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An Introduction to Fuzzy Soft Graph

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Abstract. The notions of fuzzy soft graph, union, intersection of two fuzzy soft graphs are introduced in this paper and a few properties relating to finite union and intersection of fuzzy soft graphs are established here.

1. Introduction

In 1999, D.Molodtsov[4] introduced the notion of soft set theory to solve imprecise problems in economics, engineering and environment. He has shown several applications of this theory in solving many practical problems. There are many theories like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of rough sets, etc. which can be considered as mathematical tools to deal with uncertainties. But all these theories have their own inherent difficulties. The theory of probabilities can deal only with possibilities. The most appropriate theory to deal with uncertainties is the theory of fuzzy sets, developed by Zadeh[9] in 1965. But it has an inherent difficulty to set the membership function in each particular cases. Also the theory of intuitionistic fuzzy set is more generalized concept than the theory of fuzzy set, but also there have same difficulties. The soft set theory is free from above difficulties. In 2001, P.K.Maji, A.R.Roy, R.Biswas [12, 13] initiated the concept of fuzzy soft sets which is a combination of fuzzy set and soft set. In fact, the notion of fuzzy soft set is more generalized than that of fuzzy set and soft set. Thereafter many researchers have applied this concept on different branches of mathematics, like group theory [1, 6], decision making problems, relations [3, 5], topology [14, 15, 16] etc..

In 1736, Euler first introduced the concept of graph theory. The theory of graph is extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operation research, optimization and computer science, etc.. In 1975, Rosenfeld [2] introduced the concept of fuzzy graphs. Thereafter many researchers have

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generalized the different notions of graph theory using the notions of fuzzy sets [7, 8, 10, 17].

In this paper, our aim is to introduce the notion of fuzzy soft graph and then a few operations, like union, intersections of two fuzzy soft graphs. Also it is seen that the collection of fuzzy soft graphs is closed under finite union and intersection.

2. PRELIMINARIES

Definition 2.1 ([2]). Let V be a nonempty finite set and $\sigma : V \to [0,1]$. Again, let $\mu : V \times V \to [0,1]$ such that

$$\mu(x,y) \ \leq \ \sigma(x) \wedge \sigma(y) \quad \forall \ (x,y) \in V \times V.$$

Then the pair $G := (\sigma, \mu)$ is called a **fuzzy graph** over the set V. Here σ and μ are respectively called fuzzy vertex and fuzzy edge of the fuzzy graph (σ, μ).

A fuzzy graph $G := (\sigma, \mu)$ over the set V is called **strong fuzzy graph** if

$$\mu(x,y) \ = \ \sigma(x) \wedge \sigma(y) \quad \forall \ (x,y) \in V \times V.$$

Definition 2.2 ([2]). Let $H := (\rho, \nu)$ and $G := (\sigma, \mu)$ be two fuzzy graphs over the set V. Then H is called the **fuzzy subgraph** of the fuzzy graph G if

$$\rho(x) \leq \sigma(x) \quad \text{and} \quad \nu(x,y) \leq \mu(x,y) \quad \forall \ x, y \in V.$$

Definition 2.3 ([11]). Let $G_1 := (\sigma_1, \mu_1)$ and $G_2 := (\sigma_2, \mu_2)$ be two fuzzy graphs over the set V. Then the **union** of G_1 and G_2 is another fuzzy graph $G_3 := (\sigma_3, \mu_3)$ over the set V, where

$$\sigma_3 = \sigma_1 \vee \sigma_2 \quad \text{and} \quad \mu_3 = \mu_1 \vee \mu_2,$$
$$i.e. \ \ \sigma_3(x) = \max\{\sigma_1(x), \sigma_2(x)\} \quad \forall \ x \in V$$
$$\text{and} \quad \mu_3(x,y) = \max\{\mu_1(x,y), \mu_2(x,y)\} \quad \forall \ x, y \in V.$$

Definition 2.4 ([11]). Let $G_1 := (\sigma_1, \mu_1)$ and $G_2 := (\sigma_2, \mu_2)$ be two fuzzy graphs over the set V. Then the **intersection** of G_1 and G_2 is another fuzzy graph $G_3 := (\sigma_3, \mu_3)$ over the set V, where

$$\sigma_3 = \sigma_1 \wedge \sigma_2 \quad \text{and} \quad \mu_3 = \mu_1 \wedge \mu_2,$$
$$i.e. \ \ \sigma_3(x) = \min\{\sigma_1(x), \sigma_2(x)\} \quad \forall \ x \in V$$
$$\text{and} \quad \mu_3(x,y) = \min\{\mu_1(x,y), \mu_2(x,y)\} \quad \forall \ x, y \in V.$$

Let U be an initial universal set and E be a set of parameters. Let I^U denotes the collection of all fuzzy subsets of U and $A \subseteq E$.

