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AN INDEPENDENT METHOD FOR DETERMINING THE AGE OF THE UNIVERSE

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ABSTRACT

The oldest white dwarf stars in the solar neighborhood are still observable, and their luminosity distribution can be used to determine directly the age of the Galactic disk and, somewhat less directly, the age of the universe. The observed number of white dwarfs in a given volume of space increases monotonically with decreasing luminosity, as expected from cooling rate considerations (hotter objects cool more quickly). However, their number drops abruptly at a luminosity of $\log(L/L_\odot) \approx -4.5$, due to the finite age of our Galaxy.

Comparing this sudden drop in the observed luminosity distribution with the best theoretical evolutionary white dwarf models available to us, and allowing for a mean pre–white-dwarf lifetime of 0.3 Gyr, we derive an age for the Galactic disk of

$$9.3 \pm 2.0 \text{ Gyr}.$$

To obtain the age of the universe, we must add the time between the big bang and the first appearance of stars in the Galactic disk. Rather than assume a particular cosmological model, we prefer to choose a value (and stated error) that can include all of the currently reasonable models describing this early era. On this basis we estimate the age of the universe to be 10.3 ± 2.2 Gyr.

This new technique provides a way of determining the age of the Galactic disk that is largely independent of all previous methods. Further, its current uncertainties can be materially reduced by direct measurements of the rate at which variable white dwarf stars cool through the three known instability strips in the H-R diagram.

Subject headings: cosmology — stars: evolution — stars: white dwarfs

I. WHITE DWARFS AND THE AGE OF THE GALACTIC DISK

Evolutionary models of white dwarf stars are much simpler than those of main-sequence stars. The analytical model of their cooling developed by Mestel (1952) gives a direct and simple relation between their age and luminosity. Maarten Schmidt (1959) proposed that the age of the Galactic disk could be determined by measuring the luminosity of the coolest white dwarf stars, then determining their age by comparison with Mestel's cooling theory, after adding the time for main-sequence evolution.

This direct and simple approach was frustrated for two reasons. First, no significant downturn in the space density of white dwarfs at low luminosities was then observed: the white dwarf luminosity distribution continued to rise at the lowest measured luminosities. Second, Mestel and Ruderman (1967) and Van Horn (1968) found that the interiors of low-luminosity white dwarf stars should crystallize. They showed that the reduced heat capacity of the crystalline interior would cause them to cool to invisibility very rapidly; the resultant blurring of the age-luminosity relation called the technique into serious question.

This situation has recently changed dramatically in several ways:

We now observe a very real shortfall in the number of white dwarf stars below $\log L/L_\odot = -4.5$ (cf. Liebert 1980; Liebert, Dahn, and Sion 1983).

Theorists now agree that the onset of rapid cooling is not reached for the dominant stellar mass in the population of white dwarfs (0.6 $M_\odot$) until well below the luminosity of the observe shortfall (Shaviv and Kovetz 1976; Iben and Tutukov 1984; Iben and McDonald 1985).

The observed stability of the oscillation in the variable white dwarf G117-B15A (Kepler *et al.* 1986) shows that the onset of rapid cooling cannot begin significantly earlier than the current models predict.

The direct measurement of white dwarf evolutionary time scales is not only possible (cf. Robinson and Kepler 1980) but has been successfully determined for one star (Winget *et al.* 1985); thus the theoretical ages can be directly tested.

II. THE OBSERVED WHITE DWARF LUMINOSITY DISTRIBUTION

Greenstein (1971) suggested that there might be a deficiency in the number of cool degenerate stars compared with

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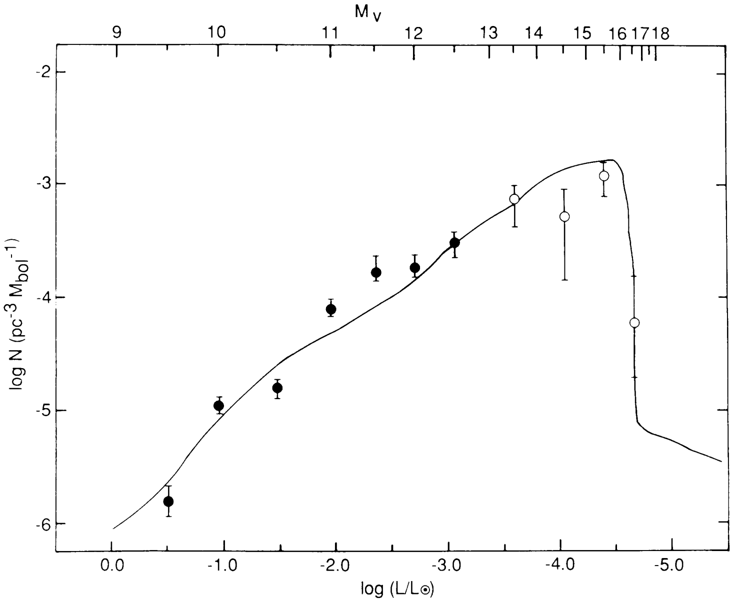

