

# AN INITIAL COMPLEX SYSTEMS ANALYSIS OF THE RISKS OF BLACKOUTS IN POWER TRANSMISSION SYSTEMS

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## Introduction

Electric power transmission systems are a key infrastructure and blackouts of these systems have major direct and indirect consequences on the economy and national security. In particular, electric power blackouts have cascading effects on other vital infrastructures. While it is useful to analyze the detailed causes of individual blackouts, in this paper we focus on the intrinsic dynamics of series of blackouts and how this complex system dynamics impacts the assessment and mitigation of blackout risk. Indeed, the mitigation of failures in complex systems needs to be approached with care.

To motivate our work we consider the statistics of series of blackouts. The North American Electrical Reliability Council (NERC) has a documented list summarizing major blackouts of the North American power transmission system from 1984 to 1998 [NERC]. One might expect a probability distribution of blackout sizes to fall off exponentially (as, for example, in a Weibull distribution). However, analyses of the NERC data [Carreras00, Carreras01a, Chen01, CarrerasPES] show that the probability distribution of the blackout sizes does not decrease exponentially with the size of the blackout, but rather has a power law tail. For example, load shed is one measure of blackout size and Figure 1 plots on a log-log scale the empirical probability distribution of load shed in the North American blackouts. The fall-off with blackout size is close to a power dependence with an exponent of about -1.1. (An exponent of -1 would imply that doubling the blackout size only halves the probability.) Thus the NERC data suggests that large blackouts are much more likely than might be expected.

The NERC blackout data seems to be the best available but the statistics have limited resolution since the data is limited to only 15 years. Thus the NERC data suggests rather than proves the existence of the power tails. Modeling and simulation of the complex system dynamics is indicated. As described below, progress has been made in modeling the overall forces shaping the dynamics of series of blackouts. Simulations on artificial power networks using the OPA model [Dobson01, Carreras01b, Carreras02] can yield power tails that are consistent with the NERC data as shown in Figure 1.

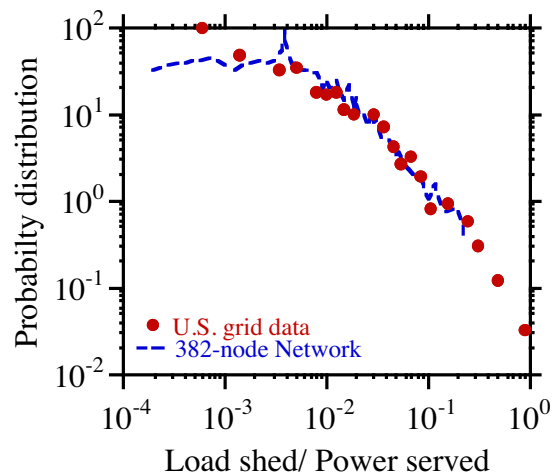


Figure 1: Blackout probability distribution vs. blackout size.

The presence of power tails has a profound effect on risk analysis for larger blackouts and the main purpose of this paper is to outline some of these effects and suggest ideas towards quantifying and mitigating the risks of larger blackouts from a complex systems perspective. Indeed, power laws are a characteristic property of complex system dynamics.

## Blackout risk analysis and power tails

To evaluate the risk of a blackout, we need to know both the frequency of the blackout and its costs. It is difficult to determine blackout costs and there are several approaches to estimate these costs, including customer surveys, indirect analytic methods and estimates for particular blackouts [Billinton96]. The estimated direct costs to electricity consumers vary by sector and increase with both the amount of power interrupted and the duration of the blackout. [Billinton87] defines an interrupted energy assessment rate IEAR in \$/kWh that is used as a factor multiplying the unserved energy to estimate the blackout cost. That is, for a blackout with size measured by unserved energy  $S$ ,

$$\text{direct costs} = (\text{IEAR}) S \quad \$ \quad (1)$$

There are substantial nonlinearities and dependencies not accounted for in (1) [Billinton96, Caves89, Kariuki96], but

expressing the direct costs as a multiple of unserved energy is a commonly used crude approximation. However, studies of individual large blackouts suggest that the indirect costs of large blackouts, such as those resulting from social disorder, are much higher than the direct costs [Billinton96; p.12, Corwin78]. Also, the increasing and complicated dependencies of other infrastructures on electrical energy tend to increase the costs of all blackouts [Rinaldi01, NERC01].

