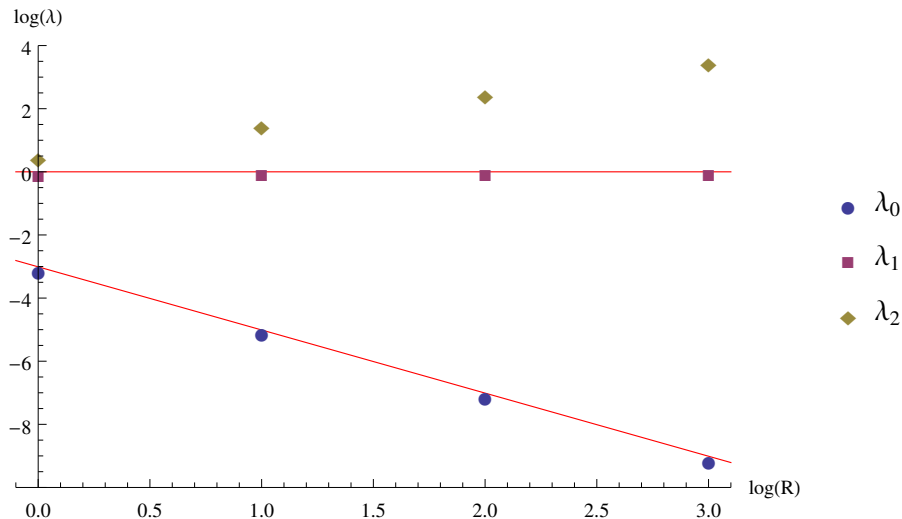
# Numerical test of the expectations I

The lowest eigenvalues as a function of  $N$ , for  $R = r = \phi = 1$ :



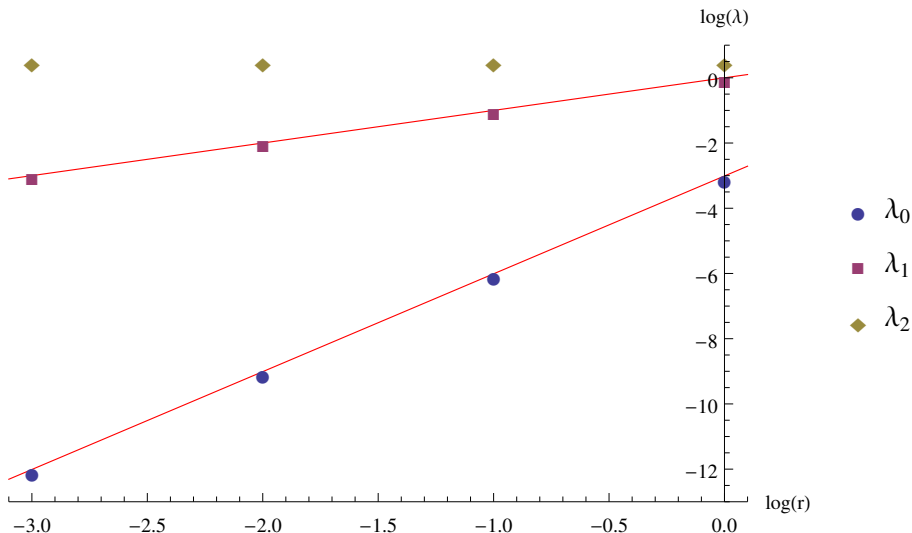
# Numerical test of the expectations II

The lowest eigenvalues as a function of  $R$ , for  $r = \phi = 1$ ,  $N = 16$ :



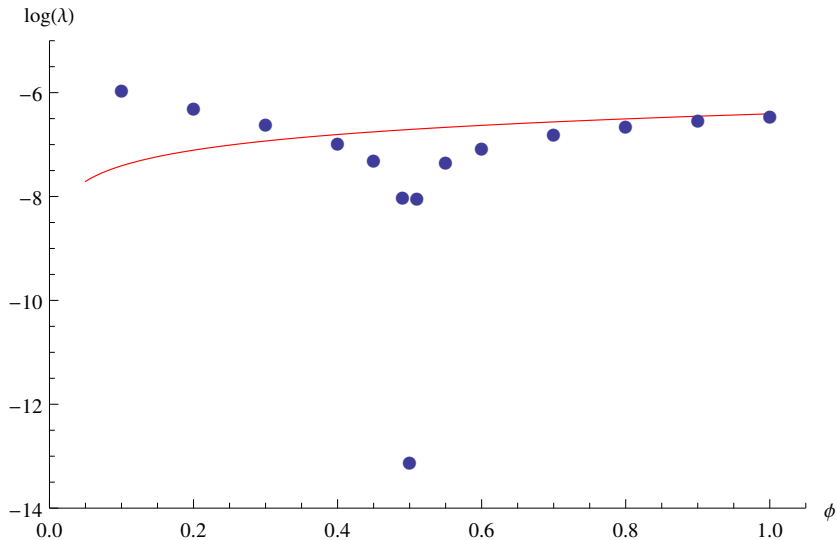
# Numerical test of the expectations III