Definition 2.5 ([15]). Let $A \subseteq E$. Then the mapping $F_A : E \to I^U$, defined by $F_A(e) = \mu^e_{F_A}$ (a fuzzy subset of U), is called **fuzzy soft set** over (U, E), where $\mu^e_{F_A} = \overline{0}$ if $e \in E \setminus A$ and $\mu^e_{F_A} \neq \overline{0}$ if $e \in A$ and $\overline{0}$ denotes the null fuzzy set.

The set of all fuzzy soft sets over (U, E) is denoted by $FS(U, E)$.

Definition 2.6 ([15]). The fuzzy soft set $F_\phi \in FS(U,E)$ is called **null fuzzy soft set** and it is denoted by Φ. Here $F_\phi(e) = \overline{0}$ for every $e \in E$.

Definition 2.7 ([15]). Let $F_E \in FS(U,E)$ and $F_E(e) = \overline{1}$ for all $e \in E$. Then F_E is called absolute fuzzy soft set. It is denoted by $\widetilde{E}$.

Definition 2.8 ([15]). Let $F_A, G_B \in FS(U,E)$ and $A \subseteq B$. If $F_A(e) \subseteq G_B(e)$ for all $e \in A$, i.e., if $\mu^e_{F_A} \subseteq \mu^e_{G_B}$ for all $e \in A$, i.e., if $\mu^e_{F_A}(x) \leq \mu^e_{G_B}(x)$ for all $x \in U$ and for all $e \in A$, then F_A is said to be fuzzy soft subset of G_B, denoted by $F_A \sqsubseteq G_B$.

Definition 2.9 ([15]). Let $F_A, G_B \in FS(U,E)$. Then the union of F_A and G_B is another fuzzy soft set H_C defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in C$, where $C = A \cup B$. Here we write $H_C = F_A \sqcup G_B$.

Definition 2.10 ([15]). Let $F_A, G_B \in FS(U,E)$. Then the intersection of F_A and G_B is another a fuzzy soft set H_C, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in C$, where $C = A \cap B$. Here we write $H_C = F_A \sqcap G_B$.

3. Fuzzy Soft Graph

Definition 3.1. Let $V = \{x_1, x_2, \cdots, x_n\}$(non-empty set), E(parameters set) and $A \subseteq E$. Also let

(i) $\rho : A \longrightarrow F(V)$(Collection of all fuzzy subsets in V)

$e \mapsto \rho(e) = \rho_e$(say)

and $\rho_e : V \longrightarrow [0,1]$

$x_i \mapsto \rho_e(x_i)$

(A,ρ) : Fuzzy soft vertex.

$(ii)\mu : A \longrightarrow F(V \times V)$(Collection of all fuzzy subsets in $V \times V$)

$e \mapsto \mu(e) = \mu_e$(say)

and $\mu_e : V \times V \longrightarrow [0,1]$

$(x_i, x_j) \mapsto \mu_e(x_i, x_j)$

(A,μ) : Fuzzy soft edge.

Then $((A,\rho),(A,\mu))$ is called **fuzzy soft graph** if and only if $\mu_e(x_i,x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ $\forall e \in A$ and $\forall i,j = 1,2,\cdots,n$ and this fuzzy soft graph is denoted by $G_{A,V}$.

Example 3.1. Consider a fuzzy soft graph $G_{E,V}$, where $V = \{x_1,x_2,x_3\}$ and $E = \{e_1,e_2,e_3\}$. Here $G_{E,V}$ is described by Table 1 and $\mu_e(x_i,x_j) = 0$ for all $(x_i,x_j) \in V \times V \setminus \{(x_1,x_2),(x_1,x_3),(x_2,x_3)\}$ and for all $e \in E$.

Definition 3.2. The fuzzy soft graph $H_{A,V} = ((A,\zeta),(A,\nu))$ is called a **fuzzy soft subgraph** of $G_{A,V} = ((A,\rho),(A,\mu))$ if $\rho_e(x_i) \geq \zeta_e(x_i)$ for all $x_i \in V, e \in A$ and $\nu_e(x_i,x_j) \leq \mu_e(x_i,x_j)$ for all $x_i, x_j \in V, e \in A$.

