AN INTERACTIVE FUZZY SATISFICING METHOD USING AUGMENTED MINIMAX PROBLEMS AND ITS APPLICATION TO ENVIRONMENTAL SYSTEMS

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FOREWORD

The International Institute for Applied Systems Analysis (IIASA) has been a leader in the area of multiobjective decisionmaking under conditions of uncertainty, and we have long been aware of Professor Sakawa's interesting work in this area. Indeed, he was invited to present a lecture here in late 1985, addressing many of the same issues raised in the appended paper.

Here, Professor Sakawa and his coauthor propose a new interactive fuzzy satisficing method for multiobjective nonlinear programming, in the case where the decisionmaker holds fuzzy goals for each of the objective functions. On the basis of this innovative method, the authors wrote a time-sharing computer program to implement man—machine interactive procedures, and this program is applied to the industrial pollution control problem, in Osaka City, Japan. Thus, the authors' ideas are in the spirit of the research that IIASA is currently carrying out on the role of analysis in the solution of real-world problems.

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An Interactive Fuzzy Satisficing Method Using Augmented Minimax Problems and Its Application to Environmental Systems

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Abstract—A new interactive fuzzy satisficing method for multiobjective nonlinear programming is presented, by considering that the decisionmaker (DM) has fuzzy goals for each of the objective functions. Through the interaction with the DM, the fuzzy goals of the DM are quantified by eliciting corresponding membership functions. In order to generate a candidate for the satisficing solution (Pareto optimal) after determining the membership functions, if the DM specifies his / her reference membership values, the augmented minimax problem is solved. The DM is thus supplied with the corresponding Pareto optimal solution together with the trade-off rates between the membership functions. Then by considering the current values of the membership functions as well as the trade-off rates, the DM acts on this solution by updating his / her reference membership values. In this way the satisficing solution for the DM can be derived efficiently from among a Pareto optimal solution set by updating his / her reference membership values. On the basis of the proposed method, a time-sharing computer program is written to implement man–machine interactive procedures. An application to the industrial pollution control problem in Osaka City in Japan is demonstrated together with the computer output.

I. Introduction

IN MULTIOBJECTIVE decisionmaking problems (MDMP), multiple objectives are usually noncommensurable and cannot be combined into a single objective. Moreover, the objectives usually conflict with each other in that any improvement of one objective can be achieved only at the expense of another. Consequently, the aim is to find a compromise or satisficing solution of a decisionmaker (DM), which is also Pareto optimal based on his/her subjective value judgement [1], [2], [6], [9]–[12], [14], [40]. Three most promising types of approaches for the determination of a compromise or satisficing solution of MDMP have been developed, which are (1) goal programming approaches [3], [4], [15], [16], [20]; (2) interactive approaches [8], [24]–[29], [33], [34], [36], [37], [38], [44]; and (3) fuzzy programming approaches [13], [19], [30]–[32], [41]–[43].

The goal-programming approaches, which assume that the DM can specify his/her goals of the objective functions, first appeared in 1961 text by Charnes and Cooper [3] in order to deal with multiobjective linear programming

(MOLP) problems. Subsequent works on goal programming approaches have been numerous including [4], [15], [16], and [20].

The interactive approaches, which assume that the DM is able to give some preference information on a local level to a particular solution, were first initiated by Geoffrion *et al.* [8] and further developed by many researchers such as [24]–[29], [33], [34], [36–38], [44].

The fuzzy programming approaches, which assume that the fuzzy goals of the DM can be quantified by eliciting his/her membership functions, were first introduced by Zimmermann [41] in solving MOLP problems and further extended by several investigators such as [13], [19], [30]–[32], [42], [43].

Naturally, each of these approaches has its own advantages and disadvantages relative to the others. Therefore in this paper we present a new interactive fuzzy satisficing method by incorporating the desirable features of both the goal programming approaches and the interactive approaches into the fuzzy approaches. After determining the membership functions for each of the objective functions through the interaction with the DM, if the DM specifies his/her reference membership values, the augmented minimax problem is solved, and the DM is supplied with the corresponding Pareto optimal solution and the trade-off rates between the membership functions. Then by considering the current values of the membership functions together with the trade-off rates, the DM responds by updating his/her reference membership values and the satisficing solution for the DM can be derived efficiently from among a Pareto optimal solution set. On the basis of the proposed method, a time-sharing computer program is written in Fortran to implement man–machine interactive procedures. The industrial pollution control problem in the industrialized areas near Osaka City in Japan is formulated, and the interaction processes are demonstrated along with the computer outputs.

II. Interactive Fuzzy Satisficing Decisionmaking

In general, the multiobjective nonlinear programming (MONLP) problem is represented as the following

vector-minimization problem:

$$\min f(x) \triangleq \left(f_1(x), f_2(x), \cdots, f_k(x) \right)^T$$

subject to $x \in X = \left\{ x | x \in E^n, g_j(x) \leq 0, j = 1, \cdots, m \right\}$

$$(1)$$

where x is an n-dimensional vector of the decision variables $f_1(x), \cdots, f_k(x)$ are k distinct objective functions of the decision vector x, $g_1(x), \cdots, g_m(x)$ are m inequality constraints, and X is the feasible set of constrained decisions.

Fundamental to the MONLP is the Pareto optimal concept, which is also known as a noninferior solution. Qualitatively, a Pareto optimal solution of the MONLP is one where any improvement of one objective function can be achieved only at the expense of another. Mathematically, a formal definition of a Pareto optimal solution to the MONLP is given in the following.

Definition 1 (Pareto optimal solution): A decision $x^* \in X$ is said to be a Pareto optimal solution to the MONLP, if and only if there does not exist another $x \in X$ such that $f_i(x) \leq f_i(x^*)$, $i = 1, \cdots k$, with strict inequality holding for at least one i.

In practice, however, since only local solutions are guaranteed in solving a scalar optimization problem by any standard optimization technique, unless the problem is convex, we deal with local Pareto optimal solutions instead of global Pareto optimal solutions. The concept of local Pareto optimal solutions was first introduced by Geoffrion [7].

