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An Improved Method to Estimate the Fractal Dimension of Color Images

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Abstract

Fractal Dimension (FD) is an important feature of fractal geometry has many applications in various fields including image processing, image analysis, texture segmentation, shape classification and identifying the image features such as roughness & smoothness of an image. There are many techniques to estimate the dimension of fractal surface. The famous technique to calculate fractal dimension is the grid dimension method, which is popularly known as box counting method and some of the other improved methods like differential box counting and improved differential box counting method are used to estimate fractal dimension of gray scale images. The usual way of estimating the roughness or FD of color image involves two steps: i) converting color image to grayscale ii) finding the roughness of generated gray scale image. But due to this conversion, significant color information is lost and leads to inaccurate roughness estimation. To avoid such inaccuracy this paper proposes the development and study of novel technique for estimating fractal dimension of color images. In this study, the improved differential box counting method is applied to the 24 bit representation of RGB color images to extract the roughness of color images. The validation of the proposal is performed by generating twelve different synthesized color images in terms of small variation of intensity value in RGB space and compared with previously three well defined existing methods that are weighted sum, average and desaturation method. The results showed that our proposed method is able to capture the accurate sharp variation of roughness as compared to the existing methods.

Keywords- fractal dimension; improve Differential box counting; Gray scale images.

1. INTRODUCTION

Fractal geometry was invented by Mandelbrot [1] to express self-similar sets called fractals. Fractal geometry comes into play where traditional Euclidean geometry fails to express the standard or split set of usual features as well as difficult objects [2]. It gives a arithmetical model for several complex real world expected objects. Self-similarity is significant assets of fractal theory, basically it was used to calculate fractal dimension (FD). The concept of fractal dimension is mainly applied in segmentation and texture examination and also helpful for image examination and categorization [3-7]. Pentland [2] also gives the idea about smoothness vs. roughness of image surface, where the smooth image indicates the fractal dimension of two and rough image like salt and pepper surface shows the fractal dimension as three.

Many researchers contributed their effort on estimating fractal dimension in the field of fractal geometry. Thus, diverse concepts in this regard have been projected to calculate FD of an image. Gangepain and Roques-Carmes [8] reported the well-liked method called reticular cell-counting technique which enhanced the idea of probability theory discuss by Voss [9]. Later, Keller et al. [10, 11] gives additional modification by a way of linear interpolation. Again voss realised and classify these concepts into 3 key categories: which are basically called box-counting methods, variance methods, and lastly spectral methods [12]. Among these methods the box counting method is mostly used in case of fractal dimension estimation because due to its ease and easy implementation for accurate estimation.[13]. Therefore in this regard there are numerous realistic box counting methods has been proposed to estimate FD[8, 14-18]. The rest of this paper is prepared as follows. Section 2, illustrate the vital definition of fractal measurement and also introduced Improved DBC to compute FD of gray scale image. Section 3 describes about Gray scale and RGB color images. Section 4 describes our proposed method and methodology adopted. Section 5 represents the experimental results and finally section 6 shows concluding remarks.

2. Basic fractal dimension calculation

2.1. Fractal dimension calculation

The fractal measurement is an essential characteristic of fractal for the reason that it has got information regarding their geometric structure. Fractal dimension of the whole images is the overall distribution of image pixels, more purposely, the roughness of the whole image. The fundamental rule of computing fractal dimension of a whole image depends upon the theory of self-similarity. As per the concepts of self-similarity we can say that the fractal is usually irregular or not exact geometric form that can be wrecked down in lesser pieces so that each is related to the original form.

Fractal dimension can be represented in terms of D of a set X is defined by equation (1). $D = \log(N)/\log(1/r)$ (1)

Where N is the entire number of dissimilar copies related to X and X is scaled down by a fraction of 1/r.

There are many method were proposed in this regard. However we have discussed some of the few methods which are related to our research work in terms of estimate roughness of gray scale image and color images. Therefore in this regard we have categorised into two part called estimation of roughness for gray scale image and estimation of roughness for color images.