Example 3.2. Consider $V = \{x_1,x_2,x_3\}$ and $E = \{e_1,e_2,e_3\}$. Here, a fuzzy soft graph $H_{A,V}$ is given by Table 2 and $\nu_e(x_i,x_j) = 0$ for all $(x_i,x_j) \in$

TABLE 1

ρ	x_1	x_2	x_3
e_1	.5	.2	0
e_2	.4	.7	.6
e_3	.5	.9	.3

μ	(x_1, x_2)	(x_1, x_3)	(x_2, x_3)
e_1	.2	0	0
e_2	.4	.4	.6
e_3	.2	.1	.3

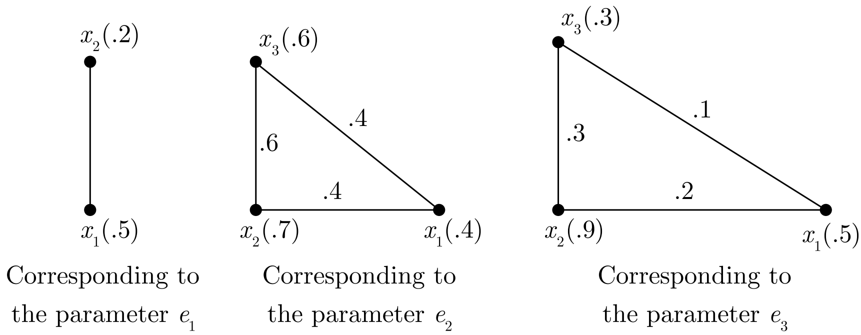

FIGURE 1. Fuzzy Soft Graph.

$V \times V \setminus \{(x_1, x_2), (x_1, x_3), (x_2, x_3)\}$ and for all $e \in E$. $H_{A,V}$ is a fuzzy soft subgraph of $G_{A,V}$, where $G_{A,V}$ given in the Example (3.1).

TABLE 2

ζ	x_1	x_2	x_3
e_1	.3	.1	0
e_2	0	.4	.3
e_3	.5	0	.2

ν	(x_1, x_2)	(x_1, x_3)	(x_2, x_3)
e_1	.1	0	0
e_2	0	0	.3
e_3	0	.1	0

Definition 3.3. The fuzzy soft subgraph $H_{A,V} = ((A, \zeta), (A, \nu))$ is said to be **spanning fuzzy soft subgraph** of $G_{A,V} = ((A, \rho), (A, \mu))$ if $\rho_e(x_i) = \zeta_e(x_i)$ for all $x_i \in V, e \in A$.
In this case, the two fuzzy soft graphs have same fuzzy soft vertex set, they differ only in the arc weights.

Example 3.3. Consider $V = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$. Here, a fuzzy soft graph $H_{A,V}$ is given by Table 3 and $\nu_e(x_i, x_j) = 0$ for all $(x_i, x_j) \in V \times V \setminus \{(x_1, x_2), (x_1, x_3), (x_2, x_3)\}$ and for all $e \in E$. $H_{A,V}$ is a spanning fuzzy soft subgraph of $G_{A,V}$, where $G_{A,V}$ given in the Example (3.1).

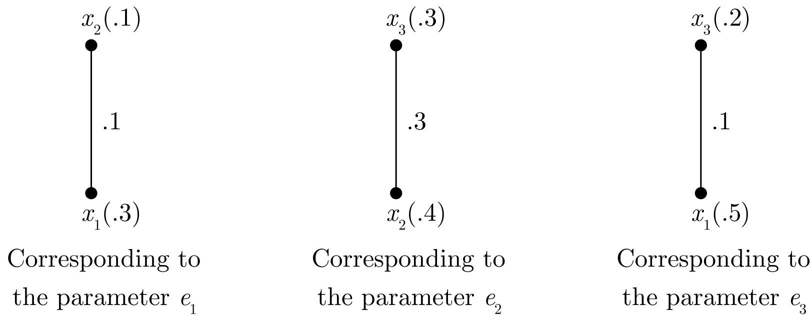

Figure 2. Fuzzy Soft Subgraph.

Table 3

ζ	x_1	x_2	x_3
e_1	.5	.2	0
e_2	.4	.7	.6
e_3	.5	.9	.3

ν	(x_1, x_2)	(x_1, x_3)	(x_2, x_3)
e_1	.1	0	0
e_2	.3	.2	.5
e_3	.1	.1	.2

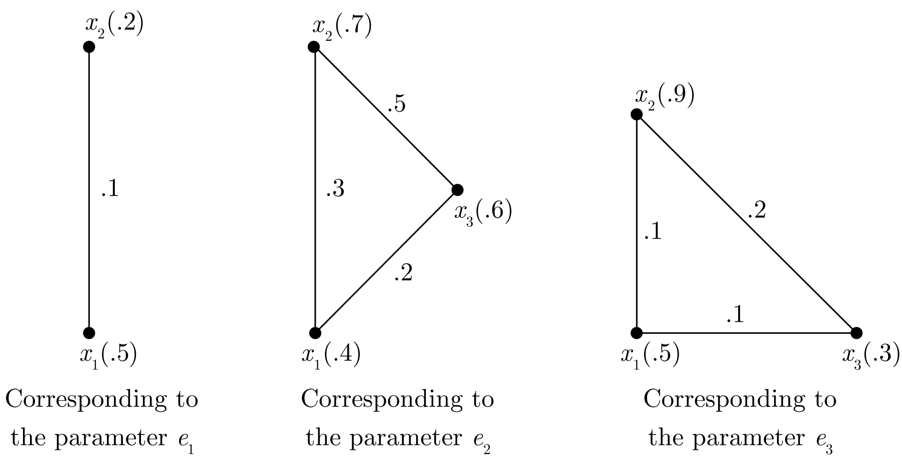

Figure 3. Spanning Fuzzy Soft subgraph.

