## AN INTEGRATING PHOTOMETER FOR GLOW LAMPS AND SOURCES OF LIKE INTENSITY.

BY CHARLES P. MATTHEWS.

In a previous paper ${ }^{1}$, I have described an equipment designed for the photometric study of arc lamps. The most valuable feature of this equipment lies in its ability to yield a value of the mean spherical luminous intensity from a single photometer setting. It is the purpose of this paper to describe an apparatus possessing this same valuable feature, and several others worthy of note, when used for the photometry of the incandescent lamp and other sources of the same order of brightness.

Designed especially for incandescent lamp measurements, the apparatus has several functions and might have been styled a " universal glow lamp photometer." It is capable of use for all photometric measurements on the glow lamp that do not lie in the province of spectro-photometry. To particularize, it may be used as follows:
(1.) As a simple photometer for any unidirectional measurements, such as occur in standardizations, ratings and candlepower distributions.
(2.) As an integrating instrument for the direct determination of mean horizontal, mean spherical, mean hemispherical or mean zonular candle-power.
(3.) As an integrating instrument for the direct determination of the spherical reduction factor; that is, the ratio-mean spherical: mean horizontal candle-power.

1. Transactions, xviii, 1901.

The Importance of the Mean Spherical Value.
The fact has long been recognized ${ }^{2}$ that the only strictly fair basis for the comparison of incandescent lamps, is that of the total flux of light emitted, a quantity proportional to the mean spherical candle-power. It is possible by altering the configuration of the filament to concentrate luminous flux in particular solid angles at the expense of flux in other angles. Hence, two lamps of equal rated candle-power may yield total light flux in quite different amounts. On the other hand, if two lamps have initially the same mean spherical candle-power, their relative value is determined simply by their power consumption and sustained candle-power. The vexed question of what is the most useful light may well be left to the purchaser, who can select the type of filament best adapted to his own needs. The question is comparable to that which asks: What is the best diet for man?

## Existing Methods for the Determination of the Mean Spherical Value.

Of the methods in use in the determination of the mean spherical candle-power, the most accurate is that in which measurements are made at equal angular intervals through $180^{\circ}$ in a plane passing through the axis of symmetry of the filament, the lamp being rotated meanwhile about this axis. From the readings so obtained, the mean spherical value may be found either by formula or the graphical construction known as the Rousseau diagram. ${ }^{3}$

The method employed in the Franklin Institute tests of 1884 involves the mean of 38 candle-power values taken in such directions at to give a nearly uniform space distribution. The mean of these values is the result sought.

Both of the foregoing methods involve a large number of readings-so large, in fact, that their application to the practical rating or extended study of lamps is out of the question.

A third method consists in the use of the spherical reduction factor appropriate to the type of filament under consideration. Unfortunately, this factor is not a constant for any given type

[^0]of lamp. With some types, the method yields a good result; with others the variations are such as to render the results only roughly approximate. A disadvantage is the large and increasing number of types on the market and the necessity for determining and keeping account of the corresponding constants.

Liebenthal found as the result of an extended series of measurements that the mean of the intensities taken at $51.8^{\circ}$ north polar distance and $51.8^{\circ}$ south polar distance on a spinning lamp is a value approximating the mean spherical intensity regardless of the type of lamp. The errors resulting from the application of this method range from $-.1 \%$ to $+3.9 \%$. This is probably the most accurate of the simpler approximative methods.

## Theory and Description of the Apparatus.

From what has been said, it would appear that there is need of a photometer capable of giving the mean spherical candlepower of an incandescent lamp with the ease and celerity obtainable in the ordinary photometric measurement. With this need in mind, I have designed and had constructed the apparatus decribed below.

The theoretical basis of the design is the approximate equation for the mean spherical intensity

$$
\begin{equation*}
I_{\mathrm{ms}}=\frac{\pi}{2 n} \sum_{0}^{\pi} I_{\theta} \sin \theta \tag{1}
\end{equation*}
$$

wherein $I_{\theta}$ is the intensity of a ray making an angle $\theta$ with a vertical passing through the light center and $n$ the number of terms in the summation. In order that equation (1) may apply to a glow lamp, it is necessary to spin the lamp precisely as is commonly done in determining mean horizontal candle-power.

