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An inflationary inventory model with time dependent demand with Weibull distribution deterioration and partial backlogging under permissible delay in payments

by

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Abstract: This paper proposes to present a general inventory model with due consideration to the factors of time dependent partial backlogging and time dependent deterioration. It also takes into account the impact of inflation, time-dependent demand and permissible delay in payments.

Keywords: inflation, partial backlogging, Weibull distribution, permissible delay in payments.

1. Introduction

It is sometimes assumed by recent authors that the shortages are either wholly backlogged or completely lost. Dye (2002) considered an inventory model with stock-dependent demand and partial backlogging. Chakrabarty et al. (1998) extended the Philip's model (1974). Skouri and Papachristos (2003) determined optimal time of an EOQ model with deteriorating items and time-dependent partial backlogging. This paper attempts to apply the results of Yan and Cheng (1998) in the case where the backlogging rate is a time-dependent function. In this connection we may mention a three parameter distribution for describing deterioration depending on time. Deterioration can not be avoided in business scenarios. Rau et al. (2004) presented an integrated inventory model to determine economic ordering policies of deteriorating items in a supply chain management system. Teng and Chang (2005) determined economic production quantity in an inventory model for deteriorating items.

Deterioration refers to decay, damage or spoilage. In respect of items of foods, films, drugs, chemicals, electronic components and radio-active substances, deterioration may happen during normal period of storage and the loss is to be taken into account where we analyze inventory systems. Dave and Patel (1983) put forward an inventory model for deteriorating items with time proportional demand, instantaneous replenishment and no shortage. Roychowdhury and Chaudhury (1983) proposed an order level inventory model considering a finite rate of replenishment and allowing shortages. In their models Mishra (1975), Deb and Chaudhuri (1986) assumed that deterioration rate is time dependent. An extensive summary in this regard was made by Raafat (1991). Berrotoni (1962) discussed some difficulties of fitting empirical data to mathematical distribution. It may be said that the rate of deterioration increases with age. It may be inferred that the work of Berrotoni (1962) inspired Covert and Philip (1973) to develop an inventory model for deteriorating items with Weibull distribution by using two parameters. Mandal and Phaujdar (1989) however, assumed a production inventory model for deteriorating items with uniform rate of production and stock dependent demand. Some valuable works in this area were also done by Padmanabhan and Vrat (1995), Ray and Chaudhuri (1997), Mondal and Moiti (1999).

Today, inflation has become a permanent feature of the economy. Many researchers have shown the inflationary effect on inventory policy. Biermans and Thomas (1977), Buzacott (1975), Chandra and Bahner (1988), Jesse et al. (1983), Mishra (1979) developed their inventory models assuming a constant inflation rate. An inventory model with deteriorating items under inflation when a delay in payment is permissible is analyzed by Liao et al. (2000). Bhahmbhatt (1982) developed an EOQ model under a variable inflation rate and marked-up price. Ray and Chaudhuri (1997) presented an EOQ model under inflation and time discounting allowing shortages.

Both in deterministic and probabilistic inventory models of classical type it is observed that payment is made to the supplier for goods just after getting the consignment. But actually nowadays a supplier grants some credit period to the retailer to increase the demand. In this respect Goyal (1985) just formulated an EOQ model under some conditions of permissible delay in payment. An EOQ model for inventory control in the presence of trade credit is presented by Chung and Huang (2005). The optimal replenishment policy for EOQ models under permissible delay in payments is also discussed by Chung et al. (2002) and Cung and Huang (2003). In recent times to make the real inventory systems more practical and realistic, Aggarwal and Jaggi (1995) extended the model with a constant deterioration rate. Hwang and Shinn (1997) determined lot-sizing policy for exponential demand when delay in payment is permissible. Shah and Shah (1998) then prepared a probabilistic inventory model with a cost in case delay in payment is permissible. After that Jamal et al. (1997) developed further following the lines of Aggarwal and Jaggi's (1995) model to take into consideration for shortage and make it more practical and acceptable in real situation.

