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AN IMPLEMENTATION OF EXPECTED UTILITY THEORY IN DECISION BASED DESIGN

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ABSTRACT

The development of a design science rests on the ideal that design is anchored in a set of fundamental axioms similar to the more 'traditional' sciences of mathematics and physics. However, the axioms upon which a design science is constructed must reflect that design is a science of the artificial. It is our contention that such axioms may exist in Decision-Based Design as those formulated by von-Neumann and Morgenstern for developing utilities under conditions of risk. In this paper we have a very narrow focus: evaluating a proposed framework for applying these axioms in the context of a simple design problem through the use of Monte Carlo simulation and expected utility theory.

GLOSSARY

Decision "...we define a decision as an irrevocable allocation of resources" (Hazelrigg, 1996). "There are two important characteristics of a decision:

- A *decision* is made at an instant in time.
- A *decision* must be made based on the information available at the time it is made."

Decision-Based Design Our fundamental paradigm for designing and creating design methods, rooted in the notion that the principal role of engineers, in the design of an artifact, is to make decisions (Shupe, 1988; Mistree, Smith et al., 1990).

NOMENCLATURE

d	Conductor diameter	(m)
s(m)	The option set (set of alternatives)	
m_i	A particular option (alternative)	
x	Vector of design variables	
a	Vector of design attributes	
y	Vector of exogenous variables	
t	Time	
p(t)	Price as a function of time	($)
q	Demand function	
N	Number of years	(-)
L	Length of cable	(m)
R	Resistivity	(Ω·m)
T	Equivalent operating time	(h/yr)
i	Interest rate	(%)
P	Cost of energy	($/kWh)
D	Demand charge	($/kWh)
c	Yearly increase in cost	(%)
V	Voltage	(V)
I_{0stat}	Standard current	(A)
LF	Load Factor	(-)
a	Yearly load increase	(%)
b	Demand charge increase	(%)
ρ	Density	(kg/m^3)

1. FRAME OF REFERENCE: AN AXIOMATIC FRAMEWORK FOR DESIGN

In this paper we explore a model for design in which decisions are made according to a set of governing axioms. We do this to anchor our view of Decision-Based Design (DBD) in decision theory. The notion that design is not simply *ad hoc*, but a science of the artificial is presented formally by Simon (1996). Our view of DBD has evolved to a state in which a fine balance exists between the principles of Living Systems Theory (Miller, 1978) and the notion of Decision Support Problems in design (Mistree, Smith et al., 1990).

Decision-Based Design is a term coined to emphasize a different perspective from which to develop methods for design. The principal role of a designer, in Decision-Based Design (DBD), is to make decisions. But while our work thus far has provided formalism to DBD, we have yet to include the rigor of decision theory and the von-Neumann and Morgenstern axioms in our work. Towards bringing these axioms to the forefront of DBD, in this paper we explore *expected utility theory* (see Section 1.1) and its application in an example problem. In doing so, we we venture away from the ideal **assumptions** that designs and designers:

- **operate under conditions of complete certainty and**
- **select and refine designs to meet a common measure of performance defined in terms of functional requirements**.

We replace these assumptions by answering the questions:

1. *How can we represent and handle the uncertainty in a system?*
2. *How can designs be refined and selected using expected utility theory?*

In this paper we have a very narrow focus. We make the assumption that uncertainty can be modeled as risk by defining *a priori* probability distributions over uncertain events. Thus, we answer the first question with a brief description of uncertainty modeled as risk (Section 1.1) and its incorporation into a proposed framework for engineering design (Section 2.2). We then explore in depth the proposed framework for engineering design in the context of an example problem to answer the second question (Section 3). Our intent is not to categorize the merits of utility theory as an encompassing method in design; we seek only to evaluate *the proposed framework* for engineering design.

In this section we give an introduction to the concepts of risk and the role of utility theory in engineering design. Section 2 of this paper is an overview on the concepts of decision and value analysis wherein the framework for the example problem is introduced. In Section 3 we give the logistics of the framework as it is applied to a case study in electrical power transmission. The final section is an overview of the future work necessary for a complete implementation of the framework outlined in this paper. We begin with a description of risk that addresses, in part, our relaxation of both of the ideal assumptions above.

1.1. The Concept of Risk

Over 200 years ago Daniel Bernoulli proposed a solution to a problem of decision making under risk that has since become known as Bernoulli's paradox. Simply stated, the problem (formulated by Daniel's cousin Nicholas) is to determine the amount of money that a person would be willing to play to enter a game where a fair coin is tossed until on the *n*th flip it lands on heads. The prize for playing is $\$2^n$. The expected monetary value (EMV) of this game is of course infinite (see Equation 1).

$$\sum_{n=1}^{\infty}(\text{probability of n})(\text{prize for n}) = \sum_{n=1}^{\infty}\frac{1}{2^n}2^n = \infty \qquad \textbf{(1)}$$

Contrary to this outcome, very few people are willing to pay large amounts of money to play this game. A hypothesized reason is that people perceive the *risk* associated with the game and consequently alter their behavior. Bernoulli formalized this discrepancy between the EMV and the behavior of individuals in terms of utility as the expected utility **hypothesis**: Individuals make decisions with respect to investments in order to *maximize expected utility* (Bernoulli, 1954). The expected utility hypothesis is a description of human behavior. We are interested in a normative theory, thus the expected utility hypothesis is applicable in design as von Neumann and Morgenstern's expected utility theory, i.e., that designers *should* choose a design that maximizes their expected utility. Note that the expected utility theories by Bernoulli and von Neumann and Morgenstern are *not* equivalent. The former defines preference from utility while *the latter defines valid utilities from preference*.

Designers are typically taught that design is defining a set of *functional requirements* and subsequently providing an artifact that satisfies those functional requirements (Pahl and Beitz, 1988; Suh, 1990). While this is true at a very low level of abstraction, we must ask ourselves if we design to meet functional requirements or if we design for some other purpose. In the context of an enterprise products are introduced or modified almost exclusively to maintain or improve the *financial* position of the firm. In this context design has very little to do with functional requirements *per se*, and instead involves producing or refining artifacts to *make money for the firm* (Goldratt and Cox, 1992).