FIG. 1.—The white dwarf luminosity distribution. The circles represent the observed number of white dwarfs in each luminosity bin; the solid line shows the theoretical distribution. The vertical axis, Φ, is log N ($\mathrm{pc}^{-3} M_{\mathrm{bol}}^{-1}$).

the expectations of theory. Subsequent systematic searches for faint white dwarfs indeed revealed a dramatic shortfall in their number, although at somewhat lower temperatures and luminosities than expected (cf. Liebert 1980; Sion 1986). Figure 1 shows the currently observed white dwarf luminosity distribution (hereafter Φ_{obs}); it is evident that a real shortfall occurs between $\log(L/L_\odot) \approx -4.35$ and -4.65.

The values displayed for Φ_{obs} are derived from several sources. Hydrogen atmosphere (DA) white dwarfs are in the clear majority for effective temperatures exceeding about 10,000 K and the results of Fleming, Liebert, and Green (1986) are used for $M_V \leq +13$. However, non-DAs contribute to the total for these hot stars, and we have adjusted our numbers upward accordingly, using statistics from Liebert *et al.* (1986).

For the cooler objects that best define the shortfall we have derived Φ_{obs} from white dwarfs identified in the Luyten Half Second (LHS) proper motion catalog, using the "$1/V_{\mathrm{max}}$" method first applied to stellar samples by Schmidt (1975), and recently to a sample of white dwarfs by J. Liebert and C. Dahn (in preparation). Our application of the method assumes that the LHS catalog is complete down to a limiting apparent magnitude of $M_v = 18$ in the range $0''.8$–$2''.0\ \mathrm{yr}^{-1}$, an assumption which we were able to test from the derived sample itself. We found no kinematical bias (i.e., no correlation between the derived mean tangential velocities and absolute magnitudes). This method permits a useful number to be derived for the faintest white dwarfs of known parallax: LP 131 − 66 and LP 701 − 29, which have $M_V > 16.0$.

The same method, applied to M dwarf stars taken from the same catalog, does not reveal any shortage of stars between $M_V = 16$ and $M_V = 18$ such as we find for the white dwarfs. The luminosity function thus derived for M dwarfs agrees with the color-selected sample of Gilmore and Reid (1984). The LHS sample includes over 50 low-luminosity M dwarfs fainter than $M_V = +15$, extending to $M_v = +19.5$. The known sample of white dwarfs now includes some 80 stars with $M_v > +14$, but none with an accurate parallax with $M_v > +16.6$. Moreover, while numerous cool white dwarfs have also been found as common proper motion companions to known nearby stars, the failure to find white dwarf companions fainter than +16.5 (Liebert *et al.* 1979) or as massive, unseen companions in perturbation analyses (Borgmann and Lippincott 1983; Shipman 1983) strengthens the case further. We are therefore confident that the precipitous drop in our numbers for white dwarfs with $M_v > 16$ is real.

For comparison with the theoretical luminosity distribution, we have converted the magnitude scale to $\log L/L_\odot$ using the stellar atmospheric calculations of Wesemael (1981) for the hot stars and Kapranidis (1985) and Shipman (1979) for the cooler ones. On this scale, the shortfall in Φ_{obs} occurs between $\log L/L_\odot$ of -4.35 and -4.65. The two lowest luminosity objects (LP 131 − 66 and LP 701 − 29) lie at -4.50 ± 0.15, and they define the shortfall. It is reassuring that the value of T_{eff} for LP 701 − 29 derived by Kapranidis and Liebert (1986) yields "normal" values of radius and mass of $\sim 0.01\ R_\odot$ and $\sim 0.6\ M_\odot$ when combined with the luminosity found here.

The existence of a tail (the point representing the lowest luminosity stars) in the distribution Φ_{obs}, and the absence of a "bump" just before it, rule out explanations of the shortfall which produce a "pile-up" of stars at the faint end of the distribution, including repeated episodes of nuclear burning (Starrfield and Sparks 1979), release of gravitational energy by the separation of carbon and oxygen upon crystallization (Stevenson 1979; Mochkovitch 1983), and heating by monopole-induced catalysis of nuclei (Freese 1984).