Let the frequency of a blackout with unserved energy  $S$  be  $F(S)$  and the cost of the blackout be  $C(S)$ . The risk of a blackout is the product of blackout frequency and cost:

$$\text{risk} = F(S) C(S)$$

The NERC data indicates a power law scaling of blackout frequency with blackout unserved energy as

$$F(S) \sim S^\alpha$$

where  $\alpha$  ranges from -0.6 to -1.9 (see Appendix 1 for a more detailed model). If we take  $\alpha = -1.2$ , and only account for the direct costs in  $C(S)$  according to (1), then

$$\text{risk} \sim S^{-0.2}$$

This indicates a moderate decrease in risk as blackout size increases. However, if we also account for the indirect costs of large blackouts, we expect much increased costs for larger blackouts relative to smaller blackouts. Then we can conclude that, although large blackouts are rarer than small blackouts, the risk of large blackouts is at least as great as the risk of small blackouts.

In contrast, consider the same risk calculation if the blackout frequency decreases exponentially with size so that

$$F(S) = A^{-S}$$

If we account for direct costs only, then

$$\text{risk} \sim S A^{-S}$$

and risk peaks for blackouts of some intermediate size and decreases exponentially for larger blackouts. Then, unless the peak risk occurs for blackouts comparable to the network size, we expect the risk of larger blackouts to be much smaller than the peak risk. It is plausible that this conclusion holds even if the indirect blackout costs are accounted for.

There is some uncertainty in assessing blackout costs, and especially the costs of large blackouts. However, the analysis above suggests that, when all the costs are considered, power tails in the blackout size frequency distribution can cause the risk of large blackouts to exceed the risk of the more frequent small blackouts. This is strong motivation for investigating both the cascading processes that typically occur in large blackouts and the global dynamics of *series* of blackouts that can cause power tails.

We now put the issue of power tails in context by discussing other aspects of blackout frequency that impact risk. The power tails are of course limited in extent in a practical power system by a finite cutoff near system size corresponding to the largest possible blackout. More importantly, the frequency of smaller blackouts and hence the shape of the frequency distribution away from the tail impacts the risk. Also significant is the absolute frequency of blackouts. When we consider the effect of mitigation on blackout risk, we need to consider changes in both the absolute frequency and the shape of the blackout frequency distribution.

### Mitigating failures in complex systems

Large disruptions can be intrinsic to the global system dynamics as is observed in systems displaying Self-Organized Criticality (SOC) [Bak87, Drossel92, Bak96, Jensen98]. A SOC system is one in which the nonlinear dynamics in the presence of perturbations organizes the overall average system state near to a critical state that is marginal to large disruptions. These systems are characterized by a spectrum of spatial and temporal scales of the disruption that exist in remarkably similar forms in a wide variety of different physical systems.

Systems that operate near criticality have power tails: the frequency of large disruptions decreases as a power function of the disruption size. This is in contrast to Gaussian systems or failures following a Weibull distribution in which the frequency decays exponentially with disruption size. Therefore, the application of traditional risk evaluation methods to such systems can underestimate the risk of large disruptions.

The success of mitigation efforts in SOC systems is strongly influenced by the dynamics of the system. One can understand SOC dynamics as including opposing forces that drive the system to a “dynamic equilibrium” near criticality in which disruptions of all sizes occur (see [Carreras00] for an explanation in a power systems context). Power tails are a characteristic feature of this dynamic equilibrium. Unless the mitigation efforts alter the self-organization forces driving the system, the system will be pushed to criticality. To alter those forces with mitigation efforts may be quite difficult because the forces are an intrinsic part of our society. Then the mitigation efforts can move the system to a new dynamic equilibrium while remaining near criticality and preserving the power tails. Thus, while the absolute frequency of disruptions of all sizes may be reduced, the underlying forces can still cause the relative frequency of large disruptions to small disruptions to remain the same.

Moreover, in some cases, efforts to mitigate small disruptions can even *increase* the frequency of large disruptions. This occurs because the large and small disruptions are not independent but are strongly coupled by the dynamics. Before discussing this in the more complicated case of power systems, we will illustrate this phenomenon with a forest fire model [Drossel92].