A 'design' therefore has a monetary present value (PV) for the firm. It is our contention that this PV is computable for simple designs, and that this PV should be the measure by which these designs are evaluated. By making decisions at the function level of abstraction, if such a thing can be defined, we are implicitly assuming that static functional requirements are the basis for a design that has a positive PV for the firm.

With the relaxation of Assumption 1, that designs and designers operate under conditions of complete certainty, the measure of PV is probabilistic. We need a mechanism by which decisions on PV account for risk. This is the expected utility theorem formulated by von Neumann and Morgenstern (1947).

1.2. State of the Art: The Role of Utility Theory in Design

Utility theory has been addressed in design during the last five years by several authors, for example (Otto and Antonsson, 1993; Thurston, Carnahan et al., 1994; Antonsson and Otto, 1995). These authors have focused almost exclusively on replacing multiobjective optimization functions in design with multiattribute utility analysis (MUA) (Keeney and Raiffa, 1993) or by other methods. Thurston and Carnahan (1994) focus on specifying a multiattribute utility function and the conditions needed to meet the condition that the attributes are Mutually Utility Independent (MUI). Otto and Antonsson (1993) implement the Method of Imprecision (MoI), constructed from fuzzy set theory, as an entirely separate method. The multiattribute utility function is a fundamentally correct extension to traditional optimization provided that the attributes are MUI, not a trivial condition to meet. But its use is more a response to multi-objective optimization in design whereby the objective function is directly translated to a multiattribute utility function in response to functional requirements. This neglects an important aspect of utility theory: that utility functions are necessary only in the presence of risk (Park and Sharp-Bette, 1990).

In this paper, we use a utility function defined for the PV of a design alternative. Our argument for doing so is based on the idea that making decisions at the function level of abstraction requires use of MUA. The implicit assumption at this level of abstraction, however, is that satisfaction of the functional requirements is a necessary condition for a positive PV of an artifact. We choose to model the relationship between the attributes of an artifact and a market demand for that artifact; something not readily accounted for at the function level of abstraction. Of course, if the objectives of the firm are to provide employment, maintain worker happiness, etc., a multiattribute utility function is still needed at the value level of abstraction. In Figure 1, we show this distinction in design decision making.

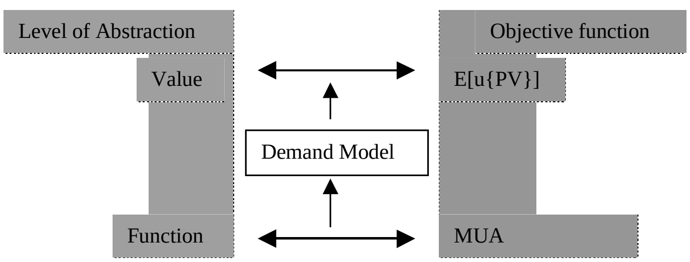

Figure 1: Levels of Abstraction and Their Respective Objective Functions

At the function level of abstraction in Figure 1, the objective function must be defined over the attributes, usually as a multiattribute utility function. Modeling market demand in terms of the artifact's attributes leads to the value level of abstraction, and the needed objective function is a utility for PV. Missing from this argument is the means by which such a mapping from the function level of abstraction to the value level of abstraction is possible. In the next section, we explain the underlying decision theory necessary for such a mapping, including a proposed framework for making decisions under risk. We view this a precursor to developing a *normative* theory of design. Implicit in the development of this theory is the central notion that the principal role of designers in Decision-Based Design (DBD) is to make decisions (Shupe, 1988; Mistree, Smith et al., 1990).

2. FUNDAMENTALS OF DECISION AND VALUE ANALYSIS

Decisions in engineering design fall almost exclusively in the domain of making choices under conditions of uncertainty and risk. In this paper, uncertainty is modeled using probabilities and values. Thus, decision making under uncertainty is modeled as decision making under risk. As a precursor to decision making under risk, we first review values (see Section 2.1). We then review a proposed framework for engineering design in which designers account for risk through the use of expected utility theory (Section 2.2). Making decisions in this framework requires three things: (1) options, (2) expectations, and (3) values. Options are the elements of the choice. A designer chooses an option in the form of a design alternative. An expectation comprises the range of possible outcomes of a decision paired with their probabilities of occurrence. Determination of expectations on each option is the realm of modeling (Hazelrigg, 1996).

2.1. Values

Values are used in decision making to rank order alternatives. Similar to numerical optimization, we make use of a numerical function to rank order for two reasons: (1) it is too cumbersome to make an *ad hoc* assessment of the comparative merits of all design alternatives, and (2) the comparison is generally too complex to make accurately and consistently without the use of a mathematical function. The mathematical requirements for a value function that rank orders all alternatives are stringent (Hazelrigg, 1996).

Von-Neumann and Morgenstern (vN-M) have formalized the mathematical requirements for a value function in their axiomatic treatment of utility (von Neumann and Morgenstern, 1947). What they showed is the following (Luce and Raiffa, 1957):

> If a person is able to express preferences between every pair of gambles, where gambles are taken over some basic set of alternatives, then one can introduce utility associations to the basic alternatives in such a manner that, if the person is guided solely by the utility expected value, he/she is acting in accord with his/her true tastes – provided only that there is an element of consistency in his/her tastes[1].

In the context of utility theory, a von Neumann-Morgenstern lottery is shown in Figure 2. In this sense, the options are defined as A_1, A_2, …, A_r where without loss of generality we can assume that $A_1 \succ A_2 \succ \ldots \succ A_r$. The circle represents a gamble, where A_1 occurs with probability p_1 and A_r occurs with probability 1- p_1.

[1] "element of consistency" - This statement is meant to convey that preference and indifference rankings must be transitive.