To see how equation (1) may be made the basis of an integrating photometer, let us consider a source of lights (Fig. 1), and a photometer screen P whose plane extended contains the effective light center of $s$. We will hereafter denote the center of the photometer screen by $p$ and the effective light center by $q$. For convenience we will further assume that $p$ and $q$ lie in the same horizontal plane to which the plane of P is normal, and we will call the line $p q$ the axis of the system. Now consider two mirrors whose planes are vertical and make an angle of $90^{\circ}$ with each other. Let the centers of these mirrors, designated by $a$ and $b$, come into the horizontal plane in such positions that the lines $q a$ and $p b$ are equal and respectively normal to the axis
of the system. Let $a^{\prime}$ and $b^{\prime}$ be the centers of a second pair of mirrors occupying a position such as would be found by swinging $a$ and $b$, without mutual displacement, upward about $q p$ as an axis until $q a$ makes the acute angle 0 with the vertical. Having thus located $a^{\prime}$ and $b^{\prime}$ angularly, we may now assume that some radial movement of this pair of mirrors is possible. The eye placed at $p$ will see virtual images of the source in horizontal and $\theta$ aspects respectively. The images $s^{\prime}$ and $s^{\prime \prime}$ may be regarded as producing jointly an illumination on the photometer screen of

$$
\begin{equation*}
i_{0}+i_{\theta}=\frac{K_{0} I_{0}}{d_{0}^{2}}+\frac{C K_{\theta} I_{\theta}}{d_{\theta}^{2}} \tag{2}
\end{equation*}
$$



Fig. 1.
where $K_{0}$ and $K_{\theta}$ are the reflection co-efficients of the pairs of mirrors, $d_{0}, d_{\theta}$, the distances from source to screen by way of the mirrors and $C$ a factor varying with the incidence of the light upon the photometer screen $P$. Now if $n$ pairs of mirrors be placed similarly to $a, b$ and $a^{\prime}, b^{\prime}$, but spaced at equal angular intervals of $\Delta \theta$ such that $n \Delta O=\pi$, we shall have as the resulting illumination

$$
\begin{equation*}
\sum_{0}^{\pi}(i)=\sum_{0}^{\pi} \frac{C K_{\theta} I_{\theta}}{d_{\theta}^{2}} \tag{3}
\end{equation*}
$$

If by radial adjustment of the mirror pairs we make

$$
\begin{equation*}
\frac{C K_{H}}{d_{\theta}^{2}}=\frac{K_{0} \sin \theta}{d^{2}{ }_{0}} \tag{4}
\end{equation*}
$$

then

$$
\begin{equation*}
\sum_{0}^{\pi}(i)=\frac{K_{0}}{d_{0}^{2}} \sum_{0}^{\pi} I \sin \theta \tag{5}
\end{equation*}
$$

That is to say, the total illumination of the screen is proportional to the mean spherical intensity of the source. (See equation 1.) To evaluate this intensity, it is necessary merely to balance this illumination against that due to a source of known intensity at a known distance.

Fig. 2 shows the disposition of twelve pairs of mirrors, $m_{1}, m_{2}$, etc., in order to produce the desired results. If the illumination of a given surface varied exactly as the cosine of the incidence of


Fig. 2.
the light upon that surface, and if all mirrors used were of equal reflecting power, then there would be no need of radial adjustment of the mirror pairs, for in such case $S C=\sin \theta$ and $K_{\theta}=$ $K_{0}$. But the so-called cosine law is only approximately true and mirrors vary in reflecting power, hence it is necessary to compensate for discrepancies in $C$ and $K_{\theta}$ by slight changes in $d_{\theta}$. The extent to which this correction is of importance depends, of course, upon the nature of the photometer screen. The plaster of Paris surface of the Lummer-Brodhun screen obeys the cosine law with exactness up to an incidence of $50^{\circ}$, but beyond this point a divergence of increasing magnitude occurs (Fig. 3). Hence, for the Lummer-Brodhun screen the only adjustment of
mirrors necessary is that to overcome variations in their reflection co-efficients except for angles greater than $50^{\circ}$. With mirrors cut from one sheet of glass, the correction for variation in reflecting power is often negligible. With the Bunsen screen, correction must be made for all angles of incidence. For example, the second curve in Fig. 3 is the result of measurements made upon ordinary draughting paper from which the Bunsen screen is often made. Here the departure from the cosine relation is noticeable from the very beginning and becomes as high as $15 \%$ at $75^{\circ}$ incidence. The third curve in the same figure shows the results obtained with ordinary glazed writing paper. The cosine relation is not even roughly approximate is this case.

