

An integrated single-vendor single-buyer production-inventory model for items with imperfect quality and inspection errors

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ABSTRACT

In this paper, we develop an integrated vendor-buyer production-inventory model for items with imperfect quality and inspection errors. The production process is imperfect and produces a certain number of defective items with a known probability density function. We consider the policy in which the delivery quantity to the buyer is identical at each shipment. Once the buyer receives the lot, a 100% screening process of the lot is conducted, and the screening process and demand proceed simultaneously. The screening process is not perfect. The inspector may incorrectly classify a non-defective item as defective, or incorrectly classify a defective item as non-defective. The expected integrated total annual cost of the vendor and the buyer is derived and a solution procedure is provided to find the optimal solution. Numerical examples show that the integrated model gives an impressive cost reduction in comparison to an independent decision by the buyer.

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1. Introduction

With the growing focus on supply chain management in the past few decades, firms have been attempting to achieve greater collaborative advantages with their supply chain partners. Since firms realize that inventory across the supply chain can be more efficiently managed through greater cooperation and better coordination, integrated inventory management has recently received a great deal of attention. Goyal (1976) considered the joint optimization problem of a single vendor and a single buyer, in which he assumed that the vendor's production rate is infinite. Banerjee (1986) assumed a finite rate of production and developed a joint economic-lot-size model for the product with a lot-for-lot shipment policy. Goyal (1988) developed a joint total relevant cost model for a single-vendor single-buyer production inventory system in which each production batch was shipped to the buyer in smaller lots of equal size. Goyal and Gupta (1989) reviewed the related literature in integrated vendor-buyer inventory models. Aderohunmu et al. (1995) showed that a significant cost reduction could be achieved to the advantage of both the vendor and the buyer in a just-in-time environment when timely cost information was shared between the parties.

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Lu (1995) considered the single-vendor multi-buyer integrated inventory problem with the objective of minimizing the vendor's total cost subject to the maximum costs which the buyers were prepared to incur. Goyal (1995) suggested an approach that was capable of obtaining lower total joint relevant costs of the single-vendor single-buyer production inventory systems; i.e., the ratio between the $(i+1)$ st shipment and the i th shipment is equal to the ratio between the vendor's production rate and the demand rate of the buyer. Hill (1997) illustrated that, in general, neither of the two policies described in Lu (1995) and Goyal (1995) would be optimal. Ha and Kim (1997) developed an integrated just-in-time lot-splitting model of facilitating multiple shipments in small lots of a single-vendor-single-buyer system under deterministic conditions for a single product, and showed that when the integrated optimal policy was adopted by both the vendor and the buyer in a cooperative manner, both parties could benefit. Viswanathan (1998) presented the results of a detailed numerical study that analyzed the relative performance of the two different strategies of equal- and unequal-sized batch shipments as described in Lu (1995) and Goyal (1995). The numerical study showed that neither strategy dominated the other for all problem parameters. In other words, the best policy for the model depended on the specific problem parameters.

Hill (1999) determined the optimal production and inventory policy for a vendor manufacturing to supply to a single buyer. The policy turned out to be a combination of Goyal's (1995) policy and an equal shipment size policy. Hoque and Goyal (2000) extended the idea of Goyal and Szendrovits (1986) and developed an optimal solution procedure for the single-vendor single-buyer integrated production–inventory system with both equal and unequal sized shipments, in which capacity constraint of the transport equipment was considered. Goyal and Nebebe (2000) considered the problem of determining economic production and the shipment policy of a product supplied by a vendor to a buyer. The objective was to minimize the total joint costs incurred by the vendor and the buyer. Wu and Ouyang (2003) considered the integrated single-vendor single-buyer inventory system with shortage. They proposed an algebraic procedure to find the optimal order quantity, the maximum shortage level, and the optimal number of deliveries from the vendor to the buyer per order of the integrated total cost of the vendor and the buyer without using differential calculus. Ouyang et al. (2004) considered a stochastic lead time demand and assumed that shortages could happen during lead time, and lead time could be reduced at an added cost.

Chang et al. (2006) studied the lead time and ordering cost reduction problem in the single-vendor single-buyer integrated inventory model. Hill and Omar (2006) revisited the single-vendor single-buyer integrated production inventory problem by allowing the holding cost to decrease down the supply chain. Hoque and Goyal (2006) developed a heuristic solution procedure to minimize the total cost of setup or ordering, inventory holding and lead time crashing for an integrated inventory system under controllable lead time between a vendor and a buyer. Sarmah et al. (2006) reviewed literature dealing with buyer vendor coordination models that have used quantity discount as a coordination mechanism under a deterministic environment and classified the various models. Siajedi et al. (2006) presented a new methodology to obtain the joint economic lot size in the case where multiple buyers were demanding one type of item from a single vendor. Chan and Kingsman (2007) proposed a coordinated single-vendor multi-buyer supply chain by synchronizing delivery and production cycles. Ertogral et al. (2007) incorporated transportation cost explicitly into a joint vendor-buyer inventory model under equal-shipment policy. Zhou and Wang (2007) developed a more general production-inventory model for a single-vendor-single-buyer integrated system. Their model neither requires that the buyer's unit holding cost be greater than the vendor's nor assumes the structure of shipment policy. Other studies on the joint economic lot sizing problem up to the year 2007 can be found in Ben-Daya et al. (2008). Chung (2008) showed that Wu and Ouyang's (2003) proof of a global cost minimum was incomplete. He then justified the algorithm described in Wu and Ouyang (2003). Ben-Daya and Al-Nassar (2008) developed a cost minimization supply chain coordination model for a three-layer supply chain

involving suppliers, manufacturers, and retailers, and derived a solution procedure for the model. Huang et al. (2010a, 2010b), by using different cost structures for the two studies, incorporated order-processing cost reduction and permissible delay in payments into the integrated vendor-buyer inventory policy.

The classical economic order quantity (EOQ) model assumes that items produced are of perfect quality, which is usually not the case in real production. Porteus (1986) introduced a model that showed a significant relationship between quality and lot size. Lee and Rosenblatt (1987) addressed the problem of joint control of production cycles or manufacturing quantities, and maintenance by inspections. Schwaller (1988) extended the EOQ model by adding the assumptions that defective items of a known proportion were present in incoming lots, and that fixed and variable inspection costs were incurred in finding and removing these defective items. It was also assumed that the supplier reimbursed the buyer for any defective items found and removed. Ben-Daya and Hariga (2000) studied the effect of imperfect production processes on the economic lot sizing policy (ELSP). They developed a mathematical model for ELSP, taking into account the effect of imperfect quality and process restoration. Salameh and Jaber (2000) developed an economic order quantity model where a random proportion of the items in a lot are defective.

