# An Extension-Based Approach to Belief Revision in Abstract Argumentation

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## Bringing Together Two Fields

**Abstract Argumentation**

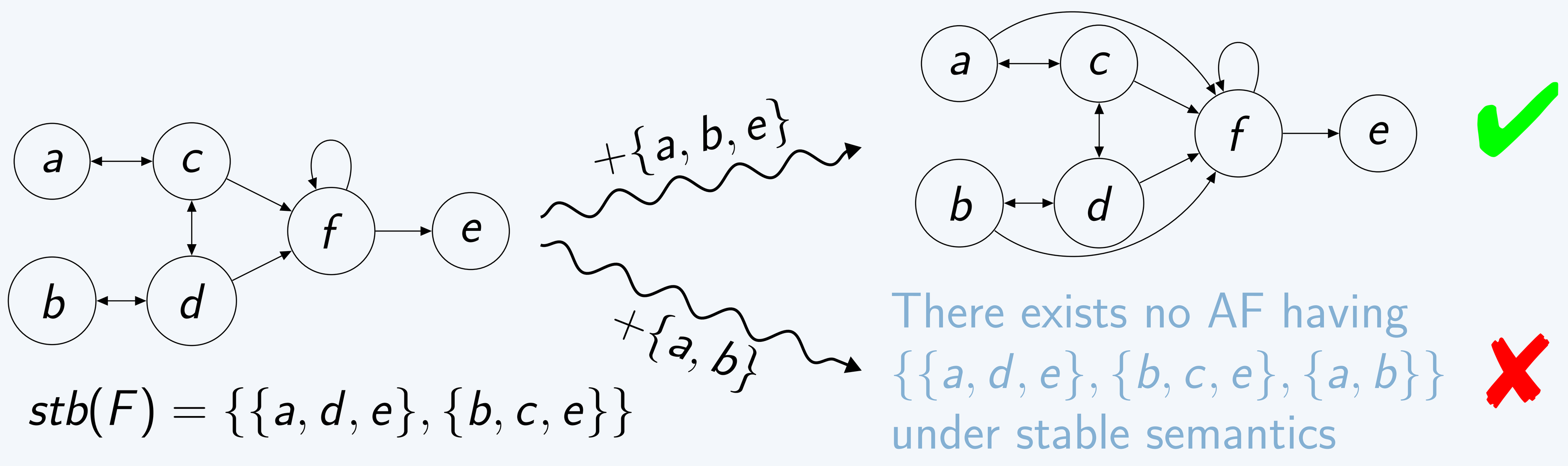
An argumentation framework (AF) is a pair $F = (A, R)$ where $A$ is a set of arguments, and $R \subseteq A \times A$ is the attack relation representing conflicts. [3]
A semantics $\sigma$ selects sets of jointly acceptable arguments, $\sigma(F) \subseteq 2^A$.
Stable semantics: $S \in stb(F)$, if $S$ is conflict-free and attacks all $(A \setminus S)$.
$\Sigma_\sigma = \{\sigma(F) \mid F \in AF_\mathfrak{A}\}$ contains all sets of extensions realizable under $\sigma$. [4]

**Belief Revision**

The problem we tackle here is how to revise an AF when some new information is provided. By revision we mean an operation that incorporates the new information while bringing minimal change to the extensions of the original AF. To the best of our knowledge, this has first been considered for AFs explicitly in [1]. Our approach is inspired by recent work on belief revision in the Horn fragment of propositional logic [2].

## Problem

Adding a new extension to an existing argumentation framework:




Our results apply to proper I-maximal semantics $\sigma$, i.e. for each $\mathbb{S} \in \Sigma_\sigma$:

1. for all $S_1, S_2 \in \mathbb{S}$: $S_1 \subseteq S_2$ implies $S_1 = S_2$,
2. for all $\emptyset \neq \mathbb{S}' \subseteq \mathbb{S}$: $\mathbb{S}' \in \Sigma_\sigma$,
3. for all $\subseteq$-incomparable extensions $S_1, S_2$: $\{S_1, S_2\} \in \Sigma_\sigma$.

Major semantics (stable, preferred, semi-stable, stage) are proper I-maximal.

## Representation Result for Revision by a Propositional Formula

defining $\sigma(F \star_\sigma \varphi) = \min([\varphi], \preceq_F)$ satisfies P$\star$1 - P$\star$6 given faithful, $\sigma$-compliant $\preceq_F$

**Faithful assignments to $\sigma$-compliant rankings $\preceq_F$**

Given semantics $\sigma$, a faithful assignment maps every AF $F$ to a total pre-order $\preceq_F$ on $2^\mathfrak{A}$ such that, for any $E_1, E_2 \in 2^\mathfrak{A}$ and $F, F_1, F_2 \in AF_\mathfrak{A}$, it holds that:

(i) if $E_1, E_2 \in \sigma(F)$, then $E_1 \approx E_2$,
(ii) if $E_1 \in \sigma(F)$ and $E_2 \notin \sigma(F)$, then $E_1 \prec_F E_2$,
(iii) if $\sigma(F_1) = \sigma(F_2)$, then $\preceq_{F_1} = \preceq_{F_2}$.