Definition 3.4. The **underlying crisp graph** of a fuzzy soft graph $G_{A,V}$ $= ((A, \rho), (A, \mu))$ is denoted by $G^* = (\rho^*, \mu^*)$ where

$$\rho^* = \{x_i \in V \ : \ \rho_e(x_i) > 0 \quad \text{for} \quad \text{some} \ \ e \in E\},$$

$$\mu^* = \{(x_i, x_j) \in V \times V \ : \ \mu_e(x_i, x_j) > 0 \quad \text{for} \quad \text{some} \ \ e \in E\}.$$

Definition 3.5. A fuzzy soft graph $G_{A,V} = ((A, \rho), (A, \mu))$ is called **strong fuzzy soft graph** if $\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j)$ for all $(x_i, x_j) \in \mu^*, e \in A$

and is a **complete fuzzy soft graph** if $\mu_e(x_i, x_j)$
$= \rho_e(x_i) \wedge \rho_e(x_j)$ for all $x_i, x_j \in \rho^*, e \in A$.

Example 3.4. For example of strong fuzzy soft graph, consider $V = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$. $G_{A,V}$ is given by Table 4 and $\mu_e(x_i, x_j) = 0$ for all $(x_i, x_j) \in V \times V \setminus \{(x_1, x_2), (x_1, x_3),$
$(x_2, x_3)\}$ and for all $e \in E$. Here, $G_{A,V}$ is a strong fuzzy soft graph.

TABLE 4

ρ	x_1	x_2	x_3
e_1	.5	.2	0
e_2	.4	.7	.6
e_3	.5	.9	.3

μ	(x_1, x_2)	(x_1, x_3)	(x_2, x_3)
e_1	.2	0	0
e_2	.4	.4	.6
e_3	.5	.3	.3

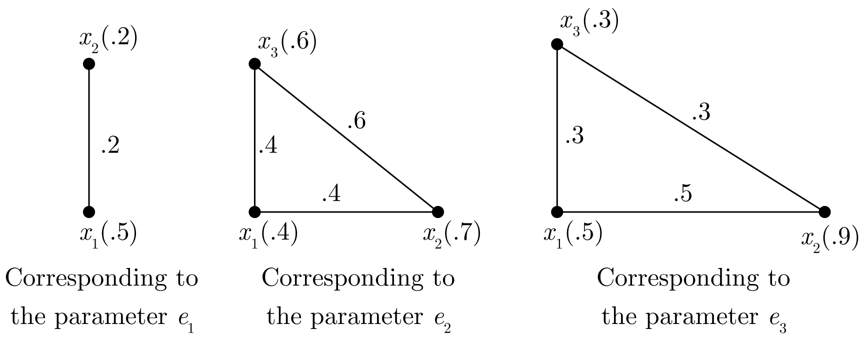

FIGURE 4. Strong Fuzzy Soft graph.

Example 3.5. For example of complete fuzzy soft graph, consider $V = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$. $G_{A,V}$ is given by Table 5 and $\mu_e(x_i, x_j) = 0$ for all $(x_i, x_j) \in V \times V \setminus \{(x_1, x_1), (x_1, x_3),$
$(x_3, x_3)\}$ and for all $e \in E$. Here, $G_{A,V}$ is a complete fuzzy soft graph.

TABLE 5

ρ	x_1	x_2	x_3
e_1	.2	0	.6
e_2	.5	0	0
e_3	.0	0	.8

μ	(x_1, x_1)	(x_1, x_3)	(x_3, x_3)
e_1	.2	.2	.6
e_2	.5	0	0
e_3	0	0	.8

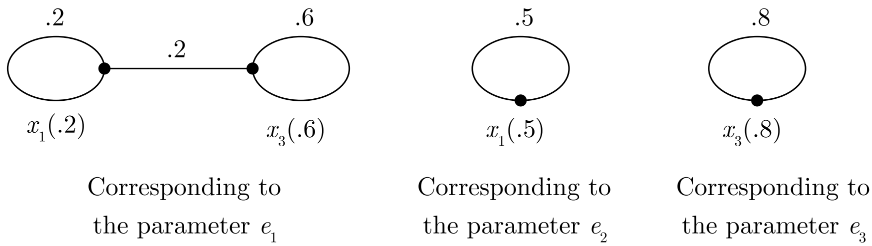

FIGURE 5. Complete Fuzzy Soft graph.