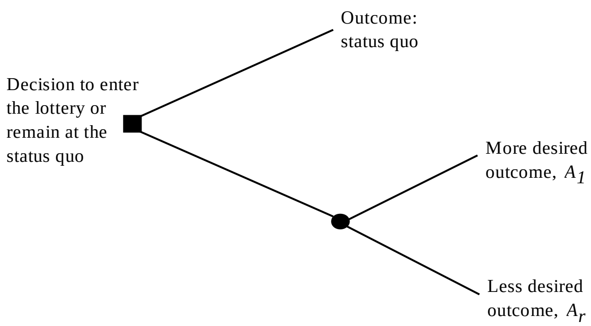

Figure 2: A von Neumann-Morgenstern Lottery

What do the von Neumann-Morgenstern axioms mean in engineering design? We refer to the following interpretation of the axioms (Luce and Raiffa, 1957):

> Suppose that one has to make a choice between a pair of lotteries that are each composed of complicated alternatives. Because of their complexity it may be extremely difficult to decide which one is preferable [and this is usually the case in engineering design]. A natural procedure, then, is to analyze each lottery by decomposing it into simpler alternatives, to make decisions as to preference among these alternatives, and to agree upon some consistency rules that relate the simpler decisions to the more complicated ones. In this way, a consistent pattern is imposed upon the choices between complicated alternatives.

This is fundamentally the notion of utility within engineering design, and the consistency rules are the vN-M axioms which are as follows (Luce and Raiffa, 1957; Hazelrigg, 1996):

1. All outcomes of a vN-M lottery (options) can be ordered in terms of the decision maker's preferences, and that ordering is transitive.
2. Any compound lottery, that is, any lottery that has an outcome another lottery, can be reduced to a simple lottery that has among its outcomes all the outcomes of the compound lottery with their associated probabilities of occurrence.
3. If the outcomes of a lottery, A_1, A_2, ... , A_r are ordered from most desired to least desired (respectively), then there exists a number u, such that one is indifferent between an outcome A_i, and $[u_i A_1, (1\text{-}u_i) A_r]$.
4. For any lottery such as that given in axiom 3, with u_i specified, there exists an outcome $[u_i A_1, (1\text{-}u_i) A_r]$ that can be substituted for A_i, and the preferences of the decision maker will remain unchanged.
5. The decision maker's preferences and indifferences among lotteries are transitive.
6. Given two lotteries, each with only two outcomes, and which differ only in terms of the probabilities of the outcomes, the lottery in which the probability of the more desired outcome is larger is the preferred lottery.

These six axioms are the basis for decisions in utility theory. The formal *expected utility theorem* based on these axioms and the notation $L = (p_1 A_1, \ldots, p_r A_r)$ and $L' = (p_1' A_1, \ldots, p_r' A_r)$ is as follows (Luce and Raiffa, 1957):

> If the preference or indifference relation ($\succ$ or $\sim$) satisfies assumptions 1 through 6, there are numbers u_i associated with the basic prizes A_i such that for two lotteries L and L′ the magnitudes of the expected values $(p_1 u_1 + p_2 u_2 + \ldots + p_r u_r)$ and $(p_1' u_1 + p_2' u_2 + \ldots + p_r' u_r)$ reflect the preferences between the lotteries.

How can expected utility theory be implemented in engineering design? In the next section we review a proposed framework for engineering design in which designers make decisions based on the expected utility theorem.

2.2. A Proposed Framework for Engineering Design

The six vN-M axioms and their associated concepts for utility have been expressed in a framework for systems design (Hazelrigg, 1996). *The most important aspect of this framework is the use of expected utility theory (derived from the vN-M axioms) to refine and select designs.* The framework (see Figure 3) begins with an *option set shown as a darkened box*, s(m), consisting of all the *configurations*. Each configuration m_i (the box above the s(m) box in Figure 3) is represented by a parametric design vector **x** shown to the right of configuration m_i in Figure 3.

The vector of design variables is a designer's representation of the system. It is not useful for describing to a client the attributes of a product. For example, in designing a car, a system variable, x_i, may be the compression ratio in the

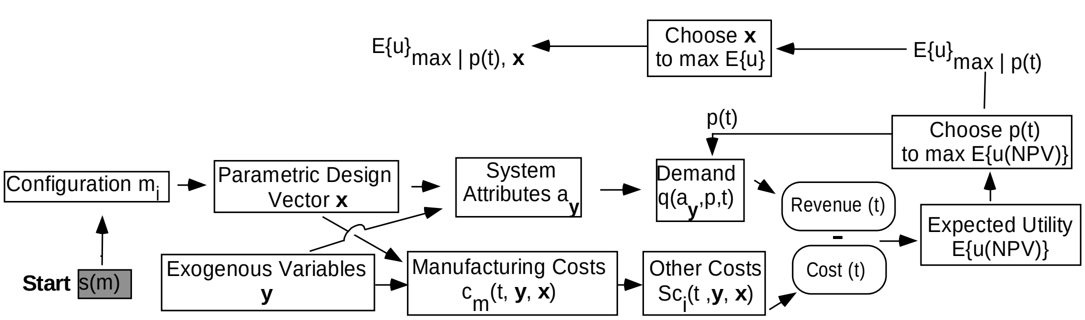

Figure 3: A Framework for Decision-Based Design. Adapted from (Hazelrigg, 1996)

engine. Consumers, however, are usually more concerned with the attributes of a particular system configuration. Thus, we change the design variables into a vector of system attributes, **a**, that are recognizable to consumers (shown as the box labeled System Attributes in Figure 3). The design variables and the attributes in general will not have a 1 to 1 correspondence.

To represent the uncertainty in **x** and the noise variables in the system we introduce a vector of exogenous variables, **y**. Exogenous variables affect **a** which are thus characterized as $\mathbf{a}_y$ (see Figure 3), that is the vector **a** subject to the vector of exogenous variables, **y**.