2. Notations

$q(t)$ = Inventory level at time t

$S = q(0)$ = Stock level at the beginning of each cycle after fulfilling backorders

H = Length of the planning horizon

K = Constant rate of inflation (\$/\$/ unit time)

$C(t)$ = Unit purchase cost for an item bought at time t, i.e., $C(t) = C_o.e^{KT}$, where C_o is the unit purchase cost at time zero

h = Holding cost (\$/unit/year) excluding interest charges

C_o = Unit purchase cost

C_2 = Shortage cost (\$/unit/time)

C_3 = The ordering cost/cycle

i_e = Interest earned (\$/time)

i_p = Interest charged (\$/time)

M = Permissible delay in settling the accounts

T_1 = Time at which shortages start ($0 \leqslant T_1 \leqslant T$)

T = Length of a cycle

$TCU(T_1, T)$ = The average total inventory cost per unit time

$TCU_1(T_1, T)$ = The average total inventory cost per unit time for $T_1 \geqslant M$ (Case I)

$TCU_2(T_1, T)$ = The average total inventory cost per unit time for $T_1 \leqslant M$ (Case II)

3. Assumptions

(i) The inventory system involves only one item.

(ii) The rate of replenishment is instantaneous.

(iii) A fraction $z(t)$ of the on hand inventory deteriorates per unit time where $z(t) = \alpha\beta t^{\beta-1}$, $0 < \alpha << 1$, $t > 0$, $\beta \geqslant 1$.

(vi) Shortages are allowed and the backlog rate is defined to be $R(t)/1+\delta(T-t)$ when inventory is negative. The backlogging parameter δ is a positive constant.

(v) The demand rate $R(t)$ at any time t is given by $R(t) = a + bt$ where a and b are non-negative constants.

4. The problem formulation

The mathematical models are derived under two different circumstances:

Case I: The permissible delay in payment, M, is less than the period of having inventory stock in hand, T_1.

Case II: The permissible delay in payment M is greater than T_1.

4.1. The mathematical model

During the time $[0, T_1]$ the instantaneous inventory level at time t will satisfy the following differential equations

$$\frac{dq}{dt} + \alpha\beta t^{\beta-1} q = -(a + bt) \qquad (0 \leqslant t \leqslant T_1) \tag{1}$$

where $0 < \alpha << 1$ and $\beta \geqslant 1$

with the boundary condition $q(T_1) = 0\,.$ (1a)

Again during the time $[T_1, T]$ the instantaneous inventory will satisfy the following differential equation

$$\frac{dq}{dt} = -\frac{a + bt}{1 + \delta(T - t)} \qquad (T_1 \leqslant t \leqslant T) \tag{2}$$

where $q(T_1) = 0\,.$ (2a)

The solution of (1) using boundary condition (1a) is

$$\begin{aligned} q(t) =& (1 - \alpha t^{\beta})\Big\{\Big(aT_1 + \frac{bT_1^2}{2} + \frac{a\alpha T_1^{\beta+1}}{\beta+1} + \frac{b\alpha T_1^{\beta+2}}{\beta+2}\Big) \\ &- \Big(at + \frac{bt^2}{2} + \frac{a\alpha t^{\beta+1}}{\beta+1} + \frac{b\alpha t^{\beta+2}}{\beta+2}\Big)\Big\}\,. \end{aligned} \tag{3}$$

The solution of (2) using boundary condition (2a) is

$$\begin{aligned} q(t) =& \Big(\frac{a}{\delta} + \frac{b(1+\delta T)}{\delta^2}\Big)\big\{\log[1 + \delta(T - t)] - \log[1 + \delta(T - T_1)]\big\} \\ &+ \frac{b}{\delta}(t - T_1)\,. \end{aligned} \tag{4}$$

4.2. Case (I) ($M < T_1$) (payment before depletion)

The total variable cost is comprised of the sum of the ordering cost, holding cost, backorder cost, deterioration cost and interest payable minus the interest earned. They are grouped together after evaluating the above cost individually.

