The analysis reported in this article was supported by the National Science Foundation under grant Nos. REC 0231037 and REC 9902982, and by the Office of Educational Research and Improvement under grant number R305A60007. The opinions expressed do not necessarily reflect the views of the Foundation or OERI. We are grateful to Jo Boaler, Jim Greeno, Na'ilah Nasir, and Jana Vishnovska for their constructive critiques of a previous draft of this article.

An Interpretive Scheme for Analyzing the Identities that Students Develop in Mathematics Classrooms

Paul Cobb Lynn Liao Hodge
Vanderbilt University

Abstract

We propose an interpretive perspective for analyzing the identities that students develop in mathematics classrooms that can inform instructional design and teaching. We develop the rationale for such a scheme by clarifying its potential significance, particularly with respect to issues of equity. We then introduce the three key constructs of normative identity, personal identity, and core identity, and operationalize them by illustrating how they can be used to conduct empirical analyses. In doing so, we address a range of issues that include both the relative merits of a situated perspective on identity and the relation between identity and the major structural features of a society such as stratification by class or by race and ethnicity. In the final part of the article, we step back to consider the potential usefulness of the interpretive scheme.

Our purpose in this article is to propose an interpretive scheme for analyzing the identities that students develop in mathematics classrooms that can inform instructional design and teaching. We also consider how empirical analyses might be conducted from the interpretive perspective that we develop by focusing on an illustrative case in which a group of middle school students received contrasting forms of mathematics instruction in two different classrooms. The notion of identity has become increasingly prominent in the mathematics education research literature in recent years, particularly in analyses that are concerned with equity in students' access to significant mathematical ideas (Abreu, 1995; Boaler, 2000, 2002; Cobb & Hodge, 2002; Gutstein, 2002a, 2002b; Nasir, 2002; Sfard, 2002). Part of the appeal of this construct is that it enables researchers to broaden the scope of their analyses beyond an exclusive focus on the nature of students' mathematical reasoning by also considering the extent to which students have developed a commitment to and have come to see value in mathematics as it is realized in the classroom.[1] The notion of identity as it is used in mathematics education therefore encompasses a range of issues that are typically subsumed under the heading of affective factors. These include students' persistence, interest in, and motivation to learn mathematics.

As an illustration of an investigation in which identity was used as an explanatory construct, Boaler and Greeno (2000) interviewed students from four high schools who were enrolled in advance placement calculus classes in which they were expected to complete tasks by applying methods and strategies presented by the teacher. Many of these students indicated both that they found their experiences of engaging in mathematical activity in these classes distasteful and that they had come to dislike mathematics and would choose not to study it further. Boaler and Greeno's analysis of these student interviews revealed that the students' viewed themselves as having to give up agency and creativity if they were to become mathematical persons. Boaler and Greeno account for this finding by arguing that for these students, the identity that they would have to develop in order to become mathematical persons was in conflict with who the students viewed themselves to be and who they wanted to become. In giving this explanation, Boaler and Greeno distinguish between the *normative identity* as a doer of mathematics established in the students' classrooms on the one hand and the students' *core identities* on the other hand (cf. Gee, 2001, 2003). We will develop these notions of the normative identity established in specific classrooms and students' core identities in subsequent sections of this article. For the present, it suffices to note that Boaler and Greeno isolate the irreconcilable differences that the students experienced between their core identities and the normative identities established in their classrooms as the source of their alienation from mathematics. As a significant majority of these students were female, Boaler and Greeno contend that forms of instruction in which students are expected to learn and apply methods introduced by the teacher generate gender inequities in mathematics.

As a second illustration, Martin (2000) proposes the construct of mathematical identity as part of his analysis of mathematically successful and failing African American students. He defines mathematical identity as "one's beliefs about one's own mathematical abilities, about the instrumental importance of mathematics, about opportunities and constraints to participate in mathematics, and one's motivation to obtain mathematical knowledge" (p. viii). He then goes on to argue that students' mathematical identities are shaped in sociohistorical, community, school, and interpersonal contexts, a process that he terms students' mathematical socialization. This formulation enables him to compare succeeding and failing African American students' mathematical identities, and to investigate the extent to which these contrasts reflect differences in the various contexts of their mathematical socialization, including that of a historical legacy of denied opportunity in mathematics.

In the course of his analysis, Martin identifies two distinct groups of students in the school in which he conducted his study. In one group that he calls the dominant group, learning mathematics had a negative connotation and the students in this group promoted norms of underachievement that involved resisting mathematics instruction, frequently by being disruptive. In contrast, the students in the second group, most of whom were succeeding in mathematics, had high levels of confidence in their mathematical ability, viewed their teachers positively, and regarded achievement in mathematics as necessary to achieve their long-term goals that involved careers in high status occupations. With regard to the constructs that we will develop in this article, these students' envisioned life trajectories are an aspect of their core identities, of who the students viewed themselves to be and who they wanted to become. Martin's focus on what he terms the students' mathematical identities is consistent with our view that it is both possible and useful to distinguish between students' core identities and the *personal identities* that they develop as they participate in (or resist) the activities of particular groups and communities, including those of the mathematics classroom. Martin's analysis indicates that in developing their personal identities as doers of mathematics, the succeeding students had reconciled their core identities with the normative identities established in their classrooms by viewing mathematics achievement as a means of social and economic advancement. As Martin's primary focus is on the succeeding students, he provides little information that allows us to infer the failing students' core identities. However, the oppositional personal identities that they were developing indicate deep and irreconcilable conflicts between their core identities and the normative classroom identities as doers of mathematics. In developing these oppositional identities, the students were active contributors to the process that delimited their access to significant mathematical ideas (cf. Willis, 1977).

In addition to providing an initial orientation to research on identity, Boaler and Greeno's (2000) and Martin's (2000) seminal investigations will serve as points of reference as we present the interpretive scheme that we propose for analyzing and accounting for the personal identities that students develop mathematics classrooms. We first develop the rationale for such a scheme by describing a range of challenges posed by current work and then clarify its potential significance, particularly with respect to issues of equity. Next, we give an overview of the illustrative case on which we will draw in which a single group of students were members of different mathematics classrooms. Against this background, we introduce the key constructs of normative identity, personal identity, and core identity, and operationalize them by illustrating how they can be used to conduct empirical analyses. In doing so, we address a range of issues that include both the relative merits of a situated perspective on identity and the relation between identity and the major structural features of a society such as stratification by class or by race and ethnicity. In the final part of the article, we then step back to consider the potential usefulness of the interpretive scheme.

A Crisis of Identity

Our attempt to develop an interpretive scheme that it useful for our purposes as mathematics educators was motivated in part by three difficulties apparent in research on identity. In our view, these difficulties together constitute a crisis that requires we scrutinize and, if necessary, rethink basic suppositions and assumptions. The first difficulty concerns a contradiction that is apparent in many analyses of identity developed from a situated perspective. Boaler and Greeno's (2000) analysis is representative in this regard in that they argue that the personal classroom identities that students develop are an aspect of their participation in the patterns of activity that typify particular classrooms. Consequently, as Boaler has noted elsewhere, the identities that students develop in the mathematics classroom may be very different from those that they develop in other classrooms (Boaler, Wiliam, & Zevenbergen, 2000). This claim that the students' identities are highly contextual appears to be at odds with Boaler and Greeno's postulation that the students in their study had also developed a view of themselves and who they want to become that was relatively stable and that spanned their participation in different groups and communities. This apparent inconsistency is significant given the central role that this seemly transcontextual sense of self plays in Boaler and Greeno's analysis. We, like Boaler and Greeno, take a situated perspective and will address this apparent paradox when we develop the key constructs of the interpretive scheme that we propose.

The second difficulty concerns the limited guidance that current formulations provide to instructional designers and to teachers. Boaler and Greeno's (2000) and Martin's (2000) investigations make a foundational contribution by demonstrating the importance of identity as a focus of research in mathematics education and by opening up new, potentially productive lines of inquiry. However, the relatively global way in which they analyze the classroom environments in which students' develop their personal identities as doers of mathematics limits the extent to which their analyses can inform practice. For example, Martin restricts his account of the classrooms he observed to a description of the teachers' attempts to initiate the renegotiation of social norms with their students, and the ways in which some of the students resisted. He offers little in the way of detail about the teachers' instructional practices beyond this focus on norms of participation that are not specific to mathematics. Furthermore, although he explicitly defines the notion of mathematical identity, this construct appears to play only a minor role in his explanation of why some of the students are successful in their mathematics classes and others are not. His primary conclusion instead focuses exclusively on the successful students' core identities. He argues that these students' success is attributable to the strong sense of self and personal agency that they have developed in opposition to both the norms of underachievement of the dominant group and negative elements of the local community. This explanation seems to imply that they but not the failing students adhere to what Mehan, Hubbard, and Villanueva (1994) refer to as the achievement ideology wherein hard work and individual effort are valued and will be rewarded in a society that will give them assess to future educational and economic opportunities.[2] Although this explanation has merit given the institutional arrangements of the school and instructional settings in which Martin conducted his investigation, it leaves instructional designers and mathematics teachers in a position of relative powerlessness. Left unexamined are the origins of the dominant group and the possibility of cultivating some of these students' interest in and valuing of mathematics by changing what it might mean to know and do mathematics in school.[3]

The characterization that Boaler and Greeno provide of the classrooms that they studied also focuses on the general norms of participation as well as on the relative openness of the instructional tasks. The norms that they document include the extent to which students were expected to collaborate to develop mathematics ideas and strategies, and to explain and justify their mathematical reasoning in classroom discussions. The resulting descriptions of the classroom learning environments enable the reader to discern the extent to which they are compatible with current reform recommendations. Boaler and Greeno relate their analyses of classroom learning environments to individual student interviews designed to document whether the students have come to value engaging in mathematical activity in the classroom and whether they perceive themselves to be co-developers of mathematical ideas and methods with their teachers. Their findings are valuable in demonstrating the relative advantages of instruction that is compatible with current reform recommendations in supporting students' development of positive personal identities as doers of mathematics. However, the accounts that they provide of classrooms are not specific to mathematics and do not touch on either the specifically mathematical norms of argumentation or the forms of mathematical reasoning that have become normative. One can, for example, imagine how the constructs that they employ could be used to analyze the learning environments established in science or in social studies classes. As a consequence, their analyses provide little guidance to teachers who subscribe to what might be termed the reform philosophy and who seek to improve their instructional practices, or to instructional designers who seek to develop instructional materials and associated resources for such teachers. Boaler (2002) in fact anticipated our argument in a recent article by calling for investigations of instructional practices that promote equity to "pay attention to a level of detail in the enactment of teaching that has been lacking from many analyses" (p. 243). The construct of the normative classroom identity as a doer of mathematics that we will introduce can be viewed as a response to this appeal for an appropriate level of detail.

The third difficulty apparent in research on identity concerns the multiple and often conflicting ways in which the term is currently used in both the mathematics education and the more general literature. As we have seen, Martin (2000) is careful to explicitly define his construct of mathematical identity as a cluster of beliefs. He clarifies that these beliefs are influenced by what he refers to as various forces that correspond to the sociohistorical, community, school, and interpersonal contexts that his study encompasses. This formulation involves what Lave (1991) calls the cognition plus perspective in which theorists analyze the social and cultural world as a network of factors that shape and influence the development internal knowledge structures and beliefs. In contrast, Boaler and Greeno (2000) have developed a perspective on identity in which people are viewed as becoming who they are by being able to participate in the practices of a particular group or community. In formulating this perspective, they explicitly question the assumption made by cognition plus theorists that social and cultural processes can be partitioned off from internal concepts and beliefs and treated as external condition for them. They instead contend that people's identities extend out into the social and cultural worlds of the groups and communities with which they have come to identify. They and Martin are therefore speaking about decidedly different notions when they use the term identity.

The divergence in viewpoints becomes even more apparent when we look beyond mathematics education. For example, anthropologists use the term cultural identity to designate a sense of self that is viewed as forming in relation to the major structural features of a society, that develops though socialization practices that inculcate central cultural values, and that is stable and enduring (cf. Holland et al., 1998). Ogbu (1999) exemplifies this anthropological perspective in his analysis of the collective identity within an African American community. He characterizes this collective identity as oppositional and argues that it was formed in relation to the minority status forced on community forbearers and the mistreatment to which they were subjected. In contrast to this formulation that takes societal macrostructures as its point of reference, Gee (2001, 2003) has proposed a perspective on identity in relation to broad Discourse groups within society. He defines a Discourse as follows:

> Discourses are sociohistorical coordinations of people, objects (props), ways of talking, acting, interacting, thinking, valuing, and (sometimes) writing and reading *that allow for the display and recognition of socially significant identities*, like being a (certain type of) African American, boardroom executive, feminist, lawyer, street-gang member, theoretical physicist, 18th-century midwife, 19th-century modernist, Soviet or Russian, schoolchild, teacher, and so on through innumerable possibilities. If you destroy a Discourse (and they do die), you also destroy its cultural models, situated meanings, and its concomitant identities. (Gee, 1997, pp. 255-256, italics added)

As an example that is closer to the experience of most mathematics educators, the various Standards documents produced by the National Council of Teachers of Mathematics (NCTM, 1989, 1991, 2000) can be viewed as proposing an educational Discourse. Teachers who are members of local professional teaching communities might also identify with and develop a sense of belonging to a broader Discourse community of mathematics education reformers. It is important to note that in equating identity with engagement in a particular Discourse, Gee (1997, 2001, 2003) defines it in terms of performance in society that results in being recognized as a certain kind of person such as a mathematics education reformer.

Whereas Gee focuses on relatively broad groups or communities, Wenger (1998) has developed a view of identity that that takes local communities of practice as its point of reference. In contrast to Discourse communities, members of a community of practice directly engage in activities together as they use a shared repertoire of tools to pursue a joint enterprise. A professional teaching community in a school or district might, for example, constitute a community of practice (Franke & Kazemi, in press; Stein, Silver, & Smith, 1998). Boaler and Greeno's (2000) analysis illustrates this perspective in which identity is analyzed as a form of individuality that develops as people engage with others in contributing to the evolution of communal practices. Wenger (1998) also emphasizes that, in his view, communal norms or standards for justifying actions orient members to look at the world in a particular way. In his view, identification is integral to the process developing this communal perspective and the forms of competence that it entails, and thus to the process of becoming a member of the community.

