AN EXTENSION OF THE ICP ALGORITHM CONSIDERING SCALE FACTOR

Shaoyi Du[1], *Nanning Zheng*[1], *Shihui Ying*[2], *Qubo You*[1], *Yang Wu*[1]

[1]Institute of Artificial Intelligence and Robotics
[2]Research Center of Applied Mathematics
Xi'an Jiaotong University, Xi'an, Shaanxi Province 710049, P.R. China
[1]{sydu, nnzheng, qbyou, ywu}@aiar.xjtu.edu.cn, [2]shhying@mail.xjtu.edu.cn

ABSTRACT

The ICP algorithm is accurate and fast for registration between two point sets in a same scale, but it doesn't handle the case with different scales. This paper instead introduces a novel approach named the Scaling Iterative Closest Point (SICP) algorithm which integrates a scale matrix with boundaries into the original ICP algorithm for scaling registration. This method uses a simple iterative algorithm with the SVD algorithm and the properties of parabola incorporated to compute the translation, rotation and scale transformations at each iterative step, and its convergence is rapid with only a few iterations. The SICP algorithm is independent of shape representation and feature extraction; thereby it is general for scaling registration. Experimental results demonstrate its robustness and fast speed compared with the standard ICP algorithm.

***Index Terms*—**ICP, SVD, parabola, scaling registration

1. INTRODUCTION

Image registration is a demanding task in computer vision and image process. The Iterative Closest Point (ICP) algorithm [1, 2, 3] is an advanced approach for this problem for its good accuracy and fast speed which has been widely used in a variety of fields such as medical images, document images, fingerprint images and face images etc. To speed up the traditional ICP algorithm, an increasing group of scholars have studied it. Jost et al. [4] combined a coarse to fine multi-resolution technique with the neighbor search algorithm into ICP to improve the registration. Moreover, many scholars have introduced other methods into ICP for its more robustness. Invariant features were proposed by Sharp et al. [5] to decrease the probability of being trapped in a local minimum, and genetic algorithms and evaluation metric were introduced by Silva et al. [6] into ICP for more precise registration.

The original ICP algorithm doesn't take scale factor into account in the Least Squares (LS) problem, while the scale factor always exists in registration. Zha et al. [7] used extended signature images to estimate the scale and applied it to traditional ICP for registration, while Zinßer et al. [8] directly estimated the scale in the ICP algorithm. Obviously the scale is a scalar that can only register two isotropic point sets, but not anisotropic ones which exists widely in scaling registration. To solve it, our approach introduces a scale matrix directly into the LS problem with the constraint condition that the scale matrix is bounded. The reason for adding this constraint condition is to avoid the phenomenon happening that points of a set converge to a point of the other set. This constraint optimization problem is solved by the Scaling Iterative Closest Point (SICP) algorithm which is an extension of the ICP algorithm. At each iterative step of this new algorithm, the translation, rotation and scale transformations are computed through a new and fast iterative algorithm. Accordingly, the SICP algorithm has the similarly fast speed to the ICP algorithm. This new presented algorithm has been tested in experiments and the experimental results demonstrate that our presented algorithm is a fast and robust technique to solve scaling registration problems caused by the scale factor, and it can be used widely in practice.

This paper is organized as follows. In section 2, a general LS problem has been stated and the ICP algorithm has been reviewed briefly. In section 3, an optimization problem is described with a constraint condition that the scale matrix is bounded, and a proposed method — the SICP algorithm is given. Following that is section 4 in which the proposed technique is evaluated on the experiments and a conclusion is finally drawn in the last section.

2. PROBLEM STATEMENT AND THE ICP ALGORITHM

2.1. Problem Statement

The registration of m-D point sets is a difficult problem. To solve this, a general statement is described first as follows. Given two point sets in $\mathbb{R}^m$, one denotes a model shape $M \triangleq \{\vec{m}_i\}_{i=1}^{N_m}$ and the other is a data shape $P \triangleq \{\vec{p}_i\}_{i=1}^{N_p}$ ($N_m \geq N_p$). To register between two m-D point sets is to find an transformation T, with which P is registered to be in the best alignment with M, so the formulation is based on the following LS problem:

$$\min_{\mathrm{T}, j\in\{1,2,\cdots,N_m\}} \Big(\sum_{i=1}^{N_p} \left\| \mathrm{T}(\vec{p}_i) - \vec{m}_j \right\|_2^2 \Big) \tag{1}$$

2.2. The ICP Algorithm

Let T of (1) be translation and rotation transformations, hence the registration between two point sets is

$$\min_{\mathbf{R}, \vec{t}, j\in\{1,2,\cdots,N_m\}} \Big(\sum_{i=1}^{N_p} \left\| (\mathbf{R}\vec{p}_i + \vec{t}) - \vec{m}_j \right\|_2^2 \Big) \qquad s.t. \quad \mathbf{R}^{\mathrm{T}}\mathbf{R} = \mathbf{I}_m, \quad \det(\mathbf{R}) = 1 \tag{2}$$

where $\mathbf{R} \in \mathbb{R}^{m\times m}$ is a rotation matrix, $\vec{t} \in \mathbb{R}^m$ is a translation vector.