The holding cost HC during $[0,T_1]$ is

$$HC = h\sum_{n=0}^{m-1} C(nT)\int_0^{T_1} q(t)dt$$

$$= hC_0\Big(\frac{e^{KH}-1}{e^{KT}-1}\Big)\Bigg[\int_0^{T_1}(1-\alpha t^{\beta})\Big\{\Big(aT_1+\frac{bT_1^2}{2}+\frac{a\alpha T_1^{\beta+1}}{\beta+1}+\frac{b\alpha T_1^{\beta+2}}{\beta+2}\Big)$$

$$-\Big(at+\frac{bt^2}{2}+\frac{a\alpha t^{\beta+1}}{\beta+1}+\frac{b\alpha t^{\beta+2}}{\beta+2}\Big)\Big\}dt\Bigg]$$

$$= hC_0\Big(\frac{e^{KH}-1}{e^{KT}-1}\Big)\Bigg[\frac{aT_1^2}{2}+\frac{bT_1^3}{3}+\alpha T_1^{\beta+3}\Big\{\frac{b}{\beta+2}-\frac{b}{2(\beta+1)}{}_1$$

$$+\frac{\beta}{2(\beta+3)}-\frac{b}{(\beta+2)(\beta+3)}\Big\}+\frac{a\alpha\beta T_1^{\beta+2}}{(\beta+1)(\beta+2)}\Bigg] \tag{5}$$

(ignoring the higher order of α).
The number of deteriorated items during $[0,T_1]$ is

$$= q(0)-\int_0^{T_1}(a+bt)dt = S-\Big(aT_1+\frac{bT_1^2}{2}\Big) = \frac{a\alpha T_1^{\beta+1}}{\beta+1}+\frac{b\alpha T_1^{\beta+2}}{(\beta+2)}\,. \tag{6}$$

The deterioration cost DC is

$$DC = C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\left\{\frac{a\alpha T_1^{\beta+1}}{\beta+1}+\frac{b\alpha T_1^{\beta+2}}{(\beta+2)}\right\}. \tag{7}$$

The shortage cost SHC during $[0,T]$ is

$$SHC = C_2.C_o\Big(\frac{e^{KH}-1}{e^{KT}-1}\Big)\int_{T_1}^{T} q(t)dt = C_2.C_o\Big(\frac{e^{KH-1}}{e^{KT}-1}\Big)\int_{T_1}^{T}\Big[\Big\{\frac{a}{\delta}+\frac{b(1+\delta T)}{\delta^2}\Big\}$$

$$\{\log|1+\delta(T-t)|-\log|1+\delta(T-T_1)|\}+\frac{b}{\delta}(t-T_1)\Big]dt$$

$$= C_2.C_o\Big(\frac{e^{KH}-1}{e^{KT}-1}\Big)\Big[\Big(\frac{a}{\delta}+\frac{b(1+\delta T)}{\delta^2}\Big)\Big\{\frac{1}{\delta}\log|1+\delta(T-T_1)|-(T-T_1)\Big\}\Big]$$

$$+\frac{b}{2\delta}(T-T_1)^2\,. \tag{8}$$

The interest earned IE_1 during time $[0,T]$ is

$$IE_1 = i_e.C_o\frac{e^{KH}-1}{e^{KT}-1}\int_0^{T_1}(T_1-t)(a+bt)dt = i_e.C_o\frac{e^{KH}-1}{e^{KT}-1}\Big(\frac{aT_1^2}{2}+\frac{bT_1^3}{6}\Big) \tag{9}$$