The forest fire model has trees that grow with a certain probability, lightning which strikes (and therefore lights fires)

with a certain probability and fires that spread to neighboring trees (if there are any) also with a given probability. The opposing forces in the forest are tree growth and fires, which act to increase and decrease the density of trees respectively. The forest settles to a dynamic equilibrium with a characteristic average density of trees. The rich dynamics of this model system has been extensively studied [Drossel92].

In our version of the forest fire model there are two types of forests. The first type is an uncontrolled forest in which the fires are allowed to burn themselves out naturally. The second type of forest has an efficient fire-fighting brigade that can extinguish small fires with a high probability. At first this appears to be a good thing; after all, we want to decrease damaging fires. However, in the longer run the effect of the fire fighting is to increase the density of flammable material (trees). Therefore when one fire is missed or a few start at once (from multiple lightning strikes), the fire brigade is overwhelmed and a major conflagration results. (This seems to be the cause of the large fires in the southeastern United States in 2001.) The enhanced probability of large fires can be seen in Figure 2 in which the frequency distribution of fire sizes is plotted for the two different situations. In the case where the small fires are efficiently extinguished, the large fire tail of the distribution is *significantly increased* over the case with no mitigation. This type of behavior is typical because in a complex system, there is a strong nonlinear coupling between the effect of mitigation and the frequency of the occurrence. Therefore even when mitigation is effective and eliminates the class of disruptions that it was designed for, it can have unexpected effects such as an increase in the frequency of other disruptions. As a result, the overall risk may be worse than the case with no mitigation.

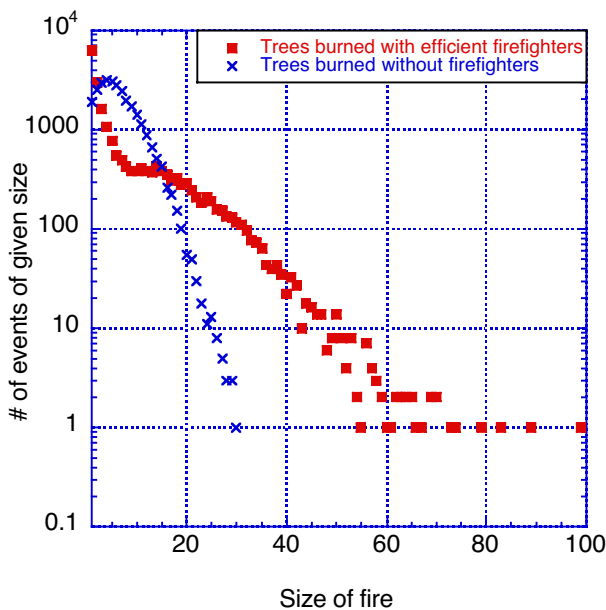


Figure 2: Frequency of forest fire sizes with and without fire fighting.

### Mitigating blackouts in power transmission systems

To study the dynamics of series of blackouts, we developed the ORNL-PSerc-Alaska (OPA) model [Dobson01, Carreras01b] that is described in more detail in Appendix 2. The OPA model shows how the slow opposing forces of load growth and network upgrades in response to blackouts can self organize the power system to dynamic equilibrium. Cascading blackouts are modeled by overloads and outages of lines determined in the context of LP dispatch of a DC load flow model. This model shows dynamical behaviors characteristic of complex systems and has a variety of transition points as power demand is increased [Dobson02, CarrerasCH]. The OPA model allows us to test some of the general complex systems ideas discussed above in the context of power transmission systems.

In the OPA model, overloaded transmission lines outage with a certain probability. To experiment with possible mitigation effects, we consider two types of mitigation:

- 1) Reducing the probability that an overloaded line outages. This strengthens the transmission lines. For example, this could roughly represent the effect of increased emergency ratings so that an overloaded line would be more likely to be able to operate while the overload was resolved by the operators. For our calculations we decreased the probability of an overloaded line suffering an outage by a factor of 15.
- 2) Requiring a certain minimum number of transmission lines to overload before any line outages can occur. This could represent operator actions that can effectively resolve overloads in a few lines, but are less effective for overloads in many lines. Our calculations used 30 for the minimum number of lines.