After the configuration is represented in terms of attributes a valid input to a demand function exists, which in turn gives the revenue of a particular configuration. This demand function is the market response to a particular configuration and ideally allows designers to find a revenue based on particular design variables. In the demand function, $q(a_y, p, t)$, p and t are price and time respectively (alternatively we could have expressed demand as $q(a_y, p(t))$). Here we see a deviation from the way in which we typically evaluate a design. Instead of measuring the design on some compound scale, we are concerned primarily with the revenue of the system, which is a function of the price, time (for the time-value of money) and the attributes. This addition is the revenue of configuration m_i as a function of time (shown as the box Revenue in Figure 3).

For the purposes of finding the present value (PV) of a configuration and subsequently an expected utility, both revenue and cost are needed. In terms of this design framework in Figure 3 the PV of a design option with continuous compounding is given by

$$PV = \int_0^\infty e^{-rt} (\mathrm{Re}\, v(t) - Cost(t))dt \qquad \textbf{(2)}$$

where r is the discount rate (Minimum Attractive Rate of Return) and t is time. Note that the costs associated with configuration m_i have not been explicitly accounted for. These costs consist of the normal entities we would associate with design and manufacture of a product including labor, machines, materials, disposal, etc. In addition life cycle issues are associated as a cost with configuration m_i. Costs are subject to the same uncertainty as the configuration itself.

Calculation of demand in Figure 3 requires knowledge of the elasticity of demand and therefore price as a function of time, p(t). Using the designer's utility for money, we can perform an optimization to maximize utility (derived from PV) with respect to p(t), a variable under our control.

This simple maximization is a subset of the more general maximization of expected utility. By changing the design vector **x** in Figure 3 the design is optimized with respect to *expected utility*. Note that the chain of computations required here is extensive; it is unlikely that the average workstation has the capability to solve these optimizations in a reasonable amount of time. However, as computational power continues to increase the arguments for using this proposed framework become more compelling. In the next section this framework is applied to a simple example problem in designing a conductor for electrical power transmission.

3. IMPLEMENTING THE PROPOSED FRAMEWORK IN A DESIGN EXAMPLE: POWER TRANSMISSION

In this section we implement the proposed framework for engineering design outlined in Section 2.2 and shown in Figure 3 as an example problem in power transmission conductor selection. In Section 3.1 we give an overview of the mathematical modeling for the example problem. The computer instantiation of this modeling is given in Section 3.2, and in Section 3.3 we present the results of implementing the proposed framework for engineering design in our power transmission problem. The designer's utility function is determined from a procedure outlined in Keeney and Raiffa (1993).

3.1. Introduction and Modeling of Power Transmission

A variety of factors are important in the design of a transmission line including reliability, conductor size, material and configuration, corona effects, radio and television interference, audible noise, electrostatic and electromagnetic effects, etc. (Crawford, Huntzinger et al., 1978). At least some of these factors have opposing impact on a design. In this example, we consider the design of the towers and the line configuration on those towers as separate from the design of the conductor. The setting for the problem is thus stated as:

> Find a suitable conductor material and configuration to tie a main power grid @69kV with a small town approximately 20km away.

A schematic for the transmission line is shown in Figure 4.

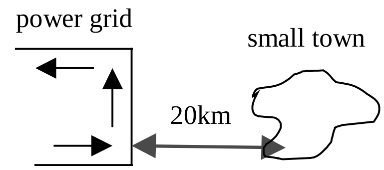

Figure 4: Power Line to Connect Small Town and Power Grid

From this basic problem statement the option set, s(m) in Figure 3 is specified as the set of conductors defined by a single stranded wire (no bundled conductors) made of either copper or aluminum (see Figure 5). In this case, the wire is defined entirely by a specification of the material type and the conductor diameter, *d*. Material type is a discrete variable while the conductor diameter is a continuous variable. Thus a particular configuration for the wire is specified as {(aluminum, copper), *d*} where we specify either aluminum or copper and a particular value for the diameter. Relative to Figure 3 this information, material and diameter, represent a design configuration m_i, shown for a particularly diameter in Figure 5.

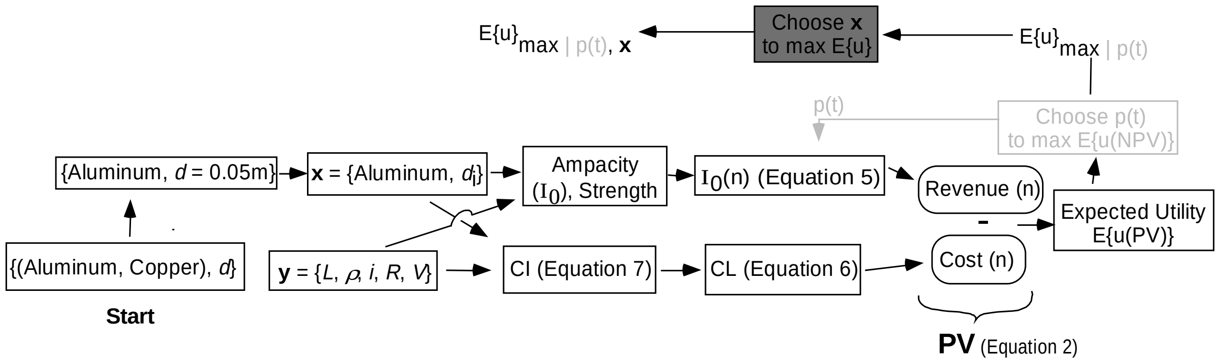

Figure 5: Framework for Power Problem in the Proposed Framework. Adapted from (Hazelrigg, 1996)

We next model the exogenous variables in the system defining the vector **y** in Figure 5. Because our intent here is to demonstrate application of this framework, modeling the exogenous variables is simplified to include only the length of the cable (L), the density of the cable material (ρ), the minimum attractive rate of return (i), the cable resistivity at 20C (R), and the voltage of the circuit (V).