The interest payable IP_1 per cycle for the inventory not being sold after due date M:

$$
\begin{aligned}
IP_1 &= i_p.C_o \frac{e^{KH}-1}{e^{KT}-1} \int_M^{T_1} q(t)dt \\
&= i_p.C_o \frac{e^{KH}-1}{e^{KT}-1} \Bigg[\Big(aT_1 + \frac{bT_1^2}{2} + \frac{a\alpha T_1^{\beta+1}}{\beta+1} + \frac{b\alpha T_1^{\beta+2}}{(\beta+2)} \Big) \\
&\quad \Big\{ (T_1 - M) - \frac{\alpha}{(\beta+1)} (T_1^{\beta+1} - M^{\beta+1}) \Big\} - \frac{a}{2}(T_1^2 - M^2) - \frac{b}{6}(T_1^3 - M^3) \\
&\quad + \frac{a\alpha\beta}{(\beta+1)(\beta+2)} (T_1^{\beta+2} - M^{\beta+2}) + \frac{b\alpha\beta}{2(\beta+2)(\beta+3)} (T_1^{\beta+3} - M^{\beta+3}) \\
&\quad + \frac{a\alpha^2}{2(\beta+1)^2} (T_1^{2\beta+2} - M^{2\beta+2}) + \frac{b\alpha^2}{(\beta+2)(2\beta+3)} (T_1^{2\beta+3} - M^{2\beta+3}) \Bigg] . \qquad (10)
\end{aligned}
$$

The total variable cost, TVC_1, is defined as

$$TVC_1 = C_3 + HC + SHC + IP_1 + DC - IE_1 \ . \qquad (11)$$

From equations (5)-(10), we obtain TVC as

$$
\begin{aligned}
TVC_1 &= C_o \Big(\frac{e^{KH}-1}{e^{KT}-1} \Big) \Bigg[\frac{aT_1^2}{2}(h - i_e - i_p) + \frac{bT_1^3}{6}(2h - i_e - i_p) + C_3 \\
&\quad + \alpha T_1^{\beta+3} \Big\{ \frac{2\beta^2 + 4\beta(b+1) + 8b + b\beta i_p}{2(\beta+2)(\beta+3)} - \frac{b}{2(\beta+1)} \Big\} \\
&\quad + \frac{\alpha T_1^{\beta+2}}{\beta+2} \Big\{ \frac{\beta(ah + ai_p + b) + b}{\beta+1} \Big\} + C_2 \Big(\frac{a\delta + b + b\delta T}{\delta^2} \Big) \Big\{ \frac{1}{\delta} \ln |T - T_1| - (T - T_1) \Big\} \\
&\quad + \frac{bC_2}{2\delta}(T - T_1)^2 + \frac{a\alpha T_1^{\beta+1}}{\beta+1} + i_p \Big(aT_1 + \frac{bT_1^2}{2} + \frac{a\alpha T_1^{\beta+1}}{\beta+1} + \frac{b\alpha T_1^{\beta+2}}{\beta+2} \Big) \\
&\quad \times \Big\{ (T_1 - M) - \frac{\alpha}{\beta+1} (T_1^{\beta+1} - M^{\beta+1}) \Big\} + \frac{ai_p M^2}{2} + \frac{bi_p M^3}{6} - \frac{ai_p \alpha\beta M^{\beta+2}}{(\beta+1)(\beta+2)} \Bigg] .
\end{aligned}
\qquad (12)
$$

The total variable cost per unit time, TCU, during the cycle period $[0, T]$ is given by

$$TCU_1 = \frac{TVC_1}{T} = \frac{K + HC + DC + SHC + IP_1 - IE_1}{T} \tag{13}$$

$$\begin{aligned}TCU_1 = C_o\Big(\frac{e^{KH}-1}{e^{KT}-1}\Big)\Bigg[&\frac{aT_1^2}{2}(h-i_e-i_p) + \frac{bT_1^3}{6}(2h-i_e-i_p) + C_3\\
&+\alpha T_1^{\beta+3}\Big\{\frac{2\beta^2+4\beta(b+1)+8b+b\beta i_p}{2(\beta+2)(\beta+3)} - \frac{b}{2\beta+1)}\Big\}\\
&+\frac{\alpha T_1^{\beta+2}}{\beta+2}\Big\{\frac{\beta(ah+ai_p+b)+b}{\beta+1}\Big\}+C_2\Big(\frac{a\delta+b+b\delta T}{\delta^2}\Big)\Big\{\frac{1}{\delta}\ln|T-T_1|-(T-T_1)\Big\}\\
&+\frac{bC_2}{2\delta}(T-T_1)^2 + \frac{a\alpha T_1^{\beta+1}}{\beta+1} + i_p\Big(aT_1 + \frac{bT_1^2}{2} + \frac{a\alpha T_1^{\beta+1}}{\beta+1} + \frac{b\alpha T_1^{\beta+2}}{\beta+2}\Big)\\
&\times\Big\{(T_1-M)-\frac{\alpha}{\beta+1}(T_1^{\beta+1}-M^{\beta+1})\Big\}+\frac{ai_pM^2}{2}+\frac{bi_pM^3}{6}-\frac{ai_p\alpha\beta M^{\beta+2}}{(\beta+1)(\beta+2)}\Bigg]/T\,.\end{aligned} \tag{14}$$