The ICP algorithm [1] achieves registration with good accuracy and fast speed, and it mainly has two steps.

Firstly, build up the set of correspondences:

$$c(i) = \underset{j\in\{1,2,\cdots,N_m\}}{\arg\min} \Big(\left\| (\mathbf{R}_k \vec{p}_i + \vec{t}_k) - \vec{m}_j \right\|_2^2 \Big) \tag{3}$$

Secondly, compute the new transformation between two point sets $\{\mathbf{R}_k \vec{p}_i + \vec{t}_k\}_{i=1}^{N_p}$ and $\{\vec{m}_{c(i)}\}_{i=1}^{N_p}$ by minimizing squared distance:

$$(\mathbf{R}^*, \vec{t}^*) = \underset{\mathbf{R}, \vec{t}}{\arg\min} \Big(\sum_{i=1}^{N_p} \left\| \mathbf{R}(\mathbf{R}_k \vec{p}_i + \vec{t}_k) + \vec{t} - \vec{m}_{c(i)} \right\|_2^2 \Big) \tag{4}$$

Update $\mathbf{R}_{k+1}$ and $\vec{t}_{k+1}$:

$$\mathbf{R}_{k+1} = \mathbf{R}^*\mathbf{R}_k, \qquad \vec{t}_{k+1} = \mathbf{R}^*\vec{t}_k + \vec{t}^* \tag{5}$$

3. THE SICP ALGORITHM

3.1. The SICP Algorithm

It is known that the ICP algorithm is a fast and accurate approach for registration between two point sets. However, the scale factor may exist in two point sets. In practice, we always need to consider the following LS problem:

$$\begin{aligned} &\min_{\substack{s, \mathbf{R}, \vec{t} \\ j\in\{1,2,\cdots,N_m\}}} \Big(\sum_{i=1}^{N_p} \left\| (\mathbf{R}\mathbf{S}\vec{p}_i + \vec{t}) - \vec{m}_j \right\|_2^2 \Big) \\ &s.t. \ \mathbf{R}^{\mathrm{T}}\mathbf{R} = \mathbf{I}_m, \quad \det(\mathbf{R}) = 1 \\ &\qquad \mathbf{S} = diag(s_1, s_2, \cdots, s_m), \ s_j \in \bigcup_k [a_{jk}, \ b_{jk}] \end{aligned} \tag{6}$$

where $\mathbf{S}$ is a scale factor and its boundaries can be estimated by the characteristics of the data sets, such as their covariance matrices.

Actually, we can solve this problem in the way the ICP algorithm does by iteration. At each iterative step, two steps are mainly included:

Step 1, build up the set of correspondences by the current transformation $(\mathbf{S}_k, \mathbf{R}_k, \vec{t}_k)$:

$$c(i) = \underset{j\in\{1,2,\cdots,N_m\}}{\arg\min} \Big(\left\| (\mathbf{R}_k \mathbf{S}_k \vec{p}_i + \vec{t}_k) - \vec{m}_j \right\|_2^2 \Big) \tag{7}$$

Step 2, assume $\mathbf{S} = diag(s_1, s_2, \cdots, s_m)$, compute the new transformation $(\mathbf{S}_{k+1}, \mathbf{R}_{k+1}, \vec{t}_{k+1})$:

$$(\mathbf{S}_{k+1}, \mathbf{R}_{k+1}, \vec{t}_{k+1}) = \underset{s_j \in \bigcup_k [a_{jk}, b_{jk}], \mathbf{R}, \vec{t}}{\arg\min} \Big(\sum_{i=1}^{N_p} \left\| \mathbf{R}\mathbf{S}\vec{p}_i + \vec{t} - \vec{m}_{c(i)} \right\|_2^2 \Big) \tag{8}$$

3.2. Computation of Scale, Rotation and Translation

To compute the scale, rotation and translation transformations in step 2, the following Lemma is given:

Lemma: Given two m-D point sets $\{\vec{q}_i\}_{i=1}^N$ and $\{\vec{n}_i\}_{i=1}^N$, the function $F(\vec{t}) = \sum_{i=1}^{N} \left\| \vec{q}_i + \vec{t} - \vec{n}_i \right\|_2^2$ has the minimum value when $\vec{t} = \frac{1}{N}\sum_{i=1}^{N}\vec{n}_i - \frac{1}{N}\sum_{i=1}^{N}\vec{q}_i$.