Definition 2 (local Pareto optimal solution): A decision $x^* \in X$ is said to be a local Pareto optimal solution to the MONLP if and only if there exists an $r > 0$ such that x^* is Pareto optimal in $X \cap N(x^*, r)$, i.e. there does not exist another $x \in X \cap N(x^*, r)$ such that $f_i(x) \leq f_i(x^*)$, $i = 1, \cdots, k$, with strict inequality holding for at least one i, where $N(x^*, r)$ denotes the set $\left\{ x | x \in E^n, \|x-x^*\| < r \right\}$.

Usually, local Pareto optimal solutions consist of an infinite number of points, and some kinds of subjective judgement should be added to the quantitative analyses by the DM. The DM must select his/her local compromise or satisficing solution from among local Pareto optimal solutions.

In this paper, assuming that the DM has imprecise or fuzzy goals for each of the objective functions in the MONLP, we propose a new interactive fuzzy satisficing method.

In a minimization problem, a fuzzy goal stated by the DM may be to achieve "substantially less" than A. This type of statement can be quantified by eliciting a corresponding membership function.

In order to elicit a membership function $\mu_{f_i}(x)$ from the DM for each of the objective functions $f_i(x)$, $i = 1, \cdots, k$ we first calculate the individual minimum f_i^{min} and maximum f_i^{max} of each objective function $f_i(x)$ under given constraints. By taking account of the calculated individual minimum and maximum of each objective function, the DM must determine his/her subjective membership func-

tion $\mu_{f_i}(x)$ which is a strictly monotonically decreasing function with respect to $f_i(x)$. Here, it is assumed that $\mu_{f_i}(x) = 0$ or $\rightarrow 0$, if $f_i(x) \geq f_i^0$ and $\mu_{f_i}(x) = 1$ or $\rightarrow 1$, if $f_i(x) \leq f_i^1$, where f_i^0 is an unacceptable level for $f_i(x)$, and f_i^1 is a totally desirable level for $f_i(x)$ within f_i^{min} and f_i^{max}.

After determining the membership functions for each of the objective functions, in order to generate a candidate for the satisficing solution, which is also local Pareto optimal, the DM is asked to specify his/her reference levels of achievement of the membership functions, called reference membership values, which can be viewed as an obvious extension of the idea of the reference point of Wierzbicki [37]. For the DM's reference membership values $\hat{\mu}_{f_i}$, $i = 1, \cdots k$, the corresponding local Pareto optimal solution, which is in a sense close to his/her requirement (or better, if the reference membership values are attainable), can be obtained by solving the following augmented minimax problem, where it is assumed that the difference $(\hat{\mu}_{f_i} - \mu_{f_i}(x))$ is of equal importance to the DM. Thus

$$\min_{x \in X} \left\{ \max_{1 \leq i \leq k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x) \right) + \rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x) \right) \right\}$$

$$(2)$$

or equivalently

$$\left. \begin{array}{l} \min_{v, x \in X} v + \rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x) \right) \\ \text{subject to} \\ \hat{\mu}_{f_i} - \mu_{f_i}(x) \leq v, \qquad i = 1, \cdots, k \end{array} \right\}$$

$$(3)$$

or

$$\left. \begin{array}{l} \min_{w, x \in X} w \\ \text{subject to} \\ \hat{\mu}_{f_i} - \mu_{f_i}(x) \leq w - \rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x) \right), i = 1, \cdots, k. \end{array} \right\}$$

$$(4)$$

The term *augmented* is adopted because the term

$$\rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x) \right)$$

is added to the usual minimax problem, where ρ is a sufficiently small positive scalar. Thus the augmented minimax problem is a natural extension of the usual minimax problem. Such an augmented minimax problem can be viewed as a modified fuzzy version of the augmented weighted Tchebycheff norm problem of Steuer and Choo [36] or Choo and Atkins [5].

It should be emphasized here that the augmented minimax problem is simply used as a means of generating a local Pareto optimal solution, and if the DM is not satisfied with the current local Pareto optimal solution, it is possible for him/her to improve the solution by updating his/her reference membership values.

The relationships between the local optimal solutions of the augmented minimax problem and the local Pareto

optimal concept of the MONLP can be characterized by the following theorems.

Theorem 1: If x^* is a local optimal solution to the augmented minimax problem for some $\hat{\mu}_f = (\hat{\mu}_{f_1}, \cdots, \hat{\mu}_{f_k})$, then x^* is a local Pareto optimal solution to the MONLP.

Proof: Assume that x^* is not a local Pareto optimal solution to the MONLP, then there exists $\bar{x} \in X \cap N(x^*, r)$ such that $f(\bar{x}) \leqslant f(x^*)$ or equivalently $\mu_f(\bar{x}) \geqslant \mu_f(x^*)$ or $\hat{\mu}_f - \mu_f(\bar{x}) \leqslant \hat{\mu}_f - \mu_f(x^*)$, where $\mu_f(x) = (\mu_{f_1}(x), \cdots, \mu_{f_k}(x))$. Then it holds that

$$\max_{1 \leqslant i \leqslant k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(\bar{x}) \right) \leqslant \max_{1 \leqslant i \leqslant k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x^*) \right)$$

$$\rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(\bar{x}) \right) < \rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x^*) \right).$$

This means that

$$\max_{1 \leqslant i \leqslant k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(\bar{x}) \right) + \rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(\bar{x}) \right)$$

$$< \max_{1 \leqslant i \leqslant k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x^*) \right) + \rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x^*) \right)$$

which contradicts the fact that x^* is a local optimal solution to the augmented minimax problem (2). Hence x^* is a local Pareto optimal solution to the MONLP.

Theorem 2: If $x^* \in X$ is a local Pareto optimal solution to the MONLP with $0 < \mu_{f_i}(x^*) < 1$ holding for all i, then there exists $\hat{\mu}_f = (\hat{\mu}_{f_1}, \cdots, \hat{\mu}_{f_k})$ such that x^* is a local optimal solution to the augmented minimax problem.