The perspectives that Ogbu, Gee, and Wenger take involve a situated rather than a cognition plus approach and characterize identity in relation to participation in the activities of a group or community. An important difference between their positions concerns the nature and scope of the group or community that they take as primary. Sfard (2002) also addresses issues of identity from a situated perspective but takes the immediate communicational context as her point of reference, thereby bringing the detailed choreography of communicative interactions to the fore. In developing her communicational perspective, she equates conceptualizing with communicating and defines the identity of a person as a story told about that person. She then goes on to distinguish between first person identities (stories the person tells about him or herself), second person identities (stories about the person told to the person), and third person identities (stories told about the person by a second person to a third person). In developing her viewpoint, Sfard emphasizes that the story told and thus the identity of the person is influenced by the immediate communicational context. For example, teachers might, in all sincerity, tell different stories about their instructional practices to each other during the meeting of a professional teaching community and to a school administrator who is observing their classroom. As Sfard makes clear, the identity of a person is, in her formulation, a process that is sensitive to the vicissitudes of communicative interactions. Consequently, she challenges the view that people have an intrinsic sense of self that they might be said to possess and instead contends that they maintain an enduring sense of identity by treating certain elements of the stories that they tell and hear as saying the same thing.

Our goal as we navigated this array of alternative and sometimes conflicting perspectives was not to try and determine which viewpoint gets identity right. Instead, we viewed these various perspectives as possible sources of ideas while developing an analytic approach that is tailored to our concerns and interests as mathematics educators. A key criterion for us was therefore that the interpretive scheme should result in analyses of the identities that students develop in mathematics classrooms that can feed back to inform instructional design and teaching. A second criterion was the interpretive scheme should enable us to account without contradiction for both students' relatively stable sense of who they are and who they want to become, and their development of different senses of self in different instructional settings. As will become apparent, the tripartite scheme of normative classroom identity, personal classroom identity, and core identity that, we claim, satisfies these criteria draws most heavily on the perspectives advanced by Wenger and by Gee.

The Significance of Students' Personal Classroom Identities as Doers of Mathematics

As we have noted, the notion of identity provides a way of accounting for students' persistence, interest in, and motivation to engage in mathematical activity as it is constituted in the classroom. As a consequence, the development of students' classroom identities and of their mathematical reasoning are intimately related. Nasir (2002) describes this interrelation in the following terms:

> [On the one hand,] as members of communities of practice experience changing (more engaged) identities, they come to learn new skills and bodies of knowledge, facilitating new ways of participating which, in turn, helps to create new identities relative to their community… [On the other hand,] increasing identification with an activity or with a

> community of practice motivates new learning. In this sense, identities can act as a motivator for new learning, prompting practice participants to seek out and gain the new skills they need to participate in their practice more effectively. (p. 239-240)

This interrelation serves to emphasize the importance of cultivating students' development of a sense of affiliation with mathematical activity as a goal for both instructional design and teaching. However, in our view, it is a mistake to take a purely instrumental stance towards the cultivation of students' personal identities as doers of mathematics by regarding it solely as a means to the end of supporting their development of increasingly sophisticated forms of mathematical reasoning. We contend that students' development of a sense of affiliation with mathematical activity should instead be treated as an important instructional goal in its own right. Although many students will not pursue career trajectories beyond school that involve direct engagement in mathematical activity, it is nonetheless reasonable to propose that mathematical literacy include an empathy for and sense of affiliation with mathematics together with the desire and capability to learn more about mathematics when the opportunity arises.

Looking beyond general issues of mathematical literacy, the importance of cultivating students' personal identities as doers of mathematics becomes all the more apparent when we consider possible sources of inequities in students' access to significant mathematical ideas. To develop our argument, we draw on and extend a distinction that D'Amato (1992) makes between two ways in which learning in school can have value to students. D'Amato refers to the first of these ways as extrinsic value or structural significance in that achievement in school has instrumental value as a means of attaining other ends such as entry to college and high-status careers or acceptance and approval in the household and other social networks. D'Amato contrasts this source of value with what he terms intrinsic value or situational significance in which students view their engagement in classroom activities as a means of maintaining valued relationships with teachers and peers, and of gaining access to experiences of mastery and accomplishment.

Ogbu (1992, 1999), D'Amato (1992), Erickson (1992), and Mehan et al. (1994) all note that students' access to the structural rationale for attempting to achieve in mathematics classes varies as a consequence of family history, race or ethnic history, class structure, and caste structure within society.

> Where school success has been associated with social mobility, as in the case of the middle and upper classes, the need to succeed in school [and in mathematics in particular] is emphasized in home-life networks, and children take for granted the value to their futures and to present social relationships of positive teacher evaluations and other markers of school success … School, however, tends to have little credible structural significance for [what Ogbu terms] castelike minority children (Ogbu, 1978) and for the majority of children of lower socioeconomic strata. (D'Amato, 1992, p. 191).

As Erickson (1992) observes, this perspective on achievement and motivation in school emphasizes that they are explicitly political processes "in which issues of institutional and personal legitimacy, identity, and economic interest are central" (p. 33).

> Students in school, like other human beings, learn constantly. When we say they are "not learning" what we mean is that they are not learning what the school authorities, teachers, and administrators intend for them to learn as a result of intentional instruction … Learning what is deliberately taught can be seen as a form of political assent. Not learning can be seen as a form of political resistance. (Erickson, 1992, p. 36)

These considerations indicate the importance of understanding how issues of legitimacy, identity, and interest are framed and understood by students as they engage in instructional activities in local classroom communities.

As a starting point in this endeavor, we extend D'Amato's (1992) analysis by differentiating between two ways in which learning mathematics have situational significance for students. One the one hand, students might reconcile their core identities with general school norms, thereby identifying with and gaining experiences of mastery and accomplishment from academic success in all subject matter areas including mathematics. This appears to be the case with the succeeding students who participated in Martin's (2000) study. On the other hand, students might reconcile their core identities specifically with the normative identity as a doer of mathematics established in one or more of their mathematics classes and come to appreciate and value engaging in mathematical activity in those classes. In this latter case, the students have come to identify with mathematics as it is realized in their classrooms.

This elaboration of D'Amato's analysis gives rise to the six possible cases shown in Table 1 when we cross whether students have access to a structural rationale for learning mathematics with whether engaging in classroom mathematical activity has situational significance for them. Consider first the top left-hand cell in which students do not have access to a structural rationale and classroom mathematical activity does not have situational significance for them. The students in the dominant group that Martin (2000) identified would seem to illustrate this case. Martin distinguished between these students and the succeeding students who participated in his study in terms of whether they had come to identify with academic success in general, rather than in terms of whether classroom mathematical activity had situational significance for them. Moving to the top right-hand cell, we note that the students in Boaler and Greeno's (2000) study who experienced a conflict between the normative identities established in their mathematics classes and the kinds of persons they wanted to become did not overtly resist instruction but instead continued to cooperate with their teachers despite their increasing alienation from mathematics. To account for this response, we speculate that most of Boaler and Greeno's advanced calculus students had access to a structural rationale for learning in school whereas most of the students in Martin's dominant group did not. In addition, we conjecture that most of Martin's succeeding students also had access to a structural rationale (see Table 1) and that this was reflected in their adherence to the achievement ideology. As Martin documents, striving to achieve in school was, for these students, a means of maintaining valued relationships with teachers and peers as well as of gaining access to experiences of mastery and accomplishment.

<table>
<tr><td></td><td></td><th colspan="2">Access to Structural Rationale</th></tr>
<tr><td></td><td></td><th>No</th><th>Yes</th></tr>
<tr><th rowspan="3">Access to Situational Rationale</th><td>No</td><td>Resistance
(e.g., Martin, 2000)</td><td>Alienation
(e.g., Boaler & Greeno, 2000)</td></tr>
<tr><td>Yes: Achieving in school</td><td></td><td>Identification with academic success
(e.g., Martin, 2000)</td></tr>
<tr><td>Yes: Engaging in Mathematical Activity</td><td>Identification with mathematical activity (e.g., Gutstein, 2002a,b)</td><td>Identification with mathematical activity (e.g., Boaler & Greeno, 2000)</td></tr>
</table>

Table 1: Hypothetical Relations Between Students' Access to a Structural and a Situational Rationale and the Nature of Their Engagement in Classroom Mathematical Activity

None of the illustrations that we have given thus far in this article involves a case in which students came to identify with mathematics as it was realized in their classroom rather than with academic success more generally. To rectify this omission, we draw on Gutstein's (2002a, b) reports of a classroom design experiment in which he served as the teacher of a group of Latino students from a poor Chicago community throughout their seventh and eighth grade years. Briefly, Gutstein describes how he used a textbook series compatible with current reform recommendations as the basis for instruction but supplemented it by developing instructional activities that addressed issues of social justice, racism, and inequality (e.g., the gentrification of the students' neighborhood and the concomitant loss of low-income housing, the distribution of wealth both within the U.S. and globally by continent). Gutstein uses classroom observations, student surveys, student journals, and video-recorded presentations of class projects to document that most of the students came to appreciate and value engaging in mathematical activity as it was constituted in his classroom. This indicates that they were not merely cooperating with Gutstein by attempting to act in accord with his expectations but had come to identify with mathematical activity as it was realized in his classroom. In our terms, they had reconciled their core identities with the normative classroom identity as a doer of mathematics and became who they were as individuals in the classroom as they participated in and contributed to the development of communal norms and practices. As there is no indication that Gutstein's students subscribed to the achievement ideology or came to identify with academic success more generally, we view this as a case in which students who did not have access to a structural rationale were given access to a situational rationale for learning mathematics (see Table 1).

The final case to consider is that in which students who have access to a structural rationale come to value and appreciate engaging in classroom mathematical activity. We again refer to Boaler and Greeno's (2000) analysis as they also interviewed advanced placement calculus students in two schools in which mathematics classes were generally consistent with current reform recommendations and involved both student collaboration and whole class discussions of students' solutions. The interviews again focused on the students' perceptions of their mathematics classes, the nature of their experiences in the classes, and their valuations of those experiences. Boaler and Greeno's analysis of these interviews indicates that most of the students in these discussion-based classes valued both the creativity involved in completing the relatively open-ended tasks that were assigned as well as the opportunities to collaborate with other students. Thus, in contrast to their peers who were in classes where they were expected to apply methods introduced by the teacher, these students seem to have reconciled who they viewed themselves to be and who they wanted to become (i.e., their core identities) with the normative identities as doers of mathematics established in their classrooms.

Taken collectively, the various cases that we have considered and that are summarized in Table 1 indicate the significance of the personal classroom identities that students develop from the point of view of equity. The students in Martin's dominant group did not have access to significant mathematical ideas. This was also the case for the students in Boaler and Greeno's study who found engaging in classroom mathematical activity distasteful as they said that they intended to terminate their study of mathematics. In our terms, the students in both these groups had not been able to reconcile their core identities with the normative identity as a doer of mathematics established in their classrooms. Martin proposes an approach for giving such students' access to significant mathematical ideas that involves fostering their adherence to the achievement ideology. He draws on his analysis of the succeeding students in his study to recommend 1) that parents and community members stress in both word and action the importance of mathematical knowledge, and 2) that students be helped to develop strong personal agency and to associate the learning of mathematics with attempts to succeed and take advantage of what life has to offer. As Hall (2002) notes, this proposed approach locates the source of the difficulty primarily with the students and their community and does not touch on either the mathematical ideas that orient instruction or the nature of mathematical activity as it is realized in the classroom.

Gutstein's (2002a, b) and Boaler and Greeno's (2000) analyses both indicate a second approach the goal of which to give students access to a situational rationale for learning mathematics in school. This approach scrutinizes what it might mean to know and do mathematics in school and strives to make it possible for students to reconcile their core identities with the normative identity as a doer of mathematics that is constituted in the classroom. The interpretive scheme that we will elaborate is designed to support efforts of this type that focus on instructional design and teaching at the classroom level. As we will clarify in the final section of this article, we view the twin approaches of giving students access to the achievement ideology and to a situational rationale as complementary rather than in opposition. We therefore concur with Martin's (2000) conclusion that what he terms community forces can make it difficult for changes in curricula and pedagogy to be realized in the classroom as intended. However, it does not follow from this observation that we should eschew the analysis of teaching and learning of mathematics at the classroom level. In our view, Martin's observation instead implies that efforts to cultivate students' appreciation and valuing of mathematics should be part of a larger endeavor that extends beyond the classroom to the school and community. A comprehensive approach of this type builds on and extends research on learning, teaching, and instructional design, thereby offering the prospect that an explicit focus on issues of equity might become more central to what might be termed mainstream research in mathematics education than is currently the case.

Before turning to the interpretive scheme, we need to give one caveat about the various cases that we have discussed. Although the cases as we have summarized them in Table 1 have heuristic value, they are overly simplified in that they reflect a sociostructurally determinist position. For example, the Table implies that as a consequence of sociostructural processes such as race or ethnic history, class structure, and family history, none of the students in Martin's dominant group had access to structural rationale, and that this determined their resistance to instruction. Similarly, the Table implies that all the succeeding students in Martin's study had access to structural rationale, and that this determined their identification with academic success. As Martin's analysis makes clear, this direct, causal account belies the complexity of the situation. In accounting for the succeeding students' identification with academic achievement, he does demonstrate the importance of both role models that the students wanted to emulate and homes and communities that stressed the value of education. However, he also documents that these means of support were important for different students in different ways. In addition, he draws attention to the succeeding students' agency by documenting that as part of the process of fashioning positive academic identities, they had established both short and long-term goals that would not be possible without education.