Now the problem is:

min TCU_1
subject to the constraint $0 < T \leqslant 1$.

5. Case (II) ($T_1 < M$) (payment after depletion)

The ordering cost C_3, the holding cost HC, the shortage cost SHC, the deterioration cost DC during the cycle period $(0, T)$ are the same as in case I. The payable per cycle is $P_T = 0$ when $T_1 < M < T$ because the supplier can be paid in full at time M, the permissible delay. The interest earned per cycle is

$$\begin{aligned}IE_2 &= i_eC_o.\frac{e^{KH}-1}{e^{KT}-1}\Bigg\{\int_0^{T_1}(T_1-t)(a+bt)dt + (M-T_1)\int_0^{T_1}(a+bt)dt\Bigg\}\\
&= i_eC_o.\frac{e^{KH}-1}{e^{KT}-1}\Big\{\Big(\frac{aT_1^2}{2}+\frac{bT_1^3}{6}\Big)+(M-T_1)\Big(aT_1+\frac{bT_1^2}{2}\Big)\Big\}\,.\end{aligned} \tag{15}$$

The total variable cost, TVC_2 is defined as

$$TVC_2 = C_3 + HC + SHC + DC - IE_2$$

with

$$TVC_2 = C_o\Big(\frac{e^{KH}-1}{e^{KT}-1}\Big)\Bigg[\frac{bT_1^3}{6}(2h-i_e-3)+C_3+\frac{T_1^2}{2}\Big\{a(h-i_e-2)+bM\Big\}$$
$$+aMT_1+\frac{bC_2}{2\delta}(T-T_1)^2+\alpha T_1^{\beta+3}\Big\{\frac{\beta^2+2\beta(b+1)+4b}{(\beta+2)(\beta+3)}-\frac{b}{2(\beta+1)}\Big\}$$
$$+\frac{a\alpha T_1^{\beta+1}}{\beta+1}+\frac{\alpha T_1^{\beta+2}\{\beta(ah+b)+b\}}{(\beta+1)(\beta+2)}$$
$$+C_2\Big(\frac{a\delta+b+b\delta T}{\delta^2}\Big)\Big\{\frac{1}{\delta}\{\ln|T-T_1|-(T-T_1)\Big\}\Bigg]. \tag{16}$$

The total variable cost per unit time $TCU_2(T_1, T)$ is

$$TCU_2 = \frac{TVC_2}{T} = \frac{C_3+HC+DC+SC-IE_2}{T}. \tag{17}$$

Now the problem is:

min TCU_2

subject to the constraint $\quad 0 \leqslant T \leqslant 1$.

6. The basic algorithm (Genetic Algorithm)

Genetic Algorithm

Genetic Algorithms are a class of adaptive search techniques based on the principle of population genetics.The algorithm is an example of a search procedure that uses random choice as a tool to guide a highly exploitative search through a coding of parameter space. Genetic Algorithms work according to the principles of natural genetics on a population of string structures representing the problem variables. All these features make genetic algorithm search robust, allowing them to be applied to a wide variety of problems.

Implementing GA

The following aspects are involved in the proposed GA to solve the problem:

(1) Parameters
(2) Chromosome representation
(3) Initial population production
(4) Evaluation
(5) Selection
(6) Crossover
(7) Mutation
(8) Termination.