Proof: Assume that x^* is not a local optimal solution to the augmented minimax problem (2) for any $\hat{\mu}_f$ satisfying

$$\hat{\mu}_{f_1} - \mu_{f_1}(x^*) = \cdots = \hat{\mu}_{f_k} - \mu_{f_k}(x^*).$$

Then there exists $\bar{x} \in X \cap N(x^*, r)$ such that

$$\max_{1 \leqslant i \leqslant k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x^*) \right) + \rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(x^*) \right)$$

$$\geqslant \max_{1 \leqslant i \leqslant k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(\bar{x}) \right) + \rho \sum_{i=1}^{k} \left(\hat{\mu}_{f_i} - \mu_{f_i}(\bar{x}) \right).$$

This implies that

$$\max_{1 \leqslant i \leqslant k} \left(\mu_{f_i}(x^*) - \mu_{f_i}(\bar{x}) \right) + \rho \sum_{i=1}^{k} \left(\mu_{f_i}(x^*) - \mu_{f_i}(\bar{x}) \right) \leqslant 0.$$

Now if either any $\mu_{f_i}(x^*) - \mu_{f_i}(\bar{x})$ is positive or all $\mu_{f_i}(x^*) - \mu_{f_i}(\bar{x})$, $i = 1, \cdots, k$, are zero, this inequality would be violated for sufficiently small positive ρ. Hence

$$\mu_{f_i}(x^*) - \mu_{f_i}(\bar{x}) \leqslant 0, \qquad i = 1, \cdots, k$$

must hold. Since by the assumption $0 < \mu_f(x^*) < 1$, we have $f(x^*) \geqslant f(\bar{x})$, which contradicts the fact that x^* is a local Pareto optimal solution to the MONLP and the theorem is proved.

As can be seen from the above proofs, it should be noted here that an obvious advantage of the augmented minimax problem over the usual minimax problem is that local Pareto optimality is guaranteed even if the uniqueness assumption for the solution is absent, because of the presence of the augmented term.

The geometric interpretation of the augmented minimax problem is as follows. Let us assume that (w^*, x^*) be an optimal solution to the augmented minimax problem (4) with all the inequality constraints of (4) active. Then the intersection of the k active inequality constraints in the membership function space can be obtained by solving the following simultaneous equation:

$$\begin{bmatrix} 1+\rho & \rho & \cdots & \rho \\ \rho & 1+\rho & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1+\rho \end{bmatrix} \begin{bmatrix} \mu_{f_1}(x) \\ \mu_{f_2}(x) \\ \vdots \\ \mu_{f_k}(x) \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\mu}_{f_1} + \rho \sum_{i=1}^{k} \hat{\mu}_{f_i} - w^* \\ \hat{\mu}_{f_2} + \rho \sum_{i=1}^{k} \hat{\mu}_{f_i} - w^* \\ \cdots\cdots\cdots\cdots\cdots \\ \hat{\mu}_{f_k} + \rho \sum_{i=1}^{k} \hat{\mu}_{f_i} - w^* \end{bmatrix}. \quad (5)$$

Denoting the left-hand matrix by A, its determinant becomes $\det(A) = 1 + k\rho \neq 0$. Also, denoting the adjugate matrix of A by $[\tilde{a}_{ij}]$, we have $\tilde{a}_{ii} = 1 + (k-1)\rho$, $\tilde{a}_{ji} = -\rho$. Therefore the solution to the simultaneous equation (5) is represented by

$$\mu_{f_i}(x) = \left(\sum_{j=1}^{k} \tilde{a}_{ji} \left(\hat{\mu}_{f_j} + \rho \sum_{l=1}^{k} \hat{\mu}_{f_l} - w^* \right) \right) / \det(A)$$

or, using the above

$$\mu_{f_i}(x) = \hat{\mu}_{f_i} - w^* / (1 + k\rho). \quad (6)$$

Added insight can be obtained by comparing the isoquants of the minimax problem and the augmented minimax problem as depicted in Fig. 1, where, in general

$$\theta = \tan^{-1} \left(\sqrt{k-1} \, \rho / (1+\rho) \right). \quad (7)$$

This relation shows that θ is monotone increasing with respect to ρ. Thus for sufficiently small positive scalar, augmented minimax problems overcome the possibility to generate weak Pareto optimal solutions as was shown in Theorems 1 and 2. Hence augmented minimax problems are attractive for generating Pareto optimal solutions even if appropriate convexity assumptions are absent.

To verify the relation (7), we merely observe that the cosine of the angle θ between the normal vector $(-\rho, \cdots, -\rho, -1-\rho, -\rho, \cdots, -\rho)$ of the isoquant of the augmented minimax problem and the normal vector $(0, \cdots, 0, -1, 0, \cdots, 0)$ of the isoquant of the minimax problem is given by $\cos\theta = (1+\rho) / \sqrt{1 + 2\rho + k\rho^2}$.
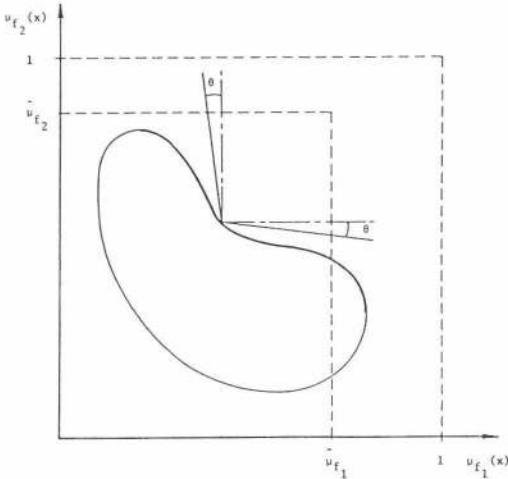
Fig. 1. Isoquants of the minimax problem and the augmented minimax problem.

Naturally, ρ should be a sufficiently small, but computationally significant, positive scalar. However, for practical purposes, a computationally significant lower bound ρ may be

$$\rho = 10^{-(a-b)} \qquad (8)$$

where a is the precision figure of the computer, and b is the figure of each membership values which the DM can distinguish. In most cases, a computationally significant value of $\rho = 10^{-3} \sim 10^{-5}$ should suffice.

Now given the local Pareto optimal solution for the reference membership values specified by the DM by solving the corresponding augmented minimax problem (3), the DM must either be satisfied with the current local Pareto optimal solution, or act on this solution by updating his/her reference membership values.