A pre-order $\preceq$ is $\sigma$-compliant if for every consistent formula $\varphi$ it holds that $\min([\varphi], \preceq) \in \Sigma_\sigma$.

**Revision postulates**

(P$\star$1) $\sigma(F \star_\sigma \varphi) \subseteq [\varphi]$.
(P$\star$2) If $\sigma(F) \cap [\varphi] \neq \emptyset$ then $\sigma(F \star_\sigma \varphi) = \sigma(F) \cap [\varphi]$.
(P$\star$3) If $[\varphi] \neq \emptyset$ then $\sigma(F \star_\sigma \varphi) \neq \emptyset$.
(P$\star$4) If $\varphi \equiv \psi$ then $\sigma(F \star_\sigma \varphi) = \sigma(F \star_\sigma \psi)$.
(P$\star$5) $\sigma(F \star_\sigma \varphi) \cap [\psi] \subseteq \sigma(F \star_\sigma (\varphi \wedge \psi))$.
(P$\star$6) If $\sigma(F \star_\sigma \varphi) \cap [\psi] \neq \emptyset$ then
$\sigma(F \star_\sigma (\varphi \wedge \psi)) \subseteq \sigma(F \star_\sigma \varphi) \cap [\psi]$.

$\star_\sigma$ satisfying P$\star$1 - P$\star$6 gives rise to faithful, $\sigma$-compliant $\preceq_F$

## Representation Result for Revision by an Argumentation Framework

defining $\sigma(F *_\sigma G) = \min(\sigma(G), \preceq_F)$ satisfies A$*$1 - A$*$6 + *Acyc* given I-fathful $\preceq_F$

**I-faithful assignments to rankings $\preceq_F$**

Given semantics $\sigma$, an I-faithful assignment maps every AF $F$ to an I-total pre-order $\preceq_F$ on $2^\mathfrak{A}$ such that, for any $\subseteq$-incomparable $E_1, E_2 \in 2^\mathfrak{A}$ and $F, F_1, F_2 \in AF_\mathfrak{A}$, it holds that:

(i) if $E_1, E_2 \in \sigma(F)$, then $E_1 \approx E_2$,
(ii) if $E_1 \in \sigma(F)$ and $E_2 \notin \sigma(F)$, then $E_1 \prec_F E_2$,
(iii) if $\sigma(F_1) = \sigma(F_2)$, then $\preceq_{F_1} = \preceq_{F_2}$.

**Revision postulates + *Acyc***

(A$*$1) $\sigma(F *_\sigma G) \subseteq \sigma(G)$.
(A$*$2) If $\sigma(F) \cap \sigma(G) \neq \emptyset$, then $\sigma(F *_\sigma G) = \sigma(F) \cap \sigma(G)$.
(A$*$3) If $\sigma(G) \neq \emptyset$, then $\sigma(F *_\sigma G) \neq \emptyset$.
(A$*$4) If $\sigma(G) = \sigma(H)$, then $\sigma(F *_\sigma G) = \sigma(F *_\sigma H)$.
(A$*$5) $\sigma(F *_\sigma G) \cap \sigma(H) \subseteq \sigma(F *_\sigma f_\sigma(\sigma(G) \cap \sigma(H)))$.
(A$*$6) If $\sigma(F *_\sigma G) \cap \sigma(H) \neq \emptyset$, then
$\sigma(F *_\sigma f_\sigma(\sigma(G) \cap \sigma(H))) \subseteq \sigma(F *_\sigma G) \cap \sigma(H)$.
(*Acyc*) If for $0 \leq i \leq n$, $\sigma(F *_\sigma G_{i+1}) \cap \sigma(G_i) \neq \emptyset$ and
$\sigma(F *_\sigma G_0) \cap \sigma(G_n) \neq \emptyset$ then $\sigma(F *_\sigma G_n) \cap \sigma(G_0) \neq \emptyset$.

$*_\sigma$ satisfying A$*$1 - A$*$6 + *Acyc* gives rise to I-faithful $\preceq_F$

## Future Work

- Extend results to semantics which are not proper I-maximal.
- Identify operators based on $\sigma$-compliant rankings for specific semantics $\sigma$.
- Extend insights to a broader theory of belief change within fragments.
- Apply findings to other belief change operations, e.g. iterated belief revision.
- Take the syntactic form of the AF into account.

## References

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