2.2.Estimation of roughness for gray scale image

Fractal dimension of images are usually to determine the surface roughness and consequently it represents the variation among different grey levels that are establish on the image, the value of N has to be computed using method like box-counting and the fractal dimension is estimated accordingly. There are many method were proposed in this regard. Most popular method is differential box counting method [14]. However, we have given the concept of improve differential box counting method [18] since it removes some demerits of differential box counting method [14] that is

- Over counting the amount of boxes covering the image intensity surface.
- Under counting the amount of boxes may occur at the boundary of the neighbouring box blocks.

2.2.1. Improved differential box counting method

Chen.et.al [18] suggested improve DBC method for computing FD of gray scale image, For this reason author took into account the difference of boxes where the greatest and least intensity value falls. In this regard they took gray scale image of range M×M in a three dimensional surface plane, where x and y plane represents the pixel location and third plane called z indicates gray level of an image. Then the entire no of pixels has been scaled down block of size l x l where $M/2 \geq l \geq 1$ and l is an integer denoting box size. Afterwards we have to compute r = l /M. For every scaled down block, there is a pillar of boxes of size s x s x s' , where s' represents height of each box, G/s' =M/s and G is the entire amount of gray levels. Let the least and greatest gray levels are I_{min} and I_{max} respectively in the (i, j)th block. Then the total quantity of boxes is essential to cover up in z direction is nr_{old} and after shifting the δ positions from nr_{old}, nr_{new} is calculated. Maximum of n_{rold} and n_{rnew} is taken as nr. nr=max(nr_{old} , nr_{new}). nr_{old} or nr_{new} is calculated as follow:

$$n_r(i,j) = ceil\left(\frac{I_{\max} - I_{\min} + 1}{s'}\right) if \ldots I_{\max} \neq I_{\min} \quad (2)$$
$$n_r(i,j) = 1 \ldots otherwise$$

And N_r will be calculated as $N_r = \sum n_r(i,j)$ (3)

The fractal dimension (D_{IDBC}) is calculated with regression plot between log(N) vs log(1/r).

2.3.Estimation of roughness for color image

Color imagery is interrelated to greyscale images, where pixels are arranged or stored in terms of grid. Conversely, in case of color image each pixel or intensity has three sampled pixel called R, G and B values are associated with it. In this regard few attempts have been made. Unlike Ivanovici and Richard [23] has proposed an efficient method called probabilistic algorithm in terms of 5-D vector to compute color fractal dimension, In order to achieve this they took a positive square of cube size L in the x and y plane and they calculate the no of intensity points that drop inside a 3D cube of size L cantered in the present pixel. Then they count the intensity or pixel as F=f(x,y,r,g,b) for which the Euclidian space to the middle of the 3D cube would be lesser than the value of L. Another researcher Nikolaidis et al. [24] has work on same 5-D vector based on box merging method to estimate the same. Therefore in this regard they partitioned the each axis of the image into s partitions to produce E dimensional grid.

3. Gray scale and RGB color image

A digital image is a two dimensional illustration of its pixel intensities. A grayscale image has each pixel with single channel of intensity ranging from black to white. But the color image has each pixel with multiple channels of intensities. RGB color image has 3 channels of intensities that are Red, Green and Blue.

3.1. Gray scale

A grayscale image has each pixel with single channel of intensity ranging from black to white. Basically in these types of images each pixel containing only one sample point, in other wards we can say that, it can contains only intensity information. Greyscale is a assortment of shades of gray not including noticeable color. In case of gray scale image, it contains total 256 numbers of different shades of colors are available in it. The scope of the color in 8bit varies between 0 to 255, where 0 represents the color of black and 255 represents the color as white and the number 127 represents the color of gray. In this type of format image basically we are using variety class of array depending on its size like unit 8 and 16, int16 and single or double array. If we consider the single or double arrays then the scope values are from [0, 1]. For array of class uint8 the scope ranges from 0 to 255. Similarly in case of uint16 array the scope varies from 0 to 65535 and also in case of array int16 the varies the ranges from 32768 to 32767.