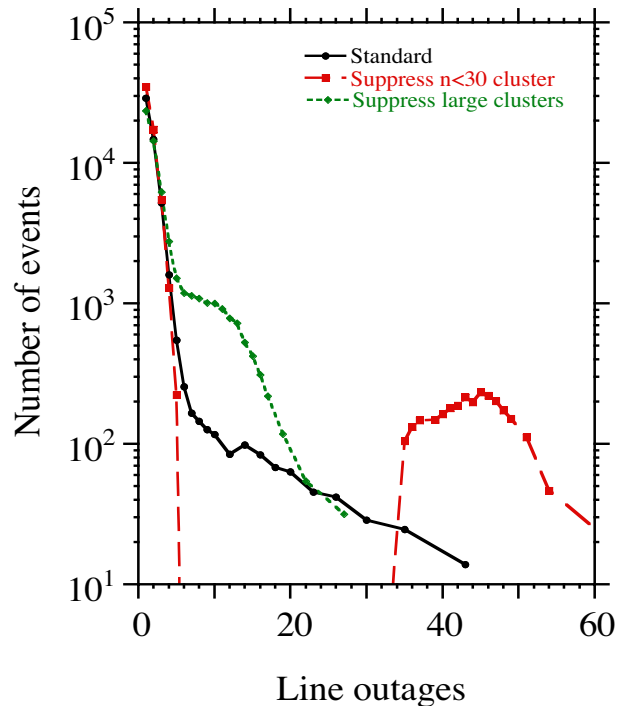
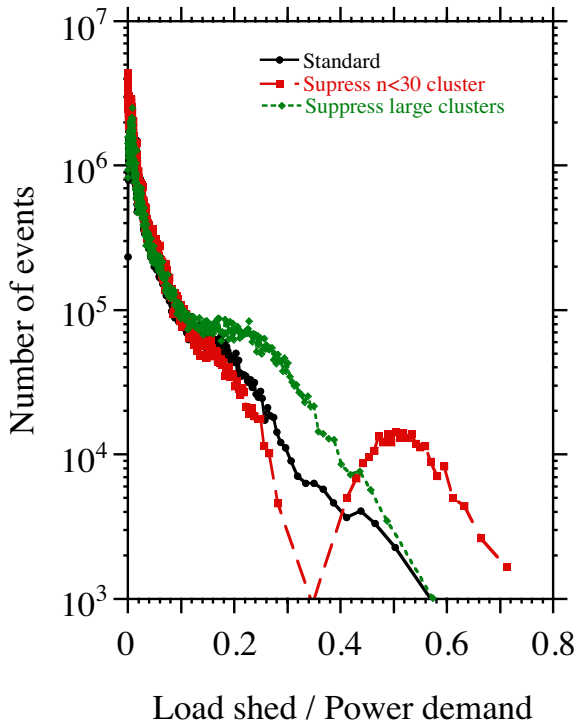


Figure 3: Effect of mitigation on the frequency distribution of number of line outages in blackouts.

We used the OPA model on a 190 node tree network to evaluate these types of mitigation. Figure 3 plots the logarithm of the number of blackouts as a function of the number of line outages for a time period of 80000 days in steady state. The logarithmic vertical scale emphasizes the rarer large blackouts, but this is appropriate given the risk analysis presented above. We can see that with no mitigation, there are blackouts with line outages ranging from zero to 43. When we decrease the line outage probability, blackouts with more than 27 line outages are eliminated. However, the total number of blackouts has increased by 9%. In the second case, all blackouts with less than 30 line outages have been eliminated, but we have a large increase in blackouts with more than 30 line outages.

In Figure 4, we have plotted the same three cases showing the distribution of load shed. Since large power shed is associated with a large number of line outages, Figures 3 and 4 give similar results. These results are broadly similar to the forest fire results described in the previous section.

