

# **An Introduction to Computer Simulation Methods**

Applications to Physical Systems

Second Edition

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**Addison-Wesley Publishing Company**

Reading, Massachusetts • Menlo Park, California • New York  
Don Mills, Ontario • Wokingham, England • Amsterdam • Bonn  
Sydney • Singapore • Tokyo • Madrid • San Juan • Milan • Paris

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