Huang (2002) extended the integrated vendor-buyer inventory model by accounting for imperfect quality items. He considered the situation where the delivery quantity sent to the buyer was identical for each shipment. Goyal et al. (2003) used the Goyal and Cárdenas-Barrón (2002) model to determine an optimal integrated vendor-buyer inventory policy for an item with imperfect quality. Huang (2004) extended the model of Ha and Kim (1997) by incorporating imperfect items into the production-inventory model. Lo et al. (2007) assumed a varying rate of deterioration, partial backordering, inflation, imperfect production processes, and multiple deliveries, and developed an integrated production and inventory model from the perspectives of both the manufacturer and the retailer.

Cárdenas-Barrón (2009) developed an economic production quantity (EPQ) inventory model with planned backorders for determining the economic production quantity and the size of backorders for a single product, which was made in a single-stage manufacturing process that generated imperfect quality products and required that all defective products be reworked at the same cycle. Sana (2010) investigated an EPL (Economic Production Lot size) model in an imperfect production system in which the production facility could shift from an 'in-control' state to an 'out-of-control' state at any random time. He formulated the model assuming that a certain percent of total product was defective (imperfect) in an 'out-of-control' state. This percentage also varied with production rate and production run time. Chung (2011) revisited the work of Cárdenas-Barrón (2009) and developed the sufficient and necessary condition for the existence of the solution satisfying the first derivative condition of the annual total relevant cost and presented a concrete solution procedure to find the optimal solution. Liao and Sheu (2011) described an integrated EPQ model that incorporated EPQ and maintenance programs. For more recent works on inventory models with imperfect quality items, we refer the readers to Khan et al. (2011a).

The Salameh and Jaber (2000) model assumed that there is no human error in the screening process. Raouf et al. (1983) studied human errors in inspection. They came up with one of the first inspection plans with misclassifications for multi-characteristic critical components. Duffuaa and Khan (2002) proposed a general inspection plan for quality assurance of critical multi-characteristic components. They extended the Raouf et al. (1983) inspection plan for the case of six types of misclassification errors, where an inspector could classify an item to be good, rework or scrap. Duffuaa and Khan (2005) carried out a sensitivity analysis to investigate the statistical and economic impact of the several types of misclassification errors on the performance measures of the inspection plan. Kok and Shang (2007) studied a single-period inventory system with inventory record inaccuracy. Yoo et al. (2009) proposed a profit-maximizing economic production quantity model that incorporated both imperfect production quality and two-way imperfect inspection; i.e., the Type I inspection error of incorrectly classifying a

non-defective item as defective and the Type II inspection error of incorrectly classifying a defective as non-defective. Lin (2009) developed a production-inventory model for a simple supply chain system with defective items and inspection errors, where he assumed that both the Type I and Type II inspection errors are known constants. Khan et al. (2011b) extended the work of Salameh and Jaber (2000) model by assuming that the inspection process was not error-free. Hsu (2012) pointed out a contradiction in Khan et al.'s (2011b) mathematical model and developed a corrected EOQ.

The purpose of this paper is to develop an integrated vendor-buyer inventory model with imperfect product quality and inspection errors. The rest of this paper is organized as follows: In section 2, the notation and assumptions used in this paper are introduced. In section 3, we develop a mathematical model that integrates the vendor's and the buyer's annual cost and takes into consideration imperfect production processes and inspection errors. Section 4 provides a numerical example and its sensitivity analysis to various parameters to illustrate important aspects of the model. Finally, in section 5 we summarize and conclude the paper and provide directions for future research.

2. Notation and assumptions

We consider a two-stage supply chain problem with a single vendor and a single buyer. The buyer has an annual demand of D units for the given product, and places regular orders of fixed size Q_p . The vendor prepares for the repeating flow of orders of size $Q_p = nQ$ from the buyer by producing items in batches of size Q_p and by planning to have each batch delivered to the buyers in n deliveries, each with a lot of Q units. The vendor fulfills the deliveries of Q units with a known and fixed lead time. Since the production system is not perfect, some of the items produced may be defective. Once the buyer receives the items, a 100% screening process is conducted. We assume the screening process and demand take place simultaneously. The screening process is also not perfect, involving Type I and Type II inspection errors. The objective is to minimize the total joint annual costs incurred by the vendor and the buyer. The following notation and assumptions are used in our model:

Q_p	the size of a production batch of items at the vendor
Q	the size of the deliveries from the vendor to the buyer
n	the number of deliveries per batch production run, a positive integer ($Q_p = nQ$)
D	the annual demand of the buyer
P	the annual production rate ($P > D$) at the vendor
x	the screening (inspection) rate
S_v	the setup cost per production run for the vendor
S_B	the ordering cost per order for the buyer
γ	the probability that an item produced is defective
$f(\gamma)$	the probability density function of γ
e_1	the probability of a Type I error (classifying a non-defective item as defective)
$f(e_1)$	the probability density function of e_1
e_2	the probability of a Type II error (classifying a defective item as non-defective)
$f(e_2)$	the probability density function of e_2
c_i	the buyer's inspection cost per unit
c_w	the vendor's unit cost for producing a defective item
c_{aB}	the buyer's cost of a post-sale defective item
c_{av}	the vendor's cost of a post-sale defective item
c_a	the cost of accepting a defective item ($c_a = c_{aB} + c_{av}$)
c_r	the cost of rejecting a non-defective item

h_v	the holding cost per unit per year for the vendor
h_B	the holding cost per unit per year for the buyer
F	the freight (transportation) cost per delivery (including the delivery of Q units from the vendor to the buyer and the returned items from the buyer to the vendor)
B_1	the number of items that are classified as defective in each delivery of Q units
B_2	the number of items that are returned from the market in each delivery of Q units
T	the time interval between successive deliveries of Q units
T_1	the period during which the vendor produces
T_2	the period during which the vendor supplies from inventory
T_c	the cycle time $= T_1 + T_2 = nT$
*	the superscript representing optimal value

Assumptions:

1. The demand rate is known, constant, and continuous.
2. The lead time is known and constant.
3. The production processes are imperfect and may produce defective items. The defective percentage γ has a probability density function $f(\gamma)$.
4. The inspection process is also imperfect. The probability of classifying a non-defective item as defective is e_1 with a probability density function $f(e_1)$.
5. The probability of classifying a defective item as non-defective is e_2 with a probability density function $f(e_2)$.
6. The buyer returns all items classified as defective and those returned from the customers to the vendor at the end of the 100% screening process, and receives a full price refund from the vendor. Thus, a defective item incurs a cost of c_w for the vendor. The vendor will sell the returned items at a discounted price to a secondary market. Therefore, c_r (the cost of rejecting a non-defective item) is the difference between the regular and the discounted prices.
7. Customers who buy the defective items will detect the quality problem and return them to the buyer and receive a good (replaced) item from the buyer. Both the vendor and the buyer incur a post-sale failure cost (e.g., loss of good will) for the items returned from the market.
8. Shortages are not allowed.
9. A single product is considered.
10. There is a single vendor and a single buyer.

3. Mathematical model

3.1 The buyer's cost per cycle

Consider a lot of size Q being delivered to the buyer. It is assumed that the lot is 100% screened at the beginning of each replenishment cycle. Each lot contains a percentage γ of defective items with the probability density function $f(\gamma)$. While screening the items, an inspector may incorrectly classify a non-defective item as defective with probability e_1 and the probability density function $f(e_1)$, or the

inspector may incorrectly classify a defective item as non-defective with probability e_2 and the probability density function $f(e_2)$. It is assumed that the buyer returns all the items classified as defective by the inspector and the items returned from the market to the vendor as a single batch at the end of the screening process and receives a full price refund from the vendor. Thus, each defective item will incur a cost of c_w and each non-defective item classified as defective will incur a cost of c_r for the vendor. For example, if the production cost is \$30/unit, the regular selling price is \$50/unit, and the discounted selling price is \$20/unit, then $c_w = \$10$ and $c_r = \$30$. For each item being returned from the market, the buyer and the vendor incur a post-sale failure cost (e.g., the loss of good will) c_{aB} and c_{av} respectively. If the vendor and the buyer do not work together in a cooperative manner towards maximizing their mutual benefits, and the buyer makes his own decisions independent of the vendor, then the vendor will produce and deliver the items to the buyer on a lot-for-lot basis. Fig. 1 illustrates the behavior of the inventory level over time for the buyer.

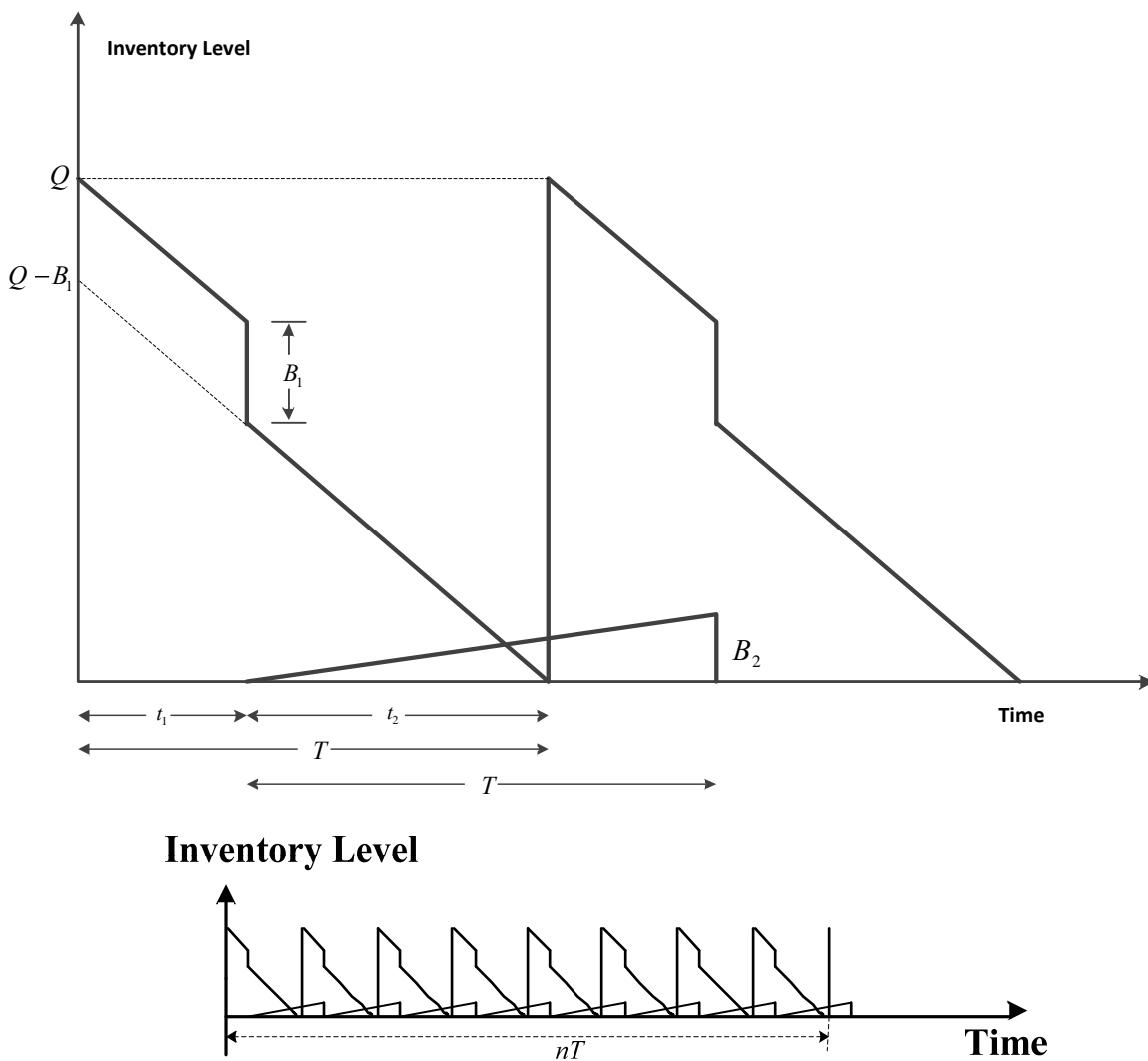


Fig.1. Behavior of the buyer's inventory level over time

By definition, we have

$$B_1 = Q(1 - \gamma)e_1 + Q\gamma(1 - e_2), \tag{1}$$

and

$$B_2 = Q\gamma e_2. \tag{2}$$

We assume that for each item returned from the market, a good item is replaced for the customer. Based on this assumption, there are two streams of demand; namely, the regular demand and the demand to replace the returned items. Let D' be the effective demand; then we have $D' = D + B_2/T$. By definition, the cycle length of each delivery of size Q is $T = (Q - B_1)/D'$. Substituting $D' = D + B_2/T$ and solving the equation, we have

$$T = \frac{(Q - B_1 - B_2)}{D} = \frac{Q(1 - \gamma)(1 - e_1)}{D}. \tag{3}$$

The holding cost per delivery cycle is

$$HC = h_B \left\{ B_1 t_1 + \frac{(Q - B_1)T}{2} \right\} + h_B \left(\frac{B_2 T}{2} \right).$$