In order to help the DM express his/her degree of preference, trade-off information between a standing membership function $\mu_{f_1}(x)$ and each of the other membership functions is very useful. Such a trade-off between $\mu_{f_1}(x)$ and $\mu_{f_i}(x)$ for each $i = 2, \cdots, k$ is easily obtainable since it is closely related to the strict positive Lagrangian multipliers of the augmented minimax problem (3). Let the Lagrangian multipliers associated with the constraints of the augmented minimax problem (3) be denoted by λ_i, $i = 1, \cdots, k$. If all $\lambda_i > 0$ for each i, then by extending the results in Haimes and Chankong [12], it can be proved that the following expression holds[1]

$$\partial \mu_{f_i}(x) / \partial \mu_{f_1}(x) = -\lambda_1 / \lambda_i, \qquad i = 2, \cdots, k. \quad (9)$$

[1]Mathematically, it is assumed that 1) (v^*, x^*) is a regular point of the constraints of the augmented minimax problem 3); 2) the second-order sufficiency conditions are satisfied at (v^*, x^*); and 3) there are no degenerate constraints at (v^*, x^*), where (v^*, x^*) is an optimal solution to the augmented minimax problem (3).

The formal proof of this relation can be found in Yano and Sakawa [39] and will therefore be omitted. However, geometrically, we can understand it as follows. In $(\mu_{f_1}, \mu_{f_2}, \cdots, \mu_{f_k}, v)$ space, the tangent hyperplane at some point on Pareto surface can be described by

$$H(\mu_{f_1}, \mu_{f_2}, \cdots, \mu_{f_k}, v)$$
$$= a_1 \mu_{f_1} + a_2 \mu_{f_2} + \cdots + a_k \mu_{f_k} + bv = c.$$

The necessary and sufficient condition for the small displacement from this point belongs to this tangent hyperplane is $\Delta H = 0$ or equivalently $a_1 \Delta \mu_{f_1} + a_2 \Delta \mu_{f_2} + \cdots + a_k \Delta \mu_{f_k} + b \Delta v = 0$. For fixed values of $\Delta \mu_{f_j} = 0$ ($j = 2, \cdots, k, j \neq i$) and $\Delta v = 0$ except μ_{f_1} and μ_{f_i}, we have $a_1 \Delta \mu_{f_1} + a_i \Delta \mu_{f_i} = 0$. Similarly, for fixed values of $\Delta \mu_{f_i} = 0$ ($i = 1, \cdots, k, i \neq j$) except μ_{f_j} and v, we have $a_j \Delta \mu_{f_j} + b \Delta v = 0$. It follows from the last two relations that

$$-\frac{\Delta \mu_{f_i}}{\Delta \mu_{f_1}} = \frac{a_1}{a_i} = \frac{(-a_1/b)}{(-a_i/b)} = \frac{(\Delta v / \Delta \mu_{f_1})}{(\Delta v / \Delta \mu_{f_i})}.$$

Consequently, it holds that

$$-\frac{\partial \mu_{f_i}}{\partial \mu_{f_1}} = \frac{(\partial v / \partial \mu_{f_1})}{(\partial v / \partial \mu_{f_i})}.$$

Now using the Lagrangian multipliers λ_i, $i = 1, \cdots, k$ associated with all the active constraints of the augmented minimax problem (3), we observe that

$$\frac{\partial v}{\partial \mu_{f_i}} = -\lambda_i, \qquad i = 1, \cdots, k. \quad (10)$$

Hence we have the result (9) as required.

It should be noted here that in order to obtain the trade-off rate information from (9), all the constraints of the augmented minimax problem (3) must be active. Therefore, if there are inactive constraints, it is necessary to replace $\hat{\mu}_{f_i}$ for inactive constraints by $\mu_{f_i}(x^*)$ and solve the corresponding augmented minimax problem (3) for obtaining the Lagrangian multipliers.

So far we have considered a minimization problem and consequently assumed that the DM has a fuzzy goal such as "$f_i(x)$ should be substantially less than a_i."

In the fuzzy approaches, we can further deal with a more general case where the DM has two types of fuzzy goals, namely fuzzy goals expressed as "$f_i(x)$ should be in the vicinity of b_i" (called fuzzy equal) as well as "$f_i(x)$ should be substantially less than a_i" (called fuzzy min). Such a generalized MONLP problem may now be expressed as:

$$\begin{array}{ll} \text{fuzzy min} & f_i(x)(i \in I) \\ \text{fuzzy equal} & f_i(x)(i \in \bar{I}) \\ \text{subject to} & x \in X \end{array} \right\} \quad (11)$$

where $I \cup \bar{I} = 1, 2, \cdots, k$.

In order to elicit a membership function from the DM for the fuzzy goal "$f_i(x)$ should be in the vicinity of b_i," it is obvious that we can use different functions to the left
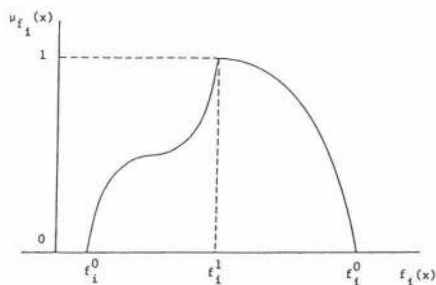
Fig. 2. An example of a fuzzy equal membership function.

and right sides of b_i. Fig. 2 illustrates the graph of the fuzzy equal membership function where the left function is hyperbolic inverse and the right function is exponential.

Having determined the membership functions for two types of fuzzy goals, if the DM specifies his/her reference membership values, the corresponding augmented minimax problem (3) is solved in order to generate a candidate for the satisficing solution.

When fuzzy equal is included in the fuzzy goals of the DM, it is desirable that $f_i(x)$ should be as close to b_i as possible. Consequently, the notion of local Pareto optimal solutions defined in terms of objective functions cannot be applied. For this reason, we introduce the concept of local M-Pareto optimal solutions which is defined in terms of membership functions instead of objective functions, where M refers to membership.

Definition 3 (local M-Pareto optimal solution): A decision $x^* \in X$ is said to be a local M-Pareto optimal solution to (11), if and only if there does not exist another $x \in X \cap N(x^*, r)$ such that $\mu_{f_i}(x) \geqslant \mu_{f_i}(x^*), i = 1, \cdots, k$, with strict inequality holding for at least one i.