The revenue and demand models are developed simultaneously, noting that the power company is a monopoly in specifying demand (this eliminates gaming from our demand function). The revenue earned for the power company is simply the power transmitted multiplied by the demand charge adjusted for projected yearly increases in load (energy use increase by the town) and a demand charge escalation factor (increase in how much the power company charges the consumers). Over a period of N years (each year n), this revenue must be discounted to account for the time value of money yielding the following simplified expression (we assume that all cash flows are at year end):

$$\sum_{n=1}^{N} \frac{N_p \cdot N_c \cdot I_0 \cdot V \cdot D \cdot 10^{-3} \cdot \left[(1+a)^2 \cdot (1+b)\right]^{n-1}}{(1+i)^n} \cdot (T) \qquad \textbf{(3)}$$

where N_p is the number of phase carriers (3), N_c is the number of circuits (1), I_0 is the maximum load on the cable during the first year (A), V is the operating line voltage (V), D is the demand charge per year $\left(\frac{\$}{kW \cdot h}\right)$, a is the increase in load per year (%), b is the demand charge escalation factor per year (%), and T is the number of years the maximum current I_0 would need to flow in order to produce the same total yearly energy as the actual variable load current. T is calculated using the approximation (Anders, 1997):

$$T = 8760h \cdot \left(0.3 \cdot LF + 0.7 \cdot LF^2\right) \qquad \textbf{(4)}$$

where LF is the load factor for the circuit defined to be $\frac{\text{the average value of the load per annum}}{\text{maximum value of the load per annum}}$ (Freeman, 1968). We assume that the maximum load and thus I_0 is governed by the conductor cross sectional area in the limiting case and by the load demand otherwise. Demand is given as:

$$I_0(n) = I_{0stat} \cdot \left[(1+a)^2\right]^{n-1} \quad n = 1, 2, ..., N \text{ years} \qquad \textbf{(5)}$$

where I_{0stat} is 350A (~70MW) and n and a are as defined. The maximum current I_0 as a function of d for aluminum is $I_0(d) = 36500d - 160$ and for copper is $I_0(d) = 40300d - 50$ with allowable current densities of 0.64A/m^2.

The cost model comprises all costing elements in Figure 3 by allowing a designer to choose a fixed cost and calculating an operating cost based on Joule losses only (a maintenance model is not included here for simplicity). Total cost is thus expressed as $CT = CI + CL$ where CT is total cost, CI is cost of the installed length of cable ($), and CL are the operating losses over the total cable life expressed in time 0 dollars (see Figure 5). Note that maximization of designer utility with respect to demand is not included in this model. Thus, p(t) is shown as greyed in Figure 5. Fixed costs of installation are taken as a designer input, while CL is calculated from the Joule losses and foregone revenue (Joule losses that could have been sold to consumers) as (Anders, 1997):

$$\begin{aligned} \mathrm{CL} &= \sum_{n=1}^{N} \frac{3 \cdot I_0^2 \cdot R \cdot L \cdot \{T \cdot P \cdot F(a,c) \cdot + D \cdot F(a,b)\}}{(1+i)^n} \\ F(a,c) &= \left[(1+a)^2 \cdot (1+c)\right]^{n-1} \\ F(a,b) &= \left[(1+a)^2 \cdot (1+b)\right]^{n-1} \end{aligned} \qquad \textbf{(6)}$$

where c is the increase in the cost of energy per year, P is the cost of energy for the power company $\left(\$/_{kW \cdot h}\right) \cdot 10^{-3}$, and all other variables are as previously defined. The installation cost is a sum of the fixed costs of installation and the material costs of the cable:

$$CI = \text{fixed cost} + \frac{\text{material costs}}{\text{kg}} \cdot \rho \qquad \textbf{(7)}$$

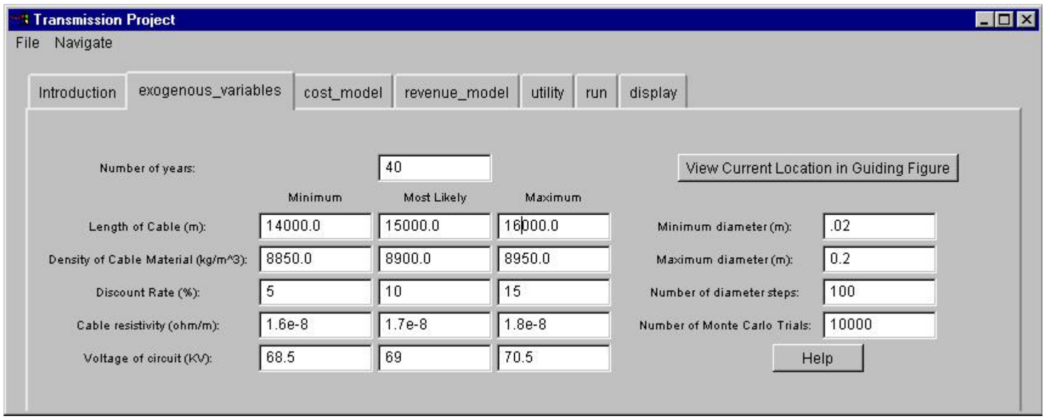

Figure 6: Exogenous Variables Input Screen

We now have models for calculating, based on a conductor configuration, the revenues and costs. What remains to be addressed is how we deal with uncertainty in these parameters.

3.2. Uncertainty Resolution and Computer Implementation

In applying the uncertainty and utility theory concepts presented in this paper to a design problem we quickly move from a model of mathematical elegance to a model with mathematical difficulty. In particular, the lack of supporting tools for the implementing the framework in Figure 3 makes such an effort daunting for all but the most simple of design problems. To solve the transmission problem (and for educational purposes) a Java program is written to implement Monte Carlo sampling for handling uncertainties in the system variables (modeled using probabilities) and time based simulation for handling computation of the revenues and costs of a conductor configuration.