Substituting t_1 with Q/x , B_1 with $Q(1 - \gamma)e_1 + Q\gamma(1 - e_2)$, B_2 with $Q\gamma e_2$ and T with $Q(1 - \gamma)(1 - e_1)/D$, the holding cost per delivery cycle is given by

$$HC = h_B \left\{ \frac{Q^2}{x} [(1 - \gamma)e_1 + \gamma(1 - e_2)] + \frac{Q^2 \{1 - (e_1 + \gamma) + \gamma(e_1 + 2e_2)\} (1 - \gamma)(1 - e_1)}{2D} \right\}.$$

After adding the ordering, transportation, inspection, and post-sale failure costs, the cost per production cycle for the buyer is

$$TC_B(n, Q) = S_B + nF + nc_i Q + nc_{ab} Q \gamma e_2 \tag{4}$$

$$+ nh_B \left\{ \frac{Q^2}{x} [(1 - \gamma)e_1 + \gamma(1 - e_2)] + \frac{Q^2 \{1 - (e_1 + \gamma) + \gamma(e_1 + 2e_2)\} (1 - \gamma)(1 - e_1)}{2D} \right\}.$$

3.2 The vendor's cost per cycle

Fig. 2 shows the vendor's holding cost per cycle can be obtained as (see, for example, Goyal et al. (2003) and Huang (2004)).

Holding cost per cycle = h_v [bold area - shaded area]

$$\begin{aligned} &= h_v \left\{ \left[nQ \left(\frac{Q}{P} + (n-1)T \right) - \frac{nQ(nQ/P)}{2} \right] - T[Q + 2Q + \dots + (n-1)Q] \right\} \\ &= h_v \left\{ \left[nQ \left(\frac{Q}{P} + (n-1) \frac{Q(1-\gamma)(1-e_1)}{D} \right) - \frac{nQ(nQ/P)}{2} \right] - \frac{Q(1-\gamma)(1-e_1)}{D} [Q + 2Q + \dots + (n-1)Q] \right\} \tag{5} \\ &= h_v \left\{ \frac{nQ^2}{P} - \frac{n^2 Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-e_1)}{2D} \right\}. \end{aligned}$$

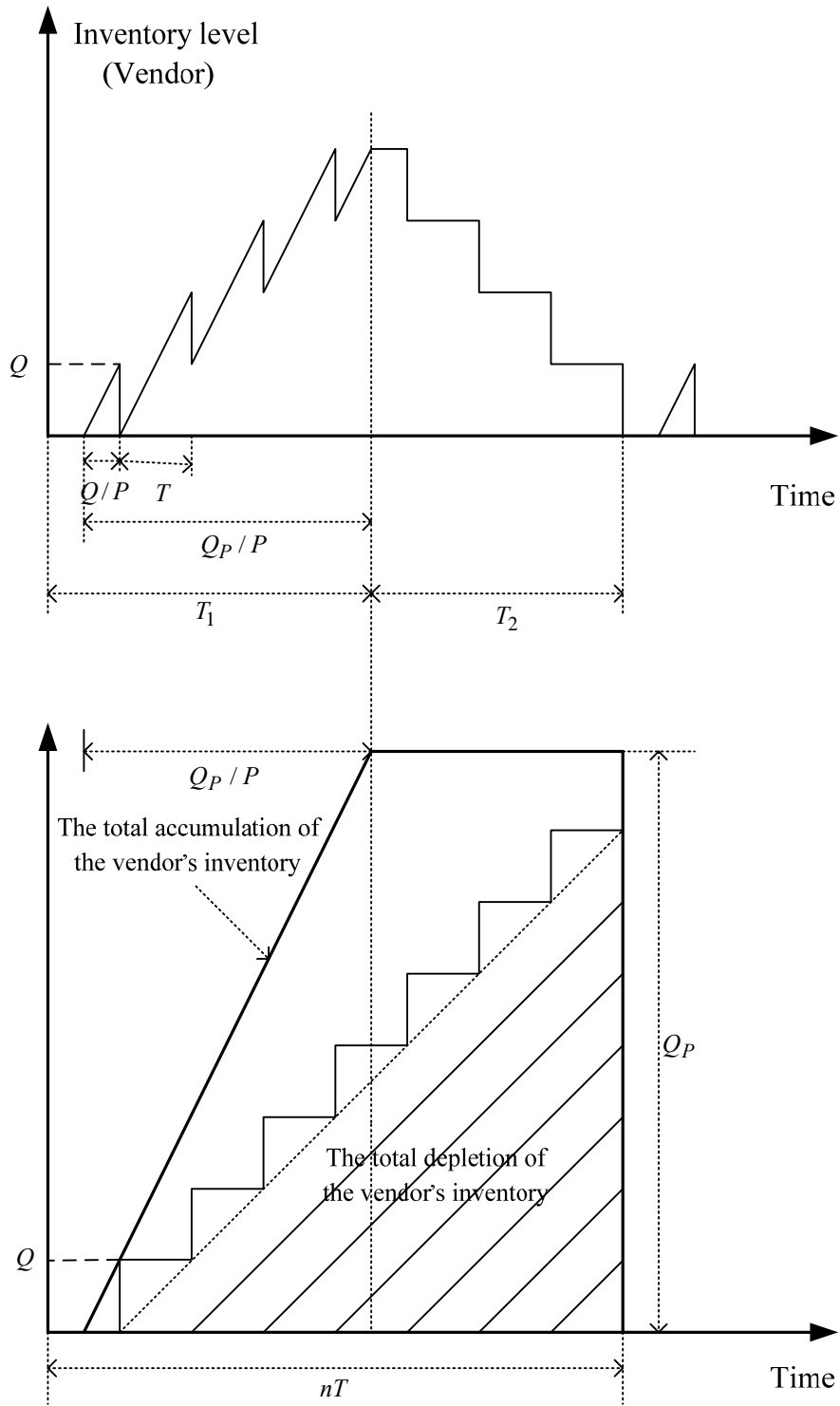


Fig. 2. Time-weighted inventory for the vendor

After adding the costs of setup, warranty, and Type I and Type II errors, the vendor's total cost per cycle $TC_v(n, Q)$ is

$$TC_v(n, Q) = S_v + nc_w Q\gamma + nc_r Q(1 - \gamma)e_1 + nc_{av} Q\gamma e_2 + h_v \left\{ \frac{nQ^2}{P} - \frac{n^2 Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-e_1)}{2D} \right\}. \quad (6)$$