The interpretive scheme that we will present acknowledges this complexity. Although we assume that students construct personal identities in immediate contexts of action and interaction such as classrooms, we also view these immediate contexts of negotiation as arenas in which sociostructural processes play out in face-to-face interaction (cf. Gutierrez, Rymes, & Larson, 1995). This position reflects the contention that it is only as people participate in the practices of various local communities that broader sociostructural processes can touch their experience (Wenger, 1998). Although analyses of identity that take either societal macrostructures or broader Discourse groups as their sole point of reference have utility at a global level, our concern is with how these processes play out in the local setting of the mathematics classroom. From the perspective that we will develop, a person's sense of what it means to be, say, a mathematics education reformer, a mathematics student, or a female African American adolescent is seen to take form within the immediate contexts of action and interaction in which he or she engages. In concert with Martin (2000), the position we take also acknowledges people's agency to make and remake the settings in which they act by improvising novel or creative responses to particular situations. However, in doing so, we also emphasize that agency is deeply cultural in that novelty and the originality stem from the manner in which people adapt and reconfigure cultural resources from a range of different practices (cf. Holland et al., 1998; Emirbayer & Mische, 1999; Varenne & McDermott, 1998).

Background to the Illustrative Case

The case that we will use to ground our presentation of the interpretive scheme involves a group of eleven eighth-grade students who received contrasting forms of mathematics instruction in two different classrooms. The eleven students attended an urban middle school that served a 40% African American population. Seven of the eleven students were African American, three were Caucasian, and one was Asian American. One of the classes was a classroom design experiment that we conducted during the last period of the school day that focused on statistical data analysis.[4] A member of the research team served as the teacher in all 41 classroom sessions that were conducted during the 14-week experiment. The second class was an algebra class that was conducted by the students' regular mathematics teacher during their mathematics period earlier in the school day.[5]

We should stress that our intent in documenting and accounting for the personal identities that the students developed in these two classes is illustrative rather than evaluative. It is important to acknowledge, for example, that the algebra teacher had to accommodate typical concerns of content coverage and accountability within the school whereas we were not subject to those constraints when we conducted the data analysis design experiment. We should also clarify that the eleven students were part of a larger group of 29 students who had participated in a prior classroom design experiment that we had conducted the previous school year when they were seventh graders. In the intervening school year, eight of the students had transferred to other schools and four had other obligations (e.g., practice for the school play or for the school band). Of the remaining 17 students, 16 volunteered to give up their activity period and eleven continued to attend throughout the 14-week experiment. The five students who dropped out, all of who were Caucasian, indicated that they were having difficulty completing their homework for other classes and wanted to use the activity period at the end of the school day for this purpose. It is, however, possible that the eleven students who continued to participate had self-selected based on the extent to which they valued what it meant to know and do mathematics in the design experiment classroom.[6] We therefore focus in the identities that the students were developing as doers of mathematics in the algebra and the design experiment classrooms solely to illustrate and operationalize the constructs of normative identity, personal identity, and core identity. To do so, we first give an overview of both the algebra class and the design experiment classes and then develop each of the three constructs in turn.

The Algebra Class

The teacher of the algebra class had 25 years classroom experience and was considered to be a successful teacher by both school administrators and her peers, in part because the majority of the students in her classes typically passed a test that determined whether they would receive a Carnegie Unit for the course. Her overall goal was to support the students' development of a range of proficiencies that included solving equations and inequalities, simplifying radical expressions and polynomials, and understanding notions such as variable, rational numbers, functions. The teacher attempted to achieve this goal by assigning problems either from the textbook or from an activity sheet. As a representative illustration, in the case of inequalities, she assigned 20 problems in which the students had to specify the range of values for which given equations such as $7x < 6x - 11$ were true. In addition to assessing her students' learning on the basis of their responses to homework problems that were assigned during each class session, the teacher also administered a weekly quiz and two or more comprehensive tests during each six-week grading period.

The organization of activities in each classroom session typically involved three phases. The teacher and students first reviewed homework problems that had been assigned during the previous class session. In the second phase, the teacher the introduced the types of problems that she planned to assign as homework in the current session. In the third phase, the students began to work on assigned homework problems either individually or in small groups. The teacher graded homework once each week and, for the other homework assignments, addressed students' questions during the initial review phase. She assisted students who indicated difficulty with a homework problem, by calling on volunteers to write their solutions on the whiteboard and then answered students' questions. During the second phase of a classroom session, the teacher demonstrated the method she expected students to use to solve the types of problems that would be assigned for homework later in the class session. In the case of an assignment that involved solving quadratic equations, for example, the teacher demonstrated a method for solving the equation $(y + 2)(3y + 5) = 0$. She then presented a second quadratic equation and asked the students to solve it. After two or three minutes, she assessed the students' learning by calling on individual students to provide particular steps of the solution. During the third phase, the students worked on problems that the teacher assigned for homework either individually or in small groups for the remainder of the session.

The Design Experiment Class

The overall goal of the eighth-grade design experiment was to support the students' development of increasingly sophisticated ways of analyzing bivariate data sets as part of the process of developing effective data-based arguments. The overarching mathematical idea that served to orient the design of the instructional activities and associated resources was that of distribution. The instructional activities were designed so that the students might view the data sets as realistic and consider the purposes for which they were to analyze them as legitimate. As an illustration, in one instructional activity the students analyzed data on carbon dioxide levels in the atmosphere for the period 1957 – 1979 as part of a series of activities that focused on global warming (see Figure 1). As a second example, the students investigated possible inequities in men's and women's salaries by analyzing data sets that showed salary against years of education for men and for women (see Figures 2a and 2b). In these and other instructional activities, the students were required to write a report in which they presented their findings to a particular audience (e.g., members of a government commission in the case of the possible inequities in salaries). The students were not assigned homework problems and there were no formal tests although tasks were frequently designed as performance assessments that provided information about the students' reasoning to inform the ongoing design process.

As the two sample activities illustrate, the students did not, for the most part, collect data but instead analyzed data sets introduced by the teacher. In the first phase of an instructional activity, the teacher and students talking through how they could generate data that would enable them to address a particular problem or issue. In these data creation discussions, the teacher and students first clarified why the problem or issue at hand was potentially significant to them or to a particular audience. Next, they discussed aspects of the phenomenon under consideration that were relevant to the issue they sought to address. Frequently, they also discussed how they would actually measure these attributes in order to generate the required data. Against this background, the teacher introduced the data the students were to analyze.

In the second phase of an instructional activity, the students analyzed the data individually or in small groups. In doing so, they used a computer-based analysis tool for creating graphical displays of data sets that was developed for the experiment. This tool provided the students with a range of options for organizing bivariate data sets that were inscribed as scatter plots. The final phase of an instructional activity consisted of a whole class discussion of the students' analyses in which a computer projection system was frequently used to support the students' explanations.

Normative Identity as a Doer of Mathematics

We introduced the notion of the normative identity jointly established by the teacher and students by drawing on Boaler and Greeno's (2000) reference to identities that students would have to develop in order to become mathematical persons in a particular classroom. The normative classroom identity is concerned with the obligations that a student has to fulfill in order to be an effective and successful mathematics student in that classroom. These obligations involve general norms for classroom participation as well as specifically mathematical norms. The approach that we will illustrate involves delineating the classroom social norms and the specifically mathematical norms that include norms or standards for mathematical argumentation, normative ways of reasoning with tools and written symbols, norms for what counted as mathematical competence and understanding, and relatedly, the normative purpose for engaging in mathematical activity. This relatively fine-grained approach moves beyond global characterizations of classrooms as traditional or reform in nature by developing analyses at what Boaler (2002) termed an appropriate level of detail.

The data that we generated to document the normative identity established in the algebra class consist of field notes of observations of each class session during the first two and the last two weeks of the design experiment, and of one class session per week during the intervening ten weeks. In the case of the design experiment classroom, the data consist of video-recordings of all 41 classroom sessions and two sets of field notes. The method that we followed to delineate both classroom social norms and specifically mathematical norms is a variant of Glaser and Strauss' (1967) constant comparative method and is specifically tailored to the systematic analysis of longitudinal data sets in mathematics education (Cobb & Whitenack, 1996). The first phase of the analysis involved working through the entire data set for each classroom in chronological order. In this phase, conjectures made about students' obligations while analyzing particular episodes were tested and, as necessary, revised while analyzing subsequent episodes. In the second phase of the analysis, we analyzed the resulting chain of conjectures and refutations to produce empirically-grounded accounts of the norms established in each classroom that consist of a network of mutually reinforcing assertions that span each data set. Elsewhere, we have described in some detail the types of evidence that we use to determine whether a particular norm has been established (Cobb et al., 2001). For our illustrative purposes, it suffices to note that the process of delineating classroom norms involves identifying patterns or regularities in the teacher's and students' ongoing interactions. The conjectures that are substantiated or refuted in the course of an analysis therefore apply not to individual students' actions but to patterns in collective activity and to students' obligations as they contribute to regeneration of these patterns.

Social Norms

The algebra class. As we have documented, the enactment of instructional activities in the algebra involved three phases: 1) A review of homework problems assigned during the previous class session, 2) the introduction of the types of tasks that would be assigned as homework in the current class session, and 3) individual or small group work in which the students began to complete the homework tasks assigned from the textbook or an activity sheet. The social norms established as instructional activities were enacted included that the students were obliged to:

- Listen and take notes in order to understand the solution methods demonstrated by the teacher.
- Ask the teacher clarifying questions in order to understand the demonstrated methods.
- Demonstrate that they understood the methods when questioned by the teacher.

The first of these norms was evident in the first phase when the teacher asked the students if they had questions about any of the homework tasks. Volunteers wrote their solutions to tasks that were identified as problematic on the whiteboard at the front of the room. The observing students were obliged to try and understand the solutions and most copied them into their notebooks. The second social norm of asking clarifying questions was also evident in this phase of lessons. In particular, if any of the observing students had difficulty in understanding one of the solutions written on the whiteboard, it was their obligation to ask clarifying questions. They invariably directed these questions to the teacher rather than to the student who had written the solution and the teacher responded by referring to the written steps of the solution method.

The norm of listening and attempting to understand was also apparent in the second phase of classroom sessions. The teacher introduced the types of tasks that would be assigned for homework by demonstrating how to solve one or more sample tasks, assigning two of three similar tasks for the students to solve, and then demonstrating the expected solution method for these tasks. During these demonstrations, the students could be observed copying the steps of the solution methods into their notebooks and the teacher typically admonished those who did not do so. Although the teacher did not encourage the students to ask questions in this phase of the lessons, it was legitimate for them to do so. When a student did ask a question, the teacher responded by restating the steps of the solution method.

The last of the students' general obligations, that of demonstrating to the teacher that they understood the demonstrated solution method, was apparent in the first two phases of lessons. In these phases, exchanges between the teacher and a student almost invariably involved the initiation, response, evaluation pattern. The teacher routinely posed questions as she demonstrated solution methods, typically by calling on particular students to give the next step of the solution method. These questions appeared to serve as an assessment function for the teacher, and the students were obliged to demonstrate their understanding by giving the response that she expected. The students were also obliged to demonstrate understanding as they completed homework assignments and weekly in-class quizzes. When the students graded each others' work, assessment was restricted to the correctness of answers, whereas when the teacher graded their work, assessment also focused on the match between the steps of students' solution methods and those that the teacher had demonstrated.

The design experiment class. The enactment of instructional activities in this class typically spanned two or more class sessions and involved three phases as described previously: 1) A whole-class discussion of the data generation process, 2) individual or small group activity in which the students worked at computers to analyze data, and 3) a whole class discussion of the students' analyses. The social norms established as instructional activities were enacted included that the students were obliged to:

- Explain and justify their reasoning.
- Ask clarifying questions in order to understand other students' explanations.
- Indicate and give reasons for disagreement with other students' arguments.

To facilitate comparison with the algebra class, we will restrict our focus to the final data analysis discussions as it was during this phase that classroom discourse centered on the process of solving instructional activities. In the course of these discussions, the teacher typically selected several individual students or groups of students to explain how they had analyzed the data. When particular students were called on, they almost invariably came to the computer projection screen at the front of the classroom and explained how they had organized and interpreted the data to arrive at their conclusion. When they had completed their explanation, several of the listening students usually raised their hands to indicate that they wanted to ask a question. A number of these questions were typically requests for clarification and indicate that the students were attempting to fulfill their obligation of understanding others' reasoning. On those occasions when none of the listening students had a question, the teacher frequently told them that she assumed that they all understood the explanation and could restate it in their own words. Consistent with this admonition, she often attempted to highlight a student's argument that was particularly relevant to her instructional agenda by asking another student to restate it.

In addition to asking for clarification, the listening students frequently challenged or suggested modifications to the analyses that were presented. In these situations, the teacher often intervened if she judged that the challenge might advance her instructional agenda by revoicing it or by posing questions to enable other students to understand the challenger's concerns (McClain, 2002). In these exchanges, the listening students were obliged to explain why they disagreed with an analysis and the explaining students were, in turn, obliged to justify their analysis. The teacher, for her part, attempted to mediate communication between the students. In addition, she expressed her institutionalized authority in action by deciding which of the students' responses to an analysis to frame as topics for further conversation and which not to pursue. In contrast to the teacher of the algebra class, she did not therefore evaluate the students' contributions explicitly. She did, however, continually attempt to influence the direction of discussions both by calling on selected students to explain their analyses and by deciding which of the students' questions and challenges to incorporate into the discussion. In the course of these discussions, the teacher was constituted as the primary social authority in the classroom as she regulated ongoing exchanges in this manner.