In specifying the variables for the models presented in Section 3.1, we should be aware of our relaxing the assumption of certainty in Section 1. We use a heuristic procedure for choosing a triangular distribution to model the uncertainty on the system variables as risk (Park and Sharp-Bette, 1990). Designers are asked to input variables to the program in the form of minimum (min), most likely (M_0) and maximum values (max) (see Figure 6) (Hillier, 1971). From these values simple

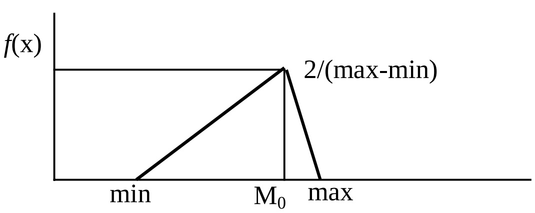

Figure 7: Triangular Distribution (Park and Sharp-Bette, 1990)

triangular distributions are constructed (see Figure 7)[2]. The density function (Equation 8) and cumulative distribution (Equation 9) are (Park and Sharp-Bette, 1990)

$$f(x)=\begin{cases}\dfrac{2(x-\min)}{(\max-\min)(M_0-\min)}, & \min\le x<M_0\\[2ex] \dfrac{2(\max-x)}{(\max-\min)(\max-M_0)}, & M_0\le x\le\max\end{cases}, \qquad (8)$$

$$F(x)=\begin{cases}\dfrac{(x-\min)^2}{(\max-\min)(M_0-\min)}, & \min\le x<M_0\\[2ex] \dfrac{(\max-x)^2}{(\max-\min)(\max-M_0)}, & M_0\le x\le\max\end{cases}, \qquad (9)$$

Similar input screens to that in Figure 6 are used for the cost and revenue models, with all uncertain variables having triangular distributions such as that in Figure 7. Online help is available to guide designers through the program.

The means by which an expected utility in Figure 5 is calculated from the input values is shown as a flowchart for the computational portion of the program in Figure 8. The main loop for the program runs on the diameter of the conductor. For each diameter and material x_i in Figure 5 (discrete step sizes are used for purposes of Monte Carlo sampling), a Monte Carlo simulation is run and for each single sample from the constructed triangular distributions of the exogenous variables, cost model variables, and revenue model variables (e.g., *i, a, b, c*, LF, V, P, D, etc.) in the Monte Carlo simulation, a time simulation is run to calculate the PV of a design option for the particular exogenous, cost and revenue model parameters sampled. The output of the time simulation is a PV for that particular sample of the cumulative distributions for all variables. This PV is converted to a utility by means of a user specified utility function. Since each output of a Monte Carlo simulation has probability 1/*maximum number of trials*, we can simply sum up these utilities calculated for each PV of the

[2] Based on a specified minimum, most likely and maximum values the maximum entropy distribution, that is the distribution that has the most uncertainty is a Beta distribution (Kapur and Kesavan, 1992). We use triangular distributions for computational simplicity.

Monte Carlo run and mulitply by 1/*maximum number of trials* to get an expected utility. Relative to Figure 5 this amounts to finding a cost and revenue for every sample of the distributions up to *maximum number of trials* (Monte Carlo), calculating a PV for each trial (a time based simulation), a utility for each PV of the time simulation, and then calculating an expected utility based on the distribution of utilities obtained for a particular diameter and material. At this point, the program stores the data, increments *d*, and performs the whole scenario again. When *d* reaches its limit for the investigation, E[u] vs. *d* are plotted and the user reads off the highest E[u] for a particular diameter.

This program differs from the proposed framework in Figure 3 in two ways. First the numerical maximization of expected utility is not done by the program but is instead performed by the designer by visually reading a maximum from a plot of E[u] vs. *d* or from the output data itself. The implication here is that we must search the entire design space and select the design with the highest expected utility. Because we have only a single design parameter for each run of the simulation (material type is specified so we vary the diameter for that material) we plot the expected utility of a conductor vs. diameter (for a given material) and select the greatest value. Second the demand charge, *D* (p(t) in Figure 5*)*, is not optimized within the program but is again user specified. The implication here is that we could in fact earn more money for each particular configuration. This affects only the absolute values of all expected utilities, not their relative values, and therefore not the conductor configuration that we select. Calculation of utilities is done prior to the study by the procedure outlined in (Keeney and Raiffa, 1993).

3.3. Results for Comparison of a Copper and Aluminum Conductor

To exemplify the use of this program and the proposed framework the values of all required inputs for selecting a design from among a copper and aluminum conductor are shown in Table 1. The utility function used for both the aluminum and copper conductors is $u(PV) = 1 - e^{-1e\text{-}7\ PV}$. For each conductor material we run the program with different material specific values and save the expected utility for that conductor material. Thus we optimize diameter for each material type and *then* select the best material and conductor diameter. In Table 1, the only differences chosen for the copper and aluminum conductors are the material specific properties of density, the cost per unit mass, and the resistivity. The other