III. THE THEORETICAL LUMINOSITY DISTRIBUTION

We base our construction of the theoretical luminosity distribution on homogeneous stellar models of pure carbon, using the equation of state and evolutionary computer code described in detail by Lamb (1974) and Lamb and Van Horn

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TABLE 1

AGES OF THE MODELS (Gyr)

$\log L/L_{\odot}$	0.4 $M_{\odot}$ C	0.6 $M_{\odot}$ C	0.8 $M_{\odot}$ C	1.0 $M_{\odot}$ C	0.6 He/C	0.6 H/He/C
−1.00	0.034	0.026	0.025	0.042	0.018	0.018
−1.50......	0.070	0.074	0.085	0.120	0.062	0.062
−2.00......	0.155	0.176	0.207	0.274	0.197	0.197
−2.50......	0.325	0.382	0.456	0.590	0.484	0.484
−3.00......	0.680	0.885	1.084	1.428	1.090	1.089
−3.25......	1.089	1.428	1.789	2.318	1.573	1.568
−3.50......	1.568	2.060	2.846	3.234	2.234	2.211
−3.75......	2.240	3.107	4.065	4.117	3.208	3.142
−4.00......	3.186	4.836	5.430	5.045	5.440	5.718
−4.25......	4.819	6.763	6.875	6.008	8.360	8.390
−4.50......	7.161	8.693	8.371	6.919	10.571	10.453
−4.75......	9.831	10.711	9.767	7.697	12.666	12.408
−5.00......	12.574	12.829	11.054	8.268	14.668	14.165

(1975). These models incorporate an accurate treatment of the principal physics of the nondegenerate envelope and the degenerate interior: specifically convection, Coulomb interactions, and crystallization. They are the simplest and most internally self-consistent models available to us.

We recognize that pure carbon models do not represent the most likely white dwarf composition. We must therefore correct for the effects on the cooling rate of overlying layers of helium (and of hydrogen for DA white dwarfs), and the presence of oxygen in the stellar core.

Table 1 shows the ages of the computed models. The addition of the lighter elements at the surface (He/C and H/He/C) slows the cooling rate for these sequences because the change in the envelope opacities alters the properties of the convection zones and the rate at which degeneracy penetrates into the envelope. The last two columns in Table 1 show that the addition of light element layers extends the computed age by about 15%.

Models of pre–white dwarf evolution suggest that the interiors of white dwarfs are probably composed of a mixture of carbon and oxygen. The presence of oxygen in the core affects their ages in three ways: it can substantially increase the release of gravitational energy if C/O phase separation occurs during crystallization, it changes the rate at which energy is lost by changing the opacity, and it changes the total amount of available energy by changing the specific heat.

The effect on the release of gravitational energy is expected to be large (Mochkovitch 1983) and would lead to a large increase in $\Phi_{\rm obs}$ at the luminosities where crystallization is expected to occur ($\log L/L_{\odot} \lesssim -4.0$ for white dwarfs of 0.6 $M_{\odot}$). The absence of such an effect in Figure 1 shows that C/O separation does not take place at luminosities high enough to affect the observed shortfall, if it takes place at all.

The effect on the opacity in the interior is small since the material is fully degenerate, and the conductive opacities of carbon and oxygen are not significantly different. The effect on the specific heat, while not negligible, is small, and we can easily estimate it. The heat capacity of the interior is largely due to the ions, since the contribution of the degenerate electrons is small. The addition of oxygen to the interior only changes the heat capacity by the ratio of the mean molecular weight of carbon over the mean molecular weight of the carbon/oxygen mixture. For a mixture of equal parts carbon and oxygen the heat capacity, and therefore the age of the white dwarf models at the luminosity of the shortfall, is decreased by about 15%.

The net effects of compositional stratification in the outer layers and of oxygen in the interior have the same magnitude, but opposite signs. The pure carbon models therefore provide the best mean estimators for the age of the white dwarf stars at the luminosity shortfall, and we estimate the internal error in this age is about 15%.

The age of a white dwarf star at a given luminosity also depends on the total stellar mass. For stars of high luminosity, the difference in cooling rate is small once neutrino losses are no longer important; it is due to the small difference in the total heat capacity, which is proportional to the total stellar mass. At low luminosities, however, the dependence on stellar mass is large. The higher central density in the more massive stars leads to an earlier onset of crystallization, which has a dramatic effect on the heat capacity. Once crystallization is nearly complete, the thermal energy content drops precipitously and rapid cooling becomes important.

We have therefore constructed the theoretical luminosity distribution by combining sequences of white dwarf models with masses 0.4, 0.6, 0.8, and 1.0 $M_{\odot}$. We first constructed the luminosity distribution for each sequence separately, normalized at $\log L/L_{\odot} = -2.0$ and weighted by the relative number of white dwarfs with masses within ± 0.1 $M_{\odot}$ of the nominal value. The weighting factors were derived from the observed mass distribution found by Weidemann and Koester (1984). They found that the DA mass distribution—although sharply peaked around 0.6 $M_{\odot}$—has a finite dispersion of $\sigma = 0.1$ $M_{\odot}$. The mass distribution of DB's is very similar (Oke, Weidemann, and Koester 1984), so we took the DA distribution to be representative of all white dwarfs. The theoretical luminosity distribution is then just the sum of the normalized luminosity distributions for each mass.