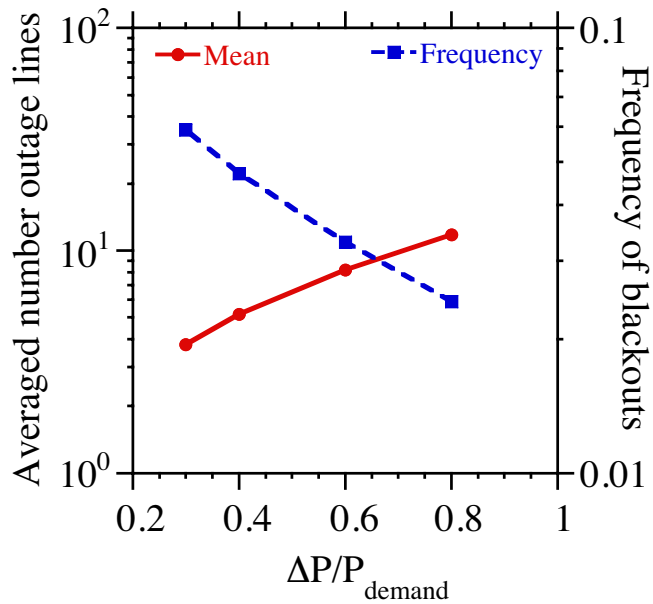
Although a detailed risk analysis along the lines suggested in the second section is needed to properly evaluate the good and bad effects of these mitigation measures on blackout risk, it is clear that both these mitigation measures have bad effects that could outweigh their good effects. Naive application of apparently sensible mitigation measures could be costly.



**Figure 4:** Effect of mitigation on the frequency distribution of fractional load shed in blackouts.

The preceding results are only meant to indicate the difficulty of identifying effective mitigation measures in a power transmission system exhibiting complex system dynamics and some of the non-intuitive consequences that can be observed. Possibly the simplest mitigation approach is the reduction of the overall number of blackouts. In the OPA

model, this can be done both by increasing the generator capacity margin and by increasing the rate of improving the transmission grid. We find that the frequency of blackouts decreases as the capacity margin increases (Figure 5). However, this only happens when this margin is greater than the standard deviation of the load demand fluctuations. When they are comparable, there are no simple mitigation measures that are effective in reducing the blackout frequency. Also note the increase in mean blackout size as blackout frequency decreases in Figure 5. The cost of these measures is likely to be high because they imply constant investments in both generation and transmission. Such investments may not be guaranteed in a deregulated open electricity market. When assessing the change in total risk caused by mitigation efforts, both the frequency of events of various sizes and their costs must be accounted for.



**Figure 5:** Blackout frequency and mean size as a function of generator capacity margin.

### Conclusion

Recent analyses of 15 years of blackout data from NERC have suggested that the frequency distribution of blackout sizes has a power tail. This power tail is consistent with the North American electric power transmission system operating as a complex system near criticality. The OPA model seeks to capture the global dynamics of series of blackouts and some OPA simulations have also shown near critical behavior and power tails that are similar to those in the NERC data.

Although there are some uncertainties in assessing blackout costs and especially the indirect costs, we argue that the presence of power tails has profound impact on blackout risk. Although larger blackouts are less frequent than smaller blackouts, the combination of the costs of large blackouts and power tails greatly increase the risk of the larger blackouts and it is plausible that the risk of large blackouts exceeds the risk of the smaller blackouts. The power tails also imply that

traditional probabilistic risk approaches that predict exponentially decaying tails are not applicable. Although the global dynamics of large blackouts involving complicated series of cascading rare events are challenging to study, our risk analysis implies that they merit research and mitigation efforts on the basis of their risk to society.

Complex system dynamics in the power transmission system also has important implications for mitigation efforts to reduce the risk of blackouts. As expected from studies of general self-organized critical systems, the OPA model shows that apparently sensible efforts to reduce the risk of smaller blackouts can sometimes increase the risk of large blackouts. This is due to the nonlinear interdependence of blackouts of different sizes caused by the dynamics. The possibility of an overall adverse effect on risk from apparently sensible mitigation efforts shows the importance of accounting for complex system dynamics when devising mitigation schemes.

Although in this paper we explore risk analysis and mitigation for electric power transmission systems, our complex system approach may find application to other large networked infrastructures, or to the extended, interdependent infrastructures of which electric power is an important part.