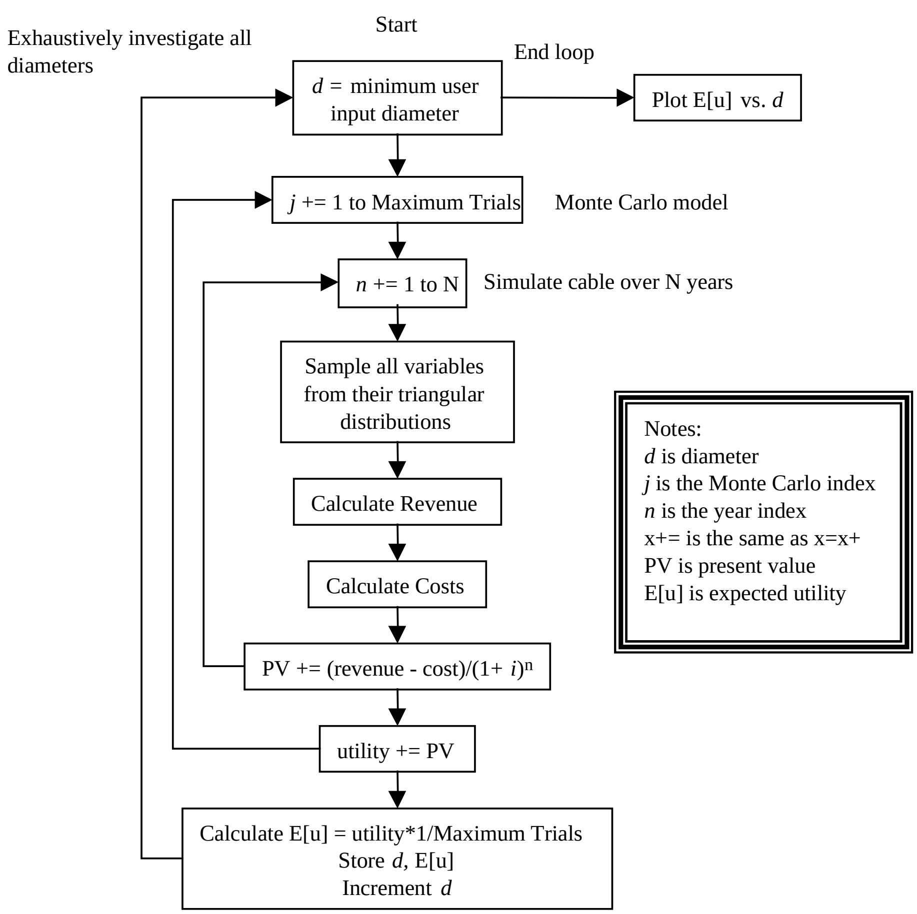

Figure 8: Flowchart for Calculating Expected Utility

values in Table 1 are chosen to reflect that this is a small town, and the power company behaves as most utilities in the United States do. The values chosen in Table 1 are illustrative of those a designer might give, but there is an important distinction here. We are relatively unconcerned with the *accuracy* of this model where accuracy is defined as the ability of the model to represent the real world on an absolute scale. Because of the modeling simplifications chosen and the uncertainty on the input variables, the output PV will not accurately reflect the true system (unless by chance). We are concerned with the model's *resolution*, where resolution is defined as the model's capability to distinguish between different designs on a relative scale (Hazelrigg, 1996). Our only objective is to have the model support us in deciding between various conductor configurations; we are not trying to produce an accurate forecast of the system performance.

The results of a particular configuration (copper or aluminum) input into the program are displayed graphically for the user. A plot of the expected utility vs. *d* for the copper values in Table 1 is shown in Figure 9. The expected utility in this case has a maximum value for a conductor diameter at around 0.12m although any value up to and including 0.2m is an acceptable diameter *based on this model.* What happens in the modeling is this: for the smaller cable diameters, the power transmitted is limited by the allowable temperature rise in the cable (in this case approximated with the simple linear relationship on current given in Section 3.3). After the cable diameter has the ampacity to carry the desired load and its anticipated growth, the expected PV of the cable is relatively constant. Because we have not included any installation or maintenance model this makes sense. With the inclusion of these models we expect the cable to be more reliable at higher diameters but more difficult to install and maintain. In Figure 9 the slight decrease in expected utility of cable diameters greater than 0.12m is due to the increased material cost of the cable itself.

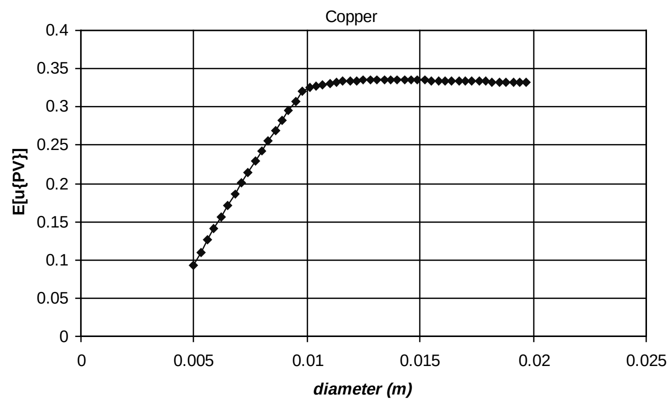

Figure 9: Expected Utility for Copper Conductor

In Figure 10, the expected utility of cables vs. *d* is shown for aluminum conductors. The aluminum conductor, due to its decreased conductivity, has the highest expected utility for a diameter of 0.0205m.

To exemplify the calculation of expected utility, the

Table 1: Illustrative Values for Copper and Aluminum

Name		Distribution (copper)			Distribution (aluminum)		
Reference	Symbol	Min	Most Likely	Max	Min	Most Likely	Max
Number of years	N	40.0			40.0		
Length of Cable	L (m)	19500.0	20000.0	20500.0	19500.0	20000.0	20500.0
Resistivity	R (Ω.m)	1.69e-8	1.7e-8	1.71e-8	2.98e-8	3.0e-8	3.02e-8
T [by way of LF]	T (h/yr)	8550.0	8600.0	8750.0	8550.0	8600.0	8750.0
Interest Rate	i (%)	5.0	10.0	15.0	5.0	10.0	15.0
Cost of Energy	P ($/kWh)	0.045	0.05	0.055	0.045	0.05	0.055
Demand Charge	D ($/kWh)	0.075	0.08	0.085	0.075	0.08	0.085
Yearly increase in Cost	c (%)	1.0	1.5	2	1.0	1.5	2
Yearly load increase	a (%)	5.0	10.0	15.0	5.0	10.0	15.0
Demand charge increase	b (%)	1.0	3.0	5.0	1.0	3.0	5.0
Fixed cost	- ($)	500000.0	1000000.0	2000000.0	500000.0	1000000.0	2000000.0
material cost	($/kg)	2.92	3.0	3.15	2.40	2.60	2.80
Density	ρ (kg/m^3)	8850.0	8900.0	8950.0	2650.0	2700.0	2750.0
Voltage	V (V)	66000.0	69000.0	73000.0	66000.0	69000.0	73000.0
I0stat	(A)	350.0			350.0		
Diameter Steps	(-)	50			50		
Monte Carlo Trials	(-)	5000			5000		
Minimum Diameter	(m)	0.005			0.005		
Maximum Diameter	(m)	0.020			0.030		

distribution of PV for the aluminum conductor at it highest expected utility (d = 0.0205m) is shown in Figure 11. A utility is calculated for each PV in this distribution, giving a distribution on utility. Expected utility is then calculated from the utility distribution. Note the large range of PV, from just over $7,000,000 to $2,500,000. These values are absolute numbers with very little meaning *except when compared to the PV distribution at $d = 0.0205 \pm \varepsilon$* where ε is the step size for diameter used in running the simulation.