Definition 3.6. Let $V_1, V_2 \subset V$ and $A, B \subset E$. Then the **union** of two fuzzy soft graphs $G^1_{A,V_1} = ((A, \rho^1_e), (A, \mu^1_e))$ and $G^2_{B,V_2} = ((B, \rho^2_e), (B, \mu^2_e))$ is defined to be $G^3_{C,V_3} = ((C, \rho^3_e), (C, \mu^3_e))$ (say), where $C = A \cup B$, $V_3 = V_1 \cup V_2$ and

$\rho^3_e(x_i) = \rho^1_e(x_i)$ for all $e \in A \setminus B$ and $x_i \in V_1 \setminus V_2$,

$= 0$ for all $e \in A \setminus B$ and $x_i \in V_2 \setminus V_1$,

$= \rho^1_e(x_i)$ for all $e \in A \setminus B$ and $x_i \in V_1 \cap V_2$,

$= \rho^2_e(x_i)$ for all $e \in B \setminus A$ and $x_i \in V_2 \setminus V_1$,

$= 0$ for all $e \in B \setminus A$ and $x_i \in V_1 \setminus V_2$,

$= \rho^2_e(x_i)$ for all $e \in B \setminus A$ and $x_i \in V_1 \cap V_2$,

$= \rho^1_e(x_i) \vee \rho^2_e(x_i)$ for all $e \in A \cap B$ and $x_i \in V_1 \cap V_2$,

$= \rho^1_e(x_i)$ for all $e \in A \cap B$ and $x_i \in V_1 \setminus V_2$,

$= \rho^2_e(x_i)$ for all $e \in A \cap B$ and $x_i \in V_2 \setminus V_1$,

and

$\mu^3_e(x_i, x_j) = \mu^1_e(x_i, x_j)$ if $e \in A \setminus B$ and $(x_i, x_j) \in (V_1 \times V_1) \setminus (V_2 \times V_2)$,

$= 0$ if $e \in A \setminus B$ and $(x_i, x_j) \in (V_2 \times V_2) \setminus (V_1 \times V_1)$,

$= \mu^1_e(x_i, x_j)$ if $e \in A \setminus B$ and $(x_i, x_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$,

$= \mu^2_e(x_i, x_j)$ if $e \in B \setminus A$ and $(x_i, x_j) \in (V_2 \times V_2) \setminus (V_1 \times V_1)$,

$= 0$ if $e \in B \setminus A$ and $(x_i, x_j) \in (V_1 \times V_1) \setminus (V_2 \times V_2)$,

$= \mu_e^2(x_i, x_j)$ if $e \in B \setminus A$ and $(x_i, x_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$,

$= \mu_e^1(x_i, x_j) \vee \mu_e^2(x_i, x_j)$ if $e \in A \cap B$ and $(x_i, x_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$,

$= \mu_e^1(x_i, x_j)$ if $e \in A \cap B$ and $(x_i, x_j) \in (V_1 \times V_1) \setminus (V_2 \times V_2)$,

$= \mu_e^2(x_i, x_j)$ if $e \in A \cap B$ and $(x_i, x_j) \in (V_2 \times V_2) \setminus (V_1 \times V_1)$.

Example 3.6. Consider $V = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and $E = \{e_1, e_2, e_3, e_4\}$. Let $V_1 = \{x_1, x_2, x_3, x_4\}$, $A = \{e_1, e_2, e_3\}$ and $V_2 = \{x_1, x_2, x_5, x_6\}$, $B = \{e_2, e_3, e_4\}$.
G^1_{A,V_1} is defined by Table 6 and $\mu_e^1(x_i, x_j) = 0$ for all $(x_i, x_j) \in V_1 \times V_1 \setminus \{(x_1, x_2), (x_2, x_3), (x_3, x_4)\}$ and for all $e \in A$.
G^2_{B,V_2} is defined by Table 7 and $\mu_e^2(x_i, x_j) = 0$ for all $(x_i, x_j) \in V_2 \times V_2 \setminus \{(x_1, x_2), (x_2, x_5), (x_5, x_6)\}$ and for all $e \in B$.
Then the union of G^1_{A,V_1} and G^2_{B,V_2} is G^3_{C,V_3} given by the Table 8 and $\mu_e^3(x_i, x_j) = 0$ for all $(x_i, x_j) \in V_3 \times V_3 \setminus \{(x_1, x_2), (x_2, x_3), (x_3, x_4), (x_2, x_5), (x_5, x_6)\}$ and for all $e \in C$.

