# BOOK REVIEW

*An Introduction to Chaotic Dynamical Systems. 2nd Edition,* by Robert L. Devaney, Addison-Wesley, 1988, 360 pp., $43.25, ISBN 0-201-13046

For the last several years, chaos has been among the most chic of mathematical areas. A best seller (Chaos, Making a New Science, by James Gleick) attracted an award from SIAM at the annual meeting in 1988 in Minneapolis. Sharing a taxi ride to the airport with a young engineering professor from Cornell, I found that his area of expertise is applications of chaos to physical systems. The next month at the Centennial AMS conference in Providence, I found that a seeming majority of the graduate students from Stanford were going to do great things in chaos. This book is thus quite "relevant". As if the chaos hook were not sufficient, the second edition has also added a treatment of fractals, (of wide interest because they provide a conceptually simple means of generating complicated visuals on work stations).

A primary attraction of chaos is the parallel drawn between chaotic physical phenomenon and simple iterations of mathematical functions. For example, the transition from laminar to turbulent flow is characterized by an increasing number of cycles, and then, as velocity is incrementally increased, a transition to chaos. The simple functional iteration $F_\lambda(x) = \lambda x ( 1 - x )$ maps the interval $[0 , 1]$ into itself provided that $0 \leq \lambda \leq 4$. If $0 \leq \lambda \leq 1$, successive iterates tend to go to the fixed point zero for all $0 \leq x \leq 1$. For $1 < \lambda < 3$, there is an attracting fixed point at $( \lambda - 1 ) / \lambda$, and zero, while a fixed point, is now repelling. As $\lambda$ grows above three," a new periodic point of period 2 is born." As $\lambda$ grows larger yet, orbits grow more complicated. Finally, for $\lambda > 2 + \sqrt{5}$, the periodic points comprize a Cantor set. The parallel to the laminar-turbulent transition is striking as this is perhaps the simplest and most satisfying parallel in the fifty or a hundred year study of the laminar turbulent transtion.

This metaphor is taken as an application and has spawned an interest in similar "applications." I have nevertheless been skeptical. Are "parallels" physics? Usually metaphors are the proper province of poets rather than scientists. Neither have I been familiar with a significant mathematics of chaos.

It is thus rather a surprise to me that Devaney's book is quite a good mathematics text. Devaney has pulled together results from the present and from the early 20th century to provide a unifying theme for many beautiful mathematical ideas. The quadratic iteration $\lambda x (1 - x)$ (with which Feigenbaum started the current chaos craze) provides a continuing thread for Devaney's exposition in the first chapter. He uses it to develop abstract mathematical concepts without assuming mathematical prerequisites. The sections in the first chapter on One-Dimensional Dynamics include Hyperbolicity, Symbolic dynamics, Topological conjugacy, Chaos, Structural stability, Sarkovskii's theorem, the Schwarzian derivative, Bifurcation theory, Morse-Smale diffeomorphisms, Homoclinic points and bifurcations, and several more. The overall theme here is provide mathematical means for disccusing the structure of a chaotic system. This chapter contains results I find quite beautiful. For instance, from the section on Sarkovskii's Theorem, we have the following theorem.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be continuous. Suppose f has a periodic point of period three. Then f has periodic points of all other periods.

The result is to me rather startling, but the proof is accessible and not long.

In the final two chapters the emphasis is again on appropriate maps for demonstrating underlying mathematical structure. The second chapter concerns Higher Dimensional Dynamic systems. Discussed here are a review section of results from calculus and linear algebra, dynamics of linear maps, the horseshoe map, hyperbolic toral automorphisms, attractors, the stable and unstable manifold theorem, etc. The final section of chapter two integrates the concepts of the chapter by applying the structures and phenomena of hyperbolic sets, homoclinic sets, bifurcations, horseshoes, etc. to the Hènon map.

Chapter Three provides a few fundamentals from complex variable theory. It reworks the quadratic iteration as a complex variable and resurrects both results of the 1920s and more current work, including Mandelbrot sets. Color plates are included.

For the book as a whole, few mathematical prerequisites are required. In theory, practically any scientist or engineer would be able to read this book, given sufficient diligence. The intelligent and diligent student will gain a good deal of mathematical sophistication in the process of understanding the material. Given the current interest in chaos, it is likely that quite a few individuals have in fact used the bridge into mathematics afforded by this book.

Misprints are few. Exercises are provided. The preface claims that it can take a student to the frontiers of research. Given a good mentor, that is probably true. However, it seems to me that some of the proofs would be easier to read if Professor Devaney tutored students who were reading the book and rewrote the proofs to avoid their confusions.

In conclusion, I am really quite enthused about this book. As a text, I put it as in the same class as **Modern Algebra** by Herstein, which was my own favorite text when an undergraduate, and which shares the same "dive in" (who needs prerequisites?) approach. This book would be appropriate for a course at an advanced undergrduate or beginning graduate level, and especially as a basic text for anyone desiring to reach a research level in chaotic dynamical systems.

For a survey of some of the applications to physics of chaotic dyanamical systems, the book **Deterministic Chaos: An Introduction**, Second Revised Edition. by Heinz Georg Schuster. VCH Publishers, New York, 1988, has received at least one good review (SIAM Review, March 1989). The SIAM review of Devaney's book (December 1987) mentions **Geometrical Methods in the Theory of Ordinary Differential Equations**, by V. I. Arnol'd, Springer, New York, 1982, and **Geometric Theory of Dynamical Systems: An Introduction**, by Palis and de Melo, Springer, New York, 1982, as more advanced texts on the subject.

GARRY HOWELL
Department of Applied Mathematics
Florida Institute of Technology
Melbourne, FL 32901, USA