Specifically Mathematical Norms

The analysis of the social norms established in the two classrooms serves to document what Erickson (1986) and Lampert (1990) term the classroom participation structure. The social norms that we have identified are not specific to mathematics but can instead apply to any subject matter area. In contrast, our intent in analyzing specifically mathematical norms is to delineate the nature or quality of mathematical activity as it was realized in each classroom as well as the normative purpose for engaging in it. The norms that proved to be relevant when specifying the normative identity as a doer of mathematics established in each classroom are 1) norms or standards for mathematical argumentation, 2) normative ways of reasoning with tools and written symbols, and 3) norms for what counted as mathematical understanding or mathematical competence. These latter norms specify how students have to engage in classroom activities in order to be recognized as mathematically competent. They therefore encompass the normative purpose for engaging in mathematical activity, namely to learn and to become mathematically competent. In focusing on the specifically mathematical norms of understanding and competence, we follow Lampert (1990) who demonstrated that what counts as mathematical understanding is constituted in the course of classroom interactions and can differ significantly between classrooms.

The algebra class. To be acceptable, an explanation had to specify the calculational steps taken to produce a result. When the teacher demonstrated a solution method, for example, she consistently explained the method as a sequence of steps that the students were obliged to follow. The students contributed to the constitution of this norm during these demonstrations by asking questions about particular solutions steps. In response, the teacher invariably restated her explanation of the calculational step. The only situations in which students were asked to explain an entire solution method occurred during the first phase of homework review after volunteer students had written their solutions on the whiteboard. On occasion, the teacher would ask one or more of these students to explain their methods and they consistently attempted to explain the calculational steps they had taken in a manner similar to that demonstrated by the teacher.

The written symbols with which the teacher and students reasoned in public classroom discourse were those of conventional algebraic notation. These notations were treated as the immediate objects of reasoning and were not constituted as signifying anything beyond themselves. Solutions as constituted in interaction between the teacher and students consisted of sequences of actions that successively transformed a given equation. In participating in the development of these solutions, the students contributed to the constitution of normative ways of reasoning as the enactment of calculational steps on written notations.

The norm of what counted as mathematical understanding that was jointly constituted by the teacher and students in the algebra classroom involved knowing how to enact the solution methods that the teacher prescribed. The strongest evidence that this was the case concerns the way in which students were positioned when they transgressed these methods. As we have noted, the teacher routinely called on students to state the next step in a solution method when she demonstrated how to solve tasks. Frequently, she did so by asking, "OK, so what do I have now?" If the student either could not answer or gave an incorrect response, the teacher consistently indicated that the student's response was inadequate and demonstrated the correct step in the solution. In such exchanges, students were positioned as not having understood the demonstrated solution method. Furthermore, the teacher was, for her part, constituted as the primary mathematical authority who regulated what counted as legitimate mathematical activity. The normative purpose for engaging in mathematical activity in this classroom was therefore to produce answers that the teacher judged correct by enacting prescribed calculational steps on written algebraic notations.

<u>The design experiment classroom</u>. With regard to the norms of what counted as an acceptable mathematical explanation, it was not necessarily sufficient for the students to explain how they had used the computer tool to create graphical displays of data sets. Instead, they were also obliged to explain their reasons for creating particular displays when this was not readily apparent to other students. Typically, in stating these reasons, the students attempted to clarify why the way they had displayed the data was relevant to the question or issue that they sought to address by conducting the analysis. Those occasions in which a student was perceived to have violated established standards of argumentation constitute relatively strong evidence in this regard. As an illustration taken from near the beginning of the eighth-grade experiment, a pair of students justified their conclusion about global warming by explaining how they had organized data on carbon dioxide levels in the atmosphere for the period 1957-1979 (see Figure 1). The first student to respond complained, " I still don't get the point. You all do it and tell us what you're doing without actually explaining why you did it." The teacher immediately legitimized this student's objection by asking, "Why did you look at those particular values? Why did you look at it that way? Is that what you are asking? Why is looking at the data that way going to help you with regard to the problem?" Exchanges of this type in which either a student or the teacher challenged the adequacy of an explanation were relatively common. In addition, the teacher was often explicit in stating her expectation that the students should clarify why they had organized the data in a particular way. For example, during the discussion of the carbon dioxide data, she interjected as a student explained his analysis, "Right now, he is just telling you what he did, but in a minute he is going to tell you why it was helpful." Observations of this type indicated to the students that they were obliged to explain why their analysis was relevant to the question at hand (e.g., understanding the phenomenon of global warming).

The teacher and students reasoned with the computer tool during the discussions of the students' analyses, frequently by annotating data displays that were projected onto a whiteboard. In these discussions, data displays were constituted as texts about the situation from which the data were generated (e.g., texts about changes in the level of carbon dioxide in the atmosphere). This was strongly indicated by the way in which analyses were consistently treated as different even if two students had created the same data display provided that they had inferred different trends or patterns from the display and had developed different insights into the phenomenon under investigation. Normative ways of reasoning with the computer tool therefore involved using it to create data displays that gave rise to such insights. Concomitantly, mathematical understanding as it was constituted in public discourse involved knowing how to identify trends and patterns in the data that gave rise to insights into the phenomenon under investigation. This was most evident when aspects of students' analyses were challenged. In these situations, they were obliged to justify their conclusions about the phenomenon under investigation by further clarifying both their reasons for creating a particular data display and the pattern they had identified. The normative purpose for engaging in mathematical activity in this classroom when both analyzing data and listening to others' explanations was therefore to gain insight into the phenomenon under investigation.

Reflections

The general classroom norms and specifically mathematical norms on which we have focused illustrate how the notion of the normative identity as a doer of mathematics jointly established by the teacher and students in a classroom can be operationalized. The sample analyses demonstrate that the normative identities or, in Boaler and Greeno's (2000) terms, who the students would have to become to be mathematical persons differed significantly between the two classes. In the case of the algebra class, the students would have to become persons who had developed a sense of affiliation with the activity of enacting prescribed calculational steps on written algebraic notations that did not signify anything beyond themselves. In contrast, in the design experiment classroom, they would have to become persons who had come to value and appreciate the activity of creating and interpreting data displays in order to identify trends and patterns that gave rise to insights into the phenomenon under investigation.

In spite of Boaler's (2002) arguments, the level of detail at which we have characterized the two classrooms might be viewed as excessive. It might, for example, be argued that the algebra class can be adequately described as a traditional class that fostered instrumental understanding and delimited the students' sense of agency, and the design experiment class as a reform class that fostered relational understanding and intellectual autonomy. To illustrate the limitation of this characterization, consider the norms of what counted as an acceptable mathematical argument established in the two classrooms. The contrast between the norm of specifying the calculational steps taken to produce a result and of explaining the reasons why data had been organized in a particular way does not correspond to Skemp's (1976) distinction between instrumental and relational understanding. This becomes apparent when we note that in many of the examples of reform classrooms to be found in the literature, students' arguments are acceptable if they merely explain the steps of their solution methods. This is the case even though, in the majority of these examples, the methods that the students explain are self-generated and appear to involve relatively sophisticated mathematical reasoning. The standard dichotomy between traditional reform classrooms therefore glosses over the contrasting norms of argumentation that we identified.

We contend that a relatively detailed analysis of norms of argumentation is relevant as we seek to document the types of persons that students would have to become to be mathematical people in particular classrooms. Following Thompson and Thompson (1996), we term the discourse in both reform classrooms where it is sufficient to explain the steps of solution methods and in the algebra classroom calculational to indicate that the issues that emerged as topics of conversation were limited to methods or strategies for producing results. In contrast, the discourse in the design experiment classroom was conceptual in that the issues that emerged as topics of conversation also included the interpretations of instructional activities that underlie particular methods and strategies, and that constituted their rationale. As we have demonstrated elsewhere (Cobb et al., 2001), discussions in which the teacher judiciously supports students' attempts to articulate their task interpretations can be extremely productive settings for their mathematical learning. The sample analyses of the algebra and design experiment classrooms indicate that an explicit focus on students' interpretations and understandings can also be relevant to their development of a sense of affiliation with mathematical activity as it is realized in the classroom.

In analyzing the normative identities established in the two classrooms, we drew attention to the manner in which both teachers were constituted as authorities in their classrooms. It is therefore tempting to assume that teachers in each classroom established the normative identity that the students were then "invited" to adopt. To counter this assumption, it is important to stress that each teacher jointly constituted the various norms on which we have focused with the students in the course of their ongoing interactions. As an illustration, we noted an incident in the design experiment classroom in which a student complained after a group had presented their analysis, " I still don't get the point. You all do it and tell us what you're doing without actually explaining why you did it." The teacher then legitimized the student's objection and the explaining students attempted to clarify why they had organized the data in a particular way. In this exchange, the objecting and explaining students and the teacher together contributed to the establishment of the norm or what counted as an acceptable mathematical explanation. As a second example, we saw that the algebra teacher did on occasion ask students to explain their methods and they consistently responded by attempting to explain the calculational steps they had taken in a manner similar to the teacher's demonstrations. In responding in this manner, the students contributed to the constitution of the norm of what counted as an acceptable explanation, as did the teacher by indicating that the students' explanations were acceptable. As these examples illustrate, students necessarily contribute to the establishment of the normative classroom identity. As a consequence, we would questions accounts in which the teacher is portrayed as inviting students to adopt a normative identity as a doer of mathematics that has been established independently of their classroom participation. Instead, in the perspective that we propose, students are seen to contribute to the initial constitution and ongoing regeneration of the normative identity, and to develop their personal classroom identities as they do so.

In this formulation, the relation between the type of person that students would have to become to be mathematical people (i.e., normative identity) and who they actually become in the classroom (i.e., personal identity) is indirect. We saw, for example, that a number of the students in the traditional classes studied by Boaler and Greeno (2000) developed identities as people who were becoming increasingly alienated from mathematics even as they contributed to the continual regeneration of the normative identities established in their classrooms. Boaler and Greeno's analysis indicates the importance of distinguishing between cases in which students have come to value and appreciate engaging in mathematical activity as it is realized in their classroom, and cases in which students are merely cooperating with their teacher by attempting to fulfill his or her expectations. This distinction between a willingness to cooperate and the development of an affiliation with classroom mathematical activity is evident in Holland et al.'s (1998) characterization of identification as a process whereby communal activities "in which one has been acting according to the directions of others becomes a world that one uses to understand and organize aspects of one's self and at least some of one's own feelings and thoughts" (p. 121). A key issue that we will necessarily have to address when we operationalize the notion of personal identity is therefore that of documenting whether and by what means students have come to identify with mathematical activity as it is realized in their classrooms. It is, however, possible to make an initial determination in this regard while analyzing the classroom participation structure. This becomes apparent when we note with Nasir (2002) that identification is one of two processes involved in the development of a sense of affiliation with the activities of a community.

Nasir (2002) follows Wenger (1998) in clarifying that the process of developing a sense of affiliation with communal activities involves negotiability as well as identification:

> *Identification* is "the process by which modes of belonging become constitutive of our identities by creating bonds or distinctions in which we become invested" (Wenger, p. 191). In other words, identification describes the process of [personal] identity construction through becoming a member of a community of practice. *Negotiability*, on the other hand, refers to "the ability, facility, and legitimacy to contribute to, take responsibility for, and shape the meanings that matter within a social configuration" (Wenger, p.197). Hence, negotiability implies an additional kind of ownership and agency that acknowledges the mutual constitution of individuals and social practices. (pp.219-220)

As Nasir notes, identification and negotiability together account for both the social structure inherent in communal activities and the individual agency inherent in initiating and contributing to the development of those communal activities. This acknowledgement of agency as well as structure implies that we can develop a crude index of whether students are merely cooperating with the teacher or have come to value and appreciate classroom mathematical activity by documenting the extent to which they initiate and sustain the explicit negotiation of mathematical meanings. It is, for example, significant that the questions the students raised in the algebra class were almost invariably concerned with clarifying aspects of the solution method that the teacher had demonstrated. In other words, their questions centered on how to act in accord with the teacher's directions. In contrast, in the design experiment class, students typically asked questions to request clarification or to challenge analyses that had been presented. Although the teacher highlighted some of the students' contributions and not others, the mathematical issues that emerged as topics of conversation were nonetheless initiated by the students for the most part and frequently resulted in protracted exchanges. Students therefore had "the ability, facility, and legitimacy to contribute to, take responsibility for, and shape" communal activities in the design experiment class but not the algebra class. This crude index gives an initial indication that the students were merely cooperating with the teacher of the algebra class but had developed a sense of affiliation with mathematical activity in the design experiment class. However, we regard inferences based on this index as speculative and consider it essential to document students' developing personal identities more directly by conducting systematic interviews that focus on students' understandings of and valuations of their classroom obligations.

Students' Developing Personal Identities

A total of 41 audio-recorded interviews were conducted to document the personal identities that the 11 students who participated throughout the design experiment were developing in both this classroom and the algebra classroom. All interviews were conducted at the school while the experiment was in progress and each student was interviewed at least twice. The students were typically interviewed in pairs or groups of three because pilot work indicated that group interviews elicited richer responses than did interviews conducted with individual students. The questions posed during the interviews focused on the students' interpretations of classroom events with a particular emphasis on their understanding and valuations of their general and specifically mathematical obligations, and on their assessments of their own and others mathematical competence.

Sfard's (2002) observation that the identity stories that people recount are influenced by the immediate communicational context serves to emphasize that an interview is a social event in which the interviewer and interviewees present themselves to each other (cf. Mishler, 1986). As a precaution, we ensured that the second author, who conducted all the interviews, was not involved in either conducting or video-recording the instructional sessions in the design experiment classroom. She introduced herself to the students by explaining that she was a former teacher in the school and wanted to talk with them to better understand their views of the lessons and the instructional activities in each classroom. Her intent in presenting herself to the students in this way was that they might view her as associated with the school rather than with the team conducting the design experiment. This strategy appears to have been effective in that the students did make a number of negative observations about the design experiment class. These criticisms concerned the length of time that some of the instructional activities focused on a single topic (e.g., global warming), and about the predictable the way in which instructional activities were enacted.