3.2. RGB color space

In generally the popular classical applied color space in graphics is the R, G and B colors, which is depends upon the combination of these three colors that is Red, Green and Blue. Combining these three colors, we can get different colors like cyan(G+B), magenta(R+B) and yellow(R+G). In case of color image the Pixel depth indicates the amount of bits used to characterize every pixel in the R, G and B space. If each and every pixel element of R, G and B is characterize by of 8 bits, then that pixel is supposed to have strength of 24 bits representation. A filled color image correspondence to a 24 bit R, G and B color image.

4. Proposed method

The method given by Ivanovici and Richard [23] called probabilistic algorithm given by Voss and another method developed by Nikolaidis et al. [24] called box merging method based on box counting mechanism are used to compute fractal dimension of RGB colour images based on the 5-D vector. In order to authenticate their method, Ivanovici and Richard [23] used the prominent algorithm called midpoint displacement method and based upon this method they generate fractal generator for R, G and B color images with the use of hurst parameter, and similarly another author Nikolaidis et al. [24] used the concept of box merging method in terms of 5-D vector and compare with some known fractal dimension image for validation. However in this letter we are addressing how to find out the roughness of color images using the three R, G and B components of the colour image based on the concept of improved differential box counting method in terms of the same 5-D vector, which roughness is lies between 2 to 5. Therefore in order to estimate roughness of color image, we first estimate roughness of each individual RGB component and subtracted its corresponding smoothness. After subtraction of smoothness we will take its fractional part and add its corresponding RGB components and finally addition of smoothness to get accurate roughness.

$$\begin{aligned} D_R &= D_{IDBC\ (R)} - 2 \\ D_G &= D_{IDBC\ (G)} - 2 \\ D_B &= D_{IDBC\ (B)} - 2 \\ D_{Color} &= 2 + D_R + D_G + D_B \end{aligned} \quad (4)$$

Again validation of our proposed we applied improved differential box counting image method is applied to the 24 bit representation of RGB color images to extract the roughness of color images. Therefore in this regard we are taking the contribution of individual RGB components in terms of six combinations that is $I_{x,y}(1)$ to $I_{x,y}(6)$ based on changing the place of RGB components to make it feasible solution and then taking the average for better estimation of roughness.

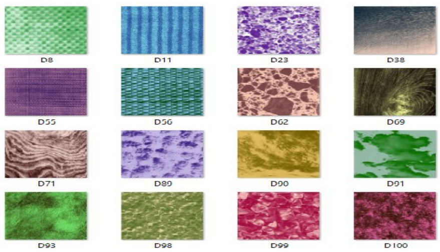
$$\begin{aligned} I_{x,y}(i) &= R_{x,y} \times 2^{16} + G_{x,y} \times 2^{8} + B_{x,y} \times 2^{0} \\ I_{x,y}(i) &= R_{x,y} \times 2^{16} + B_{x,y} \times 2^{8} + G_{x,y} \times 2^{0} \\ I_{x,y}(i) &= G_{x,y} \times 2^{16} + R_{x,y} \times 2^{8} + B_{x,y} \times 2^{0} \\ I_{x,y}(i) &= G_{x,y} \times 2^{16} + B_{x,y} \times 2^{8} + R_{x,y} \times 2^{0} \\ I_{x,y}(i) &= B_{x,y} \times 2^{16} + G_{x,y} \times 2^{8} + R_{x,y} \times 2^{0} \\ I_{x,y}(i) &= B_{x,y} \times 2^{16} + R_{x,y} \times 2^{8} + G_{x,y} \times 2^{0} \end{aligned} \qquad I_{x,y} = \frac{\sum_{i=1}^{6} I_{x,y}(i)}{3} \quad (5)$$

4.1. Methodology

As we know the color image is a hyper-surface in a color space, like RGB for instance: f(x, y) = (r, g, b). The colour image is a mapping from a 2D space of pixel positions to the 3D space of RGB colours. Therefore, in this regard we deal with a 5D Euclidian hyper-space in color images and every pixel can be view as a 5-D vector like (x, y, r, g, b). Though, unlike color edition of probability density function and box merging method, we have suggested to use the colour components method using IDBC for each of R, G and B components to find the colour fractal dimension of the colour image which will lie between 2 to 5. The roughness of such images are estimated using IDBC methodology to the 24 bit representation of RGB color images to capture the extract and sharp variation of intensity levels in color images.

5. Experimental results

Proposed method is implemented on matlab12 in windows 8 64 bit operating system, Intel (R) i7 – 4770 CPU @ 3.40 GHz. We have applied the improved differential box counting to 24 bit representation of RGB color image to estimate roughness of color image. In order to achieve we took set of 16 textured RGB color images with the similar Resolution (640x640) from the Brodatz database [22]. Further, for the validation of the proposal is performed by generating twelve different synthesized color images in terms of small variation of intensity value in RGB space and compared with previously three well defined existing methods that are weighted sum [25], average [21] and desaturation method [25] to estimate roughness as per the method proposed by us. Figure1 shows 16 different color texture image taken into consideration, Table 1 shows computation of fractal dimension of sixteen color test images using IDBC method (D_{IDBC}) using equation 3, (D_{COLOUR}) using equation 4 and (D_{24bit}) using equation 5. Figure 2 represents twelve generated synthesized image for validation purpose. Table 2 indicates Roughness (FD) estimates of generated synthesized images.



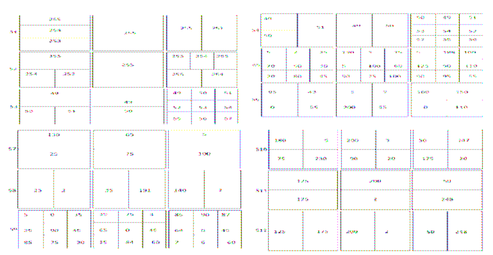


Figure 1. Sixteen color textures from Brodatz database [22] Figure 2. Twelve synthesized color images in terms of small variation of intensity value in RGB space

Image Name	D_{IDBC} (Color to Grey)	D_{COLOR}	D_{24bit}
D8	2.488201	3.5764	2.519445
D11	2.737095	4.1971	2.686333
D23	2.621135	3.8631	2.599242
D38	2.703782	4.0773	2.72196
D55	2.768397	4.3564	2.801043
D62	2.5276	3.9637	2.52517
D56	2.638384	3.5862	2.627683
D69	2.652139	3.9173	2.64732
D71	2.697293	4.0888	2.686055
D89	2.527845	3.6052	2.516926
D90	2.49472	3.4608	2.473097
D91	2.372957	3.0278	2.320186
D93	2.789447	4.3760	2.759108
D98	2.504107	3.5552	2.522483
D99	2.507724	3.5329	2.508159
D100	2.722926	4.2664	2.765416

Name(Image)	Weighted sum	Average	Desaturation	Proposed
S1	2	2	2	2.000936
S2	2	2	2	2.000819
S3	2	2	2	2.001277
S4	2	2	2	2.000472
S5	2	2	2	2.074584
S6	2	2	2	2.067523
S7	2	2	2	2.052646
S8	2	2	2	2.020867
S9	2	2	2	2.077566
S10	2	2	2	2.029978
S11	2	2	2	2.038599
S12	2	2	2	2.038599

Table 1. Roughness (FD) estimates of sixteen color test images Table 2. Roughness estimates of synthesized images

6. Conclusion

In this study, we have presented a novel method that can detect even the slightest pixel variation to estimate the roughness of RGB color images accurately, where the existing methods failed. As color image consists of more intensity levels than gray scale image, the fractal dimension method suggested by us indicates estimation of much better accuracy of roughness.

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