An Introduction to Fluid Mechanics and Transport Phenomena

### FLUID MECHANICS AND ITS APPLICATIONS

Volume 86

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Aims and Scope of the Series

The purpose of this series is to focus on subjects in which fluid mechanics plays a fundamental role.

As well as the more traditional applications of aeronautics, hydraulics, heat and mass transfer etc., books will be published dealing with topics which are currently in a state of rapid development, such as turbulence, suspensions and multiphase fluids, super and hypersonic flows and numerical modeling techniques.

It is a widely held view that it is the interdisciplinary subjects that will receive intense scientific attention, bringing them to the forefront of technological advancement. Fluids have the ability to transport matter and its properties as well as to transmit force, therefore fluid mechanics is a subject that is particularly open to cross fertilization with other sciences and disciplines of engineering. The subject of fluid mechanics will be highly relevant in domains such as chemical, metallurgical, biological and ecological engineering. This series is particularly open to such new multidisciplinary domains.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of a field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

# An Introduction to Fluid Mechanics and Transport Phenomena



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## **Preface**

This text is a brief introduction to fundamental concepts of transport phenomena within a fluid, namely momentum, heat and mass transfer. The emphasis of the text is placed upon a basic, systematic approach from the fluid mechanics point of view, in conjunction with a unified treatment of transport phenomena.

In order to make the book useful for students, there are numerous examples. Each chapter presents a collection of proposed problems, whose solutions can be found in the Problem Solutions Appendix. Also the Self Evaluation chapter gathers exercises from exams, so readers and students can test their understanding of the subject.

Most of the content can be taught in a course of 45 hours and has been employed in the course *Transport Phenomena* in Chemical Engineering at the Centro Politécnico Superior of the University of Zaragoza. The text is aimed at beginners in the subject of transport phenomena and fluid mechanics, emphasizing the foundations of the subject.

The text is divided into four parts: Fundamentals, Conservation Principles, Dimensional Analysis; Theory and Applications, and Transport Phenomena at Interfaces.

In the first part, Fundamentals, basic notions on the subject are introduced: definition of a fluid, preliminary hypothesis for its mathematical treatment, elementary kinematics, fluid forces, especially the concept of pressure, and fluid statics.

In the Conservation Principles part, the conservation equations that govern transport phenomena are presented and explained, both in integral and differential form. Emphasis is placed on practical applications of integral equations. Also, constitutive equations for transport by diffusion are contained in this part.

In the third part, Dimensional Analysis; Theory and Applications, the important tool of dimensional analysis and the laws of similitude are explained. Also the dimensionless numbers that govern transport phenomena are derived.

#### VIII Preface

The last part, Transport Phenomena at Interfaces, explains how most transport processes originate at interfaces. Some aspects of the concept of boundary layer are presented and the usage of transport coefficients to solve practical problems is introduced. Finally the analogies between transport coefficients are explained.

There are a great number of people whose help in writing this book I would like to acknowledge. First my parents, for providing an intellectually challenging environment and awakening my early interest in engineering and fluid mechanics. Professor C. Dopazo, for his inspiring passion for fluid mechanics. My family, wife and children, for their love and support. C. Pérez-Caseiras for providing ideas to strengthen the text. Many colleagues and friends, who have accompanied me during these years, especially professors T.J.R. Hughes and E. Oñate, and friends Jorge, Antonio, Connie and Ed. Finally, I would like to acknowledge the encouragement of Nathalie Jacobs. Without them, this project would not have been possible.

Zaragoza,

Guillermo Hauke May 2008

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## Nomenclature

Roman S	Roman Symbols		Dimensions
$\overline{A}$	area	$m^2$	$L^2$
c	mixture molar concentration	$\mathrm{mol/m^3}$	$NL^{-3}$
$c_A$	molar concentration of species $A$	$\mathrm{mol/m^3}$	$NL^{-3}$
$c_v$	specific heat at constant volume	J/(kg~K)	$L^2T^{-2}\Theta^{-1}$
$c_p$	specific heat at constant pressure	J/(kg~K)	$L^2T^{-2}\Theta^{-1}$
$C_{\mathrm{D}}$	drag coefficient	_	_
$\mathrm{C}_{\mathrm{f}}$	friction coefficient	_	_
D	length, diameter	m	L
$D_{AB}, D_A$	molecular mass diffusivity	$\mathrm{m}^2/\mathrm{s}$	$L^2T^{-1}$
$D_v$	power dissipated by viscous dissipation	W	$ML^2T^{-3}$
$\mathrm{Da_{I}}$	Damköhler number	_	_
e	specific internal energy	J/kg	$L^2T^{-2}$
$e_{ m tot}$	specific total energy	J/kg	$L^2T^{-2}$
Ec	Eckert number	_	_
Eu	Euler number	_	_
$oldsymbol{f}_m$	body force per unit mass	N/kg	$LT^{-2}$
$oldsymbol{f}_s$	stress at surface	Pa	$ML^{-1}T^{-2}$
$oldsymbol{f}_v$	body force per unit volume	${ m N/m^3}$	$ML^{-2}T^{-2}$