The lowest eigenvalues as a function of  $r$ , for  $r = \phi$ ,  $R = 1$ ,  $N = 16$ :



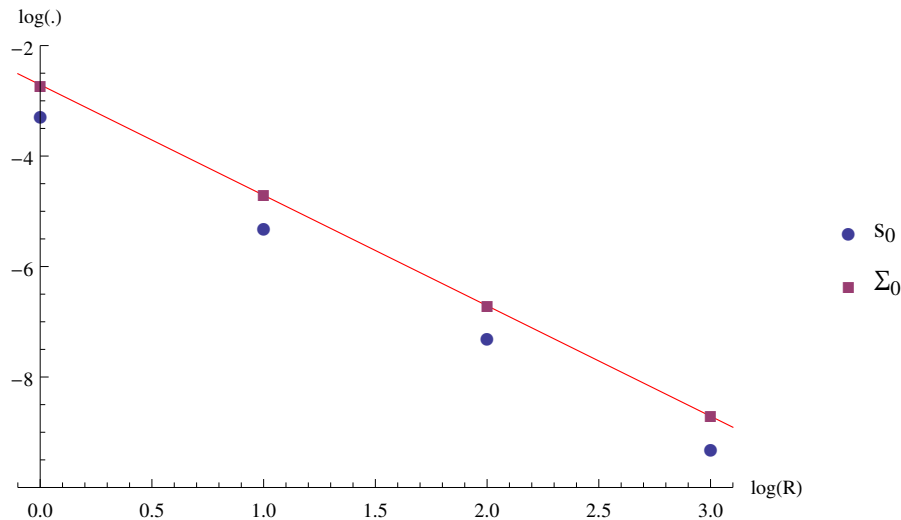
# Numerical test of the expectations IV

The lowest eigenvalue as a function of  $\phi$ , for  $R = r = 1$ ,  $N = 16$ :



# Numerical test of the expectations V

Expectation values of  $s = 1 - \sigma_3$  and  $\Sigma = 1 - \Sigma_{45}$  in the lowest eigenvalue as a function of  $R$ , for  $r = \phi = 1$ ,  $N = 16$ . The estimate is  $\Sigma \sim \frac{r^2}{2N^2R^2}$ .



# The singlet Higgs

The singlet Higgs should link  $\mathcal{D}_u$  and  $\mathcal{D}_\ell$ , so we use the ansatz

$$H_5^a = h^a S + \text{h.c.}, \quad S = \sum_n |+, p_n\rangle_u \langle q_n|_\ell$$

where

$$|+, p\rangle_u = |+\rangle|p\rangle, \quad L_3|+\rangle = \frac{N_u-1}{2}|+\rangle.$$

For suitably chosen  $p_n, q_n$ , this becomes an eigenvector of the linearized wave operator if we choose

$$h^a = h(e^8 + ie^9).$$

One verifies numerically that  $\sum_n |p_n\rangle \langle q_n|$  can be chosen to be (quasi-) localized at  $\nu_R$ , as expected. The resulting mode of the linearized wave operator is **unstable**. We assume that it is non-linearly stabilized, so that  $h$  acquires a non-zero value.

- For  $h = 0$ , we have one stack of 2 branes  $\mathcal{D}_d, \mathcal{D}_u$ , and one stack of 4 branes,  $\mathcal{D}_\ell, \mathcal{D}_B$ . Hence we have a  $U(2) \times U(4)$  symmetry. Turning on  $h$  breaks it to

$$SU(3)_c \times U(1)_Q \times U(1)_B \times U(1)_{\text{tr}}.$$

- The singlet Higgs can induce a Majorana mass for  $\nu_R$ ,

$$\text{tr}_N(\nu_R^T \gamma^0 S^* \nu_R S^*).$$

The gauge bosons are obtained from fluctuation of the  $X^\mu$ :