TABLE 6

ρ^1	x_1	x_2	x_3	x_4
e_1	.1	.5	.8	.2
e_2	.8	.2	.3	.9
e_3	.3	.8	.7	0

μ^1	(x_1, x_2)	(x_2, x_3)	(x_3, x_4)
e_1	.1	.4	.1
e_2	.2	.3	.2
e_3	.2	.5	0

TABLE 7

ρ^2	x_1	x_2	x_5	x_6
e_2	.2	.3	.9	.7
e_3	.6	.4	.5	0
e_4	0	.7	.5	.4

μ^2	(x_1, x_2)	(x_2, x_5)	(x_5, x_6)
e_2	.2	.2	.5
e_3	.4	.3	0
e_4	0	.2	.3

Proposition 3.1. *Let G^3_C be the union of the fuzzy soft graphs G^1_A and G^2_B. Then $G^3_C = G^1_A \cup G^2_B$ is a fuzzy soft graph.*

TABLE 8

ρ^3	x_1	x_2	x_3	x_4	x_5	x_6
e_1	.1	.5	.8	.2	0	0
e_2	.8	.3	.3	.9	.9	.7
e_3	.6	.8	.7	0	.5	0
e_4	0	.7	0	0	.5	.4

μ^3	(x_1, x_2)	(x_2, x_3)	(x_3, x_4)	(x_2, x_5)	(x_5, x_6)
e_1	.1	.4	.1	0	0
e_2	.2	.3	.2	.2	.5
e_3	.4	.5	0	.3	0
e_4	0	0	0	.2	.3

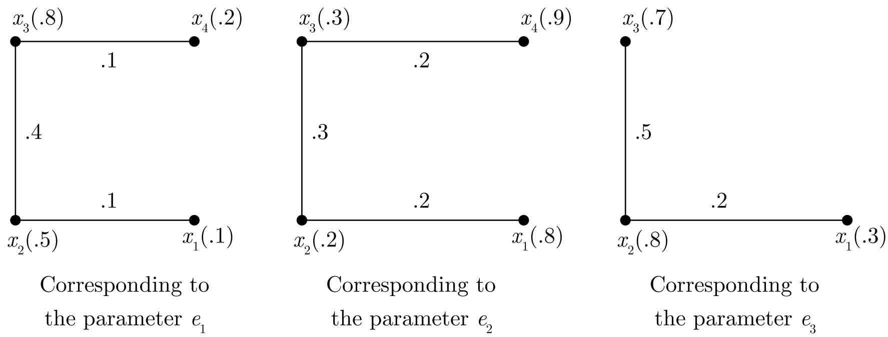

FIGURE 6. Fuzzy Soft graph 6.

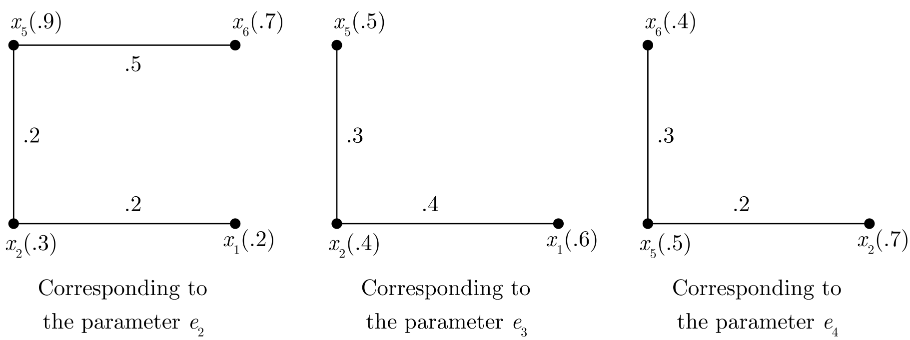

FIGURE 7. Fuzzy Soft graph 7.

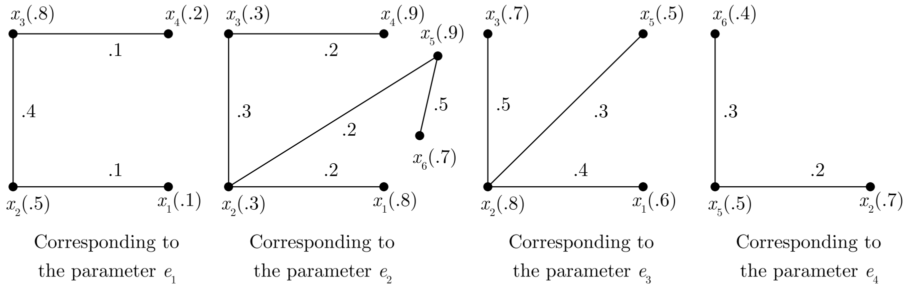

FIGURE 8. Union of the Fuzzy Soft graphs 6 & 7.