Parameters

Firstly, we set the different parameters on which the specific GA depends. These are the number of generations (MAXGEN), population size (POPSIZE), probability of crossover (PCROS), probability of mutation (PMUTE).

Chromosome representation

An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chromosomes are not effective in many non-linear problems. Since the proposed problem is highly non-linear, hence to overcome the difficulty, a real-number represention is used. In this representation, each chromosome V_i is a string of n numbers of genes G_{ij}, $(j = 1, 2, \ldots n)$ where these n numbers of genes respectively denote n number of decision variables $(X_i,\ i = 1, 2, \ldots n)$.

Initial population production

The population generation technique proposed in the present GA is illustrated by the following procedure: For each chromosome V_i, every gene G_{ij} is randomly generated between its boundary $(LB_j,\ UB_j)$ where LB_j and UB_j are the lower and upper bounds of the variables X_j, $i = 1, 2, \ldots, n$, POPSIZE.

Evaluation

Evaluation function plays the some role in GA as that, which the environment plays in natural evalution. Now, evaluation function ($EVAL$) for the chromosome V_i is equivalent to the objective function $PF(X)$. These are steps of evaluation:

Step 1. Find $EVAL(V_i)$ by $EVAL(V_i) = f(X_1, X_2, \ldots, X_n)$ where the genes G_{ij} represent the decision variable X_j, $j = 1, 2, \ldots, n$, POPSIZE and f is the objective function.

Step 2. Find total fitness of the population: $F = \sum_{i=1}^{\text{POPSIZE}} EVAL(V_i)$.

Step 3. Calculate the probability p_i of selection for each chromosome V_i as

$$Y_i = \sum_{j=1}^{i} p_j.$$

Selection

The selection scheme in GA determines which solutions in the current population are to be selected for recombination. Many selection schemes, such as stochastic random sampling, roulette wheel selection have been proposed for various problems. In this paper we adopt the roulette wheel selection process.

This roulette selection process is based on spinning the roulette wheel POPSIZE times, each time we select a single chromosome for the new population in the following way:

(a) Generate a random (float) number r between 0 to 1.

(b) If $r < Y_i$ then the first chromosome is V_i otherwise select the i^{th} chromosome V_i $(2 \leqslant i \leqslant \text{POPSIZE})$ such that $T_{i-1} \leqslant r \leqslant Y_i$.

Crossover

Crossover operator is mainly responsible for the search of new string. The exploration and exploitation of the solution space is made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generates offspring solutions by recombining both parent solution features. After selection of chromosomes for the new population, the crossover operator is applied. Here, the whole arithmetic crossover operation is used. It is defined as a linear combination of two consecutive selected chromosomes V_m and V_n and the resulting offspring's $V_m^{'}$ and are $V_n^{'}$ calculated as:

$$V_m^{'} = c.V_m + (1-c).V_n$$
$$V_n^{'} = c.V_n + (1-c).V_m$$

where c is a random number between 0 and 1.

Mutation

Mutation operator is used to prevent the search process from converging to local optima rapidly. It is applied to a single chromosome V_i. The selection of a chromosome for mutation is performed in the following way:

Step 1. Set $i \leftarrow 1$.

Step 2. Generate a random number u from the range $[0, 1]$.

Step 3. If $u <$ PMUTE, then go to Step 2.

Step 4. Set $i \leftarrow i + 1$.

Step 5. If $i \leqslant$ POPSIZE, then go to Step 2.

Then the particular gene G_{ij} of the chromosome V_i, selected by the above mentioned steps is randomly selected. In this problem, the mutation is defined as

$$G_{ij}^{\text{mut}} = \quad \text{random number from the range } [0, 1].$$

Termination

If the number of iterations is less than or equal to MAXGEN then the process is going on, otherwise it terminates. The GA's procedure is given below:

```
begin
   do {
         t ← 0
         while (all constraints are not satisfied)
         {
            initialize Population (t)
         }
         evaluate Population(t)
         while (not terminate)
         {
            t ← t + 1
            select Population(t) from Population(t-1)
            crossover and mutate Population(t)
            evaluate Population(t)
         }
         print Optimum Result
      }
end.
```

7. Numerical example and sensitivity analysis

In this paper the ordering policies have been discussed in two scenarios: payment before total depletion (Case–I) and payment after total depletion (Case–II). An example is considered to illustrate the effect of the developed model in this paper.