## XVI Nomenclature

$F, oldsymbol{F}$	force	N	$MLT^{-2}$
$oldsymbol{F}_s$	surface force	N	$MLT^{-2}$
$oldsymbol{F}_v$	body force	N	$MLT^{-2}$
$\operatorname{Fr}$	Froude number	_	_
$\boldsymbol{g}$	gravity acceleration	$\mathrm{m/s^2}$	$LT^{-2}$
$\operatorname{Gr}$	Grashof number	_	_
h	length, depth	m	L
	heat transport coefficient	$W/(m^2 K)$	$MT^{-3}\Theta$
$h_m$	mass transport coefficient	m/s	$LT^{-1}$
H	length	m	L
H	angular momentum	N m	$ML^2T^{-2}$
I	surface moment of inertia	$\mathrm{m}^4$	$L^4$
	moment of inertia	${\rm kg}~{\rm m}^2$	$ML^2$
I	identity tensor / matrix	_	_
$\boldsymbol{j}_A$	mass flux of species $A$	$\rm kg/(m^2~s)$	$ML^{-2}T^{-2}$
$\boldsymbol{j}_A'$	molar flux of species $A$	$\mathrm{mol}/(\mathrm{m}^2\;\mathrm{s})$	$NL^{-2}T^{-1}$
$\boldsymbol{j}_A^m$	mass flux of species $A$ w.r.t the molar mean velocity	$kg/(m^2 s)$	$ML^{-2}T$
$\boldsymbol{j}_A^{m\prime}$	molar flux of species $A$ w.r.t the molar mean velocity	$\mathrm{mol}/(\mathrm{m}^2\;\mathrm{s})$	$NL^{-2}T^{-1}$
$J_A$	mass flux of species $A$	kg/s	$MT^{-1}$
Kn	Knudsen number	_	_
L	length, depth	m	L
Le	Lewis number	_	_
m	mass	kg	М
$\dot{m}$	mass flux	kg/s	M/T
$M, oldsymbol{M}$	moment	N m	$ML^2/T^2$
M	molar mass of mixture	kg/kmol	$MN^{-1}$
$M_A$	molar mass of species $A$	kg/kmol	$MN^{-1}$
Ma	Mach number	_	_

$n_{\rm esp}$	number of chemical species in the mixture	_	_
$\boldsymbol{n}$	normal vector	_	_
Nu	Nusselt number	_	_
p	pressure	Pa	$ML^{-1}T^{-2}$
P	momentum	N	$MLT^{-1}$
Pe	Péclet number	_	_
$\mathrm{Pe}_{\mathrm{II}}$	Péclet II number	_	_
Pr	Prandtl number	_	_
q	heat flux vector	$\mathrm{W}/\mathrm{m}^2$	$MT^{-3}$
Q	volumetric flux	$\mathrm{m}^3/\mathrm{s}$	$L^3T^{-1}$
$\dot{Q}$	heat per unit time	W	$ML^2T^{-3}$
r, R	radius	m	L
r	position vector	m	L
Ra	Rayleigh number	_	_
Re	Reynolds number	_	_
S	surface	$\mathrm{m}^2$	$L^2$
S	Strouhal number	_	_
$S_c(t)$	control volume surface	$\mathrm{m}^2$	$L^2$
$S_f(t)$	fluid volume surface	$\mathrm{m}^2$	$L^2$
S	deformation rate	$\mathrm{s}^{-1}$	$T^{-1}$
Sc	Schmidt number	_	_
Sh	Sherwood number	_	_
St	Stanton number	_	_
t	time	s	Т
T	temperature	$^{\circ}\mathrm{C}$ or K	Θ
$\boldsymbol{u}$	velocity field	m/s	$LT^{-1}$
U	potential energy	J/kg	$L^2/T^2$
$oldsymbol{v}$	mass average velocity	m/s	$LT^{-1}$
$oldsymbol{v}_A$	velocity of species $A$	m/s	$LT^{-1}$