Observe that the set of local Pareto optimal solutions is a subset of the set of local M-Pareto optimal solutions. Using the concept of local M-Pareto optimality, the following M-Pareto version of Theorems 1 and 2 can be obtained.

Theorem 3: The $x^* \in X$ is a local M-Pareto optimal solution to (11), if and only if there exists $\hat{\mu}_f = (\hat{\mu}_{f_1}, \cdots, \hat{\mu}_{f_k})$ such that x^* is a local optimal solution to the augmented minimax problem. The proof of this theorem is much like that of Theorems 1 and 2 and thus is omitted.

Following the above discussions, we can now construct the interactive algorithm in order to derive the local satisficing solution for the DM from among the local M-Pareto optimal solution set. The steps marked with an asterisk involve interaction with the DM.

Step 0 (Individual Minimum and Maximum): Calculate the individual minimum $f_i^{\min}$ and maximum $f_i^{\max}$ of each objective function $f_i(x)$ under given constraints.

Step 1* (Membership Functions): Elicit a membership function $\mu_{f_i}(x)$ from the DM for each of the objective functions.

Step 2 (Initialization): Set the initial reference membership values $\hat{\mu}_{f_i}^{(1)} = 1$, $i = 1, \cdots, k$ and set the iteration index $r = 1$.

Step 3 (Local M-Pareto Optimal Solution): Set $\hat{\mu}_{f_i} = \hat{\mu}_{f_i}^{(r)}$, $i = 1, \cdots, k$, solve the corresponding augmented minimax problem to obtain the local M-Pareto optimal solution $x^{(r)}, f(x^{(r)})$ and the membership function value $\mu_f(x^{(r)})$ together with the trade-off rate information between the membership functions.

Step 4* (Termination or Updating): If the DM is satisfied with the current levels of $\mu_{f_i}(x^{(r)}), i = 1, \cdots, k$ of the local M-Pareto optimal solution, exit the program. Then the current local M-Pareto optimal solution $f(x^{(r)}) = (f_1(x^{(r)}), \cdots, f_k(x^{(r)}))$ is the local satisficing solution of the DM. Otherwise, ask the DM to update the current reference membership values $\hat{\mu}_{f_i}^{(r)}$ to the new reference membership values $\hat{\mu}_{f_i}^{(r+1)}, i = 1, \cdots, k$ by considering the current values of the membership functions together with the trade-off rates between the membership functions. Set $r = r + 1$ and return to Step 3. Here it should be stressed for the DM that any improvement of one membership function can be achieved only at the expense of at least one of the other membership functions.

III. An Interactive Computer Program

Fuzzy satisficing decisionmaking processes for multiobjective nonlinear programming (MONLP) problems include eliciting a membership function for each of the objective functions and reference membership values from the DM. Thus, interactive utilization of computer facilities is highly recommended. Based on the method described earlier, we have developed a new interactive computer program. Our new package includes graphical representations by which the DM can figure the shapes of his/her membership functions, and he/she can find incorrect assessments or inconsistent evaluations promptly, revise them immediately, and proceed to the next stage more easily.

Our program is composed of one main program and several subroutines. The main program calls in and runs the subprograms with commands indicated by the user. Here we give a brief explanation of the major commands prepared in our program.

MINMAX	Displays the calculated local individual minimum and maximum of each of the objective functions under the given constraints.
MF	Elicit a membership function from the DM for each of the objective functions.
GRAPH	Depicts graphically the shape of the membership function for each of the objective functions.
GO	Derives the satisficing solution for the DM from among the local M-Pareto optimal solution set by updating the reference membership values.
STOP	Exits the program.
SAVE	Saves all the necessary information, which has been put in, in a file.
READ	Restores the information which was saved in the file.

In our computer program, the DM can select his/her membership function in a subjective manner by considering the rate of increase of membership of satisfaction from among the following five types of functions: linear [41], exponential, hyperbolic [19], hyperbolic inverse and piecewise linear [13] functions. Then the parameter values are determined through the interaction with the DM. Here, except for hyperbolic functions, it is assumed that $\mu_{f_i}(x) = 0$ if $f_i(x) \geqslant f_i^0$ and $\mu_{f_i}(x) = 1$ if $f_i(x) \leqslant f_i^1$, where f_i^0 is an unacceptable level for $f_i(x)$ and f_i^1 is a totally desirable level for $f_i(x)$.

The *linear membership function* is given by

$$\mu_{f_i}(x) = \left(f_i(x) - f_i^0\right) / \left(f_i^1 - f_i^0\right).$$

The linear membership function can be determined by asking the DM to specify the two points f_i^0 and f_i^1 within $f_i^{\max}$ and $f_i^{\min}$.

The *exponential membership function* is given by

$$\mu_{f_i}(x) = a_i\left(1 - \exp\left(-\alpha_i\left(f_i(x) - f_i^0\right) / \left(f_i^1 - f_i^0\right)\right)\right).$$

The exponential membership function can be determined by asking the DM to specify the three points f_i^0, $f_i^{0.5}$ and f_i^1 within $f_i^{\max}$ and $f_i^{\min}$, where α_i is a shape parameter, and f_i^a represents the value of $f_i(x)$ such that the degree of membership function $\mu_{f_i}(x)$ is a.

The *hyperbolic membership function* is given by

$$\mu_{f_i}(x) = (1/2)\tanh\left(\alpha_i\left(f_i(x) - b_i\right)\right) + (1/2).$$

The hyperbolic membership function can be determined by asking the DM to specify the two points $f_i^{0.25}$ and $f_i^{0.5}$ within $f_i^{\max}$ and $f_i^{\min}$, where α_i is a shape parameter, and b_i is associated with the point of inflection.

The *hyperbolic inverse membership function* is given by

$$\mu_{f_i}(x) = a_i\tanh^{-1}\left(\alpha_i\left(f_i(x) - b_i\right)\right) + (1/2)$$

The hyperbolic inverse membership function can be determined by asking the DM to specify the three points f_i^0, $f_i^{0.5}$ within $f_i^{\max}$ and $f_i^{\min}$, where α_i is a shape parameter, and b_i is associated with the point of inflection.