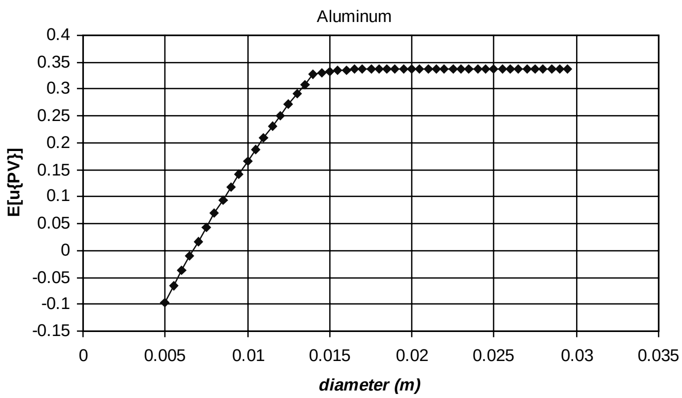

Figure 10: Expected Utility for Aluminum Conductor

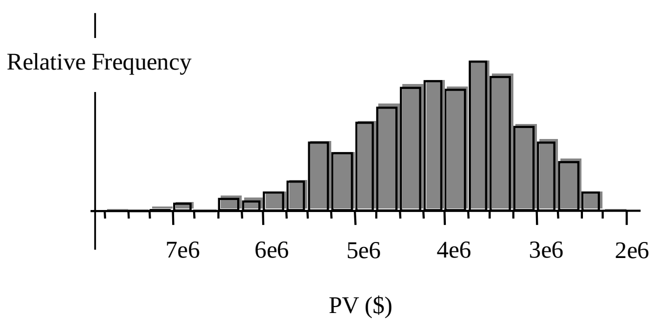

Figure 11: Distribution of PV for Aluminum Conductor *d* = 0.0205m

Comparing Figure 9 and Figure 10, we note that at their respective maximum expected utility's, it is difficult to distinguish between the copper and aluminum conductors based on this model. For the aluminum conductor the maximum expected utility is 0.338 while it is 0.334 for the copper conductor. These values are based on a distribution of PV such as that in Figure 11 composed of 5,000 values. We can construct a 95% paired t-test for the utilities based on their PV distributions with equal variances. This calculation (see Box 1) indicates that the means are indeed different and we should select the aluminum conductor with a diameter of approximately 0.0205m.

With the addition of a maintenance and cost of installation model that accounts for the material type and d, we expect the difference in the maximum expected utilities for the two materials to increase. The present model, considering only Joule losses, revenues, and conductor material cost, performs as expected producing present values that are nearly equivalent when the conductor can handle the load and the anticipated load growth.

$$|t| = \frac{\bar{x}_a - \bar{x}_c}{0.064\sqrt{0.0004}} \geq t_{0.025,9998.0}$$

$$\frac{0.338 - 0.334}{0.064(0.2)} = 3.125 > 1.96$$ where $\bar{x}_c$ is the mean of the copper utility distribution, $\bar{x}_a$ is the mean of the aluminum distribution, the sample size is 5,000 (each) and the standard deviation of the utility distribution is 0.064

Box 1: Significance Test for PVs

4. CLOSURE

In this paper we show that the proposed framework in Figure 3 can be used in engineering design. This is an important first step in establishing a 'science' of design because expected utility theory, implemented in the framework, is anchored firmly in the von Neumann and Morgenstern axioms that define valid utility functions *such as that determined for gambles among PV of conductors* (Luce and Raiffa, 1957).

In this paper we have a very narrow focus. We answer our first research question (*How can we represent and handle the uncertainty in a system?)* with a brief description of risk (Section 1.1) and its incorporation into a framework for engineering design (Section 2.2). We then explore in depth that framework in the context of an example problem to answer our second question (*How can designs be refined and selected using expected utility theory?).* Our intent in this paper is not to categorize the merits of utility theory as an encompassing method in design; we seek only to evaluate the proposed framework for engineering design.

Having shown that the framework can indeed be used for an actual design problem we turn to some of the more daunting problems facing the designer who ventures into the arena of expected utility theory. In the context of the framework we note almost immediately that all design problems cannot be reduced to a single decision, and that most design decisions are non-simultaneous (Krishnan, Eppinger et al., 1997; Smith and Eppinger, 1997; Smith and Eppinger, 1997). This implies time dependence for most design decisions, and the only logical model for non-simultaneous decisions with such dependence is *satisficing*[3] (Olander, 1975; Simon, 1996). The implications for this notion have not been thoroughly investigated as they apply to expected utility theory in design.

Additional problems with the framework are issues of size and complexity. Analysis of revenues and costs can require

3 Satisficing - not the "best" but "good enough" (use of this term, in the context of design, is attributed to Simon.)

large, computationally intensive codes. These codes may require hours or even days to run. The present framework represents an idealized view of implementing utility theory when computations take only a small amount of time (<1s). Areas of research involving meta-modeling are directly applicable to the solution of this computational difficulty (Chen, Allen et al., 1996; Simpson, Peplinski et al., 1997).

The measuring of a demand function within the framework is simple for the case study investigated (monopoly). With increasing numbers of competitors, however, the measure of a demand function places us firmly in the realm of game theory. The demand for a product is developed only in response to anticipated plays by competitors in our market of competition. Neglecting this notion will inevitably lead to serious errors since the demand function serves as a surrogate to a multi-objective optimization function. This represents perhaps the most burdensome of the problems in the framework, for introducing gaming is a monumental task. With its introduction and the inclusion of the issues on non-simultaneity, size, and complexity the framework will form the basis of a normative design theory based on a fundamental axiomatic foundation.

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