3.3 The integrated vendor-buyer inventory model

The total vendor-buyer integrated cost per production cycle is

$$\begin{aligned}
 TC_c(n, Q) = TC_v(n, Q) + TC_B(n, Q) = & S_v + S_B + nF + nc_iQ + nc_wQ\gamma + nc_aQ\gamma e_2 + nc_rQ(1-\gamma)e_1 \\
 & + nh_B \left\{ \frac{Q^2}{x} [(1-\gamma)e_1 + \gamma(1-e_2)] + \frac{Q^2 \{1 - (e_1 + \gamma) + \gamma(e_1 + 2e_2)\} (1-\gamma)(1-e_1)}{2D} \right\} \\
 & + h_v \left\{ \frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-e_1)}{2D} \right\}.
 \end{aligned} \tag{7}$$

Since the cycle time $T_c = n \frac{Q(1-\gamma)(1-e_1)}{D}$, one has

$$E[T_c] = n \frac{Q(1-E[\gamma])(1-E[e_1])}{D}. \tag{8}$$

Using the renewal reward theorem, the expected total annual cost of the vendor and the buyer is

$$\begin{aligned}
 ETC(n, Q) = \frac{E[TC_c(n, Q)]}{E[T_c]} = & \frac{(S_v + S_B + nF)D}{nQ(1-E[\gamma])(1-E[e_1])} \\
 & + \frac{D\{c_i + c_wE[\gamma] + c_aE[\gamma]E[e_2] + c_r(1-E[\gamma])E[e_1]\}}{(1-E[\gamma])(1-E[e_1])} \\
 & + h_B Q \left\{ \frac{D\{1-E[\gamma]E[e_1] + E[\gamma](1-E[e_2])\}}{x(1-E[\gamma])(1-E[e_1])} + \frac{E[A]}{2(1-E[\gamma])(1-E[e_1])} \right\} \\
 & + h_v \left\{ \frac{QD}{P(1-E[\gamma])(1-E[e_1])} - \frac{nQD}{2P(1-E[\gamma])(1-E[e_1])} + \frac{(n-1)Q}{2} \right\},
 \end{aligned} \tag{9}$$

with

$$\begin{aligned}
 A = & \{1 - (e_1 + \gamma) + \gamma(e_1 + 2e_2)\} (1-\gamma)(1-e_1) \\
 = & 1 - 2(\gamma + e_1) + 4\gamma e_1 + 2\gamma e_2(1-e_1) + \gamma^2(1-2e_1-2e_2+2e_1e_2+e_1^2) + e_1^2(1-2\gamma),
 \end{aligned}$$

and

$$\begin{aligned}
 E[A] = & 1 - 2(E[\gamma] + E[e_1]) + 4E[\gamma]E[e_1] + 2E[\gamma]E[e_2](1-E[e_1]) \\
 & + E[\gamma^2](1-2E[e_1]-2E[e_2] + 2E[e_1]E[e_2] + E[e_1^2]) + E[e_1^2](1-2E[\gamma]).
 \end{aligned}$$

Taking the first derivative of $ETC(n, Q)$ with respect to Q , we have

$$\frac{\partial ETC(n, Q)}{\partial Q} = - \frac{(S_v + S_B + nF)D}{nQ^2(1-E[\gamma])(1-E[e_1])} \tag{10}$$

$$\begin{aligned}
& + h_B \left\{ \frac{D\{(1-E[\gamma])E[e_1] + E[\gamma](1-E[e_2])\}}{x(1-E[\gamma])(1-E[e_1])} + \frac{E[A]}{2(1-E[\gamma])(1-E[e_1])} \right\} \\
& + h_v \left\{ \frac{D}{P(1-E[\gamma])(1-E[e_1])} - \frac{nD}{2P(1-E[\gamma])(1-E[e_1])} + \frac{(n-1)}{2} \right\}.
\end{aligned}$$

Taking the second derivative, we have

$$\frac{\partial^2 ETC(n, Q)}{\partial Q^2} = \frac{2(S_v + S_B + nF)D}{nQ^3(1-E[\gamma])(1-E[e_1])}. \quad (11)$$

Note that if $E[\gamma] < 1$ and $E[e_1] < 1$, we have $\frac{\partial^2 ETC(n, Q)}{\partial Q^2} > 0$, which implies that at a particular value of n , the total integrated annual cost is a convex function and there exists a unique value of Q that minimizes Eq. (9), which is given as

$$Q^*(n) = \sqrt{\frac{2(S_v + S_B + nF)D}{n \left[h_v \left((n-1)(1-E[\gamma])(1-E[e_1]) + (2-n) \frac{D}{P} \right) + h_B \left(2 \frac{D}{x} \{(1-E[\gamma])E[e_1] + E[\gamma](1-E[e_2])\} + E[A] \right) \right]}} \quad (12)$$

3.4 Solution procedure

The problem is to determine the value of n that minimizes $ETC(n, Q)$. Because the number of shipments per batch production run, n , is a discrete variable, one can find the optimal value of n from the following procedure:

1. For a range of n values, determine the corresponding $Q^*(n)$ using Eq.(12), and compute $ETC(n, Q^*(n))$ by substituting $Q^*(n)$ into Eq.(9).
2. Derive the optimal value of n , denoted by n^* , such that $ETC(n, Q^*(n)) \leq ETC(n-1, Q^*(n-1))$, and $ETC(n, Q^*(n)) \leq ETC(n+1, Q^*(n+1))$.