Fig. 2 also shows the method of balancing the illumination


Fig. 3.
Per cent. variation from cosine relation for different screens.
I.-Lummer-Brodhun screen.
II.-Unglazed paper.
III.-Glazed paper.
produced by the series of images due to the circular system of mirrors. Mirrors $M_{1}, M_{2}, M_{3}, M_{4}$ are cut from one piece of glass just as are the mirrors in the ordinary Bunsen photometer. It is not essential that the other mirrors of the system should have the same coefficient, since, as already explained, the initial adjustment corrects for failure of the cosine relaton and inequalities in the mirror coefficients at the same time. With the standard at $\mathrm{s}^{\prime}$ separated from s , the source to be tested, by an opaque screen. a balance in the illumination is obtained by moving $M_{3}$ and $M_{4}$. The method of doing this will be better under-
stood by reference to Fig. 4, which shows in elevation and plan the essential elements of the apparatus. To the right of these figures is seen the mirror system, each pair of mirrors being capable of a certain amount of radial movement for purposes of the initial adjustment of the instrument. s is the lamp to be tested, mounted upon a rotator, $\mathrm{s}^{\prime}$ is a standardized glow larnp. The mirrors $M_{3}$ and $m_{3}$, rigidly connected, may be moved along the bar by means of a rack and pinion conveniently under control of the observer at the photometer $P$. The photometer is fixed and hence the operation of making a setting is more convenient than that which obtains with the ordinary sliding form. As the design is based upon the approximate equation (1) and not upon the integral form, some error arises from this cause. With eleven pairs of mirrors the error is negligible for all practical purposes.


Fig. 4.
Fig 5 shows a photograph of the finished apparatus with all screens removed in order that the details of construction may be better seen. This particular instrument is fitted with a Lummer-Brodhun photometer screen. Each mirror bracket is provided with two pins. These pins extend through the frame of the ring radially. By means of this construction, each mirror pair may have independent radial adjustment.

We will now consider in detail the different operations to which the instrument readily lends itself.
Operation 1.-Measurement of Mean Horizontal Candle Power.
In this operation the apparatus is used as a simple photometer. Hence, all mirrors except the four horizontal ones are covered
by black screens suitably provided for the purpose. The right pair of mirrors (Fig. 4) is connected at c to the sliding rod carrying the rack. The lamp $s$ to be tested is mounted in the rotator


Fig. 5.
and driven at a speed of say 180 r. p. m., and a standardized incandescent lamp, of intensity $I_{8}$ is placed in a suitable holder, at $\mathrm{s}^{\prime}$. The rod r may be moved by the hand for a rough adjustment, and the pinion $\mathrm{P}^{\prime \prime}$ used only for the final setting. Since a
displacement $d$ of the mirror pairs means a change in the light paths of $2 d$, the moving rod is graduated in divisions one-half the unit (centimetres) in which the light paths are conveniently measured. A reading r means that the distance from $\mathrm{s}^{\prime}$ to the screen by way of the mirrors $M_{3}, M_{4}$, is $R$ centimetres. If, now the total photometric distance between the sources is 300 cm , we have for the intensity I of the light under test

$$
\begin{gather*}
I_{\mathrm{s}}=\left[\frac{300-R}{R}\right]^{2} I_{\mathrm{s}^{\prime}}  \tag{6}\\
=T_{1} I_{\mathrm{s}^{\prime}} \tag{7}
\end{gather*}
$$

where $T_{1}$ is the value of the expression in brackets and stands for " tabulated value corresponding to the reading $R$." When the apparatus is used with a standard of always the same intensity, it is a simple matter to make the instrument direct reading.