According to the lemma, if minimizing $F(\mathbf{S}, \mathbf{R}) = \sum_{i=1}^{N_p} \left\| (\mathbf{R}\mathbf{S}\vec{p}_i + \vec{t}) - \vec{m}_{c(i)} \right\|_2^2$, we get $\vec{t} = \frac{1}{N_p}\sum_{i=1}^{N_p}\vec{m}_{c(i)} - \frac{1}{N_p}\sum_{i=1}^{N_p}\mathbf{R}\mathbf{S}\vec{p}_i$. Hence,

$$\begin{aligned} F(\mathbf{S}, \mathbf{R}) &= \sum_{i=1}^{N_p} \left\| \mathbf{R}\mathbf{S}\vec{p}_i + \Big(\frac{1}{N_p}\sum_{i=1}^{N_p}\vec{m}_{c(i)} - \frac{1}{N_p}\sum_{i=1}^{N_p}\mathbf{R}\mathbf{S}\vec{p}_i\Big) - \vec{m}_{c(i)} \right\|_2^2 \\ &= \sum_{i=1}^{N_p} \left\| \mathbf{R}\mathbf{S}\Big(\vec{p}_i - \frac{1}{N_p}\sum_{i=1}^{N_p}\vec{p}_i\Big) - \Big(\vec{m}_{c(i)} - \frac{1}{N_p}\sum_{i=1}^{N_p}\vec{m}_{c(i)}\Big) \right\|_2^2 \end{aligned}$$

Let $\vec{q}_i \triangleq \vec{p}_i - \frac{1}{N_p}\sum_{i=1}^{N_p}\vec{p}_i$ and $\vec{n}_i \triangleq \vec{m}_{c(i)} - \frac{1}{N_p}\sum_{i=1}^{N_p}\vec{m}_{c(i)}$, therefore,

$$\begin{aligned} F(\mathbf{S}, \mathbf{R}) &= \sum_{i=1}^{N_p} \left\| \mathbf{R}\mathbf{S}\vec{q}_i - \vec{n}_i \right\|_2^2 \\ &= \sum_{i=1}^{N_p} (\vec{q}_i^T\mathbf{S}^T\mathbf{R}^T\mathbf{R}\mathbf{S}\vec{q}_i - \vec{n}_i^T\mathbf{R}\mathbf{S}\vec{q}_i - \vec{q}_i^T\mathbf{S}^T\mathbf{R}^T\vec{n}_i + \vec{n}_i^T\vec{n}_i) \\ &= \sum_{i=1}^{N_p} \vec{q}_i^T\mathbf{S}^2\vec{q}_i - 2\sum_{i=1}^{N_p}\vec{n}_i^T\mathbf{R}\mathbf{S}\vec{q}_i + \sum_{i=1}^{N_p}\vec{n}_i^T\vec{n}_i \end{aligned} \tag{9}$$

To minimize (9), we can recover the following partial differential equations for the estimation of registration parameters:

$$\frac{\partial F(\mathbf{S}, \mathbf{R})}{\partial \mathbf{S}} = 0 \tag{10}$$

$$\frac{\partial F(\mathbf{S}, \mathbf{R})}{\partial \mathbf{R}} = 0 \tag{11}$$

I. Suppose that $\mathbf{E}_j = diag(0, \ \cdots, \ 0, \ 1, \ 0, \ \cdots, \ 0)$, ($j = 1, 2, \cdots, m$) is a diagonal matrix where the j th element is one, but others are zero. They are the basis of the matrix $\mathbf{S}$, and then (10) can be expressed as follows:

$$\begin{aligned}\frac{\partial F(\mathbf{S},\mathbf{R})}{\partial \mathbf{S}} &= \lim_{t\to 0}\frac{F(\mathbf{S}+t\mathbf{E}_j,\mathbf{R})-F(\mathbf{S},\mathbf{R})}{t} \\ &= 2\sum_{i=1}^{N_p}\vec{q}_i^{\,T}\mathbf{S}\mathbf{E}_j\vec{q}_i - 2\sum_{i=1}^{N_p}\vec{n}_i^{\,T}\mathbf{R}\mathbf{E}_j\vec{q}_i = 0\end{aligned} \tag{12}$$