The following inventory parametric values are used $a = 500$, $b = 80$, $\alpha = 0.00010$, $\beta = 1.0$, $M = 0.1$, $\delta = 5.0$, $C_0 = 0.5$, $i_e = 0.18$, $i_p = 0.20$, $K = 0.1$, $H = 1$ year, $h =$\$ 2.00/unit, $C_3 = 100.0$, $C_2 = 0.8$/unit

To solve the problem the genetic algorithm is used. In this problem, GA consists of the parameters, POPSIZE=50, MAXGEN=50, Cross over probability=0.85, Mutation probability=0.005. The solutions of two cases for different parametric values of α, β, M and δ, are given in Table 1 and Table 2.

From the above tables the results can be discussed as follows

1. When the parameter α increases, the values of T_1 and T decrease, but the total average inventory cost (TCU) increases in both cases.
2. As the parameter β increases, the values of T_1, T decrease and total average inventory cost (TCU) increases for both cases.
3. When the parameter M increases, the values of T_1, T increase and the total average inventory cost (TCU) decreases both in Case–I and Case–II.
4. As the parameter δ increases, the cycle time T decreases but total cost (TCU) increases in both the cases.

From Table 1 and Table 2 we can infer that the parameters α, β have a bigger influence than the parameters M and δ. Therefore, the parameters α, β entail higher sensitivity towards the cycle time and total average inventory cost.

Table 1. CASE–I (Payment before depletion)

Changing parameters	Change in parameters	T_1	T	TCU_1
	0.00010	0.0853	0.0962	4673.81
α	0.00015	0.0728	0.0834	6210.16
	0.00020	0.0676	0.0740	7714.54
	1.0	0.0853	0.0962	4673.81
β	1.5	0.0722	0.0810	6530.32
	2.0	0.0636	0.0769	7046.74
	0.1	0.0853	0.0962	4673.81
M	0.3	0.0920	0.1013	3997.04
	0.5	0.1068	0.1254	2815.35
	5.0	0.0853	0.0962	4673.81
δ	6.0	0.0876	0.0938	5178.14
	7.0	0.0902	0.0925	5446.28

Table 2. CASE–II (Payment after depletion)

Changing parameters	Change in parameters	T_1	T	TCU_2
	0.00010	0.0936	0.1148	3381.50
α	0.00015	0.0841	0.1027	4205.20
	0.00020	0.0728	0.0952	4930.89
	1.0	0.0936	0.1148	3381.50
β	2.0	0.0822	0.1085	3817.55
	3.0	0.0793	0.0923	5132.07
	0.10	0.0936	0.1148	3381.50
M	0.12	0.1014	0.1365	2444.05
	0.13	0.1147	0.1476	2100.50
	5.0	0.0936	0.1148	3381.50
δ	6.0	0.0965	0.1072	4035.39
	7.0	0.0986	0.1012	4572.32

8. Conclusion

To be precise, a model has been illustrated for determination of optimal ordering time and total cost with time dependent demand for deteriorating items following the Weibull distribution. Two cases namely (I) payment before depletion ($M < T_1$) and (II) payment after depletion ($M > T_1$) have been taken into account for consideration of the model, which can help the decision maker to determine the optimal cycle time and to minimize the total average inventory cost. From the sensitivity analysis it is inferred that as the rate of deterioration (α, β) increases, the total average inventory cost increases, which is obvious. Moreover, it follows that increase in permissible delay decreases total cost i.e. there is an inverse relation between total cost and the permissible payment period. The intuitive reason behind this is that the extension in permissible payment period offers opportunity to the purchaser to earn more by investing the resource otherwise from the sale-proceed of the inventory, which results in the lower cost.

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