We analyzed the interviews by first transcribing the audiotapes and then reviewing the students' responses with respect to five themes, four of which are relevant to the documentation of their developing personal identities in each class[7]: The students' understandings of both their general obligations and their specifically-mathematical obligations, their assessments of their own competence, and their assessments of other students' competence. The process by which these themes were identified involved first grouping together students' responses in a particular interview that appeared to be related and then developing conjectures about the possible underlying issues. We tested the viability of these conjectures as we analyzed subsequent interviews. In this manner, possible themes and associated conjectures about the personal identities that the students were developing in each class were tested and revised as we worked through the interviews chronologically (cf. Glaser and Strauss, 1967). This interplay between conjectures and refutations together with the trackability of our final assertions to the audio-recordings constitute the primary means for establishing the trustworthiness of the analysis.

The Students' Understandings and Valuations of Their General Classroom Obligations

The observations the students made about their obligations in each classroom were remarkably consistent. For example, all eleven students contrasted the nature of discussions in the two classes in a similar manner. For them, discussions in the algebra class involved the teacher demonstrating how to solve particular problems. The following two comments are representative:

> S: She [algebra teacher] says it is a discussion, but it's really her talking. We listen to her and copy the stuff down. Sometimes I don't really know what I'm copying down. I just copy it down to have it. In Vandy [design experiment class], it was like we had two different discussions. One with the class and then one with your group.
> (Stacy 12-17)
>
> I: Tell me about the discussions in there [algebra class].
>
> M: We don't really discuss anything because she tells us how to do it. If I missed something I ask somebody.
> (Mike 12-16)

Consistent with our analysis of the social norms established in the algebra class, the students indicated that their primary obligation during these discussions was to listen to the teacher and learn how to enact the solution methods she introduced. In addition, they clarified that when they initiated informal exchanges with other students, their primary concern was to solicit further directions about the methods that they were expected to use.

In contrast to the emphasis that they placed on the obligation of listening to the teacher in the algebra class, all eleven indicated that they were obliged to explain their own analyses in the design experiment class and to ask questions in order to understand others' analyses. For example, two of the students described the class in the following terms:

> S: You have to do a good job explaining how you looked at the problem. That's important since you didn't talk with everybody else when you were looking at the graph. (Stacy 9-23)
>
> M: You talk about your way, or you add something to someone else's way. You can't just say that you agree or you disagree. Ms. M [the design experiment teacher] makes you explain it. You have to ask questions about things that you don't understand.
>
> I: What do you mean?
>
> M: If you, um, don't understand why someone did something you have to ask them about it. You can't just say, oh yeah, that's okay, what you did. (Megan 9-1)

The manner in which these and the other students' framed their statements (e.g. "your job", "you have to", etc.) implies that they viewed these aspects of their participation as obligations. The students' responses also indicated that they viewed these obligations of explaining their analyses and of attempting to understand others' explanations as reasonable and valued them positively. Two of the students in fact voiced some frustration that the algebra teacher did not, in their view, adequately explain the solution methods she presented.

> J: When you ask her [the algebra teacher] to work it out, she works it out, but she doesn't tell you how.
> (Janet 12-16)
>
> K: She points to it on the board and tells you the steps to do, but she doesn't explain you why she's doing those things.
> (Kate 10-14)

In summary, the students' understandings of their general obligations in the two classes are broadly consistent with our analysis of the social norms established in the two classrooms. The students described their primary responsibility in algebra class as learning the solution methods the teacher presented by listening to her. In contrast, when the interviewer questioned them about the design experiment class, they emphasized the obligations of explaining their analyses and asking questions in an attempt to understand others' explanations.

The Students' Understandings and Valuations of Their Specifically Mathematical Obligations

The students' responses to questions that probed their specifically mathematical obligations were again quite consistent and indicated that they perceived significant differences between the two classes. Their comments about what counted as an adequate explanation in the algebra class focused almost exclusively on the teacher rather than on their own activity. One student captured the general thrust of their responses when he observed:

> S: She [the algebra teacher] says this is how you do it and you do it.
> (Sean 10-1)

As we have noted, two of the students also viewed the teacher's explanations as inadequate because she did not provide a rationale for the solution methods that she demonstrated. Taken together, the students' responses indicate that an acceptable explanation in this class involved specifying the calculational steps taken to produce an answer.

We have already seen that the students emphasized their own rather than the teacher's activity when they described discussions in the design experiment class. Five of the students went on to clarify that it was not sufficient to report how they had analyzed data and stated that they also had to explain why they had analyzed the data a particular way. These five students all justified this perceived obligation in terms of the importance of other students understanding the explanation.

> V: I knew what they [the other students] did so I didn't want them to tell me what they were doing, but what were they thinking, yeah, what was your intention.
> (Valerie 10-6)
>
> S: You can't just talk about your conclusion because that doesn't let anybody know why you did things.
>
> I: Is that important?
>
> S: If you don't talk about what you were thinking about then we don't know if it all is okay … we can't figure out if it is a good point.
> (Sally 8-27)

These students' understandings of what counted as an adequate explanation are consistent with our observation that students were frequently challenged if they described how they had used the computer tool to create graphical displays of data sets but failed to explain why these displays were relevant to the question or issue under investigation. In addition, the students' responses also indicate that their valuations of this obligation were positive in that it enabled them to understand and assess the merit of others' analyses.

It was apparent from the students' responses that they also perceived significant differences in the nature of their obligations as they reasoned with tools and symbols. All eleven students indicated that the emphasis in the algebra class was on producing correct answers. Furthermore, in response to the question of what they had learned in this class, they all referred to actions on written inscriptions. The following responses are representative in this regard:

> I: What have you learned in the algebra class?
>
> B: It's a math class that's all we do, math, just straight problems. I guess I learned about … like when you have x's on both sides you want to have them on the same side to do the equation .
> (Brad 9-1)
>
> I: What is something that you learned in Mrs. W's [algebra] class?

K: I guess you can just subtract a variable from another variable if it is on a different side of the equal sign.
(Kate 8-26)

As these responses indicate, there was no indication that any of the students viewed written notations that they produced as they enacted solution methods as referring to anything beyond themselves.

In contrast, the students emphasized using the computer tool to find patterns in data displays when they were questioned about the design experiment class:

B: You have to work on the computers to find something in the data.

I: What do you mean?

B: You have to use the tool to look for stuff in the graphs.

I: That's what you have to do to do well in there?

B: Yeah, you have to look for what's consistent and then talk about it. You can use the tool to explain it. That helps me a lot.
(Brad 9-24)

I: What is some advice you would give to a new student?

S: You should study the graphs so you have something to say about them.

I: What do you mean?

S: You look for like what you think stands out in the graphs. Then you sort of use that. It's kinda easier to use the tool to explain what you saw, you know, when you tell everyone your conclusion.
(Sally 10-24)

Furthermore, their responses to the question of what they had learned in the design experiment class indicated that in analyzing data, they gained insights into the phenomenon under investigation

M: You don't really learn things like you do in other classes. Like in our algebra class, we learn things like the process or how to do things step by step. In the Vandy class [design experiment class] we learn about the world. Or what's happening.
(Mike 12-15)

I: What have you learned so far?

V: I've learned a lot about the different subjects that we talk about. You all find out about it and give us data. Like the salary stuff for men and women. I knew it was going to be different, but it's like I can prove something now and that's different than just thinking it's different.

I: Is that important, to prove it?

V: I think it just gives more to what you say. Like, plus I didn't know about how they were different so that makes me want to learn why it's so different.
(Valerie 10-23)

In speaking of "learning about the world" or "leaning about common sense things," most of the students indicated explicitly that the data displays created by using the computer tool were, for them, texts about the situations from which the data had been generated.

The students' accounts of both their general obligations and their specifically mathematical obligations in the algebra class indicate that to demonstrate mathematical understanding, they had to produce correct answers by enacting the solution methods the teacher had demonstrated.

M: You do your work and you show everything.

I: What do you have to do?

M: You have to write it all out. Like she does. Even taking the number away, like if it’s two. It's like we know it already, but we still have to show the right steps.
(Mark 12-16)

S: You have to do it all right. Like I remember on this worksheet we did if you didn't put all these wavy lines you got your answers wrong even if you had the rest of the answer and there's a lot of things that you could miss. If you forget just a little thing it's wrong. She counts off for those things. You get some credit, but mostly it's wrong.
(Stacey 12-17)

These responses are representative in that most of the students evidenced signs of frustration that the criteria for what counted as an acceptable solution were, from their perspective, somewhat arbitrary.

In contrast to their corresponding responses about the algebra class, all the students initially replied in relatively global terms when they described the process of producing acceptable solutions in the design experiment class and the interviewer had to press them to be more specific. As a consequence, the exchanges were relatively lengthy:

I: What did you learn [in the design experiment class]?

M: I learned how to work the graphs. I really did enjoy that. It is different from what we normally do in math class.

I: What do you mean work the graphs?

M: I mean analyze the graphs, compare the data, look at the medians to compare.

I: Let's say that I don't know anything about analyzing data. What do you think is important to tell me, here is what you need to know. What would you say?

M: You should look at the median and that you should compare the two.

I: Why should I look at the median?

M: Because that is the highest concentration of data.

I: If I am looking at the median, what does that tell me?

M: If you look at the median it is easier to make judgments about the graph.

I: Is the median good to use on all graphs?

M: No, but on some it is…like the speed reading graph. That wouldn't really make sense … the scores were scattered all over.
(Mike 12-16)

M: Now we know the terms like mean, median, range, and when you would want to use those terms.

I: You said you know what average is. How is it different from before? Average is something you talked about before the [design experiment] class in other classes?

M: I learned when to use it to describe the data.

I: When should you use it?

M: Most of the time I don't use the average. I like using the range. I use the range when the points are spread out. If the points are around in a really small area you probably want to use the median since that would be a better way to let someone know about the [data] points.
(Megan 12-16)

These representative responses illustrate that to demonstrate understanding in the design experiment class, the students had to know when to use particular statistics (e.g., median, range) to identify trends in data sets. It is also worth noting that the students' references to "letting someone know about the points" and "doing something with the data so someone can relate to it" indicate that they viewed themselves as developing an analysis for a particular audience. This sense of audience implies that the world in which the students acted as they analyzed data was partially organized in terms of social relationships that, for them, extended beyond the classroom.

The Students' Assessments of Their Own Competence

All eleven students said that they viewed themselves as succeeding in the design experiment class whereas only four described themselves as successful in the algebra class. Significantly, these four students included all three Caucasian students who participated in the investigation. It was also apparent from their responses that the students all gauged their success in the algebra class on the basis of the grades that they received from the teacher. In the design experiment class, however, students based their assessments of their competence on the extent to which they could contribute to class discussions in relatively substantial ways. The pilot interviews that we conducted revealed that the students also assessed their success in this manner in the prior seventh-grade design experiment even though the teacher graded homework assignments twice each week. However, the grading criteria that the teacher used were consistent with the criteria by which the students' analyses were assessed during whole class discussions and focused on the adequacy of their data-based arguments. It is therefore questionable that the contrasting ways in which the students assessed their competence in the two classes can be attributed solely to the fact that their solutions were graded in one class but not the other.

We present the following students' responses to illustrate the contrast in their assessments of their competence in each class. These assessments were consistent throughout the design experiment.

K: I'm not doing so good [in the algebra class]. It's like my grades have dropped, it's like she takes every little detail of every little problem. You are like, you leave out the x by accident it's like wrong, wrong, wrong.
(Kate 10-28)

K: I am pretty good in there [the design experiment class] since I understand what we're doing. Sometimes it's hard for me to think about the problem at the end of the day, but I usually get into it when we start talking about it.
(Kate 10-28)

M: I don't think she [the algebra teacher] thinks I'm doing the best I can. I make too many mistakes on the quizzes. They're not that hard, I just forget about stuff.
(Mike 12-15)

M: I think I'm doing okay [in the design experiment class]. Brad and I talked about our way yesterday and a lot of people were talking about it. It was a good way.
(Mike 12-15)

M: I understand it, but I am making some careless mistakes [in the algebra class]. I have a C or something. I need to do better. Mrs. W already talked to me about how I should be doing better.
(Megan 12-16)

M: Yeah, I think I'm a good student [in the design experiment class]. I come up with some good ideas.
(Megan 12-16)

S: There's a problem when no one understands [in the algebra class]. Everyone I talked to yesterday was confused about the homework. I'm not a very good student in there,

but I don't think anyone is in my class.
(Stacey 12-17)

S: Yeah, I think I am [in the design experiment class]. I can read the graph and come up with my answer.
(Stacey 12-17)

Taken together, these and the other students' responses indicate that they viewed the difficulties they experienced in the algebra class as being beyond their control. As one student put it when asked if he was doing well in the algebra class, "It's not up to me" (Brad 10-6). In contrast to this experienced lack of personal agency in the algebra class, the students spoke of knowing what they were doing, of coming up with good ideas, and of other people in the class talking about their analyses in the design experiment class. These responses suggest that the students had come to view themselves as a resource for their own learning in this class. This inference is consistent with our prior observation that the data displays they created when using the computer tool were, for them, texts about situations from which the data had been generated. It was because they could use the computer tool in this way that they could rely on their own resources to make decisions rather than, say, the judgments of the teacher.

The Students' Assessments of Others' Competence

When they were asked to indicate who was succeeding in the algebra class, all eleven students indicated specific students and most identified one student. In contrast, all but three of the students said that they viewed all their peers to be succeeding in the design experiment class. The following responses are representative in illustrating the contrasts that the students drew between the two classes:

I: Do you feel like a good student in the [design experiment] class?

K: No, I just feel regular. Equal.

I: You feel equal, the same? Doesn't anybody stand out?

K: Yeah, I think we are all equal.
(Kate 10-23)

I: Did you feel smart in the statistics class?

V: Yeah, everybody felt smart. You're like, "Hey, look what we did. We talked about global warming, we analyzed these graphs."

(Valerie 10-23)

I: Who is smart in the statistics class?

S: I think we're all smart. We talk about things so everyone knows what we're talking about. We kind of go at the same speed.

I: What about the algebra class?

S: Emily. (Sean 12-15)

I: Who is smart in the statistics class?

