An Intelligent Model for the Signorini Contact Problem in Belt Grinding Processes

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Abstract. A bottleneck in the real-time simulation of belt grinding processes is the calculation of the force distribution between the workpiece and the grinding wheel, which can be simplified by the Signorini contact problem. The Finite Element Method (FEM) is the conventional way of solving such a contact problem, but too computationally expensive to meet the real-time requirement. This paper demonstrates a new approach to model the Signorini contact problem based on learning. This new model approximates the FEM model so that it is not necessary to execute optimization for each contact in run time; hence the calculation time is dramatically reduced.

1 Background

The key link in the real-time simulation of belt grinding processes is to get the removals on the workpiece surface in time [1]. The different removals on the workpiece surface result from the different local contact forces between the workpiece surface (hard) and the elastic grinding wheel (soft).

A contact problem between an elastic body and an idealistically rigid body is named the Signorini contact problem. According to this theory the elastic body deforms in a way that tends to minimize its strain potential energy when in contact with the rigid body, requiring that some initial and boundary conditions are satisfied. Once the strain field (deformation) is known, the force distribution can be easily obtained by Hooke's law. Blum and Suttmeier [2] worked out a FEM model that considers this contact problem as the Signorini contact problem. Although having adopted an optimized mesh discretization, it still requires about 15 minutes for doing the subjected optimization of one contact situation. The calculation of the force distribution becomes a bottleneck in the real-time simulation flow.

To accelerate the calculation two branches are under research nowadays. The first branch is to optimize the mesh division; another one is to improve the convergent rate and the stability of the optimization algorithm. Both cannot avert doing the iteration steps each time when a new contact situation is presented. To overcome this a learning machine is introduced in this paper to approximate the well-established FEM model. Although an optimization process is also necessary in the training phase, it can finish the calculation of one contact situation in a

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very short time because the time-consuming transaction is put into the training phase and no longer in the run time. The Multi-Layer Perceptrons (MLP) and Support Vector Regression (SVR) are tested as the learning machines because both methods are capable of multi-dimensional regression problems.

2 Data Representation and Numerical Experiments

The input of the Signorini problem is the initial boundary condition that can be digitized in terms of the geometrical data of the workpiece. For simplification the contact area is limited to a $50mm \times 50mm$ square area and is discretized into a 50×50 mesh evenly spaced with $1mm$ interval. Thus the initial boundary condition can be written as a height matrix $\mathbf{H}$, in which each elements represent the vertical distance between the workpiece and the grinding wheel. It is not a good idea to impose all elements of the matrix $\mathbf{H}$ directly as the input to learning machine, just like what is done by FEM, because one cannot expect good results or generalization with such a high input dimension (2500). According to one assumption the contact problem can be localized to reduce the input dimension. The assumption is that *The force on one mesh point is affected only by the contact situation (heights) of its surrounding points inside a finite size area (function area).* The force of the center point is determined only by the heights in the function area, which is normally a partial contact area. Obviously, if the function area is large enough, or is the whole contact area, the assumption is undoubtedly correct. The small function leads to a low input dimension, but weakens the correctness of the assumption. Thus the function area size must not be too small to guarantee the assumption's correctness. Through training experiments the best function area size is $11mm \times 11mm$. Therefore, the learning machine takes 121 heights in the function area as input and the force on the center point of the function area as output. One contact point generates one input/output pair for training and testing. 180 characteristic contact situations are defined for training and 64 for testing and there are over 100,000 contact points in the training situations and about 50,000 in the testing situations.

Two methods can be used to further reduce the input dimension. One is the Partial Point Selection (PPS); the other one is the classical Principle Component Analysis (PCA). The PPS, as its name implies, takes only a couple of points in the function area instead of all points with preferences. PPS can easily lower the input dimension without losing much information because the workpiece surface is assumed to be continuous in every direction and varies smoothly, not sharply. Only 41 points, which locate on four (vertical, horizontal and two diagonals) lines, are selected out from all 121 points in the $11 \times 11mm^2$ function area.

For MLP, one hidden layer is used because more than one hidden layer didn't show any advantages in experiments. Only the RBF kernel is considered in the SVR. Table 1 shows the best results of different batches of training pairs. All models are managed to simulate the 64 contact situations after they are trained. The mean relative simulation error in table 1 indicates an average simulation error of 64 testing contact situations. The best result of SVR is 6.5%, which

Table 1. One model for all training sets

MLP			SVR		
Training Pairs	Neurons	Mean Relative Simulation Error	Training Pairs	nSV	Mean Relative Simulation Error
1570	10	10.5%	5160	606	8.8%
3140	11	10.2%	6880	675	6.5%
4710	15	10.8%	10264	880	8.2%

is still a little high. A training set classification strategy is applied in order to get higher precision. The training sets (contact points) are divided into 16 categories according to the relative position of contact points and the function area in a physical manner and then train a model for each category. However, the results in the adjoining area may be not as smooth as wanted. This can be overcome by an overlapping training strategy. That means that the contact points in the adjoining area of different categories should be involved in training all these categories for smoothness. There are two advantages of this classification strategy. The first one is that the learning machine can converge to a smaller error, another is that the number of neurons or support vectors is less than only one model with the same error endurance, which indicates a faster calculation. By this classification strategy the mean relative simulation error of testing situations reaches 4.1% using the SVR compared to 4.9% using the MLP. Additionally, the force distribution given by the SVR looks smoother than that by the MLP. However, the MLP model conducts the calculation much faster than that of the SVR model.

3 Conclusion

This paper demonstrates a new way to model the Signorini problem using the SVR and MLP to learn non-linear mapping rather than solving the optimization problem each time when a new contact situation is given. The experiments show that both SVR and MLP can approximate the traditional FEM model with an error below 5%. The SVR has a relatively better approximating precision than the MLP, but a longer calculating time. The calculating time is reduced to about 1 second compared to 15 minutes of original FEM model. This makes it possible to real-time simulation of belt grinding processes.

References

1. Zhang, X., Cabaravdic, M., Kneupner, K., Kuhlenkoetter, B.: Real-time simulation of robot controlled belt grinding processes of sculptured surfaces. International Journal of Advanced Robotic Systems **2** (2004) 109–114
2. Blum, H., Suttmeier, F.T.: An adaptive finite element discretisation for a simplified signorini problem. Calcolo **37** (2000) 65–77