Obviously, the mean intensity in any north or south polar zone may be found by clamping the arm of the rotator at the proper angle and spinning the lamp, readings being taken as for the mean horizontal measurement. The mean horizontal candlepower of a flame source must be found by taking the horizontal distribution step-wise since it is impracticable to rotate such a source. Equation (7) is applicable in such measurements.
Operation 2.-Distribution of Candle-Power in Vertical Planes.
The distribution of intensity of an incandescent lamp in any vertical plane is obtained with the apparatus arranged as described under Operation 1. The arm of the rotator is merely tilted about a horizontal axis so as to bring any desired aspect of the lamp toward the photometer screen. From equation (7) the different intensities are easily found.

The vertical distribution of candle-power from a flame source cannot be obtained by tilting the arm of the support. The following method is available in such cases: Mount the burner vertically at $s$, Disconnect the mirrors $M_{1}, M_{2}$ from the movable rod and push them to their place at the extreme right (Fig. 4). Now with a standard of the same order of brightness as the source to be testea, make a setting. The horizontal intensity is given by

$$
\begin{align*}
I_{\mathrm{u}} & =\left[\frac{200}{R}\right]^{2} I_{\mathrm{s}}  \tag{8}\\
& =T_{2} I_{\mathrm{s}^{\prime}} \tag{9}
\end{align*}
$$

where $T_{2}$ is the tabulated value of the expression in brackets corresponding to the reading $R$.

To obtain the intensity of the source in a direction of $\theta_{0}$ to the vertical, the horizontal and $\theta$ mirrors should both be used. This prevents the limit of the bar being reached by mirrors $M_{3}, M_{4}$. The intensity is given by

$$
\begin{equation*}
I_{\theta}=\frac{T_{2} I_{\mathrm{s}^{\prime}}-I_{0}}{\sin \theta} \tag{10}
\end{equation*}
$$

where $I_{0}$ is the intensity found in the horizontal measurement
Operation 3. Standardization of Glow Lamps.
For this operation the lamp $s$ is removed and a horizontal circular plate mounted in place of the lamp. This plate is ruled with concentric circles which facilitate the centering on the amylacetate lamp or other primary standard. The lamp to be standardized is mounted at $s^{\prime}$. With all mirrors screened except the horizontal ones, and with $\mathrm{m}_{1} \mathrm{M}_{2}$ attached to the bar, settings are made as usual. The value of $s^{\prime}$ is given by

$$
\begin{equation*}
I_{s^{\prime}}=\frac{1}{T_{1}} \tag{11}
\end{equation*}
$$

if the standard is unity.

## Operation 4. Measurement of Mean Spherical Intensity.

(a) Glow Lamp.-The lamp to be tested is mounted in the rotator and driven at a speed of about 180 r.p.m. Mirrors $M_{1} M_{2}$ are detached from the movable rod and pushed to the extreme right, in which position they may be considered a part of the system of eleven mirror pairs. With a standardized lamp at $s^{\prime}$, a setting in made in the manner already described. If $R$ is the reading, we have

$$
\begin{equation*}
I_{\mathrm{ms}}=T_{3} I_{\mathrm{s}^{\prime}} \tag{12}
\end{equation*}
$$

where $T_{3}$ is a tabulated value corresponding to the setting $R$ and $I_{s^{\prime}}$, the intensity of the standard as heretofore. Thus the operation has all the simplicity of any photometric measurement.