J: I can't think of one person. We all do a good job because she explains what we're suppose to do. We can talk to our partner about it.

I: What about the algebra class?

J: Tyler. (Janet 10-6)

I: Who is smart in the statistics class?

M: Bryan says a lot, but I think we're all pretty smart. We come up with some good ways of working with the graphs.

I: What about the algebra class?

M: In my class, I think Casey is the only one who has an A. I understand it, but I just make a lot of mistakes.
(Mike 12-15)

Given the uniformity in the students' responses, it is worth clarifying that issue of who was good a student was not discussed in the design experiment class and the teacher did not tell the students that they were all succeeding. We were in fact surprised by the students' responses, as there were significant differences in the sophistication of the data based arguments that they produced throughout the experiment (Cobb et al., 2003). We therefore interpret their judgments that they were all smart or all "went at the same speed" to mean that they viewed themselves and the other students as able to make substantial contributions to class discussions. The students' assessments are significant given that the algebra teacher and the teacher in whose classroom we conducted the prior seventh-grade design experiment described them as a diverse group in terms of their success in their mathematics classes.

Reflections

The analysis we have presented of the personal identities that the students developed in the two classrooms serves to illustrate how the notion of personal identity as a doer of mathematics can be operationalized. As we have documented, the students perceived significant differences in the nature of mathematical activity as it was realized in the two classrooms. In particular, the purpose for engaging in mathematical activity in the algebra class as they understood it was to produce correct answers by enacting prescribed methods on written notations. In contrast, the purpose in the design experiment class was to identify trends and patterns in data that give rise to insights into the phenomenon under investigation. The interviews indicate that although the students were willing to cooperate with the algebra teacher, the personal identities they had developed were those of non-mathematical people who had become disenchanted with mathematical activity as they experienced it in this class. Further, most of the students had come to view both themselves and most of their peers as lacking in competence, the development of which was beyond their control. In contrast, the interviews indicate that the personal identities the students were developing in the design experiment class were those of people who had developed a sense of affiliation with mathematical activity as it was realized in this classroom. They had also come to view both themselves and their peers as people who were developing competencies that enabled them to rely increasingly on their own resources.

In terms of the interpretive scheme that we propose, the interviews indicate that the students had reconciled their core identities with their participation in the ongoing regeneration of the normative identity in design experiment class. In contrast, they do not seem to have achieved a similar reconciliation in the algebra class. In Wenger's (1998) terms, they had come to view themselves as people who have the ability, facility, and legitimacy to contribute to, take responsibility for, and shape the meanings that matter in the design experiment class but not the algebra class. In Holland et al.'s (1998) terms, the students had come to identify with mathematical activity in the design experiment class in that it had become a social world that they used to understand and organize aspects of their selves. In contrast, their participation in the algebra class continued to involve acting according to the directions of the teacher.

We should clarify that the students' assessments of their mathematical competence in the design experiment class are consistent with analyses of their mathematical reasoning that we have reported elsewhere. The relatively impressive nature of their learning encompassed both the sophistication of the data-based arguments that they developed (Cobb et al., 2003) and the depth of their understanding of issues related to the process of generating data such as the representativeness of samples and the control of extraneous variables (Cobb & Tzou, 2000). In the account that we propose, these competencies and the students' developing sense of affiliation with mathematical activity are seen to have co-evolved. To paraphrase Nasir (2002), it was as the students developed more engaged personal identities as doers of mathematics that they learned to reason about data in increasingly sophisticated ways. These emerging mathematical competencies made it possible for them to participate in classroom activities in new ways, which in turn contributed to their sense of affiliation with mathematical activity as it was realized in this classroom. Furthermore, this increasing identification with the activity of analyzing data served to motivate their continued engagement in classroom activities, and thus their development of more sophisticated mathematical competencies.

In addition to assessing their own competences positively in the design experiment class, most of the students also indicated that they viewed all the other students to be succeeding in this class. As we noted, they appeared to base these assessments on whether they and the other students made substantial contributions to class discussions. The significance of the students' assessments becomes apparent when we observe that students who either do not have a way to participate in classroom activities or choose not to participate do not have access to mathematical ideas. The students' assessments indicate that they did not view either themselves or any of their peers as being marginalized while others were supported in the design experiment class despite the differences in the sophistication of the data based arguments that they developed. This is important as an assessment of the design experiment classroom and suggests that there was equity in the students' assess to significant mathematical ideas. Equity as we have defined it elsewhere (Cobb & Hodge, 2002) involves two closely related aspects. The first aspect is concerned with students' access to opportunities to develop forms of mathematical reasoning that have clout in terms of substantial participation either in significant out-of-school practices or in mathematics courses that have traditionally served as gatekeepers to future educational and economic opportunities. The second aspect is concerned with the cultivation of students' interest in doing mathematics both in school and in out-of-school settings and thus encompasses their development of positive personal identities as doers of mathematics.

As we have illustrated, an analysis of the normative identity established in a classroom is made from the perspective of an outside observer of classroom events. In contrast, an analysis of the personal identities that students develop as they contribute to establishment and continual regeneration of that normative identity is made from an insider's perspective that recognizes students' voices. The approach that we have illustrated for analyzing the personal identities that students develop in mathematics classrooms makes contact with the notion of mathematical identity proposed by Martin (2000), but there are four significant points of contrast. Martin, it will be recalled, defines mathematical identity as "one's beliefs about one's own mathematical abilities, about the instrumental importance of mathematics, about opportunities and constraints to participate in mathematics, and one's motivation to obtain mathematical knowledge" (p. viii). The facets of personal identity that emerged from our analysis concern students' understandings and valuations of their general and specifically mathematical classroom obligations together with their assessments of their own and others' developing mathematical competences.

The first difference is that Martin focuses on students' views about mathematics in general whereas our concern is with whether students have developed an affiliation with mathematical activity as it is realized in their classrooms. We contend that this latter orientation is appropriate given our interest in instructional design and teaching as it acknowledges that what it means to know and do mathematics can differ radically from one classroom to another. The second difference is that we give little attention to students' views about the instrumental importance of mathematics to their future educational and economic opportunities. In our view, this contrast reflects a difference in focus. Martin's analysis is concerned with students' development of positive academic identities in all subject matters including mathematics whereas our interest is to support students' development of a sense of affiliation with mathematical activity. As we have indicated, we consider these two orientations to be complementary. The third difference concerns the encompassing approach that Martin takes when he documents students' views about the opportunities and constraints to their participation in mathematics. Martin argues convincingly that students' personal identities are influenced both by their participation in the practices of their home community and by broad, historically contingent social processes. In contrast, the scope of the constructs that we have proposed to this point is restricted to ongoing classroom interactions. It is to overcome this limitation that we will shortly introduce the third aspect of the interpretative scheme by developing the notion of core identity.

The final difference concerns the contrast that we drew earlier in this article between cognition plus and situated perspectives. As we noted, Martin adopts a cognition plus perspective when he speaks of students' mathematical identities being composed of internal beliefs that are shaped by various external forces. We, in contrast, have followed Wenger (1998) and Boaler and Greeno (2000) in contending that people become who they are in particular settings as they participate in the practices of particular groups and communities. We made this situated perspective explicit in the sample analysis when we emphasized that it was as the students contributed to the continual regeneration of the normative identities established in each class that they developed their personal identities we have documented. As an illustration of this interpretive stance, we documented that the students came to view themselves and others as competent as they made substantial contributions to class discussions. If some of the students had ceased to make such contributions, they would no longer have been recognized as succeeding by others and would, in all probability, have come to view themselves as failing. This reference to how the students were recognized by others indicates our rationale for viewing their personal identities as extending out into the local social world of the classroom and as being constituted in part by how they presented themselves to and were viewed by others. The example also serves to emphasize that the personal identities that the students developed in the two classrooms were continuing accomplishments rather than fixed characteristics or traits. Cast in these terms, the personal identities that students develop as doers of mathematics are therefore ongoing processes of being particular kinds of people in the local social world of the classroom.

Core Identity

To this point, we have focused on the personal identities that students developed as they contribute to the ongoing regeneration of the normative identity as a doer of mathematics in the two classrooms classroom. In doing so, we have given little attention to what D'Amato (1992) terms sociostructural processes that encompass race or ethnic history, class structure, and caste structure within society. It is to expand the scope of the interpretive scheme that we draw on Gee's (2001, 2003) notion of core identity. Although we continue to view the classroom as the immediate context in which students construct their personal identities, we also view the classroom as an arena in which sociostructural processes play out in face-to-face interaction.

We illustrated the notion of core identity when we discussed both Martin's (2000) and Boaler and Greeno's (2000) analyses. For example, we proposed that the envisioned life trajectories of the mathematically successful students in Martin's study were aspects of their core identities. In the case of Boaler and Greeno's analysis, the distinction between the normative identity established in a classroom and students' core identities corresponds to the distinction that they drew between the identity that students have to develop in order to become mathematical persons on the one hand, and who students view themselves to be and who they wanted to become on the other hand. As we noted, the relatively stable, seemingly transcontextual sense of self to which Boaler and Greeno refer appears to sit uncomfortably with the strongly situated perspective that they developed. This same concern applies to the approach we have taken given that we have characterized students' personal identities as situated with respect to the norms and practices of the classroom.

Gee (2001) develops the notion of core identity by observing that we have each had a unique trajectory of participation in the activities of various groups and communities. As a consequence of this personal history of engagement, we have had a unique sequence of specific experiences of presenting ourselves and being recognized in particular ways, some of which have recurred. "This trajectory and the person's narrativization … of it are what constitute his or her (never fully formed and always potentially changing) 'core identity'" (p. 111).

> We have this core identity thanks to being in one and the same body over time and thanks to being able to tell ourselves a reasonably (but only reasonably) coherent life story in which we are the "hero" (or, at least, central character). But, as we take on new [personal] identities or

> transform old ones, this core identity changes and transforms as well. We are fluid creatures in the making, since we make ourselves socially through participation with others in various groups. (Gee, 2003, p. 4)

Three aspects of this definition make it particularly relevant to our purposes as mathematics educators. First, in emphasizing the role of the person in developing a life story, Gee acknowledges personal agency as well as the social structures inherent in the activities in which the person participates. It is therefore conceivable that people with similar life histories might develop markedly different core identities at any particular point in time. Second, Gee's reference to participation and to being in the same body indicates that although the life story that the person develops is of central importance, his or her core identity is not reducible to it. In this regard, core identity as Gee defines it is consistent with our characterization of personal identity as a process of being a particular kind of person. Third, in Gee's formulation, people's development of new personal identities in particular settings can involve changes in their core identities. This is important given our concern that students' development of particular identities as doers of mathematics in specific classroom settings might, over time, influence their more enduring sense of who they are and who they want to become.

We did not collect specific data sets to document the evolving core identities of the eleven students who participated in the statistics design experiment. However, we were able to make some general inferences about aspects of their core identities when we analyzed the process by which the teacher cultivated their interest in statistical data analysis (Cobb & Hodge, 2003). For the most part, the types of instructional activities in which the teacher was able to cultivate the students' interest and engagement involved the investigation of issues that had relatively wide social significance (Cobb & Hodge, 2003). Examples include investigations of the effectiveness of two treatments for AIDS patients, the impact of the introduction of airbags on automobile safety, and possible inequities in the salaries of males and females with the same number of years of education. In accounting for this finding, we noted that children typically become interested in the political and societal world around them during early adolescence (Hedegaard, 1998; Vygotsky, 1987). In other words, the types of instructional activities that proved to be productive reflected the students' experiences as young people who were beginning to develop a perspective on themselves in relation to society. These experiences in turn reflected their culturally organized participation in relatively broad societal and political processes. It would therefore appear that in developing a sense of affiliation with mathematical activity as it was realized in this class, they reconciled this and other aspects of their core identities with their ongoing participation in the regeneration of the normative identity established in the classroom. In doing so, they came to identify with the activity of generating and analyzing data as a means of developing insights into phenomena.

The data we collected during the design experiment do not enable us to document whether the students' core identities might have evolved as they developed their personal identities in this classroom. However, Gutstein's (2002a, b) findings provide an initial indication that students' development of personal identities in particular classrooms can, over an extended period of time, involve changes in their core identities. As we have noted, Gutstein conducted a design experiment in which he served as the teacher of a group of Latino middle-school students from a poor community for two years. In doing so, he supplemented the textbook series that he used with instructional activities that addressed issues of social justice, racism, and inequality. Gutstein documents that most of the students came to appreciate and value engaging in mathematical activity as it was constituted in his classroom. He then displays appropriate caution when he goes on to consider whether this sustained engagement involved transformations in his students' core identities:

> [O]ne cannot know how our almost two years together help students develop, in a long-term sense, as agents of change – nor in fact, whether helping them do so will contribute to justice in society. Life changes are hard to document, even if one followed them for years, and it is difficult to attribute students' development to any particular events, especially those in school. (2002a, p. 18)

With this caveat in place, Gutstein reports four unsolicited incidents that occurred at least a year after he had completed his design experiment in which a former student demonstrated a commitment to remedying a situation that involved a social inequity. In doing so, the students each referred to activities in which they had engaged in Gutstein's class when they described the perceived injustice and the possible means of rectifying it. In our terms, these incidents suggest that in the process of contributing to the ongoing regeneration of the normative identity in Gutstein's classroom, they might have developed a relatively enduring sense of themselves as people who are capable of contributing to the rectification of injustices.

The instructional approaches that we and Gutstein took both capitalized on students' developing sense of themselves as members of society. However, Gutstein also took account of the core identities his students were developing as they grew up in a marginalized urban community, and thus of ethnic history and class structure as they played out in his students' lives. Had either we or Gutstein attempted to systematically document students' evolving core identities, we would have found that they comprised a number of separate and only partially interwoven threads. The multifaceted and only reasonably coherent nature of students' (and our own) core identities calls into question the common assumption that students' core identities can be equated with their membership of particular ethnic groups. Our intent in raising this issue is not to deny that a sense of affiliation with the common ancestry and cultural patterns of an ethnic group can be an important source of identity (cf. Nasir & Saxe, in press). Instead, it is to suggest that we desist from merely assigning students to institutionalized ethnic categories and instead attempt to understand the activities and practices with which they have developed an enduring affiliation when we document their core identities.