Our complex system approach, which implies interdependence between large and small blackouts, should be contrasted with an approach in which large and small blackouts occur independently as uncorrelated events. The difference between the two approaches cannot be deduced from a frequency distribution of blackout sizes (for these could be the same in both approaches) but from assumptions about the dynamics governing the system that produce these statistics.

The present version of the OPA model includes very simple representations of the parts of the power transmission system, but can nevertheless as a combined model yield complicated complex system behaviors. We intend to improve the modeling and understanding of the dynamics so that effective blackout mitigation measures can be devised and assessed from a complex systems perspective.

### Acknowledgments

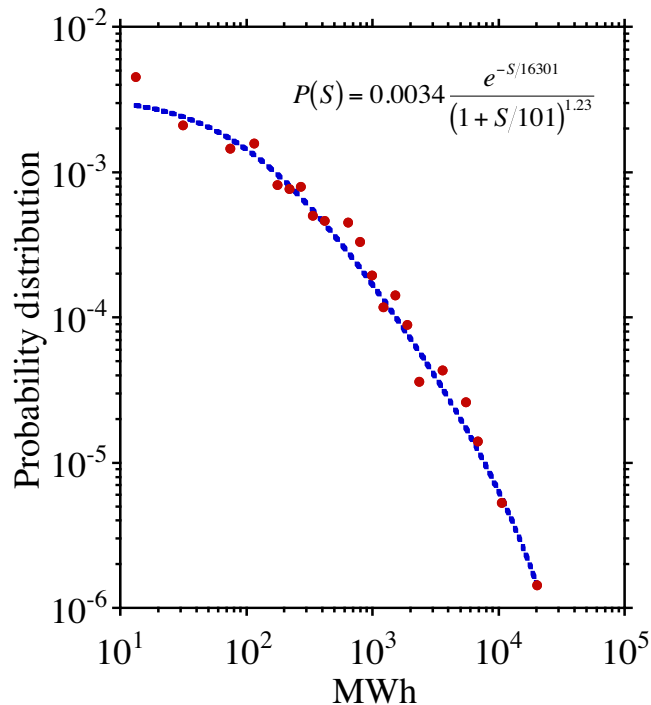
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### Appendix 1

Using the same NERC data as in [CarrerasPES], we assume the following form for the PDF of blackout size with respect to unserved energy  $S$ :

$$P(S) = \frac{ae^{-S/S_0}}{(1 + S/S_1)^\alpha} \tag{A1-1}$$

The values of  $S_0$  and  $S_1$  represent the finite-size cutoff. Fitting the functional form (A1-1) to the NERC data yields  $\alpha = 1.23 \pm 0.64$  (see Figure 6).



**Figure 6:** Blackout size probability distribution with size measured by MWh unserved.

### Appendix 2: The OPA model.

The OPA model [Dobson01, Carreras01b] represents the transmission system as a network of nodes (buses). The nodes are either loads (L), or generators (G). The power  $P_i$  injected at each node is positive for generators and negative for loads, and the maximum power injected is  $P_i^{\max}$ . The transmission line connecting nodes  $i$  and  $j$  has power flow  $F_{ij}$ , maximum power flow  $F_{ij}^{\max}$ , and the impedance of the line  $z_{ij}$ . There are  $N_N = N_G + N_L$  total nodes and  $N_l$  lines, where  $N_G$  is the number of generators and  $N_L$  is the number of loads.

The blackout model is based on the standard DC power flow equation,

$$F = AP \quad (\text{A2-1})$$

where  $F$  is a vector whose  $N_L$  components are the power flows through the lines,  $F_{ij}$ ,  $P$  is a vector whose  $N_N-1$  components are the power of each node,  $P_i$ , with the exception of the reference generator,  $P_0$ , and  $A$  is a constant matrix. The reference generator power is not included in the vector  $P$  to avoid singularity of  $A$  as a consequence of the overall power balance.

The input power demands are either specified deterministically or as an average value plus some random fluctuation around the average value. The random fluctuation is applied to either each individual load or to “regional” groups of load nodes.