Once we obtain the n^* value, the optimal size of a production batch Q_p^* is given by $Q_p^* = n^* Q^*(n^*)$.

3.5 The buyer's independent optimal solution

If the vendor and the buyer do not work together in a cooperative manner towards maximizing their mutual benefits, and the buyer makes his own decisions independent of the vendor, then the vendor will produce and deliver the items to the buyer on a lot-for-lot basis. The buyer's expected annual cost is given as

$$\begin{aligned}
ETC_B(Q) &= \frac{E[TC_B(n, Q)]}{E[T]} = \frac{(S_B + F)D}{Q(1-E[\gamma])(1-E[e_1])} + \frac{D(c_i + c_{ab}E[\gamma]E[e_2])}{(1-E[\gamma])(1-E[e_1])} \\
& + h_B Q \left\{ \frac{D\{(1-E[\gamma])E[e_1] + E[\gamma](1-E[e_2])\}}{x(1-E[\gamma])(1-E[e_1])} + \frac{E[A]}{2(1-E[\gamma])(1-E[e_1])} \right\}, \quad (13)
\end{aligned}$$

and the optimal solution is

$$Q_B^* = \sqrt{\frac{2(S_B + F)D}{h_B \left(2 \frac{D}{x} \{ (1 - E[\gamma])E[e_1] + E[\gamma](1 - E[e_2]) \} + E[A] \right)}} \tag{14}$$

The vendor's expected annual cost is given as

$$ETC_v(Q) = \frac{E[TC_c(Q)]}{E[T]} = \frac{S_v D}{Q(1 - E[\gamma])(1 - E[e_1])} + \frac{D\{c_w E[\gamma] + c_r(1 - E[\gamma])E[e_1] + c_{av} E[\gamma]E[e_2]\}}{(1 - E[\gamma])(1 - E[e_1])} + h_v \left\{ \frac{QD}{2P(1 - E[\gamma])(1 - E[e_1])} \right\} \tag{15}$$

If the decision is made solely from the buyer's perspective, then substituting Q_B^* into Eqs. (13) and (15), we obtain the buyer's and the vendor's expected annual costs respectively.

4. Numerical examples and sensitivity analysis

Consider an integrated vendor-buyer cooperative inventory model with the following parameters:

- Production rate, P = 160,000 units/year
- Demand rate, D = 50,000 units/year
- Inspection rate, x = 175,200 units/year
- Setup cost for vendor, S_v = \$300/production run
- Ordering cost for buyer, S_B = \$100/order
- Holding cost for vendor, h_v = \$2/unit/year
- Holding cost for buyer, h_B = \$5/unit/year
- Freight (transportation) cost, F = \$25/delivery
- Inspection cost, c_i = \$0.5/unit
- The cost of producing a defective item, c_w = \$50/unit
- The cost of rejecting a non-defective item, c_r = \$100/unit
- The buyer's post-sale failure cost, c_{aB} = \$200/unit
- The vendor's post-sale failure cost, c_{av} = \$300/unit

If the defective percentage and inspections errors follow a uniform distribution with

$$f(\gamma) = \begin{cases} \frac{1}{\beta}, 0 \leq \gamma \leq \beta \\ 0, otherwise \end{cases} \quad f(e_1) = \begin{cases} \frac{1}{\lambda}, 0 \leq e_1 \leq \lambda \\ 0, otherwise \end{cases} \quad f(e_2) = \begin{cases} \frac{1}{\eta}, 0 \leq e_2 \leq \eta \\ 0, otherwise \end{cases}$$

then, we have

$E[\gamma] = \int_0^\beta \gamma f(\gamma) d\gamma = \int_0^\beta \frac{\gamma}{\beta} d\gamma = \frac{\beta}{2}$, $E[\gamma^2] = \int_0^\beta \gamma^2 f(\gamma) d\gamma = \int_0^\beta \frac{\gamma^2}{\beta} d\gamma = \frac{\beta^2}{3}$, $E[e_1] = \frac{\lambda}{2}$, $E[e_1^2] = \frac{\lambda^2}{3}$, and $E[e_2] = \frac{\eta}{2}$. Specifically, if $\beta = \lambda = \eta = 0.04$, we obtain the expected total annual cost of the vendor and the buyer as a function of n and Q , which is given in Fig. 3.

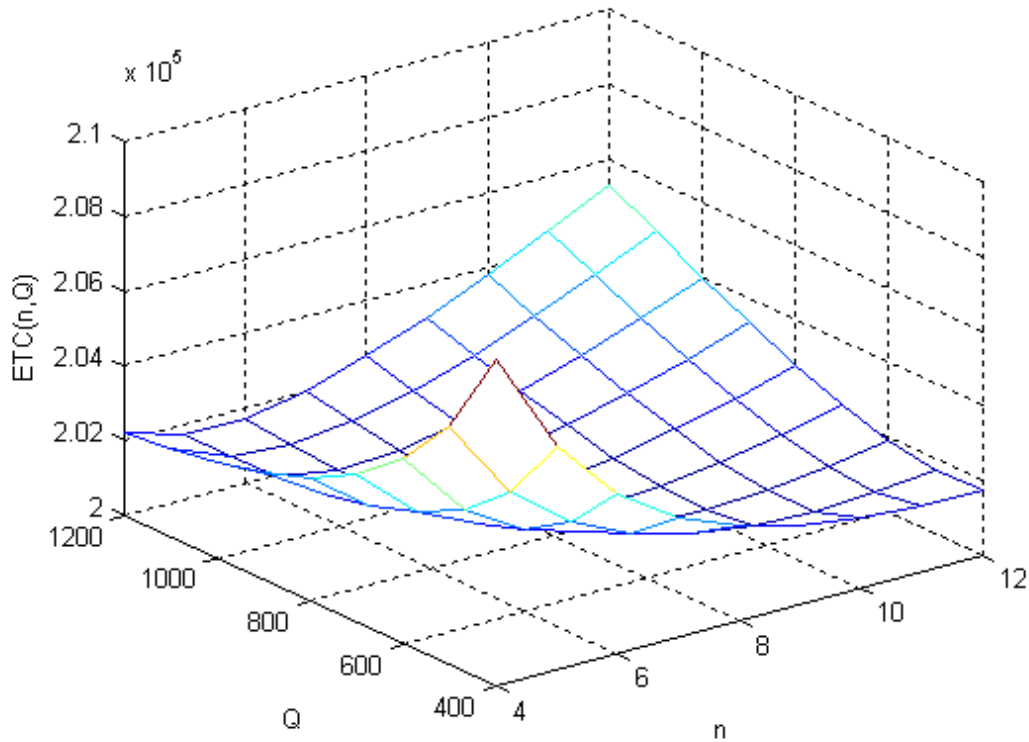


Fig. 3. The expected total annual cost of the vendor and the buyer

The three dimensional graph shows that the integrated expected total annual cost is convex, and that there exists a unique solution minimizing the integrated expected total annual cost. For the given example, the optimal solution is (see Table 1) $n^* = 7$, $Q^* = 791$, and the minimum total expected annual cost is \$201,358.50.

