An Introduction to Quantum Field Theory

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*These sections may be omitted in a one-year course emphasizing the less formal aspects of elementary particle physics.