#### XVIII Nomenclature

$oldsymbol{v}^c$	control volume velocity	m/s	$LT^{-1}$
$oldsymbol{v}^m$	molar average velocity	m/s	$LT^{-1}$
V	volume	$\mathrm{m}^3$	$L^3$
	velocity	m/s	$LT^{-1}$
$V_c(t)$	control volume	$\mathrm{m}^3$	$L^3$
$V_f(t)$	fluid volume	$\mathrm{m}^3$	$L^3$
$oldsymbol{x}$	Cartesian coordinates	m	L
	position vector	m	L
$X_A$	molar fraction of species $A$	_	_
$Y_A$	mass fraction of species $A$	_	_
$\dot{W}$	power	W	$ML^2T^{-3}$
We	Weber number	_	_
Greek Sy	ymbols		
$\alpha$	thermal diffusivity	$\mathrm{m}^2/\mathrm{s}$	$L^2T^{-1}$
	thermal diffusivity viscous boundary layer thickness	m <sup>2</sup> /s	L <sup>2</sup> T <sup>-1</sup>
α		,	
$\alpha$ $\delta$	viscous boundary layer thickness	m	L
$egin{array}{c} lpha \ \delta \ \delta_T \end{array}$	viscous boundary layer thickness thermal boundary layer thickness concentration boundary layer thick-	m m	L L
$\alpha$ $\delta$ $\delta_T$ $\delta_c$	viscous boundary layer thickness thermal boundary layer thickness concentration boundary layer thick- ness	m m m	L L L
$egin{array}{c} lpha \ \delta \ \delta_T \ \delta_c \ \eta_a \end{array}$	viscous boundary layer thickness thermal boundary layer thickness concentration boundary layer thick- ness apparent viscosity	m m m Pas or kg/(ms)	L L L
$\alpha$ $\delta$ $\delta_T$ $\delta_c$ $\eta_a$	viscous boundary layer thickness thermal boundary layer thickness concentration boundary layer thickness apparent viscosity angle	m m m Pas or kg/(ms) rad	$L$ $L$ $L$ $ML^{-1}T^{-1}$
$\alpha$ $\delta$ $\delta_T$ $\delta_c$ $\eta_a$ $\theta$	viscous boundary layer thickness thermal boundary layer thickness concentration boundary layer thickness apparent viscosity angle thermal conductivity	m m Pas or kg/(ms) rad W/(mK)	$\begin{tabular}{ll} $L$ & $L$
$\alpha$ $\delta$ $\delta_T$ $\delta_c$ $\eta_a$ $\theta$	viscous boundary layer thickness thermal boundary layer thickness concentration boundary layer thickness apparent viscosity angle thermal conductivity second viscosity coefficient	m m Pas or kg/(ms) rad W/(mK)	$\begin{tabular}{ll} $L$ & $L$
$\alpha$ $\delta$ $\delta_T$ $\delta_c$ $\eta_a$ $\theta$	viscous boundary layer thickness thermal boundary layer thickness concentration boundary layer thickness apparent viscosity angle thermal conductivity second viscosity coefficient friction factor for pipes	m m m Pas or kg/(ms) rad W/(mK) Pas	$\begin{tabular}{ll} $L$ & $L$

 ${\rm kg/m^3}$ 

 $kg/m^3$ 

 $\mathsf{ML}^{-3}$ 

 $\mathsf{ML}^{-3}$ 

mass concentration of species A

fluid density

 $\rho$ 

 $\rho_A$ 

$\sigma$	surface tension	N/m	$MT^{-2}$
	normal stress	Pa	$ML^{-1}T^{-2}$
$oldsymbol{ au}, au$	stress tensor, stress component	Pa	$ML^{-1}T^{-2}$
au'	shear stress	Pa	$ML^{-1}T^{-2}$
$oldsymbol{ au}'$	viscous stress tensor	Pa	$ML^{-1}T^{-2}$
$\phi_v$	viscous dissipation function	$\mathrm{W}/\mathrm{m}^3$	$ML^{-1}T^{-3}$
$\omega$	angular velocity	rad/s	$T^{-1}$
$\dot{\omega}_A$	chemical generation of species $A$	$\rm kg/(m^3~s)$	$ML^{-3}T^{-1}$
$\dot{\omega}_A'$	molar chemical generation of species $A$	$\text{mol/(m}^3 \text{ s})$	$NL^{-3}T^{-1}$

## Introduction

Most chemical processes, and the chemical and physical operations involved, imply a transport of momentum, heat and mass.

For example, let us consider a chemical reactor. The chemical compounds need to be transported into the reactor. Once in the reactor, the chemical concentrations will evolve according to the mass transport laws. In order to speed up mixing, agitation may be used to add velocity, vorticity, and turbulence to the fluid. Therefore, we are acting upon the velocity of the fluid, transferring momentum. Finally, by adding heat to the reactor, the temperature gradients generate energy transport from the heat source to the fluid particles, a process that is called heat transfer. As a consequence, in most chemical processes we can encounter mass, momentum and heat transport phenomena.

In general, the exchange of momentum, mass and energy are interrelated and appear together. For instance, mass and heat transfer are faster in the presence of agitation.

Furthermore, the laws and models that describe the transport of properties within a fluid are very similar. This is demonstrated by the existence of analogies between the three kinds of transport phenomena. Therefore, a unified study of all transport processes facilitates the learning process and deepens a relational understanding.

Finally, the chemical operations between solids, liquids and gases typically take place inside fluids (mainly liquids).

In brief, given that fluids are present in most chemical processes, it is vital for the chemical engineer to thoroughly understand fluid mechanics and transport phenomena.