To model the sudden drop in the luminosity distribution, we assume an epoch for the formation of the Galactic disk; the contribution of each mass to the total then falls to zero for luminosities with corresponding ages longer than the assumed age. The theoretical luminosity distribution drops dramatically, but not to zero, when the age of the dominant mass in the white dwarf population reaches the age of the disk. By matching the luminosity of the theoretical drop to that of the

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observed shortfall, and including $\pm 0.15\ L/L_\odot$ as our estimated uncertainty in its luminosity (due primarily to uncertainty in the bolometric correction), we derive a cooling age for the white dwarf population $\tau_{\rm wd} = 9.0 \pm 1.8$ Gyr.

This procedure implicitly assumes that the birth rate of white dwarfs has remained constant over the age of the Galaxy. However, our final result is quite insensitive to this assumption, since we are not trying to reproduce the observed luminosity distribution in its entirety, but only its falloff at the lowest luminosities. Any theory of bimodal star formation (e.g., Larson 1986) must accommodate the complete luminosity distribution of white dwarfs (Fig. 1); in any event, predictions from such a theory do not affect our results.

We can now use this result to estimate the age of the Galactic disk from

$$\tau_{\rm disk} = \tau_{\rm wd} + \tau_{\rm ms}\,,$$

where $\tau_{\rm ms}$ is the mean interval spent in all prior phases of evolution by the current white dwarf population. According to Iben and Renzini (1983), the results of numerical stellar evolution calculations are accurately fitted by the relation

$$M_f \approx 0.53\eta^{-0.082} + 0.15\eta^{-0.35}(M_i - 1)\,,$$

where M_i and M_f are the initial and final stellar masses, and the mass-loss parameter η is expected to lie in the range $\frac{1}{3} \leq \eta \leq 3$. If the dominant white dwarf mass at the shortfall is $\approx 0.8\ M_\odot$, as our calculations indicate, then the initial main-sequence mass should lie in the range $2 \leq M_i/M_\odot \leq 4$. Interpolating from Iben (1965, 1966, 1967), we find the time for all pre–white-dwarf stages of evolution to be $\tau_{\rm ms} \approx 0.3^{+0.8}_{-0.1}$ Gyr, and therefore the age of the Galactic disk to be $\tau_{\rm disk} = 9.3 \pm 2.0$ Gyr.

To estimate the age of the universe, we use

$$\tau_{\rm univ} = \tau_{\rm ps} + \tau_{\rm disk}\,,$$

where $\tau_{\rm ps}$ is the prestellar interval between the big bang and the formation of disk stars in our Galaxy. Although this prestellar interval depends on the cosmological model chosen, for all such models currently in vogue the time is far smaller than the disk age we have derived. We can avoid depending on any specific cosmological model by estimating the early interval and its uncertainty in a way that includes the values predicted by the different popular models: all yield ≈ 1 Gyr for the interval in question. The uncertainty is unlikely to be larger than the value itself. We add this value to the age of the disk, and therefore estimate the age of the universe as 10.3 ± 2.2 Gyr.

For a discussion of other results and methods of cosmochronology, see the excellent review by Fowler and Meisl (1986).

IV. THE POTENTIAL OF THE METHOD

We have identified several sources of uncertainty and possible error in our method, and we have tried to estimate their effects on our results. We believe that the stated uncertainties can be substantially reduced by observational and theoretical methods already in hand.

The current theoretical models can be improved as a result of direct observations. As white dwarf stars cool, they pass through three regions of pulsational instability that cause them to become variable in luminosity. The rich spectrum of observed pulsation behavior promises to reveal details of the underlying compositional layers and the core material. For example, the observed period stability of the variable G117-B15A already shows it cannot have a core of iron, and the presence of a core made entirely of oxygen will probably be tested by measurements scheduled for the current observing season. The improved theoretical models that result can then be used to calibrate the observed luminosity distribution to high precision.

As white dwarf stars cool through these strips of instability, the observed rate at which their pulsation periods change can be related directly to their rate of evolution, providing just the value needed to improve the evolutionary models. Furthermore, the coolest instability strip lies at luminosities not far above the shortfall in the observed luminosity distribution, minimizing the errors involved in extrapolating to low luminosities. Although the measurement precision required can be very high (ca. parts in 10^{-15} for the coolest instability strip), it can be achieved, since the precision of measurement improves as the square of the elapsed time. The first such measurement has already been made (Winget *et al.* 1985).

Our goal will be to improve the accuracy of both theory and observations until the dominant error in the age of the universe lies in the estimation of its prestellar history.

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