Table 1

Optimal solutions of $n, P=160,000, D=50,000, x = 175,200, S_v = 300, S_B = 100,$

$h_v = 2, h_B = 5, F=25, c_i=0.5, c_w=50, c_r=100, c_{aB}=200, c_{av}=300, \beta = \lambda = \eta = 0.04$

n	$Q^*(n)$	$ETC(n, Q^*(n))$	n	$Q^*(n)$	$ETC(n, Q^*(n))$
1	2,817.4942	206,251.9011	9	664.6448	201,424.7759
2	1,839.4721	203,281.7266	10	618.0560	201,496.0917
3	1,411.6533	202,224.2429	11	578.8818	201,583.0628
4	1,163.0292	201,736.5636	12	545.4389	201,681.3327
5	998.2423	201,497.8012	13	516.5219	201,787.8688
6	880.1603	201,389.8054	14	491.2447	201,900.5034
7*	790.9983*	201,358.5041*	15	468.9401	202,017.6520
8	721.0770	201,375.5820			

Table 2 shows the optimal solutions for different freight costs. When the freight cost increases, the optimal number of deliveries per production batch from the vendor to the buyer decreases while the optimal size of the deliveries increases. The smaller the freight cost, the larger the cost reduction of the integrated model in comparison to an independent decision by the buyer is.

Table 2

Optimal solutions for different freight cost F , $P=160,000$, $D=50,000$, $x = 175,200$, $S_v = 300$, $S_B = 100$, $h_v = 2$, $h_B = 5$, $c_i=0.5$, $c_w=50$, $c_r=100$, $c_{aB}=200$, $c_{av}=300$, $\beta = \lambda = \eta = 0.04$.

F	Buyer's independent decision			Integrated model			Cost reduction
	Q_B^*	$ETC_B(Q_B^*)$	$ETC_v(Q_B^*)$	n^*	$Q^*(n^*)$	$ETC(n^*,Q^*(n^*))$	
5	1,490.11	37,532.75	171,316.14	16	347.87	199,525.14	9,323.75
15	1,559.46	37,874.19	170,872.63	9	614.93	200,611.04	8,135.78
25	1,625.84	38,201.07	170,485.27	7	791.00	201,358.50	7,327.84
50	1,781.02	38,965.14	169,698.76	5	1,110.74	202,732.08	5,931.82
100	2,056.55	40,321.77	168,613.54	4	1,471.13	204,701.17	4,234.14

From Table 3, one can see that the smaller the vendor's holding cost is, the greater the advantages to deliver the items more frequently from the vendor to the buyer are, and the larger the cost reduction of the integrated model in comparison to an independent decision by the buyer is. Note that the optimal size of the production batch $Q_p^* = n^* Q^*(n^*)$ decreases as the vendor's holding cost increases.

Table 3

Optimal solutions for different vendor's holding cost h_v , $P=160,000$, $D=50,000$, $x = 175,200$, $S_v = 300$, $S_B = 100$, $h_B = 5$, $F=25$, $c_i=0.5$, $c_w=50$, $c_r=100$, $c_{aB}=200$, $c_{av}=300$, $\beta = \lambda = \eta = 0.04$

h_v	Buyer's independent decision			Integrated model			Cost reduction
	Q_B^*	$ETC_B(Q_B^*)$	$ETC_v(Q_B^*)$	n^*	$Q^*(n^*)$	$ETC(n^*,Q^*(n^*))$	
1	1,625.84	38,201.07	170,220.75	10	773.20	199,298.82	9,123.00
2	1,625.84	38,201.07	170,485.27	7	791.00	201,358.50	7,327.84
3	1,625.84	38,201.07	170,749.78	6	771.90	202,910.76	6,040.09
4	1,625.84	38,201.07	171,014.29	5	801.49	204,186.34	5,029.02
5	1,625.84	38,201.07	171,278.80	4	883.61	205,275.37	4,204.50

Table 4 shows that the buyer's holding cost has a reverse impact as the vendor's holding cost does on the optimal solution. As the buyer's holding cost increases, the optimal number of deliveries per production batch increases, the optimal size of the deliveries decreases, and the cost reduction of the integrated model increases.

Table 4

Optimal solutions for different buyer's holding cost h_B , $P=160,000$, $D=50,000$, $x = 175,200$, $S_v = 300$, $S_B = 100$, $h_v = 2$, $F=25$, $c_i=0.5$, $c_w=50$, $c_r=100$, $c_{aB}=200$, $c_{av}=300$, $\beta = \lambda = \eta = 0.04$

h_B	Buyer's independent decision			Integrated model			Cost reduction
	Q_B^*	$ETC_B(Q_B^*)$	$ETC_v(Q_B^*)$	n^*	$Q^*(n^*)$	$ETC(n^*,Q^*(n^*))$	
2	2,570.69	35,258.76	167,261.93	4	1,397.11	199,861.53	2,659.16
3	2,098.96	36,396.65	168,473.90	5	1,102.05	200,466.19	4,404.36
5	1,625.84	38,201.07	170,485.27	7	791.00	201,358.50	7,327.84
8	1,285.34	40,321.77	172,919.31	9	611.73	202,365.85	10,875.23
10	1,149.65	41,516.99	174,309.41	10	546.74	202,924.52	12,901.88

Table 5 shows the sensitivity analysis on the defective percentage γ (we assume that γ is uniformly distributed between 0 and β). It is interesting to note that when β increases, the optimal size of the deliveries first increases until it reaches its maximum value (at $\beta=0.3$) and then begins to decrease. The larger the β value, the greater the cost reduction of the integrated model in comparison to an independent decision by the buyer is.