As an illustration, it is tempting to assume that the classification of Gutstein's students as Latinos captures their core identities. However, Guerra's (1998) ethnographic analysis of adult members of a Latino community in Chicago gives us reason to pause. He reports that his participants' identification of themselves as Latinos was tenuous and was based almost exclusively on their familiarity with this classification through the media. In contrast, "they had an easier time imagining themselves part of Pilsen/Little Village [a Chicago neighborhood] or, in slightly more general terms, part of a group in the city that is described as 'people of Mexican origin'" (Guerra, 1998, p. 8). Guerra goes on to argue, convincingly in our view, that the diversity in people's views of themselves in the community that he studied was as great as that between different ethnic groups. In doing so, he challenges approaches that are based on the essentialist assumption that everyone who is classified within a particular ethnic category shares certain essential characteristics. As Calhoun (1996) observes, this essentialism is often linked to the suppression of some core identities (e.g., African American feminist) and to the portrayal of people within a particular category as unitary and homogeneous. The diversity that Guerra documents in his participants' core identities is not surprising when we follow Gee (2001, 2003) and attribute agency to people in fashioning their life stories.

We can further clarify our view of the relation between broad, sociostructural processes and people's ongoing fashioning of their core identities by considering those cases in which students actively resist instruction. Martin's (2000) analysis is relevant in this regard in that the students in his dominant group intentionally attempted to disrupt their mathematics teachers' instructional agendas, often with considerable success. We infer from Martin's account that these students were also disruptive in other classes. It would therefore appear that they were developing core identities in opposition not merely to the normative identities established in their mathematics classes but to the norms of the school more generally. As Martin makes clear, Ogbu's (1992, 1999) high influential analysis of the origins of students' resistance is inadequate to account for his findings.

The kernel of Ogbu's thesis is that in racially and ethnically stratified societies such as the United States, historically oppressed groups have developed a collective cultural identity in opposition to institutions such as schools that are equated with assimilation into dominant social groups. As children of oppressed minorities are socialized into this cultural identity, they become skeptical about their prospects for social advancement and have no reason to accept the achievement ideology. Ogbu therefore contends that they do not have access to a structural rationale for learning in school and resist instruction in order to maintain a sense of affiliation with their cultural group. The key point to note is that in proposing this thesis, Ogbu locates the source of students' resistance outside the classroom and the school, and views it as sociostructurally determined by the history of the group. It follow from this thesis that all students who are members of historically oppressed groups will resist instruction and that classroom interactions will unfold in predictable ways that the teacher is powerless to influence. As Martin notes, Ogbu cannot account for the manner in which the succeeding students who participated in his study had come to identify with academic achievement. In addition, he cannot account for Gutstein's (2002a, b) success in enabling his students to reconcile their core identities with their participation in the ongoing regeneration of the normative identity as a doer of mathematics in his classroom.

In proposing his sociostructural determinist thesis, Ogbu acknowledges agency at the level of the cultural group in resisting institutions associated with dominant social groups. Martin's findings indicate the importance of also acknowledging students' personal agency. Gutstein, for his part, demonstrates that the teacher is not powerless and that resistance is situational produced in classrooms as sociostructural processes play out in face-to-face interactions (cf. Mehan et al., 1994). In order to capitalize on Ogbu's crucial insight about the importance of sociostructural process while accounting for Martin's and Gutstein's findings, we draw on Willis's (1977) seminal analysis in which he sought to understand how British working class students typically end up in working class jobs. Willis's ethnographic analysis of a group of working class boys demonstrates that they were treated badly by the school in that manifestations of their working class backgrounds were devalued. However, it was not clear to Willis why the boys and their families did not demand better treatment so that they could move into the middle class. In addressing this issue, he directly challenges the view that the boys were passive bearers of a sociostructurally determined cultural identity. His analysis reveals that the boys actively constructed positive sense of their lives by drawing on a number of sources that included popular culture as well as their parents' shopfloor culture. Thus, Willis contends that their resistance was not predetermined by their socialization into a monolithic, oppositional cultural identity. Instead, the boys contributed to the reproduction of their relatively low status in society by actively fashioning oppositional core identities that involved a sense of self-worth and status.

Willis's analysis is consistent with Gee's (2001, 2003) characterization of core identity in that it acknowledges personal agency as well as a history of participation in the activities of various groups and communities. In addition, Willis emphasizes that students' fashioning of their core identities is situated with respect to their positioning in broad sociostructural processes. For example, Willis took account of the class stratification of British society when he analyzed the significance and meaning of the boys' treatment by the school. In addition, he stressed that the cultural resources on which the boys could draw as they constructed their oppositional core identities were constrained by their sociostructural position (cf. Holland et al., 1998). Thus, although the process by which the boys fashioned their core identities was unique, Willis anticipated that working class students in other British high schools might be treated similarly and that some would respond by drawing on similar cultural resources to make positive sense of their lives

Erickson (1992) notes that Willis's analysis appears to be relevant to societies such as the United States in which the major sociostructural distinctions fall along lines of race and ethnicity as well as class. In the case of the students in Martin's (2000) dominant group, for example, we would then question both the assumption that they were passive bearers of a collective cultural identity and the claim that they lacked agency and merely reflected negative elements of their community. Instead, we would view their oppositional core identities as situated with respect to both their ongoing interactions in the school and the history of their community. A series of investigations conducted by Gutierrez et al. (1995, 1999) is relevant in this regard in that they document how the processes that Wills analyzed play out at the level of immediate interactions in classrooms similar to those that Martin studied. As Gutierrez et al. clarify, students who resist instruction interactively constitute what they term a counterscript in opposition to the official classroom script. In the case of mathematics classrooms, these students jointly establish an alternative, oppositional activity as they resist contributing to the ongoing regeneration of the normative identity as a doer of mathematics inherent in the official classroom script. Furthermore, in coming to appreciate and value engaging this oppositional activity, they develop a personal identity in opposition to the normative identity established in the classroom that, over time, can involve changes in their core identities.

This perspective on resistance and on the relation between core identity and sociostructural processes offers some hope to the instructional designer and the teacher by questioning the view that interactions in mathematics classes have to unfold in a predetermined manner. D'Amato (1992), for example, draws on his experience of working with native Hawaiian students to emphasize the importance of ensuring that the general norms of participation established in the classroom fit with the structure and social dynamics of students' peer relations. In making this proposal, D'Amato notes that the balance of power is with the students in that the teacher does not control anything that they need when they have developed oppositional core identities and school does not have structural significance for them. Students therefore have no inhibitions in confronting the premises of the school openly when they conflict with the peer relations through which they make positive sense of their lives.

Looking beyond general norms of participation, the findings of both Gutstein's (2002a, b) investigation and the statistics design experiment indicate the importance of instructional design decisions in giving students access to a situational rationale for engaging in mathematical activity. As we saw, this involves capitalizing on aspects of students' core identities when developing instructional activities while simultaneously attending to the specifically mathematical norms established in the classrooms such as norms or standards for mathematical argumentation, normative ways of reasoning with tools and written symbols, norms for what counts as mathematical understanding and competence, and relatedly, the normative purpose for engaging in mathematical activity. The intent in doing so is to support students' reconciliation of their core identities with the normative identity as a doer of mathematics established in the classroom. The long-term goal of such an approach is that students might draw on their developing sense of affiliation with mathematical activity as they attempt to make positive sense of their lives.

At the same time that it offers hope, the perspective that we have outlined indicates the daunting challenges facing mathematics teachers in classrooms such as those that Martin studied. In addition having become alienated from mathematics as they have experienced it in school thus far, many of these students view the teacher as an agent of an institution to which they have developed oppositional core identities. Gutstein's findings not withstanding, we contend that it is unreasonable to expect mathematics teachers to support their students' development of positive personal identities as doers of mathematics in such circumstances unless their efforts are part of a larger endeavor that extends beyond the classroom to the school and community. We consider the comprehensive approach described by Moses to be exemplary in this regard (Moses et al., 1989; Moses & C. E. Cobb, 2001). Drawing on their experiences in the Civil Rights Movement, Moses and his colleagues argue that schools should be made accountable to who they are suppose to serve, students and their families.[8] In addition, they maintain that a continuity should be forged between home and school so that mathematics teachers can capitalize on the strengths that families can provide. As a means of achieving these goals, they clarify the importance of establishing parent groups and of encouraging parents to participate in workshops and to volunteer to assist in classrooms. In addition, they discuss the value of after-school Math Clubs and other extra-curricular activities for students. To the extent that these initiatives are successful, learning mathematics in school becomes a means by which students maintain relationships with parents as well as with peers and the staff of the Math Club.

The intent of the initiatives that Moses and colleagues describe is to provide students with access to a structural rationale for engaging in mathematical activity in the classroom. In contrast, the intent of D'Amato's (1992) proposals and of Gutstein's (2002 a, b) recommendations is to provide students with access to a situational rationale for engaging in mathematical activity. Similarly, our primary concern in developing the notions of personal identity, normative identity, and core identity has been to propose an interpretive scheme that can inform instructional designers' and teachers' efforts to support students' development of a situational rationale. In our view, a coordinated approach that attempts to give students access to both a structural and a situational rationale for engaging in mathematical activity will be required in many settings. Only then might the goal of achieving equity in students' access to significant mathematical ideas be potentially realizable. The interpretive scheme that we have proposed can best viewed as a conceptual tool that is designed to contribute to a comprehensive effort of this type.

Discussion

In the first part of this article, we clarified that an analysis of the personal identities that students develop in a particular classroom provides a way of accounting for students' persistence, interest in, and motivation to engage in mathematical activity as it is constituted in that classroom. In addition, we argued that students' development of personal identities that involve a sense of affiliation with mathematical activity should be an important instructional goal in its own right in that it relates directly to issues of equity in mathematics education. We first operationalized the notion of normative identity by presenting analyses of the algebra and design experiment classes in which the students with whom we worked participated concurrently. The approach that we illustrated was relatively fine-grained and involved delineating both classroom social norms and specifically mathematical norms that included norms or standards for mathematical argumentation, normative ways of reasoning with tools and written symbols, and norms for what counts as mathematical understanding and competence. We also stressed that the teacher and students in each classroom jointly constituted these norms in the course of their ongoing classroom interactions. This led us to reject the alternative view that students are invited to adopt a normative identity as a doer of mathematics that is established prior to their participation. We clarified that students instead develop their personal identities in the classroom as they contribute to or resist the ongoing regeneration of the normative identity. As we noted, this contention reflects a situated perspective on identity in that students are seen to become who they are in a particular classroom as they engage in or resist mathematical activity as it is realized in that classroom.

We operationalized the notion of the personal identity by drawing on small group interviews that focused on the students' interpretations of classroom events in both the algebra class and the design experiment class. The facets of the students' personal identities that emerged from our analysis of the interviews concerned their understandings and valuations of their general and specifically mathematical classroom obligations together with their assessments of their own and others' developing mathematical competences. The contrast between the personal identities that the students were developing in the algebra class and in the design experiment class served to clarify the distinction between cases in which students are merely willing to cooperate with the teacher and those in which they have come to appreciate and value classroom mathematical activity. The relations we drew between the students' contrasting personal identities and the normative identities established in the two classrooms also illustrated the value of documenting both general and specifically mathematical norms in some detail.

One of the challenges that we sought to address when developing the interpretive scheme was the apparent tension between the situated nature of the identities that students develop in particular classrooms on the one hand and their relatively enduring sense of who they were and who they wanted to become that spanned their participation in different groups and communities. It was partly for this reason that we introduced the notion of core identity which, following Gee (2001, 2003), we defined as a student's unique trajectory of participation in the activities of various groups and communities together with his or her narrativization of it. Students who have developed a sense of affiliation with mathematical activity as it is realized in the classroom could then be seen to have reconciled their core identities with their ongoing participation in the regeneration of the normative identity. In contrast, disenchantment with or alienation from mathematical activity as it is realized in the classroom indicates that students have not achieved a reconciliation even though they might continue to cooperate with the teacher. As a third possibility, we considered cases in which students develop oppositional personal identities in the classroom as they overtly resist contributing to the ongoing regeneration of the normative identity.

The introduction of the core identity as the third aspect of the interpretive scheme also enabled us to take account of broad structural features of society when analyzing the personal identities that students develop in particular classrooms. In doing so, we questioned both the assumption that core identity is synonymous ethnic identity and the view that students are passive bearers of a sociostructurally determined core identity. Instead, we took account of students' agency in constructing reasonably coherent narratives of their socially and culturally situated history of participation in activities of various groups and communities. In addition, we questioned the assumption that sociostructural processes determine how classroom interactions will unfold, and instead treated the classroom as the immediate context in which students develop their personal identities while simultaneously viewing it as an arena in which sociostructural processes play out in face-to-face interaction. From the point of view of instructional design and teaching, the challenge is then to make it possible for students to reconcile their core identities that reflect their trajectory of participation in a culturally and economically stratified society with participation in the ongoing regeneration of the normative identity in the classroom. To the extent that effort is successful, they would develop personal identities as people who have developed a sense of affiliation with mathematical activity as it is realized in the classroom.

It should be clear that the interpretive scheme for analyzing the personal identities that students are developing in the classroom does not give rise to direct instructional implications.[9] Instead, our intent in presenting the scheme has been to propose a conceptual tool that can inform both instructional design and teaching. In this regard, we note that ongoing interpretations of classroom events reflect implicit suppositions and assumptions about learning, teaching, and mathematics as well as a range of issues that are typically subsumed under the heading of affective factors. These latter issues, which have received limited attention in the mathematics education research literature, include students' attitudes towards and motivations for engaging in mathematical activity in the classroom. Our assumptions about and explanations of students' motivations and attitudes have real consequences in that they orient ongoing interpretations that inform the instructional design and pedagogical decisions that we make.