The *piecewise linear membership function* is given by

$$\mu_{f_i}(x) = \sum_{j=1}^{N_i} \alpha_{ij}\left|f_i(x) - g_{ij}\right| + \beta_i f_i(x) + \lambda_i$$

where

$$\alpha_{ij} = \left(t_{i,j+1} - t_{ij}\right)/2$$
$$\beta_i = \left(t_{i,N_i+1} + t_{i1}\right)/2,$$
$$\lambda_i = \left(s_{i,N_i+1} + s_{i1}\right)/2.$$

Here, it is assumed that $\mu_{f_i}(x) = t_{ir}f_i(x) + s_{ir}$ (i.e., t_{ir} is the slope and s_{ir} is the y-intercept) for each segment $g_{ir-1} \leqslant f_i(x) \leqslant g_{ir}$. The piecewise linear membership function can be determined by asking the DM to specify the degree of membership in each of several values of objective functions within $f_i^{\max}$ and $f_i^{\min}$.

IV. AN APPLICATION TO ENVIRONMENTAL PROBLEMS

Consider the application of the proposed method to an industrial area in Japan. The middle part of Osaka Prefecture is one of the most highly industrialized areas in Japan. Osaka City, which is the second largest industrial and commercial area in Japan, contains many small rivers which are branches of the Yodo River. The Yodo River is an important source of drinking water for Osaka's residents, but water pollution in the Yodo River basin has become increasingly serious. Air pollution is also at critical level in the greater Osaka area. In addition, the water supply capacity is limited in this area. Moreover, the limitations of land use in this area are obvious, since it is one of the most populous areas in Japan [23].

Here the industrial pollution control problem for Osaka City is formulated as the following three objective optimization problem [24], [26], [28]:

$$
\left.
\begin{aligned}
\text{maximize} \quad & f_1 = \sum_{j=1}^{n} A_j K_j^{1-b_j} L_j^{b_j} \\
\text{minimize} \quad & f_2 = \sum_{j=1}^{n} \left(\omega_{1j}/k_j\right)K_j \\
\text{minimize} \quad & f_3 = \sum_{j=1}^{n} \left(\omega_{2j}/k_j\right)K_j
\end{aligned}
\right\}
\tag{12}
$$

subject to

$$\sum_{j=1}^{n} \left(\gamma_{ij}/k_j\right)K_j \leqslant \Gamma_i, \qquad i = 1,2 \tag{13}$$

$$q_2 \leqslant \left(\sum_{j=1}^{n} K_j\right) \Big/ \left(\sum_{j=1}^{n} L_j\right) \leqslant q_1 \tag{14}$$

$$\alpha K_{j0} \leqslant K_j \leqslant \beta K_{j0} \tag{15}$$

$$\alpha' L_{j0} \leqslant L_j \leqslant \beta' L_{j0} \tag{16}$$

where

- j an industry ($j = 1,\cdots,n$; $n = 20$)
- K_j capital value (book value of tangible fixed assets) in industry j
- K_{j0} actual capital value in industry j
- L_j number of employees in industry j
- L_{j0} actual number of employees in industry j
- ω_{ij} unit load of chemical oxygen demand (COD) ($i = 1$) or sulphur dioxide (SO_2) ($i = 2$) per industrial shipment in industry j
- γ_{ij} resource coefficient for land ($i = 1$) or water ($i = 2$) per industrial shipment in industry j
- k_j capital coefficient, namely capital value per unit of shipments in industry j
- Γ_i restriction for land ($i = 1$) or water ($i = 2$)
- q_i upper ($i = 1$) or lower ($i = 2$) bound for the overall capital intensity (ratio of total capital value to total number of employees)

b_j, A_j parameters of the production function for each industry j, and

$\alpha, \beta, \alpha', \beta'$ parameters which represent friction (resistance) in the transfer of capital and labor.

The objective function f_1 is a Cobb–Douglas type of production function which is homogeneous of degree one and thus if to each factor the value of its marginal product is paid, total output is distributed between capital and labor in the production $1 - b_j$ and b_j, respectively. This value should be maximized so as to increase the total production. The objective function f_2 is the total amount of COD and should be minimized so as to decrease the water pollution. The objective function f_3 is the total amount of SO_2 and should be minimized so as to decrease the air pollution. Constraints (13) are resource constraints, each of which is a land or water resource constraint. Constraint (14) is the technical constraint, which shows capital intensity as a whole. This has been utilized to indicate the direction of technological changes occurring as a result of the reformation of the industrial structure. Constraints (15) and (16) are frictional constraints: because drastic changes in the industrial structure are not desirable, frictional coefficients are imposed to provide upper and lower bounds for each decision variable. The problem is to find the compromise or satisficing allocation of production factors (capital and labor) to each industry under constraints (13) to (16).

The resource restrictions Γ_1 and Γ_2 in the constraints (13) were assumed to be $\Gamma_1 = 232,200$, $\Gamma_2 = 200,000$. The parameters q_1 and q_2 were supposed to be 1.4 and 0.9, respectively. The parameter for capital and labor, α, α', and β, β', were assumed to be $\alpha = \alpha' = 0.903$, $\beta = \beta' = 1.070$. The parameters for A_j, b_j, i_j and ω_{ij}, γ_{ij} are shown in Table I and Table II, respectively. The code numbers of the industrial classification are explained in Table III. The sources for these data have been obtained mainly from Statistical Office of Osaka Prefecture [35], the Ministry of International Trade and Industry [21], and the Osaka Bureau of Trade and Industry [22].

In applying our computer program to this problem, suppose that the interaction with hypothetical DM establishes the following membership function and the corresponding assessment values for the three objective functions

f_1: linear, (f_1^0, f_1^1) $= (4800000, 5020000)$
f_2: hyperbolic, $(f_2^{0.25}, f_2^{0.5})$ $= (147000, 145000)$
f_3: exponential, $(f_3^0, f_3^{0.5}, f_3^1) = (110000, 104000, 102000)$.

In Fig. 3 the interaction processes using the time-sharing computer program under TSS of an ACOS-1000 digital computer at the computer center of Kobe University in Japan are explained—especially for the first iteration through the aid of some of the computer outputs. In this interaction, the initial values of the decision variables $x \triangleq (K_1, \cdots, K_{20}, L_1, \cdots, L_{20})$ are set at their lower bounds.