$$A = g(W_- t_+ + W_+ t_- + W_3 t_3) + \frac{1}{2} g' B t_Y + g_5 B_5 t_5 + g_S A_\alpha t_\alpha,$$

where, in the basis  $(\mathcal{D}_L, \mathcal{D}_Y, \mathcal{D}_Z, \mathcal{D}_\ell, \mathcal{D}_B^3)$ , with  $\mathcal{D}_L = (\mathcal{D}_w, \mathcal{D}_x)$ ,

$$t_{\pm,3} = \frac{1}{2} \text{diag}(\mathbb{1}_{N_1} \otimes \sigma_{\pm,3}, 0, 0, 0, 0_3) \quad t_Y = \text{diag}(0_2, -\mathbb{1}_{N_1}, \mathbb{1}_{N_1}, \mathbb{1}_{N_2}, -\frac{1}{3} \mathbb{1}_{N_2} \otimes \mathbb{1}_2)$$
$$t_\alpha = \text{diag}(0_2, 0, 0, 0, \mathbb{1}_{N_2} \otimes \lambda_\alpha) \quad t_5 = \text{diag}(\mathbb{1}_{N_1} \otimes \mathbb{1}_2, -\mathbb{1}_{N_1}, -\mathbb{1}_{N_1}, -\mathbb{1}_{N_2}, \frac{1}{3} \mathbb{1}_{N_2} \otimes \mathbb{1}_2)$$

with  $N_u = N_d = N_1$ ,  $N_B = N_\ell = N_2$ , and

$$g = \frac{g_{YM}}{\sqrt{N_1}}, \quad g' = \frac{g_{YM}}{\sqrt{N_1 + \frac{2}{3} N_2}}, \quad g_5 = \frac{g_{YM}}{\sqrt{8(N_1 + \frac{1}{3} N_2)}}, \quad g_S = \frac{g_{YM}}{\sqrt{N}}.$$

The  $\lambda_\alpha$  are generators of the fundamental  $\mathfrak{u}(3)$  representation. The identity generator gives  $U(1)_B$ , which is anomalous and expected to disappear from the low energy spectrum. Note that  $[t_5, \cdot]$  acts as  $B - L + \gamma_5$  on the chiral fermions, so  $U(1)_5$  is also anomalous (it is also broken by the Higgs  $\phi$ ).

As usual, the electroweak Higgses  $\phi_u = \phi_d = \phi$  induce mass terms for some gauge bosons,

$$\phi^2 N_1 \left( \frac{1}{2} g^2 (W_1^2 + W_2^2) + \frac{1}{2} g_Z^2 Z^2 + 2g_5^2 B_5^2 \right),$$

where

$$g_Z Z = g W_3 - g' B.$$

In particular, we obtain the [Weinberg angle](#)

$$\sin^2 \theta_W = \frac{1}{1 + \frac{g'^2}{g^2}} = \frac{1}{2 + \frac{2}{3} \frac{N_2}{N_1}}.$$

For  $N_1 = N_2$ , we then obtain

$$\sin^2 \theta_W = \frac{3}{8}, \quad g_s = g,$$

as in the  $SU(5)$  GUT.



The Yukawa mass terms for the would-be zero modes can be made arbitrarily small by choosing  $N_i$  large enough (provided that our heuristic estimate is correct). The mirror fermions with opposite chirality have Yukawa masses

$$m \sim g_{YM}\phi = \sqrt{2}m_W$$

However, one has to keep in mind that these are tree level masses at some high energy scale. Due to the coupling to massive Kaluza-Klein modes, one may expect that quantum effects raise this gap.

For the fluctuation of the Higgs  $\phi$ , one obtains a mass

$$m_\phi^2 = 4m_W^2 \left(1 + 2\pi^2 f''\right).$$

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## Summary:

- Configurations in the IKKT model whose low-energy physics resembles the standard model.
- Predicts mirror fermions and Kaluza-Klein towers of gauge fields.

## Issues:

- Introduced an ad-hoc supplementary term to the IKKT action, breaking 10-dimensional Poincaré and super-symmetry. Motivated by quantum effects, but can this be made precise?
- Non-linear stabilization of the scalar Higgs?
- With our configuration, only an even number of generations can be achieved.
- Low scale of the mirror fermions.

Despite these problems, one may find it remarkable that one can get standard model like low energy energy physics from an  $\mathcal{N} = 4$  supersymmetric gauge theory.

Thank you for your attention!