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# An Introduction to Optimal Control Problems in Life Sciences and Economics

*From Mathematical Models to Numerical  
Simulation with MATLAB®*

 Birkhäuser

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