Proof. Suppose that $e \in A \setminus B$ and $(x_i, x_j) \in (V_1 \times V_1) \setminus (V_2 \times V_2)$. Then by the Definition (3.6), we have

$$\begin{aligned}\mu_e^3(x_i, x_j) &= \mu_e^1(x_i, x_j)\\ &\le \min\{\rho_e^1(x_i), \rho_e^1(x_j)\} = \min\{\rho_e^3(x_i), \rho_e^3(x_j)\}.\end{aligned}$$

Now, if $e \in A \setminus B$ and $(x_i, x_j) \in (V_2 \times V_2) \setminus (V_1 \times V_1)$ then obviously,

$$\mu_e^3(x_i, x_j) = 0 \le \min\{\rho_e^3(x_i), \rho_e^3(x_j)\}.$$

Again, if $e \in A \setminus B$ and $(x_i, x_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$. Then

$$\begin{aligned}\mu_e^3(x_i, x_j) &= \mu_e^1(x_i, x_j)\\ &\le \min\{\rho_e^1(x_i), \rho_e^1(x_j)\} = \min\{\rho_e^3(x_i), \rho_e^3(x_j)\}.\end{aligned}$$

Similarly, if $e \in B \setminus A$, in all cases we have

$$\mu_e^3(x_i, x_j) \le \min\{\rho_e^3(x_i), \rho_e^3(x_j)\}.$$

Now, suppose $e \in A \cap B$ and $(x_i, x_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$. Then

$$\begin{aligned}\mu_e^3(x_i, x_j) &= \max\{\mu_e^1(x_i, x_j), \mu_e^2(x_i, x_j)\}\\ &\le \max\{\min\{\rho_e^1(x_i), \rho_e^1(x_j)\}, \min\{\rho_e^2(x_i), \rho_e^2(x_j)\}\}\\ &\le \max\{\min\{\rho_e^1(x_i), \rho_e^2(x_i)\}, \min\{\rho_e^1(x_j), \rho_e^2(x_j)\}\}\\ &\le \min\{\max\{\rho_e^1(x_i), \rho_e^2(x_i)\}, \max\{\rho_e^1(x_j), \rho_e^2(x_j)\}\}\\ &= \min\{\rho_e^3(x_i), \rho_e^3(x_j)\}.\end{aligned}$$

As in the previous, if $e \in A \cap B$ and $(x_i, x_j) \in (V_1 \times V_1) \setminus (V_2 \times V_2)$ or $(x_i, x_j) \in (V_2 \times V_2) \setminus (V_1 \times V_1)$, it can be shown that

$$\mu_e^3(x_i, x_j) \le \min\{\rho_e^3(x_i), \rho_e^3(x_j)\}.$$

Hence, $G_C^3 = G_A^1 \cup G_B^2$ is a fuzzy soft graph. □

Proposition 3.2. *If G_C^3 is the union of two fuzzy soft graphs G_A^1 and G_B^2, then both G_A^1 and G_B^2 are fuzzy soft subgraphs of G_C^3.*

Definition 3.7. Let $V_1, V_2 \subset V$ and $A, B \subset E$. The **intersection** of two fuzzy soft graphs $G^1_{A,V_1} = ((A, \rho^1_e), (A, \mu^1_e))$ and $G^2_{B,V_2} = ((B, \rho^2_e), (B, \mu^2_e))$ is $G^3_{C,V_3} = ((C, \rho^3_e), (C, \mu^3_e))$ (say), where $C = A \cap B$ and $V_3 = V_1 \cap V_2$ and $\rho^3_e(x_i) = \rho^1_e(x_i) \wedge \rho^2_e(x_i)$ for all $x_i \in V_3$ and $e \in C$, and $\mu^3_e(x_i, x_j) = \mu^1_e(x_i, x_j) \wedge \mu^2_e(x_i, x_j)$ if $x_i, x_j \in V_3$ and $e \in C$.