From (12), we get:

$$s_j = \sum_{i=1}^{N_p}\vec{n}_i^{\,T}\mathbf{R}\mathbf{E}_j\vec{q}_i \Big/ \sum_{i=1}^{N_p}\vec{q}_i^{\,T}\mathbf{E}_j\vec{q}_i \tag{13}$$

(1) If s_j is any arbitrary number, we obtain the scale of SICP with unbounded scale: $s_j = \sum_{i=1}^{N_p}\vec{n}_i^{\,T}\mathbf{R}\mathbf{E}_j\vec{q}_i / \sum_{i=1}^{N_p}\vec{q}_i^{\,T}\mathbf{E}_j\vec{q}_i$.

(2) If $s_j \in \bigcup_k [a_{jk},\, b_{jk}]$, according to (9), the function is known to be a parabola with respect to s_j and its axis parallels to vertical, so the minimum can be achieved at the point which is nearest to the vertex of this parabola, hence we get the scale s_j of the SICP algorithm:

$$s_j = \mathop{\arg\min}_{s\in\bigcup_k[a_{jk},b_{jk}]}\left| s - \sum_{i=1}^{N_p}\vec{n}_i^{\,T}\mathbf{R}\mathbf{E}_j\vec{q}_i \Big/ \sum_{i=1}^{N_p}\vec{q}_i^{\,T}\mathbf{E}_j\vec{q}_i \right| \tag{14}$$

II. For any given $\mathbf{S}$, the necessary condition of minimizing $F(\mathbf{S},\mathbf{R})$ is (11) which can't be computed easily, but according to (9), minimizing $F(\mathbf{S},\mathbf{R})$ is equivalent to maximizing $\sum_{i=1}^{N_p}\vec{n}_i^{\,T}\mathbf{R}\mathbf{S}\vec{q}_i$, which has been solved by Arun [9], thus we only give the conclusion here.

(1) Calculate $m\times m$ matrix $\mathbf{H}$ and its SVD.

$$\mathbf{H} = \frac{1}{N_p}\sum_{i=1}^{N_p}\mathbf{S}\vec{q}_i\vec{n}_i^{\,T} \tag{15}$$

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V} \tag{16}$$

(2) Calculate the rotation matrix $\mathbf{R}$.

1) If $\det(\mathbf{V}\mathbf{U}^T) = +1$, $\mathbf{V}\mathbf{U}^T$ is a rotation:

$$\mathbf{R} = \mathbf{V}\mathbf{U}^T \tag{17}$$

2) If $\det(\mathbf{V}\mathbf{U}^T) = -1$, $\mathbf{V}\mathbf{U}^T$ is a reflection:

a) If one of the singular values of $\mathbf{H}$ is zero, the desired rotation can be calculated as follows:

$$\mathbf{R} = \mathbf{V}\begin{pmatrix}\mathbf{I}_{m-1} & 0\\ 0 & -1\end{pmatrix}\mathbf{U}^T \tag{18}$$

b) If none of the singular values of $\mathbf{H}$ is zero, we go to a RANSAC-like technique.

III. Repeat the above two steps until it converges or a maximum number of iteration steps is reached, and then we can obtain the solution ($\{s_{k+1,j}\}_{j=1}^{m}$, $\mathbf{R}_{k+1}$).

IV. According to Lemma, we calculate $\vec{t}_{k+1}$:

$$\vec{t}_{k+1} = \frac{1}{N_p}\sum_{i=1}^{N_p}\vec{m}_{c(i)} - \frac{1}{N_p}\sum_{i=1}^{N_p}\mathbf{R}_{k+1}\mathbf{S}_{k+1}\vec{p}_i \tag{19}$$

4. EXPERIMENTAL RESULTS

To verify the robustness of our presented method, three experiments are tested on the following data sets: 1) the 3D point sets listed in [1], 2) certain 2D shapes in part B of CE-Shape-1 [10], 3) the bunny model of the Stanford 3D Scanning Repository [11]. The results of ICP and SICP are reported as follows in which errors are computed by RMS (Root Mean Square).

In the first experiment, we use two 3D point sets which are detailed in [1]. Set 1 has 8 points while Set 2 has 11 points. The compared results of ICP and SICP are listed in Table 1 respectively, and both algorithms converge in 5 steps.