The intensity of the standard used should be approximately that of the lamp to be tested. For example, if a $16 \mathrm{c} . \mathrm{p}$. lamp is to be tested, a standard of not less than 16 c.p. is best. With such a standard, the range of possible measurement depends upon the limit of travel of the mirrors $\mathrm{m}_{3} \mathrm{~m}_{4}$. A $16 \mathrm{c} . \mathrm{p}$. lamp will serve as a standard for the measurement of mean spherical intensities ranging from 2 to about 25 c.p., when the limit of
travel is about one metre. It is best to substitute a 32 c.p. standard for intensities much greater than 16 c.p.
(b) Flames.-To obtain the mean spherical intensity of a flame or of any source that cannot be rotated, it is necessary to repeat Operation 4 at equal angular intervals on the horizontal circle. The mean of the results may then be taken.
Operation 5. Direct Measurement of the Spherical
If the standard $s^{\prime}$ and the opaque screen o be removed, it is clear from the figure that the left side of the photometer screen will be illumined by the horizontal rays of the lamp s. In fact, if $s$ be rotated, we will have on the right side of the photometer screen an illumination proportional to the mean spherical intensity of $s$ and on the left side of the screen an illumination proportional to the mean horizontal intensity of the same source. Under these conditions the photometer setting yields the spherical reduction factor. In other words, the mean spherical intensity is measured against the mean horizontal intensity as a standard. The reduction factor is given by

$$
\begin{equation*}
f=T_{3} \tag{13}
\end{equation*}
$$

As the removal of the standard $s^{\prime}$ lengthens the distance from source to screen, it is necessary to add a constant to the reading. This is provided for by a second reading point marked R F. All readings for Operation 5 must be taken at this reference mark.
Operation 6. To Check the Horizontal Mirror Constants.
As before stated, it is essential that the four mirrors attached to the movable bar should have the same constant. To ascertain if this condition exists, remove the opaque screen and the lamp s' as in Operation 5. Connect the horizontal mirrors to the moving rod and, screening all other mirrors, take reversed photometer readings on a rotating lamp s. If the mirror coefficients are equal the mean reading will be 150 , indicating equal light paths on each side of the photometer screen. Here again the $R$ F reading mark must be used.
Operation 7. To Check the Adjustment of the Circular Mirror System.
In case of any doubt as to the accuracy of the initial adjustment of the photometer, or in case of the substitution of a new screen, it may be necessary to readjust the mirrors of the half ring. This operation is best performed as follows: Mount at s a 32 nominal c.p. lamp, and at $\mathrm{s}^{\prime}$ an 8 c.p. lamp, the latter
being in circuit with a rheostat capable of continuous variation. With mirrors $M_{1}$ and $M_{2}$ free from the moving rod, and with all other mirrors on the half-ring screened set R equal. to $100 \mathrm{c} . \mathrm{m}$. Now, while maintaining the 32 c.p. lamp at constant voltage, vary the voltage impressed on the 8 c.p. lamp until the photometer shows equal illuminaton. Note voltage on 8 c.p. lamp. Repeat measurements with reversed photometer. The lamp should finally be maintained at the mean voltage so found. Under these conditions the two lamps have a candle-power ratio of $4: 1$. Next tilt the lamp holder to an angle of $15^{\circ}$. Uncover the corresponding mirrors ( $\theta=75^{\circ}$ or $\theta=105^{\circ}$ ) and cover the horizontal ones. Under these conditions we have the same aspect of the lamp toward the photometer, but with the light incident at $15^{\circ}$. Set the bar at

$$
\begin{equation*}
R_{15}=\frac{R_{10}}{\sqrt{\cos 15^{\circ}}} \tag{11}
\end{equation*}
$$

Adjust the $15^{\circ}$ mirrors radially until an equality of illumination is obtained, then secure them by means of the set screw. This operation may be repeated until all the mirrors have been adjusted.
In conclusion, I would acknowledge my indebtedness to Messrs. D. M. Lynch and E. D. Fristoe for their painstaking labor in construction a preliminary form of this apparatus. I am also indebted to Mr. C. R. Dooley, Assistant in Electrical Engineering at Purdue University, or assistance.


[^0]:    2. In 1897 a committee of the Institute reported as follows: (Transactions, page 90.) "Although incandescent lamps are at present rated by their horizontal candle-power, yet, since the only true criterion of the total quantity of light emitted by a lamp is its mean spherical candlepower, we recommend that the rating of lamps should be based upon their mean spherical candle-power so far as is commercially practicable."
    3. Transactions, September, 1901.