Table 5

Optimal solutions when the defective percentage γ is uniformly distributed between 0 and β , $P=160,000$, $D=50,000$, $x = 175,200$, $S_v = 300$, $S_B = 100$, $h_v = 2$, $h_B = 5$, $F=25$, $c_i=0.5$, $c_w=50$, $c_r=100$, $c_{aB}=200$, $c_{av}=300$, $\lambda = \eta = 0.04$.

β	Buyer's independent decision			Integrated model			Cost reduction
	Q_B^*	$ETC_B(Q_B^*)$	$ETC_v(Q_B^*)$	n^*	$Q^*(n^*)$	$ETC(n^*,Q^*(n^*))$	
0.04	1,625.84	38,201.07	170,485.27	7	791.00	201,358.50	7,327.84
0.06	1,636.89	40,644.24	200,584.08	7	796.51	233,865.75	7,362.57
0.08	1,647.95	43,139.00	231,310.79	7	802.11	267,050.22	7,399.57
0.1	1,659.01	45,687.01	262,685.25	7	807.79	300,933.30	7,438.96
0.2	1,714.14	59,288.32	430,029.96	7	837.43	481,642.81	7,675.47
0.3	1,768.32	74,512.44	617,089.11	7	869.36	683,611.83	7,989.72
0.4	1,820.53	91,666.00	827,561.59	8	825.82	910,818.78	8,408.81

From Table 6, one can see that the Type I error e_1 (we assume e_1 is uniformly distributed between 0 and λ) has a similar impact as the defective percentage γ does on the optimal solution. When λ increases, the optimal size of the deliveries first increases until it reaches its maximum value (at $\lambda=0.3$) and then begins to decrease. The larger the λ value, the greater the cost reduction of the integrated model in comparison to an independent decision by the buyer is.

Table 6

Optimal solutions when the probability of Type I error e_1 is uniformly distributed between 0 and λ , $P=160,000$, $D=50,000$, $x = 175,200$, $S_v = 300$, $S_B = 100$, $h_v = 2$, $h_B = 5$, $F=25$, $c_i=0.5$, $c_w=50$, $c_r=100$, $c_{aB}=200$, $c_{av}=300$, $\beta = \eta = 0.04$.

λ	Buyer's independent decision			Integrated model			Cost reduction
	Q_B^*	$ETC_B(Q_B^*)$	$ETC_v(Q_B^*)$	n^*	$Q^*(n^*)$	$ETC(n^*,Q^*(n^*))$	
0.04	1,625.84	38,201.07	170,485.27	7	791.00	201,358.50	7,327.84
0.06	1,637.12	38,539.19	223,726.09	7	796.55	254,904.55	7,360.73
0.08	1,648.41	38,885.07	278,076.98	7	802.19	309,566.14	7,395.91
0.1	1,659.70	39,238.99	333,573.00	7	807.91	365,378.51	7,433.48
0.2	1,715.94	41,139.06	629,566.68	7	837.75	663,044.47	7,661.27
0.3	1,771.15	43,286.37	960,411.21	7	869.87	995,730.00	7,967.58
0.4	1,824.25	45,729.78	1,332,643.69	8	826.39	1,369,994.36	8,379.11

Table 7 shows the sensitivity analysis of the Type II error e_2 . (We assume e_2 is uniformly distributed between 0 and η). When η increases, the optimal size of the shipments decreases, and the cost reduction increases. Both of the changes are slight, indicating that the optimal solution is not sensitive to the Type II error. From the numerical examples, one can see that the integrated model results in an impressive cost reduction in comparison to an independent decision by the buyer.

Table 7

Optimal solutions when the probability of Type II error e_2 is uniformly distributed between 0 and η , $P=160,000$, $D=50,000$, $x = 175,200$, $S_v = 300$, $S_B = 100$, $h_v = 2$, $h_B = 5$, $F=25$, $c_i = 0.5$, $c_w = 50$, $c_r = 100$, $c_{aB} = 200$, $c_{av} = 300$, $\beta = \lambda = 0.04$

η	Buyer's independent decision			Integrated model			Cost reduction
	Q_B^*	$ETC_B(Q_B^*)$	$ETC_v(Q_B^*)$	n^*	$Q^*(n^*)$	$ETC(n^*, Q^*(n^*))$	
0.04	1,625.84	38,201.07	170,485.27	7	791.00	201,358.50	7,327.84
0.06	1,625.62	40,284.67	173,610.25	7	790.96	206,565.22	7,329.70
0.08	1,625.39	42,368.27	176,735.23	7	790.92	211,771.93	7,331.57
0.1	1,625.16	44,451.86	179,860.21	7	790.88	216,978.65	7,333.42
0.2	1,624.01	54,869.85	195,485.11	7	790.68	243,012.22	7,342.74
0.3	1,622.87	65,287.82	211,110.01	7	790.48	269,045.79	7,352.04
0.4	1,621.72	75,705.80	226,734.91	7	790.27	295,079.36	7,361.35

5. Conclusion and future research

In this paper, we develop an integrated vendor-buyer production-inventory model for items with imperfect quality and inspection errors. The objective is to minimize the total joint annual costs incurred by the vendor and the buyer. The production process is imperfect and produces a certain number of defective items with a known probability density function. We consider the policy in which the delivery quantity to the buyer is identical at each delivery. Once the buyer receives the lot, a 100% screening process of the lot is conducted, and the screening process and demand proceed simultaneously. The screening process is not perfect. The inspector may incorrectly classify a non-defective item as defective (a Type I inspection error), or incorrectly classify a defective item as non-defective (a Type II inspection error). The expected total integrated annual cost of the vendor and the buyer is derived and a solution procedure is provided to find the optimal solution that minimizes the expected total integrated annual cost. Numerical examples show that the integrated model gives an impressive cost reduction in comparison to an independent decision by the buyer.

This paper assumes that the defective items are sold to a secondary market at a discounted price. One possible extension of this research is to have the vendor rework the defective items and convert the defective items into good quality products to satisfy the buyer's demand. Another possible extension of our model is to consider the case where, while some customers are willing to exchange the defective items with good ones, others may be disappointed at the product quality and ask for a full price refund (i.e., the case of lost sales). Finally, we use the just in time philosophy to assume that the successive deliveries of the items from the vendor to the buyer are arranged in such a way that the items arrive at the buyer's place when the buyer's inventories from the previous delivery have just been depleted. Thus, one may extend our model by allowing shortages to occur and to be completely or partially backordered.

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