	RMS (mm)	Scale
ICP	0.4376	1
SICP ($0.9 \le s_j \le 1.1$)	0.2650	diag(0.9000,0.9722,1.0468)

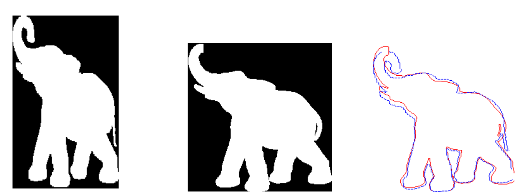
Table 1. Compared results of ICP and SICP on two 3D point sets.

From the results of Table 1, we find (RMS_{ICP}-RMS_{SICP})/RMS_{ICP} = 39.44%, meaning that SICP can get considerably better accuracy than ICP.

In the second experiment, to show the robustness for registration of 2D shapes, we compare ICP and SICP ($0.5 \le s_j \le 2$, $j = 1,2$) on two shapes of Part B of CE-Shape-1, a large 2D shapes database and the results are shown in Table 2 and Fig. 1.

	RMS	Scale
ICP	8.0546	1
SICP ($0.5 \le s_j \le 2$)	4.0564	diag(0.8484, 1.2461)

Table 2. Compared results of ICP and SICP on two 2D shapes.



(a) (b) (c)

Fig. 1. Registration result of SICP ($0.5 \le s_j \le 2$, $j = 1,2$). (a) 2D data shape. (b) 2D model shape. (c) Registration result of SICP.

Table 2 demonstrates SICP is better than ICP in two 2D shapes with different scales, and Fig. 1 shows satisfying registration result of SICP.

In the third experiment, our method is compared with ICP on the Stanford Bunny. Two range data, bun000 with 40256 points and bun045 with 40097 points, are used, in which bun045 is to register bun000. The compared results are given in Table 3 and Fig. 2.

	RMS ($\times 10^{-3}$ mm)	Scale
ICP	2.0217	1
SICP ($0.99 \le s_j \le 1.01$)	1.9639	diag(0.9900, 0.9900, 0.9900)
SICP ($0.98 \le s_j \le 1.02$)	1.9404	diag(0.9800, 0.9869, 0.9800)
SICP (s_j is any arbitrary number)	0.6854	diag(0.0033, 0.3254, 0.0034)

Table 3. Compared results of ICP and SICP on the Stanford Bunny.

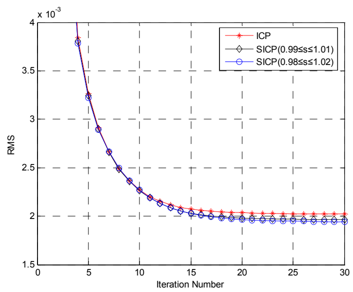
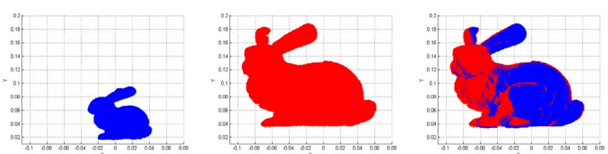


Fig. 2. The convergence of ICP and SICP on the Stanford Bunny

In Table 3, SICP with bounded scale and ICP are similar in accuracy on the Stanford Bunny. Though RMS of SICP (s_j is any arbitrary number) is 0.6854, seemingly smaller than those of others, its scale $\mathbf{S}$ is close to $0_{3\times 3}$, an unreasonable value meaning all points of bun045 converge to a very small part of bun000. Hence, the constraint condition is necessary for scaling registration problem. Moreover, as is shown in Fig. 2, SICP and ICP are quite alike in convergence at a similarly fast speed with about 1.2 sec at each iterative step.



(a) (b) (c)

Fig. 3. Registration result of SICP ($1.99 \le s_j \le 2.01$, $j = 1, 2, 3$) on bun000 and bun045 range data. (a) Bun045 with a scale of 0.5. (b) The original range data of bun000. (c) Registration result of SICP.

To evaluate the convergence of SICP with respect to the scale factor, we scale the bun045 range data by a factor of $1/(0.1n)$, $(n = 1, 2, \cdots, 50)$, then use the scaled range data to register bun000 with a constraint condition $0.1n - 0.01 \le s \le 0.1n + 0.01$. In the experiment, whatever n is, the scaled bun045 registers the bun000 quite well, which reveals that our method is robust in convergence with respect to the scale factor. Fig. 3 shows one fairly fine registration result of SICP.

5. CONCLUSION

This paper proposes a new approach for scaling registration of two m-D point sets in the way of incorporating a bounded scale matrix into the ICP algorithm. A series of experiments designed demonstrate our algorithm is more accurate in scaling registration between two m-D point sets and takes no more time in contrast to the standard ICP algorithm. For its stable convergence with respect to the scale factor, we will try to extend the framework for registration in practical use in the future.

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