The interpretive scheme that we have described offers a way of attempting to understand cases in which students openly resist engaging in mathematical activity or are merely willing to cooperate with the teacher. Furthermore, the interpretive scheme orients how we might make sense of and thus learn from cases in which students have come to identify with mathematical activity as it is realized in the classroom. In these various cases, we would account for the personal identities that students are developing by focusing on the way in which they participate in or resist the ongoing process of regenerating the normative identity as a doer of mathematics in the classroom. Our objective would be to understand how they might have reconciled their core identity with the normative identity, or to explain why they experience a conflict between their core identity and participation in the ongoing regeneration of the normative identity.

In making instructional design and pedagogical decisions on the basis of such an analysis, we would develop conjectures about the types of normative identity as a doer of mathematics with which it might be possible for students to reconcile their core identities as they participate in its initial constitution and ongoing regeneration. In addition, we would take account of the challenges that the teacher faces in initiating and guiding the interactive constitution of the conjectured normative identity. This in turn would lead to a consideration of the instructional activities and associated tools and resources that the teacher might use to achieve the dual agenda of making it possible for students to come to appreciate and value engaging in mathematical activity as it is realized in the classroom, and to develop increasingly sophisticated forms of mathematics reasoning as they do so. The importance of viewing instructional activities and associated tools as resources for the teacher to use rather than as self-contained supports for students' learning becomes apparent when we note that the manner in which they are realized in the classroom depends on both the general norms and specifically mathematical norms of participation. Elsewhere, we have argued that instructional activities, tools, and classroom norms are mutually dependent and constitute aspects of a single classroom activity system (Cobb & McClain, 2002). Cast in these terms, the intent of instructional design is to provide teachers with the resources that enable them to guide the development of their classrooms as activity systems in which students develop a sense of affiliation with mathematical activity that motivates their commitment to learn as they as they participate in and contribute to the evolution of that system.

In summary, the usefulness of the interpretive scheme stems from its potential to inform the development of classrooms as activity systems in which the participating students have access to a situational rationale for developing forms of mathematical reasoning that have clout. In our view, such classrooms are relatively equitable in that they seek to ameliorate inequities in motivations that reflect students' differential access to a structural rationale. At a recent symposium on issues of equity and diversity in mathematics education10, Kris Gutierrez urged that we scrutinize pedagogical and design proposals to determine whether the goal is to enable her and other members of marginalized groups to "become a better me, or to become like you." It should be clear that in respecting students' socially and culturally situated core identities, the interpretive approach we have proposed satisfies Gutierrez's criterion.

Notes

[1] We refer to mathematical activity *as it is realized in their classroom* to emphasize that the nature of mathematical activity can vary significantly from one classroom to the next (Boaler, 1998, 2002; Bowers & Nickerson, 2001; Cobb et al., 1992; Lampert, 1990; Nickson, 1992). Our point of reference when analyzing the identities that students develop as doers of mathematics will therefore be the specific mathematics practices established in their classroom rather than mathematics viewed as a transcendent, disembodied discourse.

[2] Mehan et al. (1994) also document that students from historically unprivileged groups who subscribe to the achievement ideology typically have realistic expectations about the barriers and prejudice with which they will have to cope.

[3] The teachers who participated in Martin's study were attempting to use instructional materials developed by the Algebra Project (Moses & Cobb, 2000). However, their efforts appear to have been largely unsuccessful due to the resistance to students in the dominant group. Gutstein (2002 a, b) documents the process by which he cultivated the mathematical interests of a class of middle school students from a poor, historically underprivileged urban community.

[4] The members of the research team that conducted the classroom design experiment were Paul Cobb, Kay McClain, Koeno Gravemeijer, Jose Cortina, Maggie McGatha, Nora Shuart, and Carrie Tzou.

[5] The eleven students were in five different sections of Algebra I that were taught by the same teacher. Observations of three of these five sections revealed that the general norms of participation and the norms for mathematical activity were very similar across these classes. We therefore refer to a single algebra class throughout this article for ease of explication.

[6] Pilot interviews conducted during the prior seventh-grade design experiment do not support this conjecture. In particular, these interviews did not reveal any systematic differences between the students who continued to participate and those who dropped out in terms of the extent to which they valued engaging in mathematical activity in the design experiment classroom (Hodge, 2001). In addition, an analysis of a second set of interviews conducted with all of the original 29 students at the end of the seventh-grade design experiment indicated that the eleven students who continued to participate were reasonably representative of the entire group in terms of the ways in which they reasoned about data (Cobb et al., 2003).

[7] The additional theme emerged in the course of the analysis concerned the students' views of the resources that supported their learning in each classroom.

[8] David Dennis, personal communication, February, 2000

[9] In two companion papers, we derive recommendations from the statistics design experiment for the process of cultivating students' mathematical interests (Cobb & Hodge, 2003) and analyze the approach we took to instructional design from the perspective of equity (Hodge, Cobb, & McClain, 2003).

[10] The Symposium on Issues of Cultural Diversity and Equity in Mathematics Education held in September 2000 at Northwestern University, Evanston, IL.

References

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. Journal for Research in Mathematics Education, 29, 41-62.

Boaler, J. (2000). Exploring situated insights into research and learning. Journal for Research in Mathematics Education, 31, 113-119.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. Journal for Research in Mathematics Education, 33, 239-258.

Boaler, J., & Greeno, J. (2000). Identity, agency, and knowing in mathematical worlds. In J. Boaler (Ed.), Multiple perspectives on mathematics teaching and learning (pp. 45-82). Stamford, CT: Ablex.

Boaler, J., Wiliam, D., & Zevernbergen, R. (2000, July). The construction of identity in secondary mathematics education. Paper presented at the Second International Conference on Mathematics, Education and Society, Montecorvo, Portugal.

Bowers, J.S., & Nickerson, S. (2001). Identifying cyclic patterns of interaction to study individual and collective learning. Mathematical Thinking and Learning, 3, 1-28.

Calhoun, C. (1996). Social theory and the politics of identity. In C. Calhoun (Ed.), Social theory and the politics of identity (pp. 9-36). Cambridge, MA: Blackwell.

Cobb, P., & Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. Mathematical Thinking and Learning, 4, 249-284.

Cobb, P., & Hodge, L. L. (2003, April). An Initial Contribution to the Development of a Design Theory of Mathematical Interests: The Case of Statistical Data Analysis. Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Cobb, P., & McClain, K. (2002). Supporting students' learning of significant mathematical ideas. In G. Wells & G. Claxton (eds.), Learning for life in the 21st Century (pp.154-166). Oxford, England: Blackwell.

Cobb, P., McClain, K., & Gravemeijer, K. (2003). Learning about statistical covariation. Cognition and Instruction, 21, 1-78.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. Journal of the Learning Sciences, 10, 113-164.

Cobb, P., & Tzou, C. (2000, April). Learning about data analysis. Paper presented at the annual meeting of the American Education Research Association, New Orleans.

Cobb, P., & Whitenack, J. (1996). A method for conducting longitudinal analyses of classroom videorecordings and transcripts. Educational Studies in Mathematics, 30, 213-228.

Cobb, P., Wood, T., Yackel,E., & McNeal, G. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. American Educational Research, 29, 573-602.

D'Amato, J. (1992). Resistance and compliance in minority classrooms. In E. Jacob & C. Jordan (Eds.), Minority education: Anthropological perspectives (pp. 181-207). Norwood, NJ: Ablex.

de Abreu, G. (1995). Understanding how children experience the relationship between home and school mathematics. Mind, Culture, and Activity, 2, 119-142.

Emirbayer, M., & Mische, A. (1998). What is agency? American Journal of Sociology, 103, 962-1023.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), The handbook of research on teaching (3rd. ed., pp. 119-161). New York: Macmillan.

Erickson, F. (1992). Transformation and school success: The policies and culture of educational achievement. In E. Jacob & C. Jordan (Eds.), Minority education: Anthropological perspectives (pp. 27-51). Norwood, NJ: Ablex.

Franke, M. L., & Kazemi, E. (in press). Teaching as learning within a community of practice: Characterizing generative growth. In T. Wood, B. Nelson, & J. Warfield (Eds.), Beyond classical pedagogy in elementary mathematics: The nature of facilitative teaching. Mahwah, NJ: Erlbaum.

Gee, J. P. (1997). Thinking, learning and reading: The situated sociocultural mind. In D. Kirshner & J. A. Whitson (Eds.), Situated cognition: Social, semiotic, and psychological perspectives (pp. 235-260). Mahwah, NJ: Erlbaum.

Gee, J. P. (1999). Identity as an analytic lens for research in education. In W. G. Secada (Ed.), Review of Research in Education (Vol. 25, pp. 99-125). Washington, DC: American Educational Research Association.

Gee, J. P. (2003). What video games have to teach us about learning literacy. New York: Palgrave/Macmillan.

Glaser, B. G., & Strauss, A. L. (1967). The discovery of grounded theory: Strategies for qualitative research. New York: Aldine.

Guerra, J. (1998). Close to home: Oral and literate practices in a transitional Mexicano community. New York: Teachers College Press.

Gutierrez, K., Baquedano-Lopez, P. & Tejeda, C. (1999). Rethinking diversity: Hybridity and hybrid language practices in the third space. Mind, Culture, and Activity, 6, 286-303.

Gutierrez, K., Rymes, B. & Larson, J. (1995). Script, counterscript, and underlife in the classroom: James Brown versus Brown v. board of education. Harvard Educational Review, 65, 444-471.

Gutstein, E. (2002a, April). Teaching and learning mathematics for social justice in an urban Latino school. Paper presented at the Research Presession of the annual meeting of the National Council of Teachers of Mathematics, Las Vegas, NV.

Gutstein, E. (2002b, April). Roads towards equity in mathematics education: Helping students develop a sense of agency. Paper presented at the annual meeting of the American Educational Research Association, New Orleans.

Hall, R. (2002). Review of Radical equations: Mathematics literacy and civil rights, and mathematics success and failure among African-American youth: The roles of sociocultural context, community forces, school influences, and individual agency. Educational Researcher, 31(7), 39-45.

Hedegaard, M. (1998). Situating learning and cognition: Theoretical learning and cognition. Mind, Culture, and Activity, 5, 114-126.

Hodge, L. L. (2001). Cultivating students' mathematical interests in the context of statistical data analysis. Unpublished doctoral dissertation, Vanderbilt University, Nashville, TN.

Hodge, L. L., Cobb, P., & McClain, K. (2003, April). Classrooms as design spaces for supporting students' identities as doers of mathematics. Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Holland, D., Skinner, D., Lachicotte, W. Jr., & Cain, C. (1998). Identity in cultural worlds. Cambridge, MA: Harvard University Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. American Educational Research Journal, 27, 29-63.

Lave, J. (1991). Situating learning in communities of practice. In L. B. Resnick, J. M. Levine, & S. D. Teasley (Eds.), Perspectives on socially shared cognition (pp. 63-82). Washington, DC: American Psychological Association.

Martin, D. B. (2000). Mathematics success and failure among African-American youth. Mahwah, NJ: Erlbaum.

McClain, K. (2002). Teacher's and students' understanding: The role of tool use in communication. Teacher's and students' understanding: The role of tool use in communication. Journal of the Learning Sciences, 11, 217-249.

Mehan, H., Hubbard, L., & Villanueva, I. (1994). Forming academic identities: Accommodation without assimilation among involuntary minorities. Anthropology and Education Quarterly, 25, 91-117.

Misher, E. G. (1986). Research interviewing: Context and narrative. Cambridge, MA: Harvard University Press.

Moses, R. & Cobb, C. (2001). Radical equations: Math literacy and civil rights. Boston: Beacon Press.

Moses, R., Kamii, M., Swap, S. M., & Howard, J. (1989). The algebra project: Organizing in the spirit of Ella. Harvard Educational Review, 59, 423-444.

Nasir, N. S. (2002). Identity, goals, and learning: Mathematics in cultural practice. Mathematical Thinking and Learning, 4, 213-248.

National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1991). Professional Standards for teaching mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2000). Principles and standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Nickson, M. (1992). The culture of the mathematics classroom: An unknown quantity? In D.A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 101-114). New York: Macmillan.

Ogbu, J. (1978). Minority education and caste: The American system in cross-cultural perspective. New York: Academic Press.

Ogbu, J. (1992). Adaptation to minority status and impact on school success. Theory into Practice, 31, 287-295.

Ogbu, J. U. (1999). Beyond language: Ebonics, proper English, and identity in a Black-American speech community. American Educational Research Journal, 36, 147-184.

Sfard, A. (2002, June). Telling identities:Conceptualizing diversity in terms of identity. Paper presented at the Fifth Congress of the International Society for Cultural Research and Activity Theory, Amsterdam, The Netherlands.

Skemp, R. (1976). Relational understanding and instrumental understanding. Mathematics Teacher, 77, 20-26.

Stein, M. K., Silver, E. A., & Smith, M. S. (1998). Mathematics reform and teacher development: A community of practice perspective. In J. G. Greeno & S. V. Goldman (Eds.), Thinking practices in mathematics and science learning (pp. 17-52). Mahwah, NJ: Erlbaum.

Thompson, A.G., & Thompson, P.W. (1996). Talking about rates conceptually, part II: Mathematical knowledge for teaching. Journal for Research in Mathematics Education, 27, 2-24.

Varenne, H., & McDermott, R. (1998). Successful failure: The schools that America builds. Boulder, CO: Westview.

Vygotsky, L.S. (1987). Thinking and speech. In R.W. Rieber & A.S. Carton (Eds.), The collected works of Vygotsky, L.S. (vol.1): Problems of general psychology. New York: Plenum.

Wenger, E. (1998). Communities of Practice. New York: Cambridge University Press.

Willis, P. (1977). Learning to labor: How working class kids get working class jobs. New York: Columbia University Press.