The generator power dispatch is solved using standard LP methods. Using the input power demand, we solve the power flow equations (A2-1), with the condition of minimizing the following cost function:

$$\text{Cost} = \sum_{i \in G} P_i(t) - W \sum_{j \in L} P_j(t) \quad (\text{A2-2})$$

We assume that all generators run at the same cost and all loads have the same priority to be served. However, we enforce a high cost for load shed by setting  $W = 100$ . This minimization is done with the following constraints:

- 1) Generator power  $0 \leq P_i \leq P_i^{\max} \quad i \in G$
- 2) Load power  $P_j \leq 0 \quad j \in L$
- 3) Power flows  $|F_{ij}| \leq F_{ij}^{\max}$
- 4) Power balance  $\sum_{i \in G \cup L} P_i = 0$

This linear programming problem is numerically solved using the simplex method.

The OPA model is characterized by two intrinsic time scales. There is a slow time scale, of the order of days to years, over which load power demand slowly increases and the network is upgraded in engineering responses to blackouts. There is also a fast time scale, of the order of minutes to hours, over which cascading overloads or outages may lead to blackout.

The slow dynamics model the growth of the demand and response to the blackout by upgrades in the grid transmission capability. The slow dynamics is carried out by a simple set of rules. At the beginning of day  $t$ , we apply the following rules:

1. Growth of the power demand. All loads are multiplied by a fixed parameter  $\lambda$  that represents the daily rate of increase in electricity demand. On the basis of past electricity consumption, we estimate that  $\lambda = 1.00005$ . This value corresponds to a yearly rate of 1.8%.

$$P_i(t) = \lambda P_i(t-1) \quad \text{for } i \in L \quad (\text{A2-3})$$

2. Power transmission grid improvement. We assume a gradual improvement in the transmission capacity of the grid in response to the outages and blackouts. This improvement is implemented through an increase of  $F_{ij}^{\max}$  for the lines that have overload during a blackout. That is,

$$F_{ij}^{\max}(t) = \mu F_{ij}^{\max}(t-1) \quad (\text{A2-4})$$

if the line  $ij$  overloads during a blackout. We take  $\mu$  to be a constant;  $\mu$  is an important control parameter in the model.

3. Maximum generator power increase.  $\Delta P$  is the total maximum generation minus the total load power demand  $P_{\text{demand}}$  and  $\Delta P/P_{\text{demand}}$  is the generator capacity margin. The maximum generation is increased in response to the load demand as follows [Carreras 02]. When  $\Delta P/P_{\text{demand}}$  is below a threshold value, we choose a generator at random until a generator is found that has incident lines with the sum of power flow line limits 20% larger than the maximum generator power. Then the maximum power at that generator is increased by an amount  $\kappa P_{\text{demand}}/N_G$ , where  $\kappa$  is a constant that is a few percent. This process is repeated until  $\Delta P/P_{\text{demand}}$  is above its threshold value.
4. Daily power fluctuations. To represent the daily local fluctuations on power demand, all load powers are multiplied by a random number  $r$ , such that  $2 - \gamma \leq r \leq \gamma$ , with  $0 \leq \gamma \leq 2$ .
5. Initial random line outage. Lines are outaged with a fixed probability. (Line outages are implemented by multiplying line impedance by a large number and dividing the line flow limit  $F_{ij}^{\max}$  by another large number. In the present calculations, these numbers are of the order of 1000.)

After applying these rules to the network parameters, we look for a solution of the power flow problem using linear programming. In solving the power dispatch problem for low load power demands, the initial conditions are chosen in such a way that a feasible solution of the linear programming problem exists. That is, the initial conditions yield a solution without line overloads and without power shed. Increases in the average load powers and random load fluctuations can cause a solution of the linear programming with line overloads or requires load power to be shed. At this point, a cascading event may be triggered.

A cascading overload may start if one or more lines are overloaded in the solution of the linear programming problem. We consider a line overloaded if the power flow through this line is within 1% of  $F_{ij}^{\max}$ . At this point, we assume that there is a probability  $p_2$  that an overloaded line will suffer an outage. If an overloaded line outages, a new solution is calculated. This process can require multiple iterations and continues until a solution is found with no more line outages.

This fast dynamics model does not attempt to capture the intricate details of particular blackouts, which may have a large



variety of complicated interacting processes also involving, for example, protection systems, dynamics and human factors. However, the fast dynamics model does represent cascading overloads and outages that are consistent with some basic network and operational constraints.

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