Pareto optimal solutions are obtained by solving the augmented minimax problem using the revised version of

TABLE I
CALCULATED VALUES OF PARAMETERS a_j, b_j, AND k_j

Industry	A_j	b_j	k_j
1	10.9000	0.1145	0.1195
2	8.6200	0.1391	0.1160
3	15.3900	0.1566	0.0716
4	6.1000	0.1779	0.1599
5	9.9900	0.1723	0.0926
6	5.4600	0.1540	0.1868
7	7.2200	0.2291	0.1824
8	7.9100	0.1294	0.1400
9	6.7300	0.1479	0.1735
10	9.5200	0.1737	0.1125
11	15.2200	0.1445	0.0670
12	6.1300	0.1865	0.1926
13	6.4900	0.1216	0.1746
14	8.1800	0.0870	0.1077
15	6.8500	0.1981	0.1486
16	7.4300	0.2000	0.1659
17	9.6700	0.1588	0.1020
18	7.3600	0.1841	0.1491
19	7.0000	0.2107	0.1394
20	8.4700	0.1677	0.1228

TABLE II
CALCULATED VALUES OF PARAMETERS ω_{ij} AND γ_{ij}

Industry	COD	SO_2	Land	Water
1	0.07875	0.00822	0.0244	0.0407
2	0.03111	0.02235	0.0718	0.1292
3	0.03110	0.02235	0.0219	0.0072
4	0.00142	0.00076	0.1024	0.0324
5	0.00142	0.00076	0.0244	0.0121
6	0.21680	0.06751	0.0487	0.1564
7	0.07133	0.05218	0.0105	0.0154
8	0.07133	0.05218	0.0429	0.0599
9	0.03466	0.01505	0.1461	0.0212
10	0.02592	0.00413	0.0553	0.0549
11	0.02592	0.00413	0.0468	0.0542
12	0.00198	0.07963	0.1087	0.0617
13	0.00587	0.02136	0.0773	0.0562
14	0.00084	0.03055	0.0354	0.0373
15	0.00116	0.00778	0.0589	0.0293
16	0.00083	0.00340	0.0464	0.0129
17	0.00105	0.00243	0.0235	0.0133
18	0.00073	0.00116	0.0702	0.0267
19	0.00367	0.00228	0.0451	0.0324
20	0.00864	0.00228	0.0354	0.0258

TABLE III
CLASSIFICATION OF INDUSTRIES

Code	Industries	Code	Industries
1	Foods	11	Leather products
2	Textile mill products	12	Clay and stone products
3	Apparel products	13	Iron and steel
4	Lumber and products	14	Nonferrous metals
5	Furnitures	15	Fabricated metal products
6	Pulp and paper products	16	Machinery
7	Printing and publishing	17	Electrical machinery
8	Chemicals and products	18	Transportation equipment
9	Coal and petroleum products	19	Precision machinery
10	Rubber products	20	Miscellaneous

```
COMMAND:
=GO

INPUT SUFFICIENTLY SMALL POSITIVE SCALAR FOR AUGMENTED TERM:
=0.001

--------------------< ITERATION 1 >----------------------

INITIATES AN INTERACTION WITH ALL THE INITIAL REFERENCE
MEMBERSHIP VALUES ARE 1

( KUHN-TUCKER CONDITIONS SATISFIED )

PARETO OPTIMAL SOLUTION TO THE AUGMENTED MINIMAX PROBLEM
FOR INITIAL REFERENCE MEMBERSHIP VALUES
-----------------------------------------------------------
        MEMBERSHIP           I     OBJECTIVE FUNCTION
-----------------------------+-----------------------------
    M(F1) =      0.5251       I   F(1) =      4915513.
    M(F2) =      0.5251       I   F(2) =       144817.
    M(F3) =      0.5251       I   F(3) =       103865.
-----------------------------+-----------------------------
    X(  1) =     28919.          X(  2) =     20749.
    X(  3) =      9132.          X(  4) =     14417.
    X(  5) =      9178.          X(  6) =     33403.
    X(  7) =     68254.          X(  8) =     78047.
    X(  9) =      1809.          X( 10) =      5520.
    X( 11) =      4026.          X( 12) =     14029.
    X( 13) =    104086.          X( 14) =     25958.
    X( 15) =     80583.          X( 16) =     87216.
    X( 17) =     32812.          X( 18) =     38813.
    X( 19) =      4896.          X( 20) =     28094.
    X( 21) =     25783.          X( 22) =     18740.
    X( 23) =     19347.          X( 24) =      8810.
    X( 25) =      8851.          X( 26) =     17157.
    X( 27) =     47008.          X( 28) =     36539.
    X( 29) =       885.          X( 30) =      4487.
    X( 31) =      5896.          X( 32) =      9062.
    X( 33) =     30980.          X( 34) =     10853.
    X( 35) =     56420.          X( 36) =     56002.
    X( 37) =     28597.          X( 38) =     19891.
    X( 39) =      4437.          X( 40) =     24280.
-----------------------------------------------------------


-----------------------------------------------------------
TRADE-OFFS AMONG MEMBERSHIP FUNCTIONS
  -DM(F2)/DM(F1) =          2.8539
  -DM(F3)/DM(F1) =          1.1151
-----------------------------------------------------------

ARE YOU SATISFIED WITH THE CURRENT MEMBERSHIP VALUES OF
THE PARETO OPTIMAL SOLUTION ?
=NO

--------------------< ITERATION 2 >----------------------

CONSIDER THE CURRENT MEMBERSHIP VALUES OF
THE PARETO OPTIMAL SOLUTION TOGETHER WITH
THE TRADE-OFFS AMONG THE MEMBERSHIP FUNCTIONS.
THEN INPUT YOUR REFERENCE MEMBERSHIP VALUES
FOR EACH OF THE MEMBERSHIP FUNCTIONS:
=0.5 0.6 0.55
```

Fig. 3. Interactive decisionmaking processes.

the generalized reduced gradient (GRG) [17] program called
GRG2 [18]. In GRG2 there are two optimality tests: 1) satisfy
the Kuhn–Tucker optimality conditions, and 2) satisfy the
fractional change condition

$$|FM - OBJTST| < EPSTOP \times |OBJTST|$$

for NSTOP time consecutive iterations, where FM is the
current objective value, and OBJTST is the objective value at
the start of the previous one dimensional search. NSTOP has
a default value of 3. In Fig. 3, it is shown that one of these
conditions is satisfied.