Example 3.7. Consider $V = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2, e_3\}$. Let $V_1 = \{x_1, x_2, x_3\}, V_2 = \{x_2, x_3, x_4\}$, $A = \{e_1, e_2\}, B = \{e_2, e_3\}$.
G^1_{A,V_1} is defined by Table 9 and $\mu^1_e(x_i, x_j) = 0$ for all $(x_i, x_j) \in V_1 \times V_1 \setminus \{(x_1, x_2), (x_1, x_3), (x_2, x_3)\}$ and for all $e \in A$.
G^2_{B,V_2} is defined by Table 10 and $\mu^2_e(x_i, x_j) = 0$ for all $(x_i, x_j) \in V_2 \times V_2 \setminus \{(x_2, x_3), (x_2, x_4), (x_3, x_4)\}$ and for all $e \in B$.
So, $V_3 = \{x_2, x_3\}$ and $C = \{e_2\}$.
Then the intersection of G^1_{A,V_1} and G^2_{B,V_2} is G^3_{C,V_3} given by the Table 11 and $\mu^3_e(x_i, x_j) = 0$ for all $(x_i, x_j) \in V_3 \times V_3 \setminus \{(x_2, x_2), (x_3, x_3), (x_3, x_2)\}$ and for all $e \in C$.

TABLE 9

ρ^1	x_1	x_2	x_3
e_1	.2	.4	.6
e_2	.3	.9	.6

μ^1	(x_1, x_2)	(x_1, x_3)	(x_2, x_3)
e_1	.2	.1	.3
e_2	.1	.2	.5

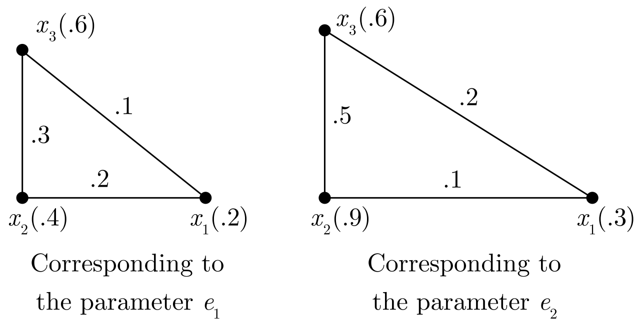

FIGURE 9. Fuzzy Soft graph 9.

TABLE 10

ρ^2	x_2	x_3	x_4
e_2	.3	.5	.4
e_3	.1	.4	.7

μ^2	(x_2, x_3)	(x_2, x_4)	(x_3, x_4)
e_2	.2	.3	.3
e_3	.05	.1	.2

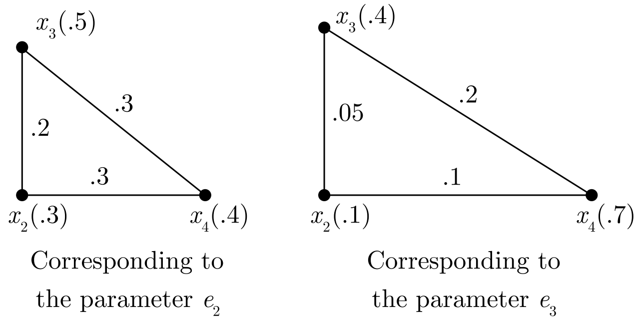

FIGURE 10. Fuzzy Soft graph 10.

TABLE 11

ρ^3	x_2	x_3	μ^3	(x_2, x_3)
e_2	.3	.5	e_2	.2

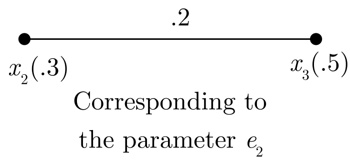

FIGURE 11. Intersection of the Fuzzy Soft graphs 9 & 10.

Proposition 3.3. *Let G_C^3 be the intersection of the fuzzy soft graphs G_A^1 and G_B^2. Then $G_C^3 = G_A^1 \cap G_B^2$ is a fuzzy soft graph.*

Proof. Let $x_i, x_j \in V_3$ and $e \in C$. Then we have

$$\begin{aligned}\mu_e^3(x_i, x_j) &= \min\{\mu_e^1(x_i, x_j), \mu_e^2(x_i, x_j)\}\\ &\leq \min\{\min\{\rho_e^1(x_i), \rho_e^1(x_j)\}, \min\{\rho_e^2(x_i), \rho_e^2(x_j)\}\}\\ &= \min\{\min\{\rho_e^1(x_i), \rho_e^2(x_i)\}, \min\{\rho_e^1(x_j), \rho_e^2(x_j)\}\}\\ &= \min\{\rho_e^3(x_i), \rho_e^2(x_j)\}.\end{aligned}$$

This proves the proposition. □

Proposition 3.4. *If G_C^3 is the intersection of two fuzzy soft graphs G_A^1 and G_B^2, then G_C^3 is fuzzy soft subgraph of both G_A^1 and G_B^2.*

4. CONCLUSION

In the definition (3.7), if we replace the set $C = A \cup B$ and $V_3 = V_1 \cup V_2$, the intersection of two fuzzy soft graphs can be defined as in the same way of the definition (3.6) and this intersection could be termed as generalized intersection of two fuzzy soft graphs. In fact, with respect to generalized intersection, propositions (3.3) and (3.4) can be easily verified.

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