An Improved Two-Dimensional Numerical Modeling Method for E-Core Transformers

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An Improved Two-Dimensional Numerical Modeling Method for E-Core Transformers

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Abstract: A new 2D modeling method for E-core transformers adds an extended flux return path to the model of the section perpendicular to the core. Averaged with a section parallel to the core modified to have the same reluctance as the 3D model, the finite-element simulation results for total magnetic energy and magnetic energy in the windings are within 0.2-5.6% error relative to the 3D model in fifteen of sixteen cases, a large improvement over existing methods. A refinement of fringing-flux calculations for gapped core legs is also presented.

I. Introduction

A. Problems with Existing Methods:

There currently does not exist a reliable method for computer modeling of E-core transformers in two dimensions (2D). Three-dimensional (3D) modeling of such transformers using finite element analysis (FEA) or other numerical methods can be highly accurate, but it is very time-consuming, making iterative simulation unfeasible. In addition to large time requirements, 3D models require very large amounts of memory. For these reasons 2D numerical modeling is more commonly used in practice.

Despite their popularity, 2D numerical models fail to account for important 3D effects. The most basic 2D simulation involves modeling a cross-section parallel to the outer core legs, as shown in Fig. 1a, which can be simulated assuming axial symmetry about the center of the core. This section is designated || in this paper. However, this approach ignores all 3D effects, which have been shown to be significant [1], [2]. One approach to accounting for 3D effects, used in [3,5,6], is to combine the results from the || model with the results from a section perpendicular to the plane of the core, as shown in Fig. 1b, designated T in this paper. This method works well for the specific situation tested in [3,5], but it fails in many other situations, as demonstrated in Section III.

A further inaccuracy in simple 2D modeling methods stems from the assumption of rotational symmetry about the center of the transformer. For a section parallel to the core, the actual shape simulated is usually one with cylindrical symmetry, in order to accurately represent the winding geometry. However, the cylindrical simulation also increases the cross-sectional area of the outer legs. As in [4], this effect can be accounted for by modifying the 2D model to have the same total reluctance as the 3D model (Fig. 2a). This equal reluctance model (designated ER in this paper) can be helpful but has limitations, in that it does not model the field in the plane perpendicular to the core.

B. Overview of the Proposed New Method:

This paper proposes a new method of modeling E-core transformers in two dimensions using a weighted average of results from two modified, mutually perpendicular sections. We want to somehow create two 2D models that will simulate the behavior in the two sections shown in Fig. 3. To model the section parallel to the core, we can use the section modified to have equal reluctance to the 3D model when simulated axisymmetrically [4], as described above and shown in Fig. 2a (ER). The other section will be a modified version of the section perpendicular to the core. In order to create it, we recognize that the major difference between the 3D and 2D models of this section is the lack of an outer flux path in the 2D model, since in the 3D model, magnetic flux is free to return though the core in another plane. Therefore our 2D model of this section should include a flux return path. However, the parallel, equal reluctance 2D model (ER, Fig. 2a) will not provide accurate results because of the proximity of the outer core to the windings. Instead, we use the model shown in Fig. 2b, which has an extended outer flux

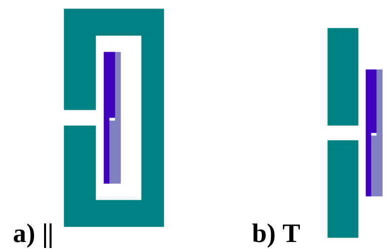

Fig. 1: The sections a) parallel (designated by || in this paper) and b) perpendicular to the core (designated T).

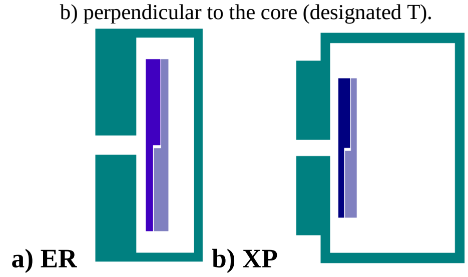

Fig. 2: a) A 2D model modified to have the same reluctance as the 3D model (designated ER, for equal reluctance). b) The new extended path model for simulation of the plane perpendicular to the core (designated XP).