In this example, at the fourth iteration, the satisficing
solution of the DM is derived and the values of the
objectives and decision variables are shown in Fig. 4. The
CPU time required in this interaction process was 8.003,
and the example session takes about ten minutes.

The satisficing allocation of capital and labor to each
industry corresponding to the results obtained by interac-

```
--------------------< ITERATION 4 >----------------------

CONSIDER THE CURRENT MEMBERSHIP VALUES OF
THE PARETO OPTIMAL SOLUTION TOGETHER WITH
THE TRADE-OFFS AMONG THE MEMBERSHIP FUNCTIONS.
THEN INPUT YOUR REFERENCE MEMBERSHIP VALUES
FOR EACH OF THE MEMBERSHIP FUNCTIONS:
=0.48 0.62 0.57

( KUHN-TUCKER CONDITIONS SATISFIED )

PARETO OPTIMAL SOLUTION TO THE AUGMENTED MINIMAX PROBLEM
FOR YOUR REFERENCE MEMBERSHIP VALUES
-----------------------------------------------------------
        MEMBERSHIP           I     OBJECTIVE FUNCTION
-----------------------------+-----------------------------
    M(F1) =      0.4568       I   F(1) =      4900487.
    M(F2) =      0.5968       I   F(2) =       144286.
    M(F3) =      0.5468       I   F(3) =       103752.
-----------------------------+-----------------------------
    X(  1) =     28579.          X(  2) =     20749.
    X(  3) =      9132.          X(  4) =     14417.
    X(  5) =      9178.          X(  6) =     33403.
    X(  7) =     68254.          X(  8) =     78047.
    X(  9) =      1809.          X( 10) =      4660.
    X( 11) =      3776.          X( 12) =     14029.
    X( 13) =    103740.          X( 14) =     25958.
    X( 15) =     80583.          X( 16) =     87216.
    X( 17) =     32812.          X( 18) =     38813.
    X( 19) =      4896.          X( 20) =     28094.
    X( 21) =     25783.          X( 22) =     18740.
    X( 23) =     19347.          X( 24) =      8810.
    X( 25) =      8851.          X( 26) =     17157.
    X( 27) =     47008.          X( 28) =     36539.
    X( 29) =       885.          X( 30) =      4487.
    X( 31) =      5896.          X( 32) =      9062.
    X( 33) =     30980.          X( 34) =     10853.
    X( 35) =     56420.          X( 36) =     56002.
    X( 37) =     28597.          X( 38) =     19891.
    X( 39) =      4437.          X( 40) =     24280.
-----------------------------------------------------------


-----------------------------------------------------------
TRADE-OFFS AMONG MEMBERSHIP FUNCTIONS
  -DM(F2)/DM(F1) =          0.9431
  -DM(F3)/DM(F1) =          1.3559
-----------------------------------------------------------

ARE YOU SATISFIED WITH THE CURRENT MEMBERSHIP VALUES OF
THE PARETO OPTIMAL SOLUTION ?
=YES
```

Fig. 4. The satisficing solution of the DM.

tion is summarized in Table IV together with the values in
1975.

The satisficing values for the objective functions can be
interpreted as compromised values of the DM between the
conflicting objectives which are the maximization of the
production function and the minimization of two environ-
mental factors (COD and SO_2). The satisficing solution for
the decision variables K_j and L_j shows the satisficing
allocation of capital and labor to each industry. These
results show that capital values in industry as a whole are
reduced compared with the values in 1975. Capital forma-
tion in the coal and petroleum industry and in the chem-
icals and related products industry is especially severely
reduced, and the nonferrous metals industry and the
fabricated metal product industry decrease their capital
formation. On the other hand, in consumer industries such
as the lumber and furniture industries, as well as in mac-
hine industries such as the electrical machinery industry,
capital formation is promoted.

V. CONCLUSION

In this paper, we have proposed an interactive fuzzy
satisficing method using the augmented minimax problems
in order to deal with the fuzzy goals of the DM in

TABLE IV
SATISFICING ALLOCATION OF CAPITAL AND LABOR

Industry	1975		Proposal	
	Capital	Labor	Capital	Labor
1	31653	22527	28579	25783
2	22981	17521	20749	18740
3	10114	18088	9132	19347
4	13479	8237	14417	8810
5	8581	8275	9178	8851
6	36996	16041	33403	17157
7	75595	43494	68254	47008
8	86440	34161	78047	36539
9	2004	827	1809	885
10	5161	4195	4660	4487
11	3764	5512	3776	5896
12	15538	8472	14029	9062
13	108036	28964	103740	30980
14	28750	10147	25958	10853
15	75339	52749	80583	56420
16	81541	52358	87216	56002
17	30677	26736	32812	28597
18	32687	18597	38813	19891
19	4577	4148	4896	4437
20	26266	22701	28094	24280

multiobjective nonlinear programming problems. In our interactive scheme, after determining the membership functions, the satisficing solution of the DM can be derived by updating his/her reference membership values based on the current values of the membership functions together with the trade-off rates between the membership functions. Furthermore, M-Pareto optimality of the generated solution in each iteration is guaranteed. Based on the proposed method, the time-sharing computer program has been written to facilitate the interaction processes.

An application to the industrial pollution control problem in Osaka City demonstrated the feasibility and efficiency of both the proposed method and its interactive computer program by simulating the responses of the hypothetical DM. Although the actual DM for the formulated problem would of course select other values of the three objectives than the ones which were selected by the hypothetical DM used in this paper, the way to iterate and calculate is essentially the same. However, further applications must be carried out in cooperation with a person actually involved in decisionmaking. From such experiences the proposed method and its computer program must be revised. We hope that the proposed method and its extension will become efficient tools for man–machine interactive fuzzy decisionmaking under multiple conflict objectives.

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