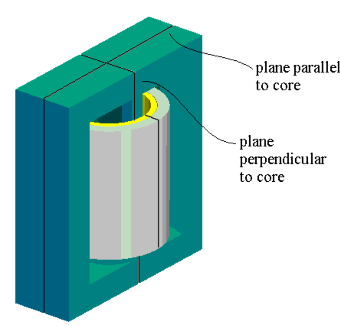

Fig. 3: The two planes we wish to simulate.

return path and is still designed to have the same reluctance as the 3D model. The extended path (XP) allows flux to return through high-permeability material without placing the material unrealistically close to the winding.

A weighted average of results from each of these two models (ER and XP; Figs. 2a and 2b) can then be used to approximate values of interest. The method is described in detail in Section II.

C. The Test Cases:

Some previous models work only for specific situations, so we wish to test a wide range of possibilities. Varying only the presence and location of gaps in the E-core transformer results in four different possible configurations: no gaps, center gap, outer gap, and gaps in all three legs. With two windings, any winding excitation may be considered as some combination of two basic excitations: current in the same directions in both windings (magnetizing excitation), and currents in opposite directions (leakage excitation).

For each of these four gap configurations and each of the two winding excitations we wish to compute two important volume integrals. The first is the total energy, useful for calculating inductance:

$$\text{total magnetic energy} = \frac{1}{2}\int_{total} B \cdot H dV$$

The second is the magnetic energy inside the two windings, which is related to eddy-current loss in the windings, as discussed, for example, in [10]:

$$\text{magnetic energy in the winding region} = \frac{1}{2}\int_{windings} B \cdot H dV$$

Each of the 2D modeling techniques, including those previously developed by others, will be tested in each situation. They will then be compared to the 3D FEA solution to determine their accuracy.

II. Detailed Description of New Method

To simulate an E-core transformer or inductor, we first construct the ER and XP sections (Fig. 2a and 2b), using the reluctance calculations detailed in Section II.A. The ER model is discussed in Section II.B, and the XP model is discussed in Section II.C. Next, these sections are simulated by finite-element or other numerical methods. The results

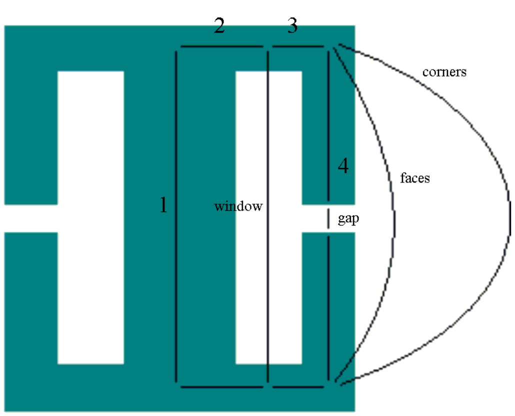

Fig. 4: Flux paths in the transformer with gapped outer legs.

from the two simulations are then averaged, using the weighting described in Section II.D.

A. Finding the Core Reluctance for Use in Drawing ER and XP Models:

Both the ER and XP models require redrawing the outer leg of the core while preserving the original reluctance. In order to do this, we must first calculate the reluctance of the original 3D structure, which is most difficult when the core has gaps in the outer legs (whether or not the centerpost is gapped). We calculate the total reluctance seen by flux passing through the outer legs based on combining the reluctances of the paths marked in Fig. 4. The reluctances through the core ($\mathcal{R}_1$ through $\mathcal{R}_4$) and the gap reluctance $\mathcal{R}_{gap}$ may be simply calculated from

$$\mathcal{R} = \frac{\ell}{\mu_0 \mu_R A}, \tag{1}$$

where ℓ is the length of the path, μ_R is the relative permeability of the material, A is the cross sectional area of the path, and μ_0 is the permeability of free space. The window reluctance, $\mathcal{R}_{window}$ is also calculated using (1), because it has only a minor effect on the total reluctance and a rough calculation is acceptable.

More difficult and important is making an estimate of the fringing flux around the gap. An exact formula for the two-dimensional fringing reluctance around a gap is derived by conformal mapping in [7]. Unfortunately, we found that for typical gapped legs in practical devices, the three-dimensional effects are significant, and the result in [7] is not accurate by itself. An alternative is to use the rough approximations in [8], but these also proved to have inadequate accuracy for this situation; they were developed long before modern computer power made higher accuracy possible. Thus, we chose to combine the use of the formula in [7] for the 2D aspects with a formula for the 3D aspects, similar to those in [8], but calibrated by 3D simulations.

The situation to be modeled is shown in Fig. 5. The method in [7] can be used to model the flux fringing path from one face to another, traveling through the darkly shaded regions in the bottom cross section in Fig. 5. However, the flux through the lightly shaded regions is a three dimensional effect that must be modeled separately. Thus, we use two parallel reluctances, $\mathcal{R}_{faces}$ and $\mathcal{R}_{corners}$, to account for the fringing flux through the darkly and lightly shaded regions, respectively. The first, $\mathcal{R}_{faces}$, can be exactly calculated, for all sides of a leg with perimeter $p = 2(d+w)$, as [7]

$$\mathcal{R}_{faces} = \frac{\pi}{p \cdot \mu_0 \left(1 + \ln \frac{\pi \ell}{2\ell_{gap}}\right)} \tag{2}$$

The dimensions for this calculation are defined in Fig. 5, with the exception of ℓ_{gap} which is defined as the length of the gap in the outer leg.

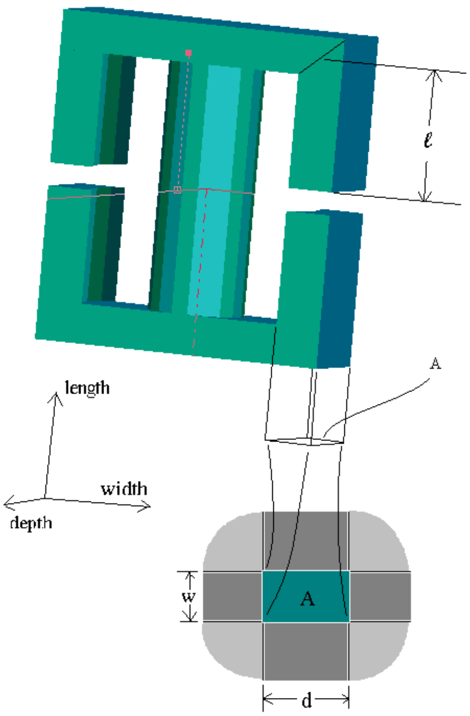

Fig. 5: Geometry for fringing reluctance calculations.

From the approach in [8] and from dimensional considerations, we conclude that the gap corner fringing reluctance can be approximated by an expression of the form

$$\mathcal{R}_{corners} = \frac{1}{\mu_0 k \ell}. \quad (3)$$

where k is a dimensionless constant to be determined. In [8], k is estimated from geometrical considerations. In order to more accurately determine the value, we performed a set of 3D simulations of simple structures, and found a value of k = 1.23.

Using a combination of (2) and (3) gives a more accurate 3D fringing flux calculation than is possible with either the method of [7] alone (because it does not account for 3D effects) or [8] alone (because, instead of the exact analysis of [7], it uses rough approximations, most of which have not been precisely calibrated by simulations or measurements).

We have now calculated all the reluctances in Fig. 4, and we can calculate the total reluctance by combining them in series and parallel. The final result for a transformer with gapped outer legs is

$$\mathcal{R}_{total,3D} = \mathcal{R}_1 + \frac{1}{2}\left(2\cdot\mathcal{R}_2 + \mathcal{R}_{window} \left\| \left(2\cdot\mathcal{R}_3 + \left(2\cdot\mathcal{R}_4 + \mathcal{R}_{gap}\right)\right\| \mathcal{R}_{faces} \right\| \mathcal{R}_{corners}\right)\right)$$

In transformers with no air gap, the reluctance calculation is more straightforward. The reluctance of a 3D model without a gap in the outer legs is

$$\mathcal{R}_{total,3D} = \mathcal{R}_1 + \frac{1}{2}\left(2\mathcal{R}_2 + \mathcal{R}_{window} \left\| \left(2\mathcal{R}_3 + \mathcal{R}_4\right)\right.\right).$$

The four numbered reluctances and $\mathcal{R}_{window}$ correspond to the flux paths shown in Fig. 6 and are the same as those discussed above for the gapped case.

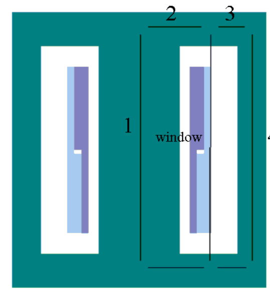

Fig. 6: Flux path definitions in the 3D gapless transformer.

B. Drawing the Equal Reluctance Model:

The equal reluctance (ER) model is derived from the model parallel to the transformer core (||) by trimming down the top, bottom, and outer legs. The three trimmed legs have a remaining thickness t, which is determined by giving the ER model the same reluctance as the 3D model, as calculated above. The calculations of reluctance for both the original 3D model and for the ER model are similar, with several important variations.

For the ER model, the reluctances $\mathcal{R}_2$ and $\mathcal{R}_3$ (as defined in Fig. 4 or Fig. 6) have a cross section that increases with radius when an axisymmetric simulation is performed. In general, the reluctance of a path of cross sectional area $A(x)$ that varies as a function of position x along a path of length ℓ may be calculated with the integral

$$\mathcal{R} = \int_0^\ell \frac{1}{\mu_0 \mu_R A(x)} dx.$$

For a radial flux path in a disk of thickness h, starting at radius R_a and ending at radius R_b, the reluctance is

$$\mathcal{R}_{top} = \mathcal{R}_{bottom} = \frac{\ln \frac{R_b}{R_a}}{\mu_0 \mu_R 2\pi h}.$$

This can be used with the appropriate radii to obtain $\mathcal{R}_2$ and $\mathcal{R}_3$.

The fringing-field calculation may be accomplished by using only (2), with a perimeter $p = 2\pi(R_o+(R_o + t))$ where R_o is the radius of the outer edge of the window and t is the thickness of the outer leg, such that $2\pi R_o$ is the inside perimeter and $2\pi(R_o + t)$ is the outer perimeter.

The reluctances can then be combined to find a total reluctance

$$\mathcal{R}_{total,ER} = \mathcal{R}_1 + \frac{1}{2}\left(2\cdot\mathcal{R}_2 + \mathcal{R}_{window} \left\| \left(2\cdot\mathcal{R}_3 + \left(2\cdot\mathcal{R}_4 + \mathcal{R}_{gap}\right)\right\| \mathcal{R}_{fringing}\right)\right).$$

We set this equal to $\mathcal{R}_{total,3D}$ and numerically solve for the value of t that provides equal reluctance.

C. Drawing the Extended Path Model:

The extended path (XP) model is created by adding an extended flux return path to the model of the section perpendicular to the transformer core (T). The first step in drawing the extended flux path is determining how far away it should be from the windings in order to accurately simulate

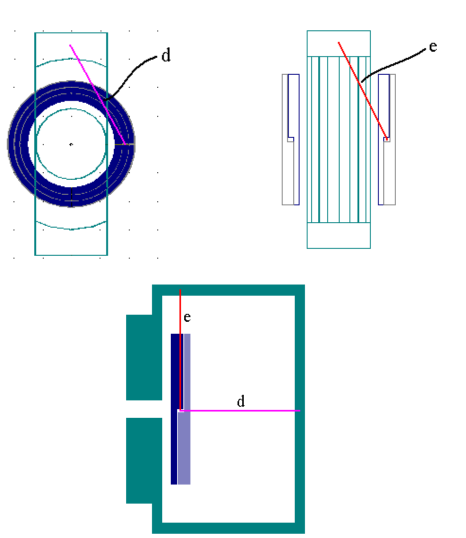

Fig. 7: Finding the dimensions of the extended path.

the 3D situation. Fig. 7 is a diagram of the geometrical equivalence used to determine the dimensions of the path. The top two diagrams show views of the 3D transformer; the distances *d* and *e* correspond to the respective distances on the 2D model at the bottom of Fig. 7.

Note that, unlike the calculations discussed in Section II.A and II.B, the determination of *d* and *e* is purely heuristic. Its justification is that it is intuitively reasonable and that the results work well (see Section III). The lack of a more precise way to determine these dimensions is not a major concern, however, because we found that minor variations in the dimensions had little effect on the simulation results.

Once *d* and *e* have been determined, the reluctance of the XP model is set equal to the calculated reluctance of the 3D model in a manner similar to that used in the ER model. Once again we use the thickness parameter *t* to set the 2D model's reluctance.

D. Determination of the Weighting Factor:

The final result is found by a weighted average of the simulation results obtained using the ER model and those obtained using the XP model. A reasonable initial approximation is to give equal weighting to each. However, this may not be the best approximation. Consider a view from above the transformer (looking down though the middle of the winding, as in Fig. 8). Presumably, the part of the winding that is covered by the top piece of the core in this view is in a situation similar to that modeled by the ER section, and the part of the winding that is visible outside the core in this view is best modeled by the XP section. Thus, we base the weighting on the areas of the two sections:

$$w = A1 \big/ (A1 + A2) .$$

Using this weighting, we define energies *E* of the combined models as

$$E_{\|T} \equiv wE_T + (1\text{-}w)E_{\|}$$

and

$$E_{ERXP} \equiv wE_{XP} + (1\text{-}w)E_{ER}.$$

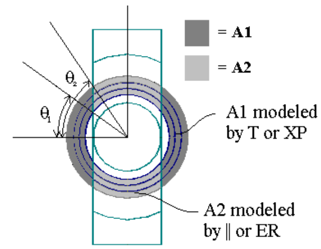

Fig. 8: Determining the weighting factor.

III. Results and Comparison of Models:

To evaluate the performance of the various modeling methods, we tested each on a two-winding transformer on an ETD-39 round-center-post EE-core. We first calculated the ER and XP models as described in Section II. The detailed calculations of these geometries for this example are provided in [9]. We then performed magnetostatic finite-element simulations[1] for the four 2D models in Fig. 1 and Fig. 2 (||, T, ER and XP) and for a full 3D model, each for four gapping configurations, each of these for magnetizing and leakage excitation. The transformer is the same as was used to compare results of 3D finite-element simulations to experimental measurements in [10]. Although the experiments in [10] were performed for only one gapping configuration (all legs gapped), the measurements for this configuration did match the 3D simulation, and so they give us added confidence that the 3D simulations are an accurate benchmark. The results are reported for the four individual sections, and for weighted combinations: The combination proposed in [3] and [5], an average of the section parallel to the core (||) with the section perpendicular to the core (T) which is abbreviated ||T; and the combination we recommend, a weighted average of the equal reluctance section (ER) and the section containing the extended flux path (XP), referred to as the ERXP model.

Fig. 9 shows the results from all 40 simulations, with winding energy and total energy for each, plus the ||T and ERXP combined results. The results for the benchmark 3D simulation are shown with darker shading on the far left of each plot for comparison. The results from the 3D model are assumed to be correct for the purposes of this paper. As noted previously, the total magnetic energy of the system and the magnetic energy in the windings are the quantities used to judge the simulations. With the graphs turned so that the majority of the writing runs in the conventional direction, the top two charts in each group of four are of models of the transformer under magnetizing excitation, and the bottom two charts in each group of four are of models of the transformer under leakage excitation. The two charts on the left in each group represent the results for the total magnetic energy of the system, and the two charts on the right show the results for the magnetic energy in the windings.

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[1] All 2D simulations were run on Ansoft's Maxwell 2D Field Simulator Version 7 software, a FEA tool for electromagnetic systems. 3D simulations used the same company's Maxwell 3D Field Simulator Version 5.

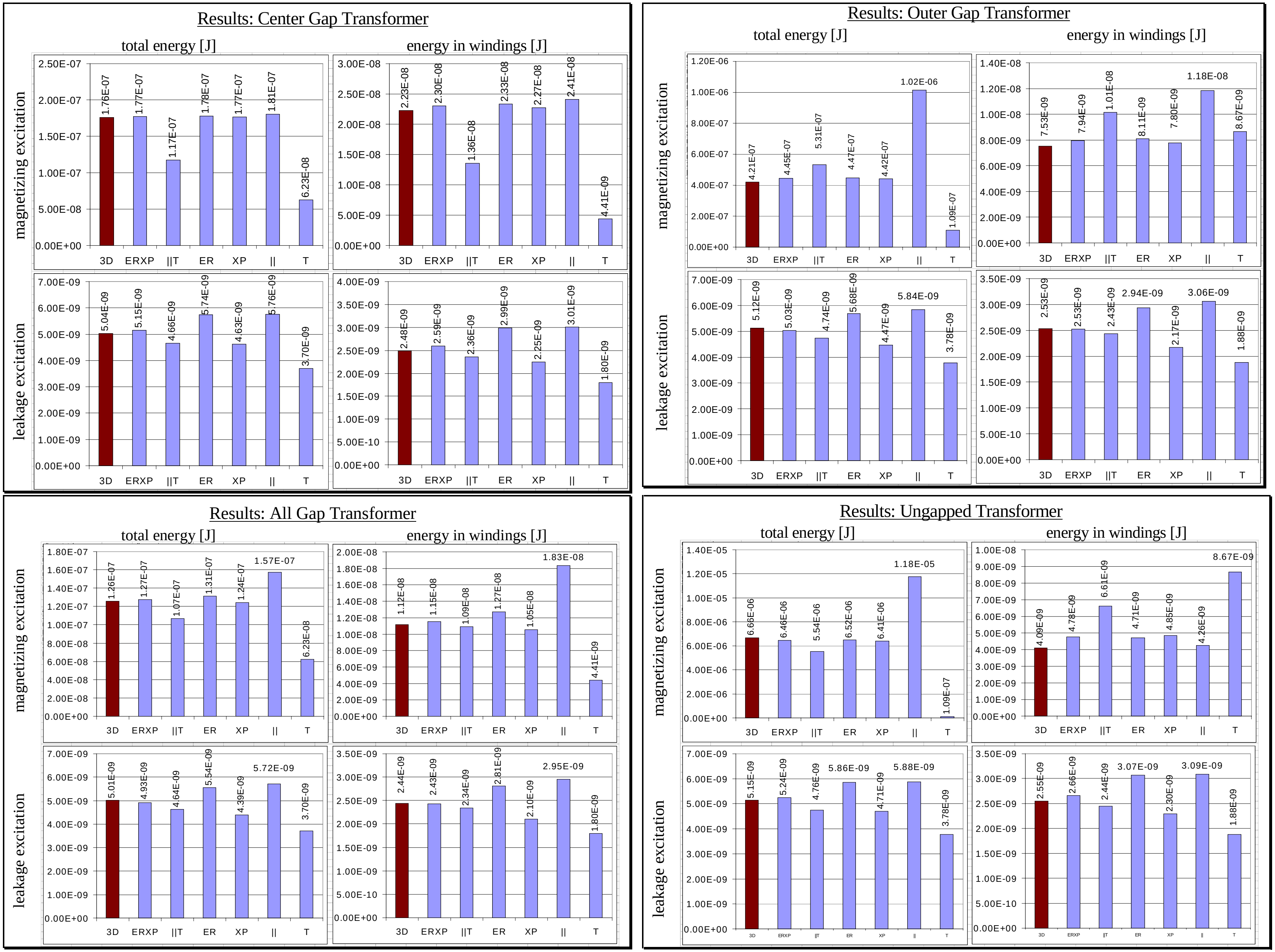

Fig. 9: Results for all cases.

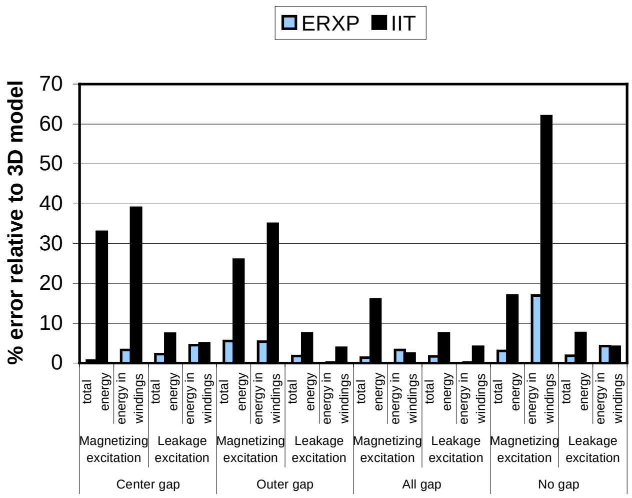

Fig. 10: Comparison of ERXP and ||T models.

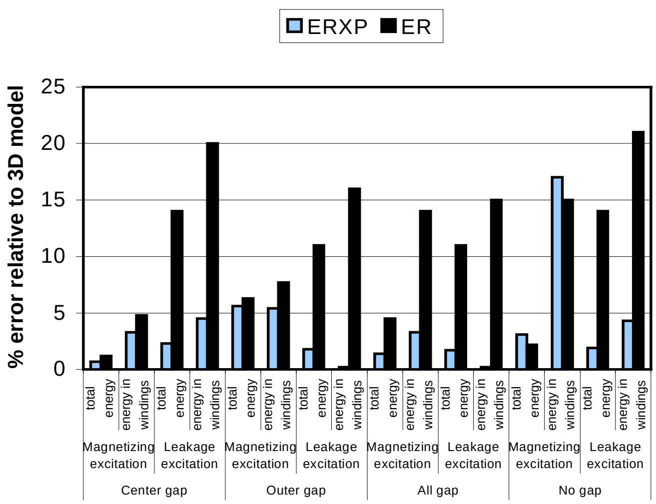

Fig. 11: Comparison of ERXP and ER models.

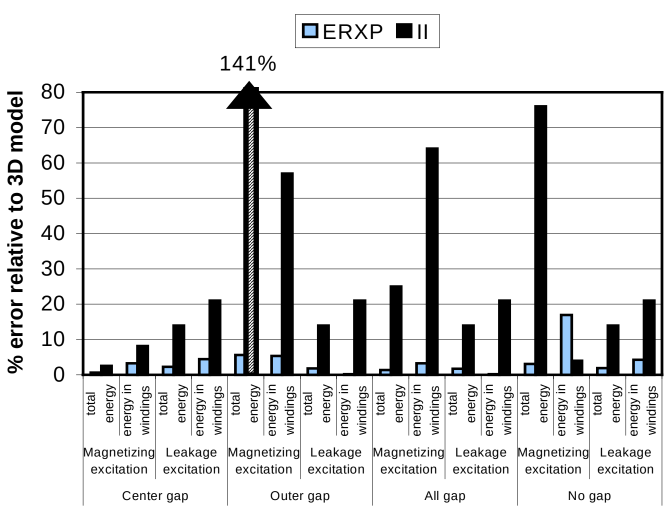

Fig. 12: Comparison of ERXP and || models.

It is clear from Fig. 9 that the ERXP model always returns a value quite close to the 3D value. In certain cases one or two other models come closer to the 3D model than the ERXP, but in other cases those same models are significantly off. The ERXP model is the most consistently accurate. The energy in the windings of the gapless transformer under magnetizing excitation is the only value the ERXP fails to calculate within a few percentage points of the 3D model. This case will be discussed more extensively below.

Another useful way to view the results is to compare two different modeling methods across all the situations modeled. Figs. 10, 11 and 12 compare the new ERXP model to the three models that have been proposed by other authors: ||, ER, and ||T. In each case the percentage errors (relative to the 3D model) of the two models being compared are graphed side by side for each of the 16 situations simulated. Note that the vertical scale varies from chart to chart.

Figs. 10, 11 and 12 further show the ERXP model to be the most consistently accurate. However, it is apparent from each that the ERXP model falls short of its usual accuracy in the case of the energy in the windings of the gapless transformer under magnetizing excitation, where it has 17% error. In this case it is still much better than the ||T model and comparable to the ER model. The || model achieves much better accuracy for this particular case, but there is no reason to expect it to work well consistently for this case. The || model predicted winding energy accurately, but gave 76% error on overall energy, so it is not accurately modeling the field configuration, and the accuracy of the winding energy prediction is most likely just a coincidence. Thus, it does not seem justified to have greater confidence in any of the other models even for the ERXP model's worst case.

Fortunately, this one case in which the ERXP model's accuracy is weak is one of the least important scenarios to simulate. It is the winding energy calculation, which is useful for predicting winding losses. But it is for the scenario of magnetizing excitation of an ungapped transformer. The magnetizing current of an ungapped transformer is typically small, and so the winding losses under magnetizing current excitation are also small enough to be negligible, or, if not entirely negligible, small enough that a 17% error in their calculation is of little significance—much less important that a similar error in other numbers would be.

Although of little practical importance, the uncharacteristic inaccuracy of the ERXP model in this one specific situation is interesting to examine. The most obvious characteristic unique to this situation is that the flux completes its circuit around the conductors without leaving the core. All other cases either contain a gap that the flux must cross or are under leakage excitation, which produces leakage flux paths that leave the core to travel between the windings. Whenever there is a portion of the flux path outside the core, the effect of this portion dominates the reluctance and any quantities that depend on reluctance, such as inductance and energy. The total magnetic energy is predicted to 3% accuracy by the ERXP model for magnetizing excitation with no gaps, indicating that our calculation to match reluctance is accurate. However, we have matched only total reluctance, rather than matching the reluctance of each segment to the corresponding reluctance in the 3D model. The MMF drop across the individual segments ($\mathcal{R}_2$, $\mathcal{R}_3$ and $\mathcal{R}_4$ in Fig. 6) establishes the boundary conditions for the winding field, and thus can be important for the winding energy. Matching reluctance segment-by-segment might produce a better match for winding energy in this case, but there are few situations in which winding loss with magnetizing excitation and no gaps is important—perhaps only with a low-permeability core.

It is also possible to use the results in Fig. 10 to determine why previous work on 2D models of E-core transformers showed good results using the ||T model [3,5]. In [3,5], the model was only tested using leakage excitation. Fig. 10 shows that for leakage excitation, the ||T results are consistently good, although not quite as accurate as the ERXP model. It is with magnetizing excitation that the ||T model fails, with typical errors in the 20 to 40% range. The poor performance of the ||T model with magnetizing excitation can be easily explained by the fact that, in the real three dimensional transformer, flux returns through the outer core legs, not through the air, as it is required to do in the T section of the ||T model.

The discussion and the example have been for round-centerpost transformers only. For square centerposts, the basic ERXP method is applicable, but the details remain to be verified. We believe that it is still best to use an axisymmetric simulation to account for the increase of winding length with radius and related effects, but it is not immediately clear whether to make the ER and XP centerposts equal to the square center post in perimeter or in area. The best choice is probably equal perimeter, to better model the winding length and the perimeter fringing effects, but it may be necessary to insert a zero-permeability "plug" in the center of the centerpost to reduce its area to match that of the square centerpost. However, for rectangular centerposts that are far from square, as in [6], a simulation in rectangular coordinates is more appropriate.

IV. CONCLUSIONS

The new ERXP model of E-core transformers is accurate to within 0.2 to 5.6% error in 15 of the 16 cases studied, as compared to the 3D model. It is useful in all four different gap configurations studied under both magnetizing and leakage excitation, and it accurately predicts both the total magnetic energy and the magnetic energy in the windings. In the gapless transformer under magnetizing excitation, its prediction of the energy in the windings was off by 17%. However, in gapless transformers, the magnetizing current is usually small compared to other currents, so the loss related to this current is very small compared to other sources of loss. Therefore the reduced accuracy of our model in this one case is not detrimental to its overall utility.

In comparison to the other models studied across all situations modeled, the ERXP model is clearly the most generally accurate method to model 3D E-core transformers in 2D. In the few cases where some other model is slightly more accurate, both the ERXP and the more accurate model have very low errors. Also, all other models failed severely in at least one important case, giving predictions that are too poor to be useful. The ERXP model allows designers to use relatively simple, fast, and inexpensive 2D modeling to accurately model 3D effects in E-core transformers.

Although we have only tested the method with one specific core and winding geometry, we have tested that geometry exhaustively, and the results match those used for experimental verification in [10]. Further testing involving simulation of different geometries could better establish the broad applicability of the method. Pending further testing, however, the method is still the most accurate, best-tested method for 2D simulation of 3D E-core transformers.

In addition, we have developed a refined fringing reluctance calculation combining the exact formula for 2D gap fringing reluctance from [7] with an approximate formula for corner fringing reluctance. Using 3D numerical simulations allowed us to calibrate the approximation to be more accurate than the formulas in [8]. This result is expected to be useful beyond its application in calculating